CHAPTER - VI

CHAPTER VI

SOFT REGULAR WEAKLY CLOSED SETS IN SOFT TOPOLOGICAL SPACES

Definition 6.1

In a Soft Topological Spaces (X, τ, E), a soft set (F, E) over X is called

i. a **soft semi open** if $(F, E) \subseteq Cl(Int(F, E))$ and **soft semi closed** if $Int(Cl(F, E)) \subseteq (F, E)$.

ii. a **soft regular open** if $(F, E) = Int(Cl(F, E))$ and **soft regular closed** if $(F, E) = Cl(Int(F, E))$.

iii. a **soft weakly closed** (briefly **sw-closed**) if $Cl((F, E) \subseteq (U, E)$ whenever $(F, E) \subseteq (U, E)$ and (U, E) is soft semi open in X.

iv. a **soft regular semi open** if there exists a soft regular open set (U, E) such that $(U, E) \subseteq (F, E) \subseteq Cl(U, E)$.

Result 6.2

i. Every soft regular semi open set in (X, τ, E) is soft semi open.

ii. If (F, E) is soft regular semi open in (X, τ, E) then $(X, E) \backslash (F, E)$ is also soft regular semi open.

Result 6.3

Let (X, τ, E) be a Soft Topological Spaces over X and (F, E) and (G, E) be a soft sets over X. Then

i. (F, E) is soft closed set if and only if $(F, E) = Cl(F, E)$

ii. $Cl((F, E) \cup (G, E)) = Cl(F, E) \cup Cl(G, E)$

iii. $Cl(Cl(F, E)) = Cl(F, E)$.

Definition 6.4

A subset A of a Topological Spaces (X, τ) is called

i. a **semi open** if $A \subseteq Cl(Int(A))$ and **semi closed** if $Int(Cl(A) \subseteq A$.

ii. a **regular open** if $A = Int(Cl(A))$ and **regular closed** if $A = Cl(Int(A))$.

iii. a **regular semi open** if there exists a regular open set U such that $U \subseteq A \subseteq Cl(U)$.

iv. a **weakly closed** (briefly **w-closed**) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

v. a **regular weakly closed** (briefly **rw-Closed**) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in (X, τ).

Definition 6.5

Let (X, τ, E) be a Soft Topological Space. A soft set (F, E) is called **Soft Regular Weakly Closed set** (briefly **SRW-Closed set**) if $Cl(F, E) \subseteq (U, E)$ whenever $(F, E) \subseteq (U, E)$ and (U, E) is soft regular semi open set in (X, τ, E).

Example 6.6

Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and

$\tau = \{\Phi, \widetilde{E}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$ where

$F_1(e_1) = \{x_1\}$ $F_1(e_2) = \{x_1\}$

$F_2(e_1) = \{x_2\}$ $F_2(e_2) = \{x_2\}$

$F_3(e_1) = \{x_3\}$ $F_3(e_2) = \{x_3\}$

$F_4(e_1) = \{x_1, x_2\}$ $F_4(e_2) = \{x_1, x_2\}$

$F_5(e_1) = \{x_2, x_3\}$ $F_5(e_2) = \{x_2, x_3\}$

$F_6(e_1) = \{x_1, x_3\}$ $F_6(e_2) = \{x_1, x_3\}$

Then (X, τ, E) is a Soft Topological Spaces. Define soft sets (G, E) and (H, E) over X such that

$G(e_1) = \{x_1, x_3\}, G(e_2) = \{x_1\}$ and $H(e_1) = \{x_1, x_2\}$, $H(e_2) = \{x_2\}$.

Here both (G, E) and (H, E) are SRW-Closed sets in (X, τ, E).

Theorem 6.7

Every soft closed set is a SRW-Closed set but not conversely.

Proof

Let (F, E) be a soft closed set in (X, τ, E) and (U, E) be soft regular semi open set such that $(F, E) \subseteq (U, E)$. Consider $Cl(F, E) = (F, E) \subseteq (U, E)$. Therefore (F, E) is SRW-Closed set.

In Example 6.6, (G, E) is a SRW-Closed set but not soft closed set.

Theorem 6.8

Every SW-Closed set is a SRW-Closed set but not conversely.

Proof

The proof follows from the definitions and the fact that every soft regular semi open set is soft semi open set.

In Example 6.6, (G, E) is a SRW-Closed set but not SW-Closed set.

Theorem 6.9

If (F, E) and (G, E) are SRW-Closed sets in (X, τ, E) then $(F, E) \cup (G, E)$ is SRW-Closed set in (X, τ, E).

