On g*s – closed sets in topological spaces

Elakkiya, M

(13PMA003)

Thesis submitted to

Avinashilingam Institute for Home Science and Higher Education for Women,

Coimbatore – 641 043

In Partial Fulfilment of the Requirements for the

Degree of Master of Science in Mathematics

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Signature of the Head of the Department

N. Balamani
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Signature of the Supervisor

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Content

CONTENT

CHAPTERS **TITLE**

Introduction

INTRODUCTION

“Pure Maths Is In Its Way, The Poetry Of Logical Ideas”

-Albert Einstein

Topology is an indispensable subject of study with open sets as well as closed sets being the most fundamental concepts in topological spaces. General Topology has shown its fruitfulness in both the pure and applied directions.

Levine [1970] introduced the concept of generalized closed sets and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, generalization of continuity and covering properties. After the introduction of generalized closed sets there are many research papers which deals with different types of generalized closed sets.

Levine [1963] introduced semi continuous functions using semi open sets. The study of semi – continuous functions is further carried out by Noiri [1986], Crossely and Hilderbrand [1972] and many others. Sundaram [1991] introduced the concept of generalized continuous functions and studied several properties related to it.

This thesis is devoted to the study of g*s – closed sets, g*s – continuous functions, g*s – closed maps and g*s – open maps, strongly g*s – continuous maps and perfectly g*s – continuus maps, g*s – connectedness, g*s – locally closed sets, g*s – irresolute maps, and g*s – homeomorphisms in topological spaces.

The following articles are chosen for discussion :

- “g*s closed sets in topological spaces” by **A. Pushpalatha and K. Anitha [2011]**.
- “Strongly g*s – continuous maps and perfectly g*s – continuous maps in topological spaces” by **A. Pushpalatha and K. Anitha [2012]**

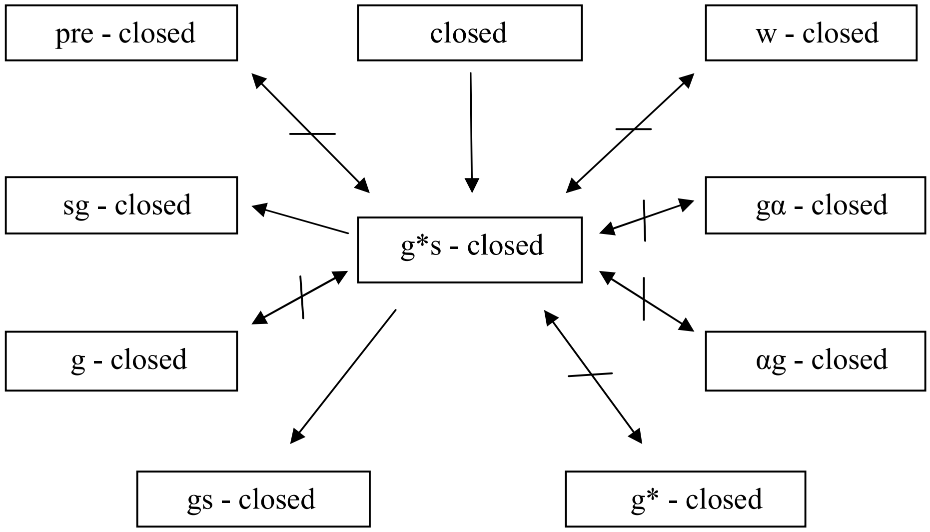
- "g*s – connectedness and g*s – locally closed sets in topological spaces" by **K. Anitha and A. Pushpalatha [2012]**.
- "g*s – irresolute maps and g*s – homeomorphisms in topological spaces" by **M. Perachi Sundari, Latha Martin [2014].**

In Chapter 1, g*s – closed sets in topological spaces due to Pushpalatha and Anitha [2011] are studied. In Section 1.1, deals with preliminary definitions that are related for our study.In Section 1.2, g*s – closed sets and g*s – open sets in topological spaces are studied. The relationship between g*s - closed sets and other generalized closed sets are discussed. Properties, characterization theorems on g*s – closed sets are discussed.

Some of the interesting results discussed in this section are as follows:

- A subset A of X is g*s-closed set in X iff scl(A)-A contains no non empty gs - closed set in X.
- A set A is g*s –open in X if and only if F ⊆ sint(A) whenever F is gs –closed in X and F ⊆ A.
- Let (X , τ) be a compact topological space and suppose that A is g*s -closed subset of X. Then A is compact.

The following diagram shows the relationship between g*s – closed sets with various closed sets.



In Section 1.3 deals with g*s – continuous functions, results on g*s – continuous functions and the relationship between g*s – continuous functions with other continuous functions.

Some of the interesting results discussed here are :

- If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is continuous then it is g*s –continuous.
- If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is g*s - continuous then it is gs – continuous.
- Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then the following statements are equivalent:
    - f is g*s –continuous
    - the inverse image of each open set in Y is g*s –open in X.

In Section 1.4, g*s – closed maps and g*s – open maps are defined and some of the characterizations and properties are discussed.

In Chapter 2, Strongly g*s – continuous maps and Perfectly g*s – continuous maps in topological spaces due to Pushpalatha and Anitha [2012] are discussed.

In Section 2.1, preliminary definitions are given. In Section 2.2 deals with T_L – space and T_A – space in terms of g*s – closed sets. In Section 2.3, Strongly g*s – continuous maps and perfectly g*s – continuous maps are studied.

- A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said be strongly g*s –continuous if the inverse image of every g*s –open set in Y is open in X.
- A map $f : X \rightarrow Y$ is said to be perfectly g*s –continuous if the inverse image of every g*s –open set in Y is both open and closed in X.

Some of the important results discussed are as follows :

- If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is strongly g*s –continuous then it is continuous.

- If a map f : (X, τ) → (Y, σ) is strongly continuous then it is strongly g*s –continuous.
- If a map f : (X, τ) → (Y, σ) from a topological space X into the topological space Y is perfectly g*s –continuous then it is strongly g*s – continuous.

In Section 2.4, g*s – compactness and some of the characterization theorems are discussed.

In Chapter 3, g*s – connectedness and g*s – locally closed sets in topological spaces due to Anitha and Pushpalatha [2012] are studied.

In Section 3.1, preliminary definitions needed for our work are collected.

