

CHAPTER VI

POSSIBILITY NEUTROSOPHIC SOFT SETS

Definition:6.1

Let U be an initial universe. Let E be a set of parameters and $N(U)$ denotes set of all neutrosophic sets of U . I^U denotes the collection of all fuzzy subsets of U . A **possibility neutrosophic soft set** $\tilde{F}_\mu$ over U is a set of ordered pairs defined by

$$\tilde{F}_\mu = \left\{ \left(e_i, \left\{ \left(\frac{x_j}{\tilde{F}(e_i)(x_j)}, \mu(e_i)(x_j) \right) : x_j \in U \right\} \right) : e_i \in E \right\}$$

or a mapping defined by $\tilde{F}_\mu : E \rightarrow N(U) \times I^U$ where $i, j \in \Lambda$, $\tilde{F}_\mu$ is a mapping given by $\tilde{F}_\mu : E \rightarrow N(U)$ and $\mu(e_i)$ is a fuzzy set such that $\mu : E \rightarrow I^U$ for each parameter $e_i \in E$, $\tilde{F}_\mu(e_i) = \{ \langle x_j, T_{\tilde{F}(e_i)}(x_j), I_{\tilde{F}(e_i)}(x_j), F_{\tilde{F}(e_i)}(x_j) \rangle : x_j \in U \}$

Indicates neutrosophic value set of parameter e_i and where $T, I, F : U \rightarrow [0,1]$ are the truth membership, indeterminacy membership, falsity membership functions of the element $x_j \in U$ respectively. For each $x_j \in U$ and $e_i \in E$,

$$0 \leq T_{\tilde{F}(e_i)}(x_j) + I_{\tilde{F}(e_i)}(x_j) + F_{\tilde{F}(e_i)}(x_j) \leq 3. \text{ Also } \mu(e_i), \text{ degrees of possibility of}$$

belongingness of elements of U in $\tilde{F}_\mu(e_i)$. so

$$\tilde{F}_\mu(e_i) = \left\{ \left(\frac{x_1}{\tilde{F}(e_i)(x_1)}, \mu(e_i)(x_1) \right), \left(\frac{x_2}{\tilde{F}(e_i)(x_2)}, \mu(e_i)(x_2) \right), \dots, \left(\frac{x_n}{\tilde{F}(e_i)(x_n)}, \mu(e_i)(x_n) \right) \right\}$$

Example:6.2

Let $U = \{x_1, x_2, x_3\}$ be a set of three cars. Let $E = \{e_1, e_2, e_3\}$ be a set of qualities where e_1 – cheap, e_2 – equipment, e_3 – fuel consumption and let $\mu : E \rightarrow I^U$. A function $\tilde{F}_\mu : E \rightarrow N(U) \times I^U$ as follows:

$$\tilde{F}_\mu = \left\{ \begin{array}{l} \tilde{F}_\mu(e_1) = \left\{ \left(\frac{x_1}{(0.5, 0.2, 0.6)}, 0.8 \right), \left(\frac{x_2}{(0.7, 0.3, 0.5)}, 0.4 \right), \left(\frac{x_3}{(0.4, 0.5, 0.8)}, 0.7 \right) \right\} \\ \tilde{F}_\mu(e_2) = \left\{ \left(\frac{x_1}{(0.8, 0.4, 0.5)}, 0.6 \right), \left(\frac{x_2}{(0.5, 0.7, 0.2)}, 0.8 \right), \left(\frac{x_3}{(0.7, 0.3, 0.9)}, 0.4 \right) \right\} \\ \tilde{F}_\mu(e_3) = \left\{ \left(\frac{x_1}{(0.6, 0.7, 0.5)}, 0.2 \right), \left(\frac{x_2}{(0.5, 0.3, 0.7)}, 0.6 \right), \left(\frac{x_3}{(0.6, 0.5, 0.4)}, 0.5 \right) \right\} \end{array} \right\}$$

we can define a function $\tilde{G}_\nu : E \rightarrow N(U) \times I^U$ as follows:

$$\tilde{G}_\nu = \begin{cases} \tilde{G}_\nu(e_1) = \left\{ \left(\frac{x_1}{(0.6, 0.3, 0.8)}, 0.4 \right), \left(\frac{x_2}{(0.6, 0.5, 0.5)}, 0.7 \right), \left(\frac{x_3}{(0.2, 0.6, 0.4)}, 0.8 \right) \right\} \\ \tilde{G}_\nu(e_2) = \left\{ \left(\frac{x_1}{(0.5, 0.4, 0.3)}, 0.3 \right), \left(\frac{x_2}{(0.4, 0.6, 0.5)}, 0.6 \right), \left(\frac{x_3}{(0.7, 0.2, 0.5)}, 0.8 \right) \right\} \\ \tilde{G}_\nu(e_3) = \left\{ \left(\frac{x_1}{(0.7, 0.5, 0.3)}, 0.8 \right), \left(\frac{x_2}{(0.4, 0.4, 0.6)}, 0.5 \right), \left(\frac{x_3}{(0.8, 0.5, 0.3)}, 0.6 \right) \right\} \end{cases}$$

Definition:6.3

Let U be an initial universe. Let $\tilde{F}_\mu, \tilde{G}_\nu$ are two possibility neutrosophic soft sets over U . Then $\tilde{F}_\mu$ is said to be a **possibility neutrosophic soft subset** of $\tilde{G}_\nu$ if,

1. $\mu(e)$ is a fuzzy subset of $\nu(e)$, for all $e \in E$.
2. $\tilde{F}$ is a possibility neutrosophic subset of $\tilde{G}$

and is denoted by $\tilde{F}_\mu \subseteq \tilde{G}_\nu$.

Example:6.4

Let $U = \{x_1, x_2, x_3\}$ be a set of three houses. Let $E = \{e_1, e_2, e_3\}$ be a set of parameters where e_1 – modern, e_2 – big, and e_3 – cheap and a possibility neutrosophic soft set $\tilde{F}_\mu$ as follows:

$$\tilde{F}_\mu = \begin{cases} \tilde{F}_\mu(e_1) = \left\{ \left(\frac{x_1}{(0.5, 0.2, 0.6)}, 0.8 \right), \left(\frac{x_2}{(0.7, 0.3, 0.5)}, 0.4 \right), \left(\frac{x_3}{(0.4, 0.5, 0.9)}, 0.7 \right) \right\} \\ \tilde{F}_\mu(e_2) = \left\{ \left(\frac{x_1}{(0.8, 0.4, 0.5)}, 0.6 \right), \left(\frac{x_2}{(0.5, 0.7, 0.2)}, 0.8 \right), \left(\frac{x_3}{(0.7, 0.3, 0.9)}, 0.4 \right) \right\} \\ \tilde{F}_\mu(e_3) = \left\{ \left(\frac{x_1}{(0.6, 0.7, 0.5)}, 0.2 \right), \left(\frac{x_2}{(0.5, 0.3, 0.8)}, 0.6 \right), \left(\frac{x_3}{(0.6, 0.5, 0.4)}, 0.5 \right) \right\} \end{cases}$$

and another possibility neutrosophic soft set $\tilde{G}_\nu$ as follows:

$$\tilde{G}_v = \left\{ \begin{array}{l} \tilde{G}_v(e_1) = \left\{ \left(\frac{x_1}{(0.6, 0.1, 0.5)}, 0.9 \right), \left(\frac{x_2}{(0.8, 0.2, 0.3)}, 0.6 \right), \left(\frac{x_3}{(0.7, 0.5, 0.8)}, 0.8 \right) \right\} \\ \tilde{G}_v(e_2) = \left\{ \left(\frac{x_1}{(0.9, 0.2, 0.4)}, 0.7 \right), \left(\frac{x_2}{(0.9, 0.5, 0.1)}, 0.9 \right), \left(\frac{x_3}{(0.8, 0.1, 0.9)}, 0.5 \right) \right\} \\ \tilde{G}_v(e_3) = \left\{ \left(\frac{x_1}{(0.6, 0.5, 0.4)}, 0.4 \right), \left(\frac{x_2}{(0.7, 0.1, 0.7)}, 0.9 \right), \left(\frac{x_3}{(0.8, 0.2, 0.4)}, 0.7 \right) \right\} \end{array} \right\}$$

$\tilde{F}_\mu$ is said to be a possibility neutrosophic soft subset of $\tilde{G}_v$

Definition:6.5

Let U be an initial universe. Let $\tilde{F}_\mu, \tilde{G}_v$ are two possibility neutrosophic soft sets over U . Then $\tilde{F}_\mu, \tilde{G}_v$ is called a **possibility neutrosophic soft equal set** if $\tilde{F}_\mu \subseteq \tilde{G}_v$ and $\tilde{F}_\mu \supseteq \tilde{G}_v$ and is denoted by $\tilde{F}_\mu = \tilde{G}_v$.