Proof

Suppose (F, E) and (G, E) are SRW-Closed sets in (X, τ, E). Then $Cl(F, E) \subseteq (U, E)$ and $Cl(G, E) \subseteq (U, E)$ where $(F, E) \subseteq (U, E)$ and $(G, E) \subseteq (U, E)$.

Hence $Cl\big((F, E) \cup (G, E)\big) = Cl(F, E) \cup Cl(G, E) \subseteq (U, E)$. That is $Cl\big((F, E) \cup (G, E)\big) \subseteq (U, E)$. Therefore $(F, E) \cup (G, E)$ is SRW-Closed set in (X, τ, E).

Remark 6.10

Intersection of two SRW Closed sets need not be a SRW-Closed set.

In Example 6.6, (H, E) and (G, E) are SRW-Closed sets in (X, τ, E). But $(H, E) \cap (G, E)$ is not SRW-Closed set in (X, τ, E).

Theorem 6.11

If a soft set (F, E) is SRW-Closed set in (X, τ, E) then the difference $Cl(F, E) \setminus (F, E)$ does not contain any non-empty soft regular semi open set in (X, τ, E).

Proof

We prove the result by contradiction. Let (U, E) be a non-empty soft regular semi open set such that

$$Cl(F, E)\backslash(F, E) \supseteq (U, E) \text{------------------------------- } (1)$$

Therefore $(U, E) \subseteq (X, E)\backslash(F, E)$ then $(F, E) \subseteq (X, E)\backslash(U, E)$. Since (U, E) is soft regular semi open set by result 6.2(ii) $(X, E)\backslash(U, E)$ is also soft regular semi open set in (X, τ, E). Since (F, E) is SRW-Closed set in (X, τ, E) and $Cl(F, E) \subseteq (X, E)\backslash(U, E)$, so $(U, E) \subseteq (X, E)\backslash Cl(F, E)$. Also by (1) $(U, E) \subseteq Cl(F, E)$. Therefore $(U, E) \subseteq Cl(F, E) \cap ((X, E)\backslash Cl(F, E)) = \phi$.This shows that (U, E) is empty, which is a contradiction.

Hence $Cl(F, E)\backslash(F, E)$ does not contain any non-empty soft regular semi open set in (X, τ, E).

Corollary 6.12

If (F, E) is SRW-Closed set in (X, τ, E) then $Cl(F, E)\backslash(F, E)$ does not contain any non-empty soft regular open set in (X, τ, E).

Proof

Follows from theorem (6.11) and the fact that every soft regular open set is soft regular semi open set.

Corollary 6.13

If (F, E) is SRW-Closed set in (X, τ, E) then $Cl(F, E)\backslash(F, E)$ does not contain any non-empty soft regular closed set in (X, τ, E).

Proof

Follows from theorem (6.11) and the fact that every soft regular open set is soft regular semi open set.

Theorem 6.14

If (F, E) is a SRW-Closed set in (X, τ, E) such that $(F, E) \subseteq (G, E) \subseteq Cl(F, E)$ then (G, E) is SRW-Closed set in (X, τ, E).

Proof

Let (F, E) be SRW-Closed set in (X, τ, E) such that $(F, E) \subseteq (G, E) \subseteq Cl(F, E)$. Let (U, E) be a soft regular semi open set of (X, τ, E) such that $(G, E) \subseteq (U, E)$. Then $(F, E) \subseteq (U, E)$.Since (F, E) is SRW-Closed set,$Cl(F, E) \subseteq (U, E)$. Now $Cl(G, E) \subseteq Cl(Cl(F, E)) = Cl(F, E) \subseteq (U, E)$. That is $Cl(G, E) \subseteq (U, E)$. Therefore (G, E) is SRW-Closed set in(X, τ, E).

Theorem 6.15

Let (F, E) is SRW-Closed set in (X, τ, E). Then (F, E) is soft closed set if and only if $Cl(F, E)\backslash(F, E)$ is soft regular semi open set in (X, τ, E).

Proof

Suppose (F, E) is soft closed set in (X, τ, E). Then $Cl(F, E) = (F, E)$ and $Cl(F, E)\backslash (F, E) = \phi$, which is a soft regular semi open set in (X, τ, E).

Conversely, suppose $Cl(F, E)\backslash(F, E)$ is soft regular semi open set in (X, τ, E). Since (F, E) is SRW-Closed set by theorem (6.11), $Cl(F, E)\backslash(F, E)$ does not contain any nonempty soft regular semi open set in (X, τ, E). Then $Cl(F, E)\backslash(F, E) = \phi$. Hence (F, E) is soft closed set in (X, τ, E).