Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test I – August 2025

Semester V

Class : III UG **Time : 2 Hours**

Branch : Special Education and Mathematics **Max.Marks : 60**

23BSMC13 - Abstract Algebra I

Course Outcomes:

CO1: Apply Lagrange's Theorem to analyze the cyclic subgroup of a group.

CO2: Explore the concepts of homomorphism and isomorphism for groups.

CO3: Develop group structures of Automorphism.

CO4: Analyse the properties of rings and ring homomorphisms.

CO5: Demonstrate examples for ideals and quotient rings.

Part A **6 x 1 = 6**

Choose the Correct Answer

1. If H is a subgroup of G with O(G)=21, then which of the following can be o(H)? CO1K1

a. 7 b. 4 c. 2 d. 6

2. If n is a positive integer and a is relatively prime to n , then CO1 K2

a. $a^{\varphi(n)} \equiv 1 \; mod \; n$ b. $a^{\varphi(n-1)} \equiv 1 \; mod \; n$ c. $a^{n-1} \equiv 1 \; mod \; n$ d. $a^{\varphi(n)} \equiv 0 \; mod \; p$

3. If G={ 1, -1, i,-i} is a group under multiplication, then the O(-1) is CO2 K1

a. 2 b. 4 c. 3 d.1

4.The identity element of the quotient group G/ N is CO2 K2

a. G b.1 c. N d. 0

5. If G is a finite group, N is a normal subgroup of G and x,y are in G, then Nxy is CO2 K2

a.xy b.N(x-y) c.Nx+Ny d.NxNy

6.Let G be a group and ϕ an automorphism of G.If a∈G is of order o(a) > 0, then CO3 K1

a.o(ϕ (a)) = o(a)+1 b. o(ϕ (a)) < o(a) c.o(ϕ (a)) > o(a) d.o(ϕ (a)) = o(a)

Part B **3 x 6 = 18**

Answer ALL questions

7. a. Prove that the relation a ≡b mod H is an equivalence relation. CO1K3

(or)

7. b. If H is a nonempty finite subset of a group G and H is closed under multiplication then prove that H is a subgroup of G. CO1K3

8. a.State and prove Fermat's theorem CO1K4

(or)

8. b. If ϕ is a homomorphism of G into $\overline{G}$, then prove that CO2K4

(i) $\phi(e) = \bar{e}$, the unit element of $\overline{G}$

(ii) $\phi(x^{-1}) = [\phi(x)]^{-1}$ for all $x \in G$.

9. a. Prove that kernel of a homomorphism is a normal subgroup of G. CO2K3

(or)

9. b. Let G be a group; Then prove that set of automorphisms of G is a group CO3K3

Part C **3 x 12 = 36**

Answer ALL questions

10. a. (i) Prove that if Hand K are subgroups of G and $o(H) > \sqrt{o(G)}$, $o(K) > \sqrt{o(G)}$ CO1K4

then $H \cap K \neq \{e\}$.

(ii) Prove that for all $a \in G, Ha = \{x \in G \text{ such that } a \equiv x \bmod H\}$

(or)

10. b. (i) Prove that HK is a subgroup of G if and only if HK = KH. CO1K4

(ii) Prove that a subgroup N of G is a normal if and only if $gNg^{-1} = N$ for every $g \in G$

11. a. Let ϕ be a homomorphism of G onto $\bar{G}$ with kernel K. Then prove that $G/K \approx \bar{G}$. CO2K5

(or)

11. b.(i) Prove that N is a normal subgroup of G if and only every left coset of N in G is a right coset of N in G

(ii) Prove that G/N is a group where G is a group and N a normal subgroup of G,.

CO2K4

12. a. Prove that group isomorphism is an equivalence relation. CO2K5

(or)

12. b. Prove that $\mathcal{I}(G) \approx G/Z$, where $\mathcal{I}(G)$ is the group of inner automorphisms of G, and Z is the center of G.

CO3K5

No. of copies: 17