Definition:6.6

Let $\tilde{F}_\mu$ be a possibility neutrosophic soft set over U . Then $\tilde{F}_\mu$ is said to be **null possibility neutrosophic soft set** denoted by ϕ_μ if for all $e \in E$, $\phi_\mu : E \rightarrow N(U) \times I^U$ such that

$$\phi_\mu(e) = \left\{ \left(\frac{x}{\phi(e)(x)}, \mu(e)(x) \right) : x \in U \right\},$$

where $\phi(e) = \{ \langle x, 0, 1, 1 \rangle : x \in U \}$ and $\mu(e) = \{ (x, 0) : x \in U \}$

Definition:6.7

Let $\tilde{F}_\mu$ be a possibility neutrosophic soft set over U . Then $\tilde{F}_\mu$ is said to be **universe possibility neutrosophic soft set** denoted by U_μ if for all $e \in E$, $U_\mu : E \rightarrow N(U) \times I^U$ such that

$$U_\mu(e) = \left\{ \left(\frac{x}{U(e)(x)}, \mu(e)(x) \right) : x \in U \right\},$$

where $U(e) = \{ \langle x, 1, 0, 0 \rangle : x \in U \}$ and $\mu(e) = \{ (x, 1) : x \in U \}$

Theorem:6.8

Let $\tilde{F}_\mu, \tilde{G}_\nu$ and $\tilde{H}_\delta$ be three possibility neutrosophic soft sets over U . Then

1. $\phi_\mu \subseteq \tilde{F}_\mu$
2. $\tilde{F}_\mu \subseteq U_\mu$
3. $\tilde{F}_\mu \subseteq \tilde{F}_\mu$
4. $\tilde{F}_\mu \subseteq \tilde{G}_\nu$ and $\tilde{G}_\nu \subseteq \tilde{H}_\delta \Rightarrow \tilde{F}_\mu \subseteq \tilde{H}_\delta$

Definition:6.9

Let $\tilde{F}_\mu$ be a possibility neutrosophic soft set over U , where $\tilde{F}_\mu$ is

$$\tilde{F}_\mu(e_i) = \{ (\tilde{F}(e_i)(x_j), \mu(e_i)(x_j)) : e_i \in E, x_j \in U \} \text{ and}$$

$$\tilde{F}(e_i) = \{ \langle x, T_{\tilde{F}(e_i)}(x_j), I_{\tilde{F}(e_i)}(x_j), F_{\tilde{F}(e_i)}(x_j) \rangle \} \text{ for all } e_i \in E, x \in U. \text{ for } e_i \in E \text{ and } x \in U.$$

1. $\tilde{F}_\mu^T$ is said to be **truth membership** part of $\tilde{F}_\mu$ is $\tilde{F}_\mu^T = \{ (\tilde{F}_{ij}^T(e_i), \mu_{ij}(e_i)) \}$ and

$$\tilde{F}_{ij}^T(e_i) = \{ (x_j, T_{\tilde{F}(e_i)}(x_j)) \}, \mu_{ij}(e_i) = \{ (x_j, \mu(e_i)(x_j)) \}$$

2. $\tilde{F}_\mu^I$ is said to be **indeterminacy membership** part of $\tilde{F}_\mu$ is $\tilde{F}_\mu^I = \{(\tilde{F}_{ij}^I(e_i), \mu_{ij}(e_i))\}$ and

$$\tilde{F}_{ij}^I(e_i) = \{(x_j, I_{\tilde{F}(e_i)}(x_j))\}, \mu_{ij}(e_i) = \{(x_j, \mu(e_i)(x_j))\}$$

3. $\tilde{F}_\mu^F$ is said to be **falsity membership** part of $\tilde{F}_\mu$ is $\tilde{F}_\mu^F = \{(\tilde{F}_{ij}^F(e_i), \mu_{ij}(e_i))\}$ and

$$\tilde{F}_{ij}^F(e_i) = \{(x_j, F_{\tilde{F}(e_i)}(x_j))\}, \mu_{ij}(e_i) = \{(x_j, \mu(e_i)(x_j))\}$$

Possibility neutrosophic soft set in the form of $\tilde{F}_\mu = (\tilde{F}_\mu^T, \tilde{F}_\mu^I, \tilde{F}_\mu^F)$

Definition:6.10

A binary operation $\otimes: [0,1] \times [0,1] \rightarrow [0,1]$ is continuous **t-norm** if $\otimes$ satisfies the following conditions

1. $\otimes$ is commutative and associative.
2. $\otimes$ is continuous
3. $a \otimes 1 = a, \forall a \in [0,1]$
4. $a \otimes b \leq c \otimes d$ whenever $a \leq c, b \leq d$ and $a, b, c, d \in [0,1]$.

Definition:6.11

A binary operation $\oplus: [0,1] \times [0,1] \rightarrow [0,1]$ is continuous **t-conorm** if $\oplus$ satisfies the following conditions

1. $\oplus$ is commutative and associative.
2. $\oplus$ is continuous
3. $a \oplus 0 = a, \forall a \in [0,1]$
4. $a \oplus b \leq c \oplus d$ whenever $a \leq c, b \leq d$ and $a, b, c, d \in [0,1]$.

Definition:6.12

Let $I^3 = [0,1] \times [0,1] \times [0,1]$ and $N(I^3) = \{(a,b,c) : a,b,c \in [0,1]\}$. Then $(N(I^3), \oplus, \otimes)$ be a **lattices** together with partial ordered relation $\leq$, where ordered relation $\leq$ on $N(I^3)$ can be defined by

$$(a,b,c) \leq (e,f,g) \Leftrightarrow a \leq e, b \geq f, c \geq g \text{ for } (a,b,c), (e,f,g) \in N(I^3)$$

Definition:6.13

A binary operation $\tilde{\otimes} : ([0,1] \times [0,1] \times [0,1])^2 \rightarrow [0,1] \times [0,1] \times [0,1]$ is continuous **n-norm** if $\tilde{\otimes}$ satisfies the following conditions

1. $\tilde{\otimes}$ is commutative and associative.
2. $\tilde{\otimes}$ is continuous
3. $a \tilde{\otimes} \hat{0} = \hat{0}, a \tilde{\otimes} \hat{1} = a, \forall a \in [0,1] \times [0,1] \times [0,1], (\hat{1} = (1,0,0)) \text{ and } (\hat{0} = (0,1,1))$
4. $a \tilde{\otimes} b \leq c \tilde{\otimes} d$ whenever $a \leq c, b \leq d$ and $a,b,c,d \in [0,1] \times [0,1] \times [0,1]$.

$$a \tilde{\otimes} b = \tilde{\otimes}(<T(a), I(a), F(a)>, <T(b), I(b), F(b)>) = <T(a) \otimes T(b), I(a) \oplus I(b), F(a) \oplus F(b)>$$

Definition:6.14

A binary operation $\tilde{\oplus} : ([0,1] \times [0,1] \times [0,1])^2 \rightarrow [0,1] \times [0,1] \times [0,1]$ is continuous **n-conorm** if $\tilde{\oplus}$ satisfies the following conditions

1. $\tilde{\oplus}$ is commutative and associative.
2. $\tilde{\oplus}$ is continuous

3. $a \oplus \hat{0} = a, a \otimes \hat{1} = \hat{1}, \forall a \in [0,1] \times [0,1] \times [0,1], (\hat{1} = (1,0,0)) \text{ and } (\hat{0} = (0,1,1))$
4. $a \oplus b \leq c \oplus d$ whenever $a \leq c, b \leq d$ and $a, b, c, d \in [0,1] \times [0,1] \times [0,1]$. here,
 $a \oplus b = \oplus(<T(a), I(a), F(a)>, <T(b), I(b), F(b)>) = <T(a) \oplus T(b), I(a) \otimes I(b), F(a) \otimes F(b)>$

Definition:6.15

Let $\tilde{F}_\mu, \tilde{G}_\nu$ be two possibility neutrosophic soft sets. The **union of two possibility neutrosophic soft sets** $\tilde{F}_\mu$ and $\tilde{G}_\nu$ over U , denoted by $\tilde{F}_\mu \cup \tilde{G}_\nu$, is defined as

$$\tilde{F}_\mu \cup \tilde{G}_\nu = \{(e_i, \{\alpha, \mu_{ij}(e_i) \oplus \nu_{ij}(e_i)\} : x_j \in U\}) : e_i \in E\}$$

where $\alpha = \frac{x_j}{(\tilde{F}_{ij}^T(e_i) \oplus \tilde{G}_{ij}^T(e_i), \tilde{F}_{ij}^I(e_i) \otimes \tilde{G}_{ij}^I(e_i), \tilde{F}_{ij}^F(e_i) \otimes \tilde{G}_{ij}^F(e_i))}$

Definition:6.16

Let $\tilde{F}_\mu, \tilde{G}_\nu$ be two possibility neutrosophic soft sets. The **intersection of two possibility neutrosophic soft sets** $\tilde{F}_\mu$ and $\tilde{G}_\nu$ over U , denoted by $\tilde{F}_\mu \cap \tilde{G}_\nu$, is defined as

$$\tilde{F}_\mu \cap \tilde{G}_\nu = \{(e_i, \{\theta, \mu_{ij}(e_i) \otimes \nu_{ij}(e_i)\} : x_j \in U\}) : e_i \in E\}$$

where $\theta = \frac{x_j}{(\tilde{F}_{ij}^T(e_i) \otimes \tilde{G}_{ij}^T(e_i), \tilde{F}_{ij}^I(e_i) \oplus \tilde{G}_{ij}^I(e_i), \tilde{F}_{ij}^F(e_i) \oplus \tilde{G}_{ij}^F(e_i))}$