In Section 3.2, deals with the g*s – connectedness

"A topological space X is called g*s –connected if X cannot be written as a disjoint union of two nonempty g*s –open sets. A subset of X is g*s –connected if it is g*s – connected as subspace of X."

Characterizations of g*s – connectedness are given as follows :

- For a topological space X. The following are equivalent :
    - X is g*s –connected.
    - The only subsets of X which are both g*s –open and g*s –closed are the empty set φ and X.
    - Each g*s continuous map of X into a discrete space Y with at least two points is a constant map.

In Section 3.3, g*s, g*s* and g*s** - closed sets in a topological spaces are defined and its properties are studied.

In Chapter 4, g*s – irresolute maps and g*s – homeomorphisms in topological spaces due to Perachi Sundari and Latha Martin [2014] are studied.In Section 4.1, g*s – irresolute maps, characterizations and properties are discussed.

The following theorems are obtained:

- If f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are both g*s –irresolute, then g ∘ f : (X, τ) → (Z, η) is g*s –irresolute.

- If a map f : (X, τ) → (Y, σ) is g*s –irresolute and a subset B of X is g*s –compact relative to X, then the image f(B) is g*s –compact relative to Y.

In Section 4.2, g*s – Hausdorff space in topological spaces and its properties are studied.

Section 4.3, deals with g*s – Homeomorphisms and g*sc – homeomorphisms in topological spaces and its properties and characterizations.

- **"**A bijection f : (X, τ) → (Y, σ) is called a g*s -homeomorphism if f is both g*s -continuous g*s –open**".**
- **"**A bijective f : (X, τ) → (Y, σ) is said to be g*sc –homeomorphism if both f and f^{-1} are g*s –irresolute**".**

Characterizations of g*s and g*sc – homeomorphisms are given as follows :

- Let f : (X, τ) → (Y, σ) be a bijective g*s –continuous map, then the following are equivalent:
  - f is g*s –open map.
  - f is g*s –homeomorphism.
  - f is g*s –closed map.
- Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are g*sc – homeomorphism.Then their composition g ∘ f : (X, τ) → (Z,n) is also g*sc –homeomorphism.

Review of Literature

REVIEW OF LITERATURE

Research on the field of generalized closed sets was developed by many authors in the last two decades. The theory was extensively developed in 1990's several new concepts were studied and investigated.

The initiation of the study of generalized closed sets was done by Levine [1970] as he considered sets whose closures belong to every open super set. The space in which the concepts of g – closed and closed sets coincide is called $T_{1/2}$ spaces. Dunham [1977] showed that $T_{1/2}$ spaces are preciously the spaces in which singletons are open or closed.

Bhattacharya and Lahiri [1987] introduced the notation of semi – generalized closed sets by replacing the closure operator in the original Levine's definition by semi closure operator and by replacing openness of the superset with semi – openness.

Arya and Nour [1990] defined the notation of generalized semi – closed sets (briefly gs - closed). Veerakumar [2000] introduced the new class of sets called g* - closed sets which is properly placed in between the class of closed and the class of g – closed sets.

Balachandran et.al [1991] introduced g – continuous functions. Sundaram [1991] introduced the concept of semi – generalized continuous maps and generalized semi – continuous maps. Devi et. al [1993] introduced semi – generalized closed maps (briefly sg – closed), generalized semi – closed maps (briefly gs – closed) and studied some of their basic properties. As applications, they showed that under the continuous, gs – closed surjection the image of a normal space is s – normal and that under semi – open, continuous, generalized semi – closed surjection the image of a regular space is s – regular. Further, they characterized the class of $T_{1/2}$ – spaces by using gs – closed sets and semi closed sets.

Levine [1960] introduced and investigated the concept of strong continuity in topological spaces. Sundaram [1991] introduced strongly g – continuous maps and perfectly g – continuous maps in topological spaces. Sundaram [1991] introduced the completely GO – compact spaces by using g – open covers.

The notion of a locally closed sets in a topological space was introduced by Kuratowski and Sierpinski [1921]. According to Bourbaki [1996] a subset of a topological space is locally closed in X if it is the intersection of an open set in X and a closed set in X. Ganster and Reilly [1989] introduced locally closed sets in topological spaces and studied three different notations of generalized continuity namely lc – continuity, lc – irresoluteness and sub lc – continuity. According to them, a subset of (X, τ) is locally closed in X if it is the intersection of an open subset of X and closed subset of X. Also they gave the decomposition that a function between two topological spaces is continuous if and only if it sub – lc continuous and nearby continuous. H. Maki, et.al [1996] introduced the concept of generalized locally closed sets and obtained different notations of generalized continuities.

Crossley and Hildebrand [1972] introduced irresolute maps. Cammamaroto and Noiri [1989] introduced almost irresolute functions in topological spaces. Maki et. al [1991] introduced generalized homeomorphisms in topological spaces. Devi et. al [1995] introduced two classes of mappings namely generalized semi homeomorphism and semi generalized homeomorphism and investigated some properties of the mappings from the quotient spaces to other spaces.

Chapter – 1

CHAPTER – 1

g*s – CLOSED SETS IN TOPOLOGICAL SPACES

Section 1.1

Preliminaries

Definition 1.1.1 [32]

A subset A of a topological space (X, τ) is called **regular closed** if A = cl(int(A)).

Definition 1.1.2 [17]

A subset A of a topological space (X, τ) is called **semi closed** if int(cl(A)) ⊆ A.

Definition 1.1.3 [26]

A subset A of a topological space (X, τ) called **α - closed** if cl(int(cl(A))) ⊆ A.

Definition 1.1.4 [23]

A subset A of a topological space (X, τ) is called **preclosed** if cl(int(A)) ⊆ A.

Definition 1.1.5 [18]

A subset A of a topological space (X, τ) is called **generalized closed** (briefly g-closed) if cl(A) ⊆ U whenever A ⊆ U and U is open in X.

Definition 1.1.6 [2]

A subset A of a topological space (X, τ) is called **generalized semiclosed** (briefly gs-closed) if scl(A) ⊆ U whenever A ⊆ U and U is open in X.

Definition 1.1.7 [5]

A subset A of a topological space (X, τ) is called **semi-generalized closed** (briefly sg-closed) if scl(A) ⊆ U whenever A ⊆ U and U is semi open in X.