Example:6.17

Let us consider the possibility neutrosophic soft sets $\tilde{F}_\mu$ and $\tilde{G}_\nu$ defined as in example 4. Suppose that t-norm is defined by $a \otimes b = \min(a, b)$ and t-conorm is defined by $a \oplus b = \max(a, b)$ for $a, b \in [0,1]$. Then,

$$\tilde{F}_\mu \cup \tilde{G}_\nu = \left\{ \begin{aligned} (\tilde{F}_\mu \cup \tilde{G}_\nu)(e_1) &= \left\{ \left(\frac{x_1}{(0.6, 0.2, 0.6)}, 0.8 \right), \left(\frac{x_2}{(0.7, 0.3, 0.5)}, 0.7 \right), \left(\frac{x_3}{(0.4, 0.5, 0.4)}, 0.8 \right) \right\} \\ (\tilde{F}_\mu \cup \tilde{G}_\nu)(e_2) &= \left\{ \left(\frac{x_1}{(0.8, 0.4, 0.3)}, 0.6 \right), \left(\frac{x_2}{(0.5, 0.6, 0.2)}, 0.8 \right), \left(\frac{x_3}{(0.7, 0.2, 0.5)}, 0.8 \right) \right\} \\ (\tilde{F}_\mu \cup \tilde{G}_\nu)(e_3) &= \left\{ \left(\frac{x_1}{(0.7, 0.3, 0.3)}, 0.8 \right), \left(\frac{x_2}{(0.5, 0.3, 0.6)}, 0.6 \right), \left(\frac{x_3}{(0.8, 0.5, 0.3)}, 0.6 \right) \right\} \end{aligned} \right\}$$

and

$$\tilde{F}_\mu \cap \tilde{G}_\nu = \left\{ \begin{aligned} (\tilde{F}_\mu \cap \tilde{G}_\nu)(e_1) &= \left\{ \left(\frac{x_1}{(0.5, 0.3, 0.8)}, 0.4 \right), \left(\frac{x_2}{(0.6, 0.5, 0.5)}, 0.4 \right), \left(\frac{x_3}{(0.2, 0.6, 0.8)}, 0.7 \right) \right\} \\ (\tilde{F}_\mu \cap \tilde{G}_\nu)(e_2) &= \left\{ \left(\frac{x_1}{(0.5, 0.4, 0.5)}, 0.3 \right), \left(\frac{x_2}{(0.4, 0.7, 0.5)}, 0.6 \right), \left(\frac{x_3}{(0.7, 0.3, 0.9)}, 0.4 \right) \right\} \\ (\tilde{F}_\mu \cap \tilde{G}_\nu)(e_3) &= \left\{ \left(\frac{x_1}{(0.6, 0.7, 0.5)}, 0.2 \right), \left(\frac{x_2}{(0.4, 0.5, 0.7)}, 0.5 \right), \left(\frac{x_3}{(0.6, 0.5, 0.4)}, 0.5 \right) \right\} \end{aligned} \right\}$$

Theorem:6.18

Let $\tilde{F}_\mu$, $\tilde{G}_\nu$ and $\tilde{H}_\delta$ be three possibility neutrosophic soft sets. Then,

1. $\tilde{F}_\mu \cap \tilde{F}_\mu = \tilde{F}_\mu$ and $\tilde{F}_\mu \cup \tilde{F}_\mu = \tilde{F}_\mu$
2. $\tilde{F}_\mu \cap \tilde{G}_\nu = \tilde{G}_\nu \cap \tilde{F}_\mu$ and $\tilde{F}_\mu \cup \tilde{G}_\nu = \tilde{G}_\nu \cup \tilde{F}_\mu$
3. $\tilde{F}_\mu \cap \phi_\mu = \phi_\mu$ and $\tilde{F}_\mu \cap U_\mu = \tilde{F}_\mu$
4. $\tilde{F}_\mu \cup \phi_\mu = \tilde{F}_\mu$ and $\tilde{F}_\mu \cup U_\mu = U_\mu$
5. $\tilde{F}_\mu \cap (\tilde{G}_\nu \cap \tilde{H}_\delta) = (\tilde{F}_\mu \cap \tilde{G}_\nu) \cap \tilde{H}_\delta$ and $\tilde{F}_\mu \cup (\tilde{G}_\nu \cup \tilde{H}_\delta) = (\tilde{F}_\mu \cup \tilde{G}_\nu) \cup \tilde{H}_\delta$
6. $\tilde{F}_\mu \cap (\tilde{G}_\nu \cup \tilde{H}_\delta) = (\tilde{F}_\mu \cap \tilde{G}_\nu) \cup (\tilde{F}_\mu \cap \tilde{H}_\delta)$
 $\tilde{F}_\mu \cup (\tilde{G}_\nu \cap \tilde{H}_\delta) = (\tilde{F}_\mu \cup \tilde{G}_\nu) \cap (\tilde{F}_\mu \cup \tilde{H}_\delta)$

Definition:6.19

A function $N:[0,1] \rightarrow [0,1]$ is called a **negation** if $N(0)=1, N(1)=0$ and N is non increasing ($x \leq y \Rightarrow N(x) \geq N(y)$). A negation is called a strict negation if it is strictly negation if it is strictly decreasing ($x < y \Rightarrow N(x) > N(y)$) and continuous.

Definition:6.20

A function $n_N:[0,1] \times [0,1] \times [0,1] \rightarrow [0,1] \times [0,1] \times [0,1]$ is called a **negation** if $n_N(\hat{0})=\hat{1}, n_N(\hat{1})=\hat{0}$ and n_N is non-increasing ($x \leq y \Rightarrow n_N(x) \geq n_N(y)$). A negation is called a strict negation if it is strictly negation if it is strictly decreasing ($x < y \Rightarrow N(x) > N(y)$) and continuous.

Definition:6.21

Let $\tilde{F}_\mu$ be a possibility neutrosophic soft set. Then the **complement of possibility neutrosophic soft set** $\tilde{F}_\mu$, denoted by $\tilde{F}_\mu^c$ is defined by

$$\tilde{F}_\mu^c = \left\{ \left(e, \left\{ \left(\frac{x_j}{n_N(\tilde{F}(e_i))}, N(\mu_{ij}(e_i)(x_j)) \right) : x_j \in U \right\} \right) : e \in E \right\} \text{ where}$$

$$(n_N(\tilde{F}_{ij}^T(e_i))) = (N(\tilde{F}_{ij}^T(e_i)), N(\tilde{F}_{ij}^I(e_i)), N(\tilde{F}_{ij}^F(e_i))) \text{ for all } i, j \in \Lambda,$$

Example:6.22

Let us consider the possibility neutrosophic soft set $\tilde{F}_\mu$ defined as in example 4. Suppose that the negation is defined by

$$N(\tilde{F}_{ij}^T(e_i)) = \tilde{F}_{ij}^F(e_i), N(\tilde{F}_{ij}^F(e_i)) = \tilde{F}_{ij}^T(e_i), N(\tilde{F}_{ij}^I(e_i)) = 1 - \tilde{F}_{ij}^I(e_i). \text{ and}$$

$N(\mu_{ij}(e_i)) = 1 - \mu_{ij}(e_i)$, respectively. Then $\tilde{F}_\mu^c$ is defined as follows

$$\tilde{F}_\mu^c = \left\{ \begin{array}{l} \tilde{F}_\mu^c(e_1) = \left\{ \left(\frac{x_1}{(0.6, 0.8, 0.5)}, 0.2 \right), \left(\frac{x_2}{(0.5, 0.7, 0.7)}, 0.6 \right), \left(\frac{x_3}{(0.8, 0.5, 0.4)}, 0.3 \right) \right\} \\ \tilde{F}_\mu^c(e_2) = \left\{ \left(\frac{x_1}{(0.5, 0.6, 0.8)}, 0.4 \right), \left(\frac{x_2}{(0.2, 0.3, 0.5)}, 0.2 \right), \left(\frac{x_3}{(0.9, 0.7, 0.7)}, 0.6 \right) \right\} \\ \tilde{F}_\mu^c(e_3) = \left\{ \left(\frac{x_1}{(0.5, 0.3, 0.6)}, 0.8 \right), \left(\frac{x_2}{(0.7, 0.7, 0.5)}, 0.4 \right), \left(\frac{x_3}{(0.4, 0.5, 0.6)}, 0.5 \right) \right\} \end{array} \right\}$$

Theorem:6.23

Let $\tilde{F}_\mu$ be possibility neutrosophic soft set. Then,

1. $\phi_\mu^c = U_\mu$
2. $U_\mu^c = \phi_\mu$

Theorem:6.24

Let $\tilde{F}_\mu, \tilde{G}_\nu$ be two possibility neutrosophic soft sets. Then,

1. $(\tilde{F}_\mu \cup \tilde{G}_\nu)^c = \tilde{F}_\mu^c \cap \tilde{G}_\nu^c$
2. $(\tilde{F}_\mu \cap \tilde{G}_\nu)^c = \tilde{F}_\mu^c \cup \tilde{G}_\nu^c$

Proof:

Let $i, j \in \Lambda$

$$\begin{aligned} (\tilde{F}_\mu \cup \tilde{G}_\nu)^c &= \left\{ e_i, \left\{ \left(\frac{x_j}{(\tilde{F}_{ij}^T(e_i) \oplus \tilde{G}_{ij}^T(e_i), \tilde{F}_{ij}^I(e_i) \otimes \tilde{G}_{ij}^I(e_i), \tilde{F}_{ij}^F(e_i) \otimes \tilde{G}_{ij}^F(e_i))}, \mu_{ij}(e_i) \oplus \nu_{ij}(e_i)) : x_j \in U \right\} : e_i \in E \right\}^c \\ &= \left\{ e_i, \left\{ \left(\frac{x_j}{(\tilde{F}_{ij}^F(e_i) \otimes \tilde{G}_{ij}^F(e_i), N(\tilde{F}_{ij}^I(e_i) \otimes \tilde{G}_{ij}^I(e_i)), \tilde{F}_{ij}^T(e_i) \oplus \tilde{G}_{ij}^T(e_i))}, N(\mu_{ij}(e_i) \oplus \nu_{ij}(e_i)) : x_j \in U \right\} : e_i \in E \right\} \end{aligned}$$

$$\begin{aligned}
&= \left\{ \left\{ e_i, \left\{ \left(\frac{x_j}{(\tilde{F}_{ij}^F(e_i) \otimes \tilde{G}_{ij}^F(e_i), N(\tilde{F}_{ij}^I(e_i)) \oplus N(\tilde{G}_{ij}^I(e_i)), \tilde{F}_{ij}^T(e_i) \oplus \tilde{G}_{ij}^T(e_i)) \right), N(\mu_{ij}(e_i)) \otimes N(\nu_{ij}(e_i)) : x_j \in U \right\} \right\} : e_i \in E \right\} \\
&= \left\{ \left\{ e_i, \left\{ \left(\frac{x_j}{(\tilde{F}_{ij}^F(e_i), N(\tilde{F}_{ij}^I(e_i)), \tilde{F}_{ij}^T(e_i)) \right), N(\mu_{ij}(e_i)) : x_j \in U \right\} \right\} : e_i \in E \right\} \\
&\cap \left\{ \left\{ e_i, \left\{ \left(\frac{x_j}{(\tilde{G}_{ij}^F(e_i), N(\tilde{G}_{ij}^I(e_i)), \tilde{G}_{ij}^T(e_i)) \right), N(\nu_{ij}(e_i)) : x_j \in U \right\} \right\} : e_i \in E \right\} \\
&= \left\{ \left\{ e_i, \left\{ \left(\frac{x_j}{(\tilde{F}_{ij}^T(e_i), \tilde{F}_{ij}^I(e_i), \tilde{F}_{ij}^F(e_i)) \right), \mu_{ij}(e_i) : x_j \in U \right\} \right\} : e_i \in E \right\}^c \\
&\cap \left\{ \left\{ e_i, \left\{ \left(\frac{x_j}{(\tilde{G}_{ij}^T(e_i), \tilde{G}_{ij}^I(e_i), \tilde{G}_{ij}^F(e_i)) \right), \nu_{ij}(e_i) : x_j \in U \right\} \right\} : e_i \in E \right\}^c \\
&= \tilde{F}_\mu^c \cap \tilde{G}_\nu^c.
\end{aligned}$$

Proof of (ii) is similar.

CHAPTER VII

INTUITIONISTIC NEUTROSOPHIC SOFT SETS

Definition:7.1

Let U be an universe of discourse. Let X be a non empty fixed set. An **intuitionistic neutrosophic set** A is an object having the form $A = \{ \langle x : T_A(x), I_A(x), F_A(x) \rangle, x \in U \}$ where $T_A(x)$ represents the degree of truth membership, $I_A(x)$ represents the degree of

indeterminacy membership and $F_A(x)$ represents the degree of falsity membership function respectively of each element $x \in X$ to the set A where

$$\min[T_A(x), I_A(x)] \leq 0.5, \min[F_A(x), I_A(x)] \leq 0.5, \min[T_A(x), F_A(x)] \leq 0.5, \forall x \in U.$$

and the function satisfies the condition

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 2$$

Note:7.2

An element x of U is called significant with respect to intuitionistic neutrosophic set A of U if the degree of truth membership $T_A(x)$, the degree of indeterminacy membership $I_A(x)$ and the degree of falsity membership function $F_A(x) \leq 0.5$. Otherwise it is called insignificant.

Example:7.3

Take the universe of discourse $U = \{x_1, x_2, x_3\}$ where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 characterizes the prices of objects. It may further assumed that the values of x_1, x_2 , and x_3 are in $[0,1]$. The degree of truth membership, the degree of indeterminacy membership and the degree of falsity membership to explain the characteristics of the objects. Suppose A is an intuitionistic neutrosophic set of U such that,

$$A = \{ \langle x_1, 0.3, 0.5, 0.4 \rangle, \langle x_2, 0.4, 0.2, 0.6 \rangle, \langle x_3, 0.7, 0.3, 0.5 \rangle \}$$

where the degree of truth membership of capability is 0.3, the degree of indeterminacy membership of capability is 0.5 and degree of falsity membership of capability is 0.4 etc.

Definition:7.4

Let U be an initial universe set and E be a set of parameters, a non empty set $A \subset E$. Let $IN(U)$ denotes the set of all intuitionistic neutrosophic sets of U . The pair (F, A) is called **intuitionistic neutrosophic soft set** (in short INSS) over U , where F is a mapping given by

$$F: A \rightarrow IN(U) \text{ and } INSS(F, A) \text{ is denoted as } \tilde{F}_A$$

Example:7.5

Let U be the set of blouses under consideration and E be the set of parameters. Let $E = \{e_1 - \text{bright}, e_2 - \text{cheap}, e_3 - \text{costly}, e_4 - \text{colorful}, e_5 - \text{cotton}, e_6 - \text{expensive}\}$. Suppose that there are four blouses in the universe U given by, $U = \{b_1, b_2, b_3, b_4\}$ and set of parameters $A = \{e_1, e_2, e_3, e_4\}$. The INSS $\tilde{F}_A$ as follows,

$F(\text{bright}) = \{ \langle b_1, 0.5, 0.6, 0.2 \rangle, \langle b_2, 0.4, 0.7, 0.2 \rangle, \langle b_3, 0.6, 0.2, 0.3 \rangle, \langle b_4, 0.7, 0.3, 0.2 \rangle \}$

$F(\text{cheap}) = \{ \langle b_1, 0.6, 0.3, 0.5 \rangle, \langle b_2, 0.7, 0.4, 0.3 \rangle, \langle b_3, 0.8, 0.1, 0.2 \rangle, \langle b_4, 0.7, 0.1, 0.3 \rangle \}$

$F(\text{costly}) = \{ \langle b_1, 0.7, 0.4, 0.3 \rangle, \langle b_2, 0.6, 0.1, 0.2 \rangle, \langle b_3, 0.7, 0.2, 0.5 \rangle, \langle b_4, 0.5, 0.2, 0.6 \rangle \}$

$F(\text{colorful}) = \{ \langle b_1, 0.8, 0.1, 0.4 \rangle, \langle b_2, 0.4, 0.2, 0.6 \rangle, \langle b_3, 0.3, 0.6, 0.4 \rangle, \langle b_4, 0.4, 0.8, 0.5 \rangle \}$

Definition:7.6

Let $\tilde{F}_A$ and $\tilde{G}_B$ be two intuitionistic neutrosophic soft sets over the common universe U . $\tilde{F}_A$ is said to be **intuitionistic neutrosophic soft subset** of $\tilde{G}_B$ if and only if,

$$3. \quad A \subset B$$

$$4. \quad \tilde{F}(e) \text{ is an intuitionistic neutrosophic subset of } \tilde{G}(e)$$

$$\text{or } T_{\tilde{F}(e)}(x) \leq T_{\tilde{G}(e)}(x), I_{\tilde{F}(e)}(x) \leq I_{\tilde{G}(e)}(x), F_{\tilde{F}(e)}(x) \geq F_{\tilde{G}(e)}(x), \forall e \in A, \forall x \in U$$

This relationship is denoted by $\tilde{F}_A \subseteq \tilde{G}_B$

$\tilde{F}_A$ is said to be **intuitionistic neutrosophic soft super set** of $\tilde{G}_B$ if $\tilde{G}_B$ is an intuitionistic neutrosophic soft subset of $\tilde{F}_A$. This relationship is denoted by $\tilde{F}_A \supseteq \tilde{G}_B$.

Definition:7.7

Let $\tilde{F}_A$ and $\tilde{G}_B$ be two intuitionistic neutrosophic soft sets over the common universe U are said to be **intuitionistic neutrosophic soft equal** if $\tilde{F}_A$ is an intuitionistic

neutrosophic soft sub set of $\tilde{G}_B$ and $\tilde{G}_B$ is an intuitionistic neutrosophic soft subset of $\tilde{F}_A$.

This relationship is denoted by $\tilde{F}_A = \tilde{G}_B$

Definition:7.8

The **complement of an intuitionistic neutrosophic soft set** $\tilde{F}_A$ is denoted by $\tilde{F}_A^c$ and is defined by $\tilde{F}_A^c = (F^c, \neg A)$ where $\tilde{F}_A^c$ is a mapping given by $\tilde{F}_A^c : \neg A \rightarrow IN(U)$ and $\tilde{F}_A^c(e)$ = intuitionistic neutrosophic soft complement with

$$T_{\tilde{F}_A^c(e)}(x) = F_{\tilde{F}(e)}(x), I_{\tilde{F}_A^c(e)}(x) = I_{\tilde{F}(e)}(x) \text{ and } F_{\tilde{F}_A^c(e)}(x) = T_{\tilde{F}(e)}(x), \forall x \in U, \forall e \in \neg A$$

Example:7.9

In example 7.5, the intuitionistic neutrosophic soft complement $\tilde{F}_A^c$ as follows,
 $F(\text{not bright}) = \{ \langle b_1, 0.2, 0.6, 0.5 \rangle, \langle b_2, 0.2, 0.7, 0.4 \rangle, \langle b_3, 0.3, 0.2, 0.6 \rangle, \langle b_4, 0.2, 0.3, 0.7 \rangle \}$
 $F(\text{not cheap}) = \{ \langle b_1, 0.5, 0.3, 0.6 \rangle, \langle b_2, 0.3, 0.4, 0.7 \rangle, \langle b_3, 0.2, 0.1, 0.8 \rangle, \langle b_4, 0.3, 0.1, 0.7 \rangle \}$
 $F(\text{not costly}) = \{ \langle b_1, 0.3, 0.4, 0.7 \rangle, \langle b_2, 0.2, 0.1, 0.6 \rangle, \langle b_3, 0.5, 0.2, 0.7 \rangle, \langle b_4, 0.6, 0.2, 0.5 \rangle \}$
 $F(\text{not colorful}) = \{ \langle b_1, 0.4, 0.1, 0.8 \rangle, \langle b_2, 0.6, 0.2, 0.4 \rangle, \langle b_3, 0.4, 0.6, 0.3 \rangle, \langle b_4, 0.5, 0.8, 0.4 \rangle \}$

Definition:7.10

An intuitionistic neutrosophic soft set $\tilde{F}_A$ over U is said to be **empty intuitionistic neutrosophic soft set** with respect to the parameter A if

$$T_{\tilde{F}(e)}(x) = 0, I_{\tilde{F}(e)}(x) = 0, F_{\tilde{F}(e)}(x) = 0 \forall e \in A, \forall x \in U.$$

It is denoted by $\tilde{0}_A$. $\therefore T_{\tilde{0}(e)}(x) = 0, I_{\tilde{0}(e)}(x) = 0, F_{\tilde{0}(e)}(x) = 0$

Example:7.11

Let $U = \{b_1, b_2, b_3, b_4\}$ be the set of four blouses and $A = \{\text{bright, cheap}\}$ be the set of parameters that characterizes the blouses. Consider the INSS $\tilde{F}_A$ which describes the cost of blouses and

$$F(\text{bright}) = \{ \langle b_1, 0, 0, 0 \rangle, \langle b_2, 0, 0, 0 \rangle, \langle b_3, 0, 0, 0 \rangle, \langle b_4, 0, 0, 0 \rangle \}$$

$$F(\text{cheap}) = \{ \langle b_1, 0, 0, 0 \rangle, \langle b_2, 0, 0, 0 \rangle, \langle b_3, 0, 0, 0 \rangle, \langle b_4, 0, 0, 0 \rangle \}$$

here the INSS $\tilde{F}_A$ is null intuitionistic neutrosophic soft set.

Definition:7.12

Let $\tilde{F}_A$ and $\tilde{G}_B$ be two INSSs over the common universe U . Then **intuitionistic neutrosophic soft union** of $\tilde{F}_A$ and $\tilde{G}_B$ is denoted by ' $\tilde{F}_A \cup \tilde{G}_B$ ' is defined by $\tilde{F}_A \cup \tilde{G}_B = \tilde{K}_D$, where $D = A \cup B$ and the truth membership, indeterminacy membership and falsity membership of $\tilde{K}_D$ are as follows:

$$T_{\tilde{K}(e)}(x) = \max(T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)), I_{\tilde{K}(e)}(x) = \min(I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x))$$

$$F_{\tilde{K}(e)}(x) = \min(F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)), \forall e \in A \cup B, \forall x \in U$$

Definition:7.13

Let $\tilde{F}_A$ and $\tilde{G}_B$ be two INSSs over the common universe U . Then **intuitionistic neutrosophic soft intersection** of $\tilde{F}_A$ and $\tilde{G}_B$ is denoted by ' $\tilde{F}_A \cap \tilde{G}_B$ ' is defined by $\tilde{F}_A \cap \tilde{G}_B = \tilde{K}_D$, where $D = A \cap B$ and the truth membership, indeterminacy membership and falsity membership of $\tilde{K}_D$ are as follows:

$$T_{\tilde{K}(e)}(x) = \min(T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)), I_{\tilde{K}(e)}(x) = \max(I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x))$$

$$F_{\tilde{K}(e)}(x) = \max(F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)), \forall e \in A \cap B, \forall x \in U.$$

Theorem:7.14

Let $\tilde{F}_A$ and $\tilde{G}_B$ be two INSSs over the common universe U . Then

- 1) $\tilde{F}_A \cup \tilde{F}_A = \tilde{F}_A$ and $\tilde{F}_A \cap \tilde{F}_A = \tilde{F}_A$
- 2) $\tilde{F}_A \cup \tilde{G}_B = \tilde{G}_B \cup \tilde{F}_A$ and $\tilde{F}_A \cap \tilde{G}_B = \tilde{G}_B \cap \tilde{F}_A$
- 3) $\tilde{F}_A \cup \tilde{0}_A = \tilde{F}_A$ and $\tilde{F}_A \cap \tilde{0}_A = \tilde{0}_A$
- 4) $(\tilde{F}_A^c)^c = \tilde{F}_A$

Theorem:7.15

Let $\tilde{F}_A$, $\tilde{G}_B$ and $\tilde{K}_D$ be three INSSs over the common universe U . Then

- 1) $\tilde{F}_A \cup (\tilde{G}_B \cap \tilde{K}_D) = (\tilde{F}_A \cup \tilde{G}_B) \cap \tilde{K}_D$
- 2) $\tilde{F}_A \cap (\tilde{G}_B \cup \tilde{K}_D) = (\tilde{F}_A \cap \tilde{G}_B) \cup \tilde{K}_D$
- 3) $\tilde{F}_A \cup (\tilde{G}_B \cap \tilde{K}_D) = (\tilde{F}_A \cup \tilde{G}_B) \cap (\tilde{F}_A \cup \tilde{K}_D)$

$$4) \tilde{F}_A \cap (\tilde{G}_B \cup \tilde{K}_D) = (\tilde{F}_A \cap \tilde{G}_B) \cup (\tilde{F}_A \cap \tilde{K}_D)$$

Example:7.16

$$\text{Let } \tilde{F}_A = \{ \langle b_1, 0.6, 0.3, 0.1 \rangle, \langle b_2, 0.4, 0.7, 0.5 \rangle, \langle b_3, 0.4, 0.1, 0.8 \rangle \},$$

$$\tilde{G}_B = \{ \langle b_1, 0.2, 0.2, 0.6 \rangle, \langle b_2, 0.7, 0.2, 0.4 \rangle, \langle b_3, 0.1, 0.6, 0.7 \rangle \},$$

$$\tilde{K}_D = \{ \langle b_1, 0.3, 0.8, 0.2 \rangle, \langle b_2, 0.4, 0.1, 0.6 \rangle, \langle b_3, 0.9, 0.1, 0.2 \rangle \}, \text{ be three INSSs of}$$

U , then

$$\tilde{F}_A \cup \tilde{G}_B = \{ \langle b_1, 0.6, 0.2, 0.1 \rangle, \langle b_2, 0.7, 0.2, 0.4 \rangle, \langle b_3, 0.4, 0.1, 0.7 \rangle \},$$

$$\tilde{F}_A \cup \tilde{K}_D = \{ \langle b_1, 0.6, 0.3, 0.1 \rangle, \langle b_2, 0.4, 0.1, 0.5 \rangle, \langle b_3, 0.9, 0.1, 0.2 \rangle \},$$

$$\tilde{G}_B \cap \tilde{K}_D = \{ \langle b_1, 0.2, 0.2, 0.6 \rangle, \langle b_2, 0.4, 0.1, 0.6 \rangle, \langle b_3, 0.1, 0.1, 0.7 \rangle \},$$

$$\tilde{F}_A \cup (\tilde{G}_B \cap \tilde{K}_D) = \{ \langle b_1, 0.6, 0.2, 0.1 \rangle, \langle b_2, 0.4, 0.1, 0.5 \rangle, \langle b_3, 0.4, 0.1, 0.7 \rangle \} \text{ -----(i)}$$

$$(\tilde{F}_A \cup \tilde{G}_B) \cap (\tilde{F}_A \cup \tilde{K}_D) = \{ \langle b_1, 0.6, 0.2, 0.1 \rangle, \langle b_2, 0.4, 0.1, 0.5 \rangle, \langle b_3, 0.4, 0.1, 0.7 \rangle \} \text{ ----(ii)}$$

From (i) and (ii)

$$\tilde{F}_A \cup (\tilde{G}_B \cap \tilde{K}_D) = (\tilde{F}_A \cup \tilde{G}_B) \cap (\tilde{F}_A \cup \tilde{K}_D)$$

Definition:7.17

Let $\tilde{F}_A$ and $\tilde{G}_B$ be two INSSs over the common universe U . Then ‘**AND**’ operation on $\tilde{F}_A$ and $\tilde{G}_B$ is denoted by ‘ $\tilde{F}_A \text{ AND } \tilde{G}_B$ ’ is defined as $\tilde{F}_A \wedge \tilde{G}_B = (\tilde{K}, A \times B)$, where the truth membership, indeterminacy membership and falsity membership of $(\tilde{K}, A \times B)$ are as follows.

$$T_{\tilde{K}(\alpha, \beta)}(x) = \min(T_{\tilde{F}(\alpha)}(x), T_{\tilde{G}(\beta)}(x)), I_{\tilde{K}(\alpha, \beta)}(x) = \min(I_{\tilde{F}(\alpha)}(x), I_{\tilde{G}(\beta)}(x))$$

$$F_{\tilde{K}(\alpha, \beta)}(x) = \max(F_{\tilde{F}(\alpha)}(x), F_{\tilde{G}(\beta)}(x)), \forall \alpha \in A, \forall \beta \in B, \forall x \in U.$$

Definition:7.18

Let $\tilde{F}_A$ and $\tilde{G}_B$ be two INSSs over the common universe U . Then ‘**OR**’ operation on $\tilde{F}_A$ and $\tilde{G}_B$ is denoted by ‘ $\tilde{F}_A \text{ OR } \tilde{G}_B$ ’ is defined as $\tilde{F}_A \vee \tilde{G}_B = (\tilde{H}, A \times B)$, where the truth membership, indeterminacy membership and falsity membership of $(\tilde{H}, A \times B)$ are as follows.

$$T_{\tilde{H}(\alpha,\beta)}(x) = \max(T_{\tilde{F}(\alpha)}(x), T_{\tilde{G}(\beta)}(x)), I_{\tilde{H}(\alpha,\beta)}(x) = \min(I_{\tilde{F}(\alpha)}(x), I_{\tilde{G}(\beta)}(x))$$

$$F_{\tilde{H}(\alpha,\beta)}(x) = \min(F_{\tilde{F}(\alpha)}(x), F_{\tilde{G}(\beta)}(x)), \forall \alpha \in A, \forall \beta \in B, \forall x \in U.$$

Theorem:7.19

Let $\tilde{F}_A$ and $\tilde{G}_B$ be two INSSs over the common universe U . Then

$$1) (\tilde{F}_A \vee \tilde{G}_B)^c = \tilde{F}_A^c \wedge \tilde{G}_B^c$$

$$2) (\tilde{F}_A \wedge \tilde{G}_B)^c = \tilde{F}_A^c \vee \tilde{G}_B^c$$

Proof:

Let $\tilde{F}_A = \{ \langle x, T_{\tilde{F}_A}(x), I_{\tilde{F}_A}(x), F_{\tilde{F}_A}(x) \rangle / x \in U \}$ and

$\tilde{G}_B = \{ \langle x, T_{\tilde{G}_B}(x), I_{\tilde{G}_B}(x), F_{\tilde{G}_B}(x) \rangle / x \in U \}$ be two INSSs over the common

universe U . Also let $(\tilde{H}, A \times B) = \tilde{F}_A \vee \tilde{G}_B$ where,

$$\tilde{H}(\alpha, \beta) = \{ \langle x, \max(T_{\tilde{F}(\alpha)}(x), T_{\tilde{G}(\beta)}(x)), \min(I_{\tilde{F}(\alpha)}(x), I_{\tilde{G}(\beta)}(x)), \min(F_{\tilde{F}(\alpha)}(x), F_{\tilde{G}(\beta)}(x)) \rangle / x \in U \}$$

$$(\tilde{F}_A \vee \tilde{G}_B)^c = \{ \langle x, \min(F_{\tilde{F}(\alpha)}(x), F_{\tilde{G}(\beta)}(x)), \min(I_{\tilde{F}(\alpha)}(x), I_{\tilde{G}(\beta)}(x)), \max(T_{\tilde{F}(\alpha)}(x), T_{\tilde{G}(\beta)}(x)) \rangle / \forall x \in U \}$$

-----**(i)**

$$\tilde{F}_A^c \wedge \tilde{G}_B^c = \{ \langle x, \min(F_{\tilde{F}^c(\alpha)}(x), F_{\tilde{G}^c(\beta)}(x)), \min(I_{\tilde{F}^c(\alpha)}(x), I_{\tilde{G}^c(\beta)}(x)), \max(T_{\tilde{F}^c(\alpha)}(x), T_{\tilde{G}^c(\beta)}(x)) \rangle / x \in U \}$$

$$= \{ \langle x, \max(T_{\tilde{F}(\alpha)}(x), T_{\tilde{G}(\beta)}(x)), \min(I_{\tilde{F}(\alpha)}(x), I_{\tilde{G}(\beta)}(x)), \min(F_{\tilde{F}(\alpha)}(x), F_{\tilde{G}(\beta)}(x)) \rangle / x \in U \}^c$$

$$= \{ \langle x, \min(F_{\tilde{F}(\alpha)}(x), F_{\tilde{G}(\beta)}(x)), \min(I_{\tilde{F}(\alpha)}(x), I_{\tilde{G}(\beta)}(x)), \max(T_{\tilde{F}(\alpha)}(x), T_{\tilde{G}(\beta)}(x)) \rangle / \forall x \in U \} \text{ ----(ii)}$$

from (i) and (ii)

$$(\tilde{F}_A \vee \tilde{G}_B)^c = \tilde{F}_A^c \wedge \tilde{G}_B^c$$

2) Let $\tilde{F}_A = \{ \langle x, T_{\tilde{F}_A}(x), I_{\tilde{F}_A}(x), F_{\tilde{F}_A}(x) \rangle / x \in U \}$ and

$\tilde{G}_B = \{ \langle x, T_{\tilde{G}_B}(x), I_{\tilde{G}_B}(x), F_{\tilde{G}_B}(x) \rangle / x \in U \}$ be two INSSs over the common universe

U . Also let $(\tilde{K}, A \times B) = \tilde{F}_A \wedge \tilde{G}_B$ where,

$$\tilde{K}(\alpha, \beta) = \{ \langle x, \min(T_{\tilde{F}(\alpha)}(x), T_{\tilde{G}(\beta)}(x)), \min(I_{\tilde{F}(\alpha)}(x), I_{\tilde{G}(\beta)}(x)), \max(F_{\tilde{F}(\alpha)}(x), F_{\tilde{G}(\beta)}(x)) \rangle / x \in U \}$$

$$(\tilde{F}_A \wedge \tilde{G}_B)^c = (\tilde{K}, A \times B)^c = \{ \langle x, \max(F_{\tilde{F}(\alpha)}(x), F_{\tilde{G}(\beta)}(x)), \min(I_{\tilde{F}(\alpha)}(x), I_{\tilde{G}(\beta)}(x)), \min(T_{\tilde{F}(\alpha)}(x), T_{\tilde{G}(\beta)}(x)) \rangle / x \in U \}$$

.....(iii)

$$\tilde{F}_A^c \vee \tilde{G}_B^c = \{ \langle x, \max(F_{\tilde{F}^c(\alpha)}(x), F_{\tilde{G}^c(\beta)}(x)), \min(I_{\tilde{F}^c(\alpha)}(x), I_{\tilde{G}^c(\beta)}(x)), \min(T_{\tilde{F}^c(\alpha)}(x), T_{\tilde{G}^c(\beta)}(x)) \rangle / x \in U \}$$

$$= \{ \langle x, \min(T_{\tilde{F}(\alpha)}(x), T_{\tilde{G}(\beta)}(x)), \min(I_{\tilde{F}(\alpha)}(x), I_{\tilde{G}(\beta)}(x)), \max(F_{\tilde{F}(\alpha)}(x), F_{\tilde{G}(\beta)}(x)) \rangle / x \in U \}^c$$

.....(iv)

$$\text{from (iii) and (iv) } (\tilde{F}_A \wedge \tilde{G}_B)^c = \tilde{F}_A^c \vee \tilde{G}_B^c$$

Definition:7.20

The necessity operation on an intuitionistic neutrosophic soft set $\tilde{F}_A$ is denoted by $\Box \tilde{F}_A$ and is defined as

$$\Box \tilde{F}_A = \{ \langle x : T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), S_{\tilde{F}(e)}(x) - T_{\tilde{F}(e)}(x) \rangle, x \in U \} \text{ where}$$

$$S_{\tilde{F}(e)}(x) = T_{\tilde{F}(e)}(x) + I_{\tilde{F}(e)}(x) + F_{\tilde{F}(e)}(x).$$

Example:7.21

Let $U = \{b_1, b_2, b_3, b_4\}$ be the set of four blouses. Let $E = \{\text{bright, cheap, costly, colorful, cotton, long sleeve, expensive}\}$ be the set of parameters and $A = \{\text{bright, cheap, costly}\}$ be the set of parameters that characterizes the blouses. Consider the INSS $\tilde{F}_A$ which describes the cost of blouses and is given by

$$F(\text{bright}) = \{ \langle b_1, 0.6, 0.2, 0.4 \rangle, \langle b_2, 0.7, 0.3, 0.2 \rangle, \langle b_3, 0.5, 0.4, 0.4 \rangle, \langle b_4, 0.6, 0.4, 0.3 \rangle$$

$$F(\text{cheap}) = \{ \langle b_1, 0.7, 0.3, 0.2 \rangle, \langle b_2, 0.8, 0.1, 0.1 \rangle, \langle b_3, 0.7, 0.5, 0.2 \rangle, \langle b_4, 0.8, 0.5, 0.1 \rangle$$

$$F(\text{costly}) = \{ \langle b_1, 0.8, 0.5, 0.1 \rangle, \langle b_2, 0.6, 0.4, 0 \rangle, \langle b_3, 0.6, 0.5, 0.2 \rangle, \langle b_4, 0.2, 0.3, 0.4 \rangle$$

Then the necessity operation on an intuitionistic neutrosophic soft set $\tilde{F}_A$ becomes as $\Box \tilde{F}_A$

$$\tilde{F}_A$$

$$\Box F(\text{bright}) = \{ \langle b_1, 0.6, 0.2, 0.6 \rangle, \langle b_2, 0.7, 0.3, 0.5 \rangle, \langle b_3, 0.5, 0.4, 0.8 \rangle, \langle b_4, 0.6, 0.4, 0.7 \rangle$$

$$\Box F(\text{cheap}) = \{ \langle b_1, 0.7, 0.3, 0.5 \rangle, \langle b_2, 0.8, 0.1, 0.2 \rangle, \langle b_3, 0.7, 0.5, 0.7 \rangle, \langle b_4, 0.8, 0.5, 0.6 \rangle$$

$$\Box F(\text{costly}) = \{ \langle b_1, 0.8, 0.5, 0.6 \rangle, \langle b_2, 0.6, 0.4, 0.4 \rangle, \langle b_3, 0.6, 0.5, 0.7 \rangle, \langle b_4, 0.2, 0.3, 0.7 \rangle$$

Theorem:7.22

Let $\tilde{F}_A$ and $\tilde{G}_B$ be two INSSs over the common universe U . Then

$$1) \quad \Box (\tilde{F}_A \cup \tilde{G}_B) = \Box \tilde{F}_A \cup \Box \tilde{G}_B$$

$$2) \quad \Box (\tilde{F}_A \cap \tilde{G}_B) = \Box \tilde{F}_A \cap \Box \tilde{G}_B$$

$$3) \quad \Box \Box (\tilde{F}_A) = \Box \tilde{F}_A$$

For any positive integer n

$$4) \quad \Box (\tilde{F}_A)^n = (\Box \tilde{F}_A)^n$$

$$5) \quad \Box (\tilde{F}_A \cup \tilde{G}_B)^n = (\Box \tilde{F}_A \cup \Box \tilde{G}_B)^n$$

$$6) \quad \Box (\tilde{F}_A \cap \tilde{G}_B)^n = (\Box \tilde{F}_A \cap \Box \tilde{G}_B)^n$$

Proof:

1) Let $\tilde{F}_A$ and $\tilde{G}_B$ be two INSSs over the common universe U . Then intuitionistic neutrosophic soft union of $\tilde{F}_A$ and $\tilde{G}_B$ is defined by $\tilde{F}_A \cup \tilde{G}_B = \tilde{K}_D$, where $D = A \cup B$

$$T_{\tilde{K}(e)}(x) = \max(T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)), I_{\tilde{K}(e)}(x) = \min(I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x))$$

$$F_{\tilde{K}(e)}(x) = \min(F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)), \forall e \in A \cap B, \forall x \in U$$

Now, $\Box (\tilde{F}_A \cup \tilde{G}_B)$ is

$$T_{\tilde{K}(e)}(x) = \max(T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)), I_{\tilde{K}(e)}(x) = \min(I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x))$$

$$F_{\tilde{K}(e)}(x) = S_{\tilde{K}(e)}(x) - \max(T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)), \forall e \in A \cap B, \forall x \in U \quad \dots\dots(i)$$

$$\Box \tilde{F}_A = \{ \langle x : T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), S_{\tilde{F}(e)}(x) - T_{\tilde{F}(e)}(x) \rangle, x \in U \}$$

$$\Box \tilde{G}_B = \{ \langle x : T_{\tilde{G}(e)}(x), I_{\tilde{G}(e)}(x), S_{\tilde{G}(e)}(x) - T_{\tilde{G}(e)}(x) \rangle, x \in U \}$$

$\Box \tilde{F}_A \cup \Box \tilde{G}_B$ is

$$T_{\tilde{K}(e)}(x) = \max(T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)), I_{\tilde{K}(e)}(x) = \min(I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x))$$

$$F_{\tilde{K}(e)}(x) = S_{\tilde{K}(e)}(x) - \max(T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)), \forall e \in A \cap B, \forall x \in U \quad \dots\dots(ii)$$

From (i) and (ii)

$$\Box (\tilde{F}_A \cup \tilde{G}_B) = \Box \tilde{F}_A \cup \Box \tilde{G}_B$$

Similarly, one can prove (2)

3) Let $\tilde{F}_A = \{ \langle x : T_{\tilde{F}_A}(x), I_{\tilde{F}_A}(x), F_{\tilde{F}_A}(x) \rangle, x \in U \}$ and $\Box \tilde{F}_A$ is

$$\Box \tilde{F}_A = \{ \langle x : T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), S_{\tilde{F}(e)}(x) - T_{\tilde{F}(e)}(x) \rangle, x \in U \}$$

$$\Box \Box \tilde{F}_A = \{ \langle x : T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), S_{\tilde{F}(e)}(x) - T_{\tilde{F}(e)}(x) \rangle, x \in U \}$$

$$\Box \Box (\tilde{F}_A) = \Box \tilde{F}_A$$

$$4) (\tilde{F}_A)^n = \{ \langle x, (T_{\tilde{F}_A}(x))^n, (I_{\tilde{F}_A}(x))^n, (F_{\tilde{F}_A}(x))^n \rangle, x \in U \}$$

$$\Box (\tilde{F}_A)^n = \{ \langle x : (T_{\tilde{F}(e)}(x))^n, (I_{\tilde{F}(e)}(x))^n, S_{\tilde{F}(e)}(x) - (T_{\tilde{F}(e)}(x))^n \rangle, x \in U \}$$

$$[\Box (\tilde{F}_A)]^n = \{ \langle x : (T_{\tilde{F}(e)}(x))^n, (I_{\tilde{F}(e)}(x))^n, S_{\tilde{F}(e)}(x) - (T_{\tilde{F}(e)}(x))^n \rangle, x \in U \}$$

$$5) (\tilde{F}_A)^n \cup (\tilde{G}_B)^n = (\tilde{F}_A \cup \tilde{G}_B)^n$$

$$\Box (\tilde{F}_A \cup \tilde{G}_B)^n = [\Box (\tilde{F}_A \cup \tilde{G}_B)]^n \quad (\text{by 4})$$

$$= (\Box \tilde{F}_A \cup \Box \tilde{G}_B)^n \quad (\text{by 1})$$

$$6) (\tilde{F}_A)^n \cap (\tilde{G}_B)^n = (\tilde{F}_A \cap \tilde{G}_B)^n$$

$$\Box (\tilde{F}_A \cap \tilde{G}_B)^n = [\Box (\tilde{F}_A \cap \tilde{G}_B)]^n \quad (\text{by 4})$$

$$= (\Box \tilde{F}_A \cap \Box \tilde{G}_B)^n \quad (\text{by 2})$$

Definition:7.23

The possibility operation on an intuitionistic neutrosophic soft set $\tilde{F}_A$ is denoted by $\Diamond \tilde{F}_A$ and is defined as

$$\Diamond \tilde{F}_A = \{ \langle x : S_{\tilde{F}_A}(x) - F_{\tilde{F}_A}(x), I_{\tilde{F}_A}(x), F_{\tilde{F}_A}(x) \rangle, x \in U \} \text{ where}$$

$$S_{\tilde{F}_A}(x) = T_{\tilde{F}_A}(x) + I_{\tilde{F}_A}(x) + F_{\tilde{F}_A}(x).$$

Example:7.24

Let $U = \{b_1, b_2, b_3\}$ be the set of three blouses. Let $E = \{\text{bright, cheap, costly, colorful, cotton, long sleeve, expensive}\}$ be the set of parameters and $A = \{\text{bright, cheap, costly}\}$ be the set of parameters that characterizes the blouses. Consider the INSS $\tilde{F}_A$ which describes the cost of blouses and is given by

$$F(\text{bright}) = \{ \langle b_1, 0.7, 0.1, 0.2 \rangle, \langle b_2, 0.8, 0.3, 0 \rangle, \langle b_3, 0.8, 0.2, 0.1 \rangle \}$$

$$F(\text{cheap}) = \{ \langle b_1, 0.5, 0.3, 0.2 \rangle, \langle b_2, 0.7, 0.5, 0.1 \rangle, \langle b_3, 0.4, 0.3, 0.2 \rangle \}$$

$$F(\text{costly}) = \{ \langle b_1, 0.8, 0.4, 0.2 \rangle, \langle b_2, 0.6, 0.1, 0.3 \rangle, \langle b_3, 0.5, 0.5, 0.1 \rangle \}$$

The possibility operation on an intuitionistic neutrosophic soft set $\tilde{F}_A$ becomes as $\diamond \tilde{F}_A$

$$\diamond F(\text{bright}) = \{ \langle b_1, 0.8, 0.1, 0.2 \rangle, \langle b_2, 1.1, 0.3, 0 \rangle, \langle b_3, 1, 0.2, 0.1 \rangle \}$$

$$\diamond F(\text{cheap}) = \{ \langle b_1, 0.8, 0.3, 0.2 \rangle, \langle b_2, 1.2, 0.5, 0.1 \rangle, \langle b_3, 0.7, 0.3, 0.2 \rangle \}$$

$$\diamond F(\text{costly}) = \{ \langle b_1, 1.2, 0.4, 0.2 \rangle, \langle b_2, 0.7, 0.1, 0.3 \rangle, \langle b_3, 1, 0.5, 0.1 \rangle \}$$

Theorem:7.25

Let $\tilde{F}_A$ and $\tilde{G}_B$ be two INSSs over the common universe U . Then

$$1) \diamond (\tilde{F}_A \cup \tilde{G}_B) = \diamond \tilde{F}_A \cup \diamond \tilde{G}_B$$

$$2) \diamond (\tilde{F}_A \cap \tilde{G}_B) = \diamond \tilde{F}_A \cap \diamond \tilde{G}_B$$

$$3) \diamond \diamond (\tilde{F}_A) = \diamond \tilde{F}_A$$

For any positive integer n

$$4) \diamond (\tilde{F}_A)^n = (\diamond \tilde{F}_A)^n$$

$$5) \diamond (\tilde{F}_A \cup \tilde{G}_B)^n = (\diamond \tilde{F}_A \cup \diamond \tilde{G}_B)^n$$

$$6) \diamond (\tilde{F}_A \cap \tilde{G}_B)^n = (\diamond \tilde{F}_A \cap \diamond \tilde{G}_B)^n$$

Proof:

1) Let $\tilde{F}_A$ and $\tilde{G}_B$ be two INSSs over the common universe U . Then intuitionistic neutrosophic soft union of $\tilde{F}_A$ and $\tilde{G}_B$ is defined by $\tilde{F}_A \cup \tilde{G}_B = \tilde{K}_D$, where $D = A \cup B$

$$T_{\tilde{K}(e)}(x) = \max(T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)), I_{\tilde{K}(e)}(x) = \min(I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x))$$

$$F_{\tilde{K}(e)}(x) = \min(F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)), \forall e \in A \cap B, \forall x \in U$$

Now, $\diamond (\tilde{F}_A \cup \tilde{G}_B)$ is

$$T_{\tilde{K}(e)}(x) = S_{\tilde{F}(e)}(x) - \min(F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)), I_{\tilde{K}(e)}(x) = \min(I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x))$$

$$F_{\tilde{K}(e)}(x) = \min(F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)), \forall e \in A \cap B, \forall x \in U \text{ -----(i)}$$

$$\diamond \tilde{F}_A = \{ \langle x : S_{\tilde{F}(e)}(x) - F_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \rangle, x \in U \}$$

$$\diamond \tilde{G}_B = \{ \langle x : S_{\tilde{G}(e)}(x) - F_{\tilde{G}(e)}(x), I_{\tilde{G}(e)}(x), F_{\tilde{G}(e)}(x) \rangle, x \in U \}$$

$\diamond \tilde{F}_A \cup \diamond \tilde{G}_B$ is

$$T_{\tilde{K}(e)}(x) = S_{\tilde{F}(e)}(x) - \min(F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)), I_{\tilde{K}(e)}(x) = \min(I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x))$$

$$F_{\tilde{K}(e)}(x) = \min(F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)), \forall e \in A \cap B, \forall x \in U \text{ -----(ii)}$$

from (i) and (ii)

$$\diamond(\tilde{F}_A \cup \tilde{G}_B) = \diamond \tilde{F}_A \cup \diamond \tilde{G}_B$$

Proof of (2) is Similar.

3) Let $\tilde{F}_A = \{ \langle x : T_{\tilde{F}_A}(x), I_{\tilde{F}_A}(x), F_{\tilde{F}_A}(x) \rangle, x \in U \}$ and $\diamond \tilde{F}_A$ is

$$\diamond \tilde{F}_A = \{ \langle x : S_{\tilde{F}(e)}(x) - F_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \rangle, x \in U \}$$

$$\diamond \diamond \tilde{F}_A = \{ \langle x : S_{\tilde{F}(e)}(x) - F_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \rangle, x \in U \}$$

$$\diamond \diamond (\tilde{F}_A) = \diamond \tilde{F}_A$$

$$4) (\tilde{F}_A)^n = \{ \langle x, (T_{\tilde{F}_A}(x))^n, (I_{\tilde{F}_A}(x))^n, (F_{\tilde{F}_A}(x))^n \rangle, x \in U \}$$

$$\diamond (\tilde{F}_A)^n = \{ \langle x : S_{\tilde{F}(e)}(x) - (F_{\tilde{F}(e)}(x))^n, (I_{\tilde{F}(e)}(x))^n, (F_{\tilde{F}(e)}(x))^n \rangle, x \in U \}$$

$$[\diamond (\tilde{F}_A)]^n = \{ \langle x : S_{\tilde{F}(e)}(x) - (F_{\tilde{F}(e)}(x))^n, (I_{\tilde{F}(e)}(x))^n, (F_{\tilde{F}(e)}(x))^n \rangle, x \in U \}$$

$$5) (\tilde{F}_A)^n \cup (\tilde{G}_B)^n = (\tilde{F}_A \cup \tilde{G}_B)^n$$

$$\diamond (\tilde{F}_A \cup \tilde{G}_B)^n = [\diamond (\tilde{F}_A \cup \tilde{G}_B)]^n \quad (\text{by 4})$$

$$= (\diamond \tilde{F}_A \cup \diamond \tilde{G}_B)^n \quad (\text{by 1})$$

$$6) (\tilde{F}_A)^n \cap (\tilde{G}_B)^n = (\tilde{F}_A \cap \tilde{G}_B)^n$$

$$\diamond (\tilde{F}_A \cap \tilde{G}_B)^n = [\diamond (\tilde{F}_A \cap \tilde{G}_B)]^n \quad (\text{by 4})$$

$$= (\diamond \tilde{F}_A \cap \diamond \tilde{G}_B)^n \quad (\text{by 2})$$

Theorem:7.26

Let $\tilde{F}_A$ be an intuitionistic neutrosophic soft set over U . Then

$$1) \diamond \Box (\tilde{F}_A) = \Box (\tilde{F}_A)$$

$$2) \Box \diamond (\tilde{F}_A) = \diamond (\tilde{F}_A)$$

Proof:

Let $\tilde{F}_A$ be an intuitionistic neutrosophic soft set over U .

$$\text{Then } \tilde{F}_A = \{ \langle x : T_{\tilde{F}_A}(x), I_{\tilde{F}_A}(x), F_{\tilde{F}_A}(x) \rangle, x \in U \}$$

$$\Box \tilde{F}_A = \{ \langle x : T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), S_{\tilde{F}(e)}(x) - T_{\tilde{F}(e)}(x) \rangle, x \in U \}$$

$$\Diamond \tilde{F}_A = \{ \langle x : S_{\tilde{F}(e)}(x) - F_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \rangle, x \in U \}$$

$$\Diamond \Box (\tilde{F}_A) = \{ \langle x : S_{\tilde{F}(e)}(x) - (S_{\tilde{F}(e)}(x) - T_{\tilde{F}(e)}(x)), I_{\tilde{F}(e)}(x), S_{\tilde{F}(e)}(x) - T_{\tilde{F}(e)}(x) \rangle, x \in U \}$$

$$= \{ \langle x : T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), S_{\tilde{F}(e)}(x) - T_{\tilde{F}(e)}(x) \rangle, x \in U \}$$

$$= \Box \tilde{F}_A$$

Proof of (2) is similar.