Definition 1.1.8 [34]

A subset A of a topological space (X, τ) called **weakly closed** (briefly w-closed) if cl(A) ⊆ U whenever A ⊆ U and U is semi open in X.

Definition 1.1.9 [24]

A subset A of a topological space (X, τ) is called **weakly generalized closed** (briefly wg-closed) if cl(int(A)) ⊆ U whenever A ⊆ U and U is open in X.

Definition 1.1.10 [19]

A subset A of a topological space (X, τ) is called **generalized α-closed** (briefly gα-closed) if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in X.

Definition 1.1.11 [20]

A subset A of a topological space (X, τ) is called **α-generalized closed** (briefly αg-closed) if αcl(A) ⊆ U whenever A ⊆ U and U is open in X.

Definition 1.1.12 [4]

A subset A of a topological space (X, τ) is called **regular w-closed** (briefly rw-closed) if αcl(A) ⊆ U whenever A ⊆ U and U is regular semiopen in X.

Definition 1.1.13 [29]

A subset A of a topological space (X, τ) is called **strongly g-closed** (briefly g* - closed) if cl(A) ⊆ U whenever A ⊆ U and U is g-open in X.

Definition 1.1.14 [16]

A function f : X →Y is called **strongly continuous** if $f^{-1}(V)$ is both open and closed in X for each subset V in Y.

Definition 1.1.15 [27]

A function f : X →Y is called **perfectly continuous** if $f^{-1}(V)$ is both open and closed in X for each open set V in Y.

Definition 1.1.16 [3]

A function f : X →Y is called **generalized continuous** (g-continuous) if $f^{-1}(V)$ is g-open in X for each open set V in Y.

Definition 1.1.17 [33]

A function f : X →Y is called **strongly g-continuous** if $f^{-1}(V)$ is open in X for each g-open set V in Y.

Definition 1.1.18 [5]

A function f : X →Y is called **semi-generalized continuous** (sg-continuous) if $f^{-1}(V)$ is sg-open in X for each open set V in Y.

Definition 1.1.19 [12]

A function f : X →Y is called **generalized semi-continuous** (gs-continuous) if $f^{-1}(V)$ is gs-open in X for each open set V in Y.

Section 1.2

g*s - CLOSED SETS AND g*s – OPEN SETS

Definition 1.2.1[9]

A subset A of a topological space (X, τ) is called **g*s-closed** if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs-open in X.

The collection of all g*s-closed sets in a topological space (X, τ) is denoted by g*s - C(X, τ).

Remark 1.2.2

A subset A of a topological space X is called g*s-open if A^c is g*s-closed set.

Example 1.2.3

Let X = {a, b, c} with τ = {φ, {a,b}, X}. Then

gs-open sets = {φ, X, {c}, {b,c},{c,a}};

gs-closed sets = {φ, X, {a}, {b}, {a,b}};

g*s-closed sets = {φ, X, {c}, {b,c}, {c,a}} and

g*s-open sets = {φ, X, {a}, {b}, {a,b}}.

Theorem 1.2.4

Every closed set in X is g*s closed set in X.

Proof:

Let A be a closed set in X. Let U be a gs-open set such that A ⊆ U. Since A is closed, cl(A) = A, cl(A) ⊆ U. But scl(A) ⊆ cl(A) ⊆ U. Therefore scl(A) ⊆ U. Hence A is g*s- closed set in X.

Remark 1.2.5

The converse of the above theorem is not true by the following example.

Example 1.2.6

Let X = {a, b, c} with τ = {φ, X, {a}, {b},{a,b}}. Then

closed sets ={φ, X, {b,c}, {a,c}, {c}} and

g*s closed sets = {φ, X, {a}, {b},{c},{b,c},{c,a}}.

Here the sets {a} and {b} are g*s- closed but not closed.

Theorem 1.2.7

Union of two g*s- closed sets is g*s- closed set.

Proof:

Let A and B be g*s closed sets in X. Let U be gs -open in X such that A⋃B ⊆ U.Then A ⊆ U and B ⊆ U. Since A and B are g*s-closed , scl(A) ⊆ U and scl(B) ⊆ U. Hence scl(A ⋃ B) = scl(A) ⋃ scl(B) ⊆ U. Therefore A ⋃ B is g*s-closed in X.

Theorem 1.2.8

Every g*s-closed set is gs-closed set in X.

Proof:

Let A be g*s-closed set in X. Let U be open set such that A ⊆ U. Since every open set is gs-open and A is g*s-closed, we have scl(A) ⊆ U. Therefore A is gs-closed in X.

Remark 1.2.9

The converse of the above theorem is not true by the following example.

Example 1.2.10

Let X = {a, b, c} with τ ={φ, X, {a}, {b,c}}, Then

gs-closed sets = {φ, X, {a}, {b}, {a,b},{b,c},{c,a}} and

g*s- closed sets = {φ, X, {a},{a,b},{b,c}}.

Here the sets {a, c} and {b} are gs – closed but not g*s -closed.

Theorem 1.2.11

Every g*s-closed set in X is a sg-closed set in X.

Proof:

Let A be g*s closed set in X. Let U be a semi-open set in X such that A ⊆ U. Since every semiopen set is gs-open and A is g*s-closed. We have scl(A) ⊆ U. Therefore A is sg-closed set in X.

Remark 1.2.12

The converse of the above theorem is not true by the following example.

Example 1.2.13

Let X = {a, b, c} with τ ={φ, X, {c}, {a, b}}. Then

sg-closed sets = {φ, X, {a}, {b}, {c}, {a, b}, {b, c}, {c, a}} and g*s-closed sets = {φ, X, {c}, {a, b}, {b, c}, {c, a}}.

Here the sets {a} and {b} are sg - closed but not g*s-closed.

Remark 1.2.14

g*s – closed sets and g – closed sets are independent.

Example 1.2.15

Let X ={a, b, c, d} with τ = {φ, X, {a}, {b}, {a, b}, {a, b, c}}. Then

g*s-closed sets = {φ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b,c}, {b,d}, {a, d}, {c,d}, {b, c, d}, {a, c, d}} and

g - closed sets: {φ, X, {d}, {a, d}, {b, d}, {c, d}, {b, c, d}, {a, c, d},{a, b, d}}.

Then the sets {a}, {b}, {c}, {a, c},{b, c} are g*s-closed sets but not g-closed sets and the set {a, b ,d} is g-closed set but not g*s-closed set.

Remark 1.2.16

g*s – closed sets and preclosed closed sets are independent.

Example 1.2.17

Let X ={a, b, c} with τ = {φ, X, {a}, {b}, {a, b}}. In this space the sets {a} and {b} are g*s-closed but not preclosed. In the topology τ = {φ , X , {a} , {b, c}}, the sets {b}, {c}, {a,b}, {a,c} are pre-closed but not g*s-closed**.**

Remark 1.2.18

g*s – closed sets and strongly g – closed (g* - closed) sets are independent.

Example 1.2.19

Let X ={a, b, c, d} with τ ={φ, X, {a},{b}, {a, b}, {a, b, c}}. Then

g*s-closed sets = {φ , X , {a} , {b} , {c} , {d} , {a , c} , {b,c} , {b , d} , {a , d} , {c, d},{b , c , d}, {a , c , d}} and

Strongly generalized closed (g* - closed) sets = {φ , X, {d} , {a,d} , {b ,d} , {c , d}, {b , c , d} , {c , d , a}, {a , b , d}}.

Here the sets {a} , {b} , {c} , {a,c} , {b , c} are g*s-closed but not strongly generalized closed (g* - closed). The set {a , b , d} is strongly generalized closed (g* - closed) but not g*s-closed.

Remark 1.2.20

g*s – closed sets and w - closed closed sets are independent.

Example 1.2.21

Let X ={a, b, c} with τ = {φ , X , {c}}.In this space the sets {a} and {b} are g*s-closed but not w-closed.

In the topology τ = {φ , X , {a} , {b , c}}, the sets {b}, {c}, {a , b}, {a , c} are w-closed but not g*s - closed set.

Remark 1.2.22

g*s – closed sets and αg - closed sets are independent.

Example 1.2.23

Let X ={a, b, c, d} with τ = {φ , X , {a} , {b} ,{a , b } , {a , b , c}}.

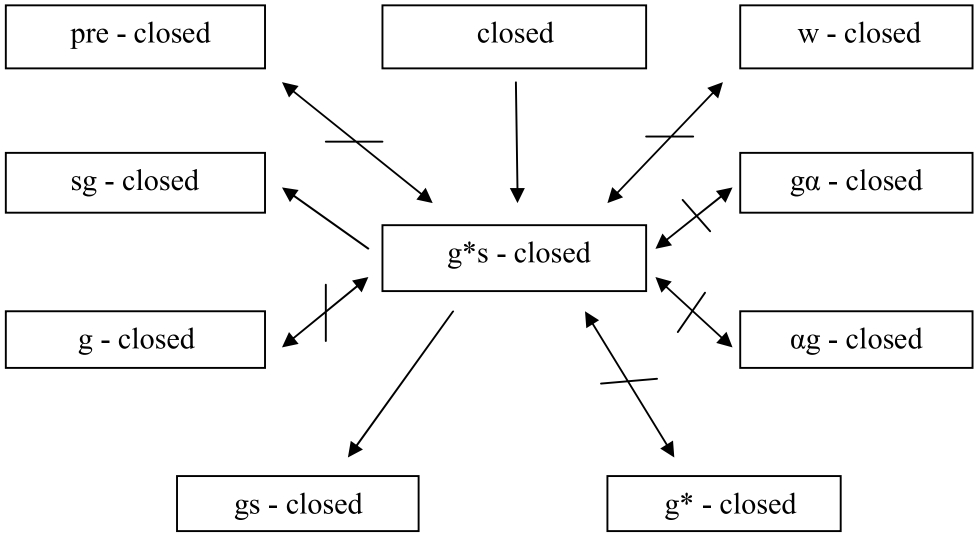
g*s - closed sets = {φ , X , {a} ,{b} , {c} , {d} , {a , c} , {a , d} , {c , d},{b , c , d}, {a , c , d}} and

αg - closed sets = {φ , X , {d}, {a,d}, {b , d} , {c , d} , {b , c , d}, {a , c , d}, {a , b , d} }.

Here the sets {a} , {b} , {c} , {a , c} , {b , c} are g*s - closed sets but not αg - closed set. The set {a , b , d} is αg - closed but not g*s - closed.

Remark 1.2.24

The following diagram summarises the above discussions :



where $A \longrightarrow B$ indicates that A implies B and $A \not\longleftrightarrow B$ indicates that A and B are independent to each other.

Theorem 1.2.25

If A is both preopen and semi closed then A is g*s –closed set.

Proof:

Assume that A is preopen and semi closed.

Preopen implies that $A \subseteq int(cl(A))$ ………… (1.1)

semi – closed implies that $int(cl(A)) \subseteq A$ …………….. (1.2)

From (1.1) and (1.2) we get. $A = int(cl(A))$ ………… (1.3)

To prove:

$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs –open in X.

Let $A \subseteq U$ and U is gs –open in X.

$scl(A) = A \cup int(cl(A))$

From (1.3) we get that, scl(A) = A ∪ A = A ⊆ U

(ie) scl(A) ⊆ U.

Hence A is g*s closed set.

Theorem 1.2.26

If A is semiclosed set then A is g*s –closed set.

Proof:

Assume that A is semiclosed set.

(ie) int(cl(A)) ⊆ A …………… (1.4)

To prove:

A is g*s –closed set.

Let A ⊆ U, U is gs –open in X.

scl(A) = A ∪ int(cl(A)) ⊆ A ∪ A.

Therefore scl(A) ⊆ A ⊆ U.

(ie) scl(A) ⊆ U.

Hence A is g*s – closed set.

Theorem 1.2.24

If A is regular open then A is g*s – closed set.

Proof:

Assume that A is regular open, then A = int(cl(A)) ………… (1.5)

To prove:

A is g*s –closed set.

Let A ⊆ U, where U is gs –open.

scl(A) = A ∪ int(cl(A)) = A ∪ A = A ⊆ U. (since from (1.5)) (ie) scl(A) $\subseteq$ U.

Therefore A is g*s –closed set.

Theorem 1.2.28

A subset A of X is g*s-closed set in X iff scl(A)-A contains no non empty gs-closed set in X.

Proof:

Necessity:

Let A be a g*s-closed set.

Let scl(A) - A contains a gs-closed set (say F).

(ie), F$\subseteq$ scl(A)-A $\Rightarrow$ F $\subseteq$ scl(A) $\cap$ A^c

$\Rightarrow$ F$\subseteq$ scl(A) and F $\subseteq$ A^c ……………. (1.6)

F $\subseteq$ A^c $\Rightarrow$ F^c $\subseteq$ A. Since F^c is gs-open and A is g*s-closed set, we have scl(A) $\subseteq$ F^c

$\Rightarrow$ F $\subseteq$ $[scl(A)]^c$ .... (1.7)

From (1.6) and (1.7)

F $\subseteq$ scl(A) and F $\subseteq$ $[scl(A)]^c$

$\Rightarrow$ F $\subseteq$ scl(A) $\cap$ $[scl(A)]^c = \varphi$.

Therefore F = φ.

Thus scl(A)-A contains no nonempty gs-closed set.

Sufficiency :

Let scl(A) – A contains no non-empty gs-closed set.

Let A $\subseteq$ U, where U is gs-open. Suppose that scl(A) is not contained in U, $U^c \cap$ scl(A) $\neq \varphi$. Then $U^c \cap$ scl(A) is a non-empty gs closed set and contained in scl(A)-A. This implies that scl(A) - A contains a non-empty gs-closed set. Which is contradiction to the fact that scl(A)-A contains no non-empty gs closed set. Therefore scl(A) $\subseteq$ U and hence A is g*s –closed set.

Theorem 1.2.29

Let (X , τ) be a compact topological space and suppose that A is g*s -closed subset of X. Then A is compact.

Proof:

Let { U_i } be an open covering of A (ie) A $\subseteq \bigcup U_i$. Since every open set is gs-open and A is g*s-closed. We get scl(A) $\subseteq \bigcup U_i$. Since a closed subset of a compact space is compact, scl(A) is compact. Therefore there exists {U_1 , U_2 , U_3 , U_4 , ……U_n} a finite subcollection of {U_i } such that scl(A) $\subseteq \bigcup U_i$.

$\Rightarrow$ A $\subseteq$ scl(A) $\subseteq \bigcup U_i$.

Therefore A is compact.

Theorem 1.2.30

Let (X ,τ) be Lindelof (countably compact) and suppose that A is g*s closed subset of X. Then A is Lindelof (countably compact).

Definition 1.2.31

A subset A of a topological space X is called g*s-open set if A^c is g*s-closed. The class of all g*s open sets in a topological space is denoted by g*s-O(X , τ).

Theorem 1.2.32

If A and B are g*s – open sets in X then A $\cap$ B also g*s – open set in X.

Proof:

Let A and B are g*s –open sets in X. Then A^c and B^c are g*s – closed sets in X. **By theorem 1.2.7,** $A^c \cup B^c$ is a g*s – closed set in X. That is $(A \cap B)^c$ is a g*s – closed set in X. Therefore A $\cap$ B is g*s – open set in X.

Theorem 1.2.33

For each $x \in X$, either $\{x\}$ is gs –closed or $\{x\}^c$ is g*s- closed in X.

Proof:

Let $x \in X$ and suppose that $\{x\}$ is not gs - closed in X. Then $\{x\}^c$ is not gs-open in X. Hence X is the only gs- open set containing $\{x\}^c$ in X. Thus semiclosure of $\{x\}^c$ is contained in X which implies that $\{x\}^c$ is g*s- closed in X.

Theorem 1.2.34

A set A is g*s –open in X if and only if $F \subseteq \text{sint}(A)$ whenever F is gs –closed in X and $F \subseteq A$.

Proof :

Assume that A is g*s –open in X and F is gs –closed set of X such that $F \subseteq A$.

Then $X - A$ is g*s –closed set in X and $X - F$ is an gs –open set in X containing $X - A$. Since $X - A$ is g*s –closed,

$$\text{scl}(X - A) \subseteq X - F$$

But, $\text{scl}(X - A) = X - \text{sint}(A)$

$$\Rightarrow X - \text{sint}(A) \subseteq X - F$$

$$\Rightarrow [\text{sint}(A)]^c \subseteq F^c.$$

Therefore $F \subseteq \text{sint}(A)$.

Conversely assume that F is contained in sint(A).

(ie) $F \subseteq \text{sint}(A)$, whenever F is contained in A and F is gs –closed in X.

Let G be an open set containing A^c. Then $G^c \subseteq \text{scl}(A)$

$$\Rightarrow G \supseteq \text{sint}(A) \Rightarrow \text{sint}(A^c) \subseteq G.$$

(ie), A^c is g*s –closed and hence A is g*s –open.

Section 1.3

g*s- CONTINUOUS FUNCTIONS

Definition 1.3.1[30]

A map f : (X, τ) → (Y, σ) from a topological spaces X into a topological space Y is called **g*s –continuous** if the inverse image of every closed set in Y is g*s –closed in X.

The next two theorems together with the examples following them shows that the g*s – continuous map is properly placed between the continuous map and gs – continuous map.

Theorem 1.3.2

If a map f : (X, τ) → (Y, σ) is continuous then it is g*s –continuous.

Proof:

Let f : (X, τ) → (Y, σ) be continuous. Let F be any closed set in Y. Then the inverse image f^{-1}(F) is closed in X. Since every closed set is g*s –closed, f^{-1} (F) is g*s –closed in X. Therefore f is g*s –continuous.

Remark 1.3.3

The converse of the above theorem need not be true as seen from the following example.

Example 1.3.4

Let X = Y = {a, b, c}, τ = {φ, X, {b} } and σ = {φ, Y, {a, b}}. Let f : (X, τ) → (Y, σ) be the identity map.Then f is not continuous, since for the closed {c} in Y, f^{-1} ({c}) = {c} is not closed in X. But f is g*s –continuous.

Theorem 1.3.5

If a map f : (X, τ) → (Y, σ) is g*s - continuous then it is gs - continuous.

Proof:

Let f : (X, τ) → (Y, σ) be g*s – continuous. Let F be any closed in Y.(ie) F is g*s – closed set is g*s – closed in Y. Then $f^{-1}(F)$ is g*s – closed in X. Since every g*s – closed set is gs - closed , $f^{-1}(F)$ is gs - closed in X. Therefore f is gs - continuous.

Remark 1.3.6

The converse of the above theorem need not be true as seen from the following example.

Example 1.3.7

Let X = Y = {a, b, c}, τ = {φ, x, {a}}, σ = {φ, y, {b}}. Let f : (X, τ) → (Y, σ) be the identity map.

Then f is gs –continuous but not g*s –continuous. Since {a, c} is closed in Y but $f^{-1}(\{a, c\}) = \{a, c\}$ is not g*s –closed in X.

Theorem 1.3.8

Let f : (X, τ) → (Y, σ) be a map. Then the following statements are equivalent:

a) f is g*s –continuous
b) the inverse image of each open set in Y is g*s –open in X.

Proof:

Assume that f : (X, τ) → (Y, σ) is g*s –continuous. Let G be open in Y. Then G^c is closed in Y. Since f is g*s –continuous, $f^{-1}(G^c)$ is g*s –closed in X. But $f^{-1}(G^c) = X – f^{-1}(G)$. Thus $f^{-1}(G)$ is g*s –open in X.

Conversely assume that the inverse image of each open set in Y is g*s –open in X. Let F be any closed set in Y. By assumption $f^{-1}(F)$ is g*s –open in X. But $f^{-1}(F^c) = X – f^{-1}(F)$. Thus $X – f^{-1}(F)$ is g*s –open in X and also $f^{-1}(F)$ is g*s –closed in X. Therefore f is g*s –continuous. Hence (a) and (b) are equivalent.

Theorem 1.3.9

Let X and Z be any topological spaces and Y be a T_{g*s} –spaces and Y be a T_{g*s} –space. Then the composition $g \circ f : (X, \tau) \to (Z, \eta)$ of the g*s –continuous map, $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ is also g*s –continuous.

Proof:

Let F be closed in Z. Since g is g*s –continuous, $g^{-1}(F)$ is g*s –closed in Y. But Y is T_{g*s} – space and so $g^{-1}(F)$ is closed. Since f is g*s –continuous, $f^{-1}(g^{-1}(F))$ is g*s –closed in X. But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$. Therefore $g \circ f$ is g*s –continuous.

Section 1.4

g*s –CLOSED MAPS AND g*s –OPEN MAPS

Definition 1.4.1[30]

A map $f : (X, \tau) \to (Y, \sigma)$ is called **g*s –closed map** if for each closed set F in X, f (F) is a g*s –closed in Y**.**

Example 1.4.2

Let $X = Y = \{a, b, c\}$ with topologies $\tau = \{\varphi, X, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{\varphi, Y, \{a\}, \{b\}, \{a, b\}\}$. Then $\tau^c = \{\varphi, X, \{a, c\}, \{c\}, \{a\}\}$ and $\sigma^c = \{\varphi, Y, \{c\}, \{b, c\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be an identity map. Then for each closed set in X, the image is g*s – closed in Y. Therefore f is g*s – closed map.

Theorem 1.4.3

If $f : (X, \tau) \to (Y, \sigma)$ is a closed map then it is g*s –closed.

Proof:

Let $f : (X, \tau) \to (Y, \sigma)$ be a closed map.Let F be any closed set in X. Then f (F) is an closed set in Y. Since every closed set is g*s –closed, f (F) is a g*s –closed set. Therefore f is a g*s –closed map.

Remark 1.4.4

The converse of the above theorem need not be true as seen from the following example.

Example 1.4.5

Let $X = Y = \{a, b, c\}$ with topologies $\tau = \{\varphi, X, \{a\}\}$ and $\sigma = \{\varphi, Y, \{b, c\}\}$.Then $C(X, \tau) = \{\varphi, X, \{b, c\}\}$, $C(Y, \sigma) = \{\varphi, Y, \{a\}\}$ and

$g^*s – C (Y, \sigma) = \{\varphi, Y, \{a\}, \{a, b\}, \{a, c\}\}$.

Let f be the identify map from X to Y. Then f is g*s –closed but not a closed map. Since the closed set $\{b, c\}$ in (X, τ), $f(\{b, c\}) = \{b, c\}$ is not closed set in Y.

Theorem 1.4.6

A map $f : (X, \tau) \to (Y, \sigma)$ is g*s –closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$ there is a g*s –open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof:

Let S be a subset of (Y, σ) and U be an arbitrary open set in X containing $f^{-1}(S)$. It is enough we produce a g*s – open set V of (Y, σ) such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Choose $V = Y – (f (X - U))$

As U is open, X – U is closed and by definition of g*s – closed map, f (X - U) is g*s – closed in Y. Hence V is a g*s – open set and $f^{-1}(S) \subseteq U$.

$X – U \subseteq X – f^{-1}(S) = f^{-1}(Y - S)$

$f (X - U) \subseteq Y – S$

$S \subseteq Y – f (X - U) = V$

Now $V = Y – f (X - U) \Rightarrow f (X - U) \subseteq Y – V$

$$\Rightarrow (X - U) \subseteq f^{-1}(Y - V) = X – f^{-1}(V)$$

$\Rightarrow f^{-1}(V) \subseteq U$

Conversely let S be closed in X. Then X – S is open. In the given criteria, put U = X – S and S = Y – f (S). As $f^{-1}(Y - f(S)) \subseteq X - S = U$, there exists a g*s – open set V of (Y, σ) such that $Y - f(S) \subseteq V$ and $f^{-1}(V) \subseteq X - S \Rightarrow S \subseteq X - f^{-1}(V)$.

Now $Y - f(S) \subseteq V \Rightarrow Y - V \subseteq f(S) \subseteq f(X - f^{-1}(V)) \subseteq Y - V$

Therefore f (S) = Y – V. Since Y – V is g*s – closed, f (S) is g*s – closed and hence f is g*s – closed map.

Theorem 1.4.7

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is closed and $h : (Y, \sigma) \rightarrow (Z, \eta)$ is g*s –closed then $h \circ f : (X, \tau) \rightarrow (Z, \sigma)$ is g*s –closed.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is a closed map and $h : (Y, \sigma) \rightarrow (Z, \eta)$ is a g*s –closed map. Let V be any closed set in X. Since $f : (X, \tau) \rightarrow (Y, \sigma)$ is closed , f (V) is closed in Y and since $h : (Y, \sigma) \rightarrow (Z, \eta)$ is g*s –closed, h(f (V)) is a g*s –closed set in Z. Therefore $h \circ f : X \rightarrow Z$ is a g*s –closed.

Theorem 1.4.8

Let (X, τ) and (Y, σ) be any topological spaces. Then,

a) If $f : (X, \tau) \rightarrow (Y, \sigma)$ is g*s – closed and A is a closed subset of (X, τ), then $f_A : (A, \tau_A) \rightarrow (Y, \sigma)$ is g*s – closed.
b) If $f : (X, \tau) \rightarrow (Y, \sigma)$ is g*s – closed and $A = f^{-1}(B)$ for some closed set B of (Y, σ), then $f_A : (A, \tau_A) \rightarrow (Y, \sigma)$ is g*s – closed.

Proof of (a):

Let A be a closed subset of (X, τ) and B be a closed set of A. Then $B = A \cap F$, for some closed set F of (X, τ) ⟹ B is closed in X also. Now, f is g*s – closed and B is closed in X ⟹ f (B) is g*s – closed in (Y, σ).

But $f(B) = f_A(B)$

$\Longrightarrow f_A(B)$ is g*s closed in (Y, σ).

$\Longrightarrow f_A$ is a g*s – closed map.

Proof of (b):

Let B be a closed subset of (Y, σ) and $A = f^{-1}(B)$

Let D be a closed set of A. Then $D = A \cap H$, for some closed set H in (X, τ).

We know, $f_A(D) = f(D)$

$= f(A \cap H)$

$= f(f^{-1}(B) \cap H)$ [By (1)]

$= B \cap f(H)$

f is g*s – closed $\Longrightarrow f(H)$ is g*s – closed.

Also B is closed set of A. Therefore $B \cap f(H)$ is g*s – closed in (Y, σ).

Hence f_A is g*s closed map.

Theorem 1.4.9

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a continuous, g*s –closed map from a normal space X onto a space Y, then Y is normal.

Proof:

Let A, B be disjoint closed sets of Y. Then $f^{-1}(A)$, $f^{-1}(B)$ are disjoint closed sets of X.

Since X is normal, then there exists a disjoint open sets U, V in X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$.

By theorem 1.4.6, there exists g*s – open G, H in Y such that $A \subseteq G$, $B \subseteq H$ and $f^{-1}(G) \subseteq U$ and $f^{-1}(H) \subseteq U$. Since U, V are disjoint, int(G) and int(H) are disjoint open sets. Since G is g*s – open and A is closed and $A \subseteq G \Longrightarrow A \subseteq int(G)$. Similarly $B \subseteq int(H)$. Hence Y is normal.

Definition 1.4.10

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called **g*s –open map** if f (U) is g*s –open in Y for every open set U in X.

Example 1.4.11

Let $X = Y = \{a, b, c\}$ with topologies $\tau = \{\varphi, X, \{b\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{\varphi, Y, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an identity map. Then for each open set in X, the image is g*s – open in Y. Therefore f is g*s – open map.

Theorem 1.4.12

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is an open map then it is g*s –open.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an open map. Let U be any open set in X. Then f (U) is an open set in Y.Therefore f (U) is g*s –open, since every open set is g*s –open. Therefore f is g*s –open.

Remark 1.4.13

The converse of the above theorem need not be true as seen from the following example.

Example 1.4.14

Let $X = Y = \{a, b, c\}$ with topologies $\tau = \{\varphi, X, \{a, b\}\}$ and $\sigma= \{\varphi, Y, \{b\}\}$.Here $g^*s – C(Y, \sigma) = \{\varphi, Y, \{b, c\}, \{a, b\}, \{b\}\}$. Then the identity function $f : (X, \tau) \rightarrow (Y, \sigma)$ is g*s –open but not open. Since for the open set {a, b} in (X, τ), $f(\{a, b\}) = \{a, b\}$ is g*s open but not open in (Y, σ). Therefore f is not an open map.

Theorem 1.4.15

For any bijection $f : (X, \tau) \rightarrow (Y, \sigma)$, the following are equivalent.

a) $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is g*s –continuous.
b) f is a g*s –open map.
c) f is a g*s –closed map.

Proof:

To prove : (a) ⟹(b):

Let U be an open set of (X, τ).

(a) ⟹ $(f^{-1})^{-1}(U) = f(U)$ is g*s – open in (Y, σ).

Hence f is a g*s – open map.

To prove: (b) ⟹ (c) :

Let F be a closed set of (X, τ).

⟹ X – F is open in (X, τ).

(b) ⟹ f (X - F) = Y – f (F) is g*s – open in (Y, σ).

⟹ f (F) is g*s – closed in (Y, σ).

⟹ f is g*s – closed map.

To prove: (c) ⟹ (a) :

Let F be closed set in (X, τ).

(c) ⟹ f (F) is g*s – closed in (Y, σ). But $f(F) = (f^{-1})^{-1}(F)$.

⟹ f^{-1} is g*s – continuous on Y.

Definition 1.4.16

Let x be a point of (X, τ) and let V be a subset of X. Then V is called a **g*s – neighborhood** of x in (X, τ) if there exists a g*s – open set U of (X, τ) such that $x \in U \subseteq V$.

Theorem 1.4.17

Let f : (X, τ) → (Y, σ) be a mapping. Then the following statements are equivalent:

a) f is g*s – open mapping
b) for a subset A of (X, τ), f (int(A)) ⊆ g*s – int(f (A))

c) for each $x \in X$ and for each neighborhood U of x in (X, τ), there exists a g*s – neighborhood W of f (x) in (Y, σ) such that $W \subseteq f(U)$.

Proof :

To prove : (a)$\Longrightarrow$ (b) :

Let A be a subset of $(X, \tau) \Longrightarrow$ int (A) is open in (X, τ).

Then (a) $\Longrightarrow$ f (int (A)) is g*s – open in (Y, σ) ………. (1.7)

We know that, $\text{int}(A) \subseteq A$

$\Longrightarrow f(\text{int}(A)) \subseteq f(A) \Rightarrow g^*s - \text{int}(f(\text{int}(A))) \subseteq g^*s - \text{int}(f(A))$ ……….. (1.8)

But (1) $\Rightarrow g^*s - \text{int}(f(\text{int}(A))) = f(\text{int}(A))$ ……… (1.9)

(1.8) and (1.9) $\Longrightarrow (f(\text{int}(A))) \subseteq g^*s - \text{int}(f(A))$.

To prove: (b) $\Longrightarrow$ (c):

Let $x \in X$ and U be an arbitrary neighbourhood of x in (X, τ). Then there exists an open set G such that $x \in G \subseteq U$.

a) $\Longrightarrow f(G) = f(\text{int}(G)) \subseteq g^*s - \text{int}(f(G))$

$\Longrightarrow f(G) = g^*s - \text{int}(f(G))$

$\Longrightarrow$ f (G) is g*s – open in (Y, σ).

A lso, $x \in G \subseteq U \Longrightarrow f(x) \in f(G) \subseteq f(U)$.

Take $W = f(G) \Longrightarrow W \subseteq f(U)$

To prove: (c) $\Longrightarrow$ (a) :

Let U be an open set in (X, τ).

Take $x \in U$ and $f(x) = y$

$\Longrightarrow y \in f(U)$, for each $x \in U$.

a) $\Rightarrow$ There exists a g*s – neighbourhood W_y of y in (Y, σ) such that $W_y \subseteq f(U)$,

$\Rightarrow$ there exists a g*s – open set V_y in (Y, σ) such that $y \in V_y \subseteq W_y$.

$\Rightarrow f(U) = \cup \{V_y / y \in f(U)\}$

We know that, union of g*s – open sets is a g*s – open set.

Hence f (U) is a g*s – closed set of (Y, σ).

$\Rightarrow$ f is g*s – open mapping.

Theorem 1.4.18

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is g*s – open if and only if any subset B of Y and for any closed set S containing $f^{-1}(B)$, there is a g*s – closed set A of Y containing B such that $f^{-1}(A) \subseteq S$.

Proof :

Similar to **Theorem 1.4.6.**

Theorem 1.4.19

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is g*s – open if and only if $f^{-1}(g^*s - cl(B)) \subseteq cl(f^{-1}(B))$ for every subset B of (Y, σ).

Proof :

Let F be a g*s – open map and B be a subset of (Y, σ). Then, $f^{-1}(B) \subseteq cl(f^{-1}(B))$. **By theorem 1.4.18,** there exists a g*s – closed set A of (Y, σ) such that $B \subseteq A$ and $f^{-1}(A) \subseteq cl(f^{-1}(B))$ by choosing S in the theorem as $cl(f^{-1}(B))$. Since A is a g*s – closed set in (Y, σ), $f^{-1}(g^*s - cl(B)) = f^{-1}(B) \subseteq f^{-1}(A) \subseteq cl(f^{-1}(B)) \Rightarrow f^{-1}(g^*s - cl(B)) \subseteq cl(f^{-1}(B))$.

Conversely, let S be any subset of (Y, σ) and F be any closed set containing $f^{-1}(S)$. Put $A = g^*s - cl(S) \Rightarrow$ A is g*s – closed set, $S \subseteq A$ and $A = g^*s - cl(S)$

$\Rightarrow f^{-1}(A) = f^{-1}(g^*s - cl(S)) \subseteq cl(f^{-1}(S)) \subseteq F$.

By the theorem 1.4.18, f is a g*s – open map.

Chapter – 2

CHAPTER - 2

STRONGLY g*s –CONTINUOUS MAPS AND PERFECTLY g*s –CONTINUOUS MAPS IN TOPOLOGICAL SPACES

Section 2.1

Preliminaries

Definition 2.1.1 [18]

A topological space X is called $T_{1/2}$ **space** if every g –closed set of X is closed in X.

Definition 2.1.2 [10]

A topological space X is called T_b **space** if every gs –closed set of X is closed in X.

Definition 2.1.3 [10]

A topological space X is called T_d **space** if every gs –closed set of X is g –closed in X.

Definition 2.1.4 [10]

A topological space X is called T_{gs} **–space** if every gs –closed set of X is sg –closed in X.

Definition 2.1.6 [35]

A topological space X is called T_p **–space** if every strongly g –closed set of X is closed in X.

Definition 2.1.7 [35]

A topological space X is called T_s **–space** if every g –closed set of X is strongly g –closed in X.

Definition 2.1.8 [33]

A function f : (X, τ) → (Y, σ) is called **Perfectly g –continuous** if $f^{-1}(V)$ is both open and closed in X for each g –open set V in Y.

Section 2.2

g*s - SEPERATION AXIOMS

Definition 2.2.1[31]

A topological space X is called a $\mathbf{T_L}$ **–space** if every g*s –closed set of X is closed in X.

Definition 2.2.2 [31]

A topological space X is called a $\mathbf{T_A}$ **–space** if every gs –closed set of X is g*s –closed in X.

Theorem 2.2.3

Every T_b - space is T_L - space.

Proof:

Assume that (X, τ) is a T_b –space. Since every g*s –closed set is gs –closed and X is a T_b - space, X is a T_L - space.

Remark 2.2.4

The converse of the above theorem need not be true as seen from the following example.

Example 2.2.5

Let X = {a, b, c} and τ = {φ, X, {c}, {a, b}, {b, c}}.Then (X, τ) is T_L –space but not T_b –space. Since C(X, τ) = {φ, X, {a}, {c}, {a, b}},

g*s –C(X, τ) = {φ, X, {a}, {c}, {a, b}} and

gs –C(X, τ) = {φ, X, {a}, {b}, {c}, {a, b}, {c, a}}.

Here the sets {b} and {a, c} are not closed in (X, τ).

Theorem 2.2.6

Every T_b – space is a T_A - space.

Proof:

Assume that (X, τ) is a T_b –space. Let A be any gs –closed set in X. Since X is a T_b - space, A is closed in X and every closed set is g*s –closed, A is g*s –closed. Hence X is a T_A - space.

Remark 2.2.7

The converse of the above theorem need not be true as seen from the following example.

Example 2.2.8

Let X = {a, b, c} with τ = {φ, X, {a, b}}.Then X is T_A - space but not T_b - space. Since C(X, τ) = {φ, X, {c}}, g*s –C(X, τ) = {φ, X, {c}, {b, c}, {a, c}} and

gs –C(X, τ) = {φ, X, {c}, {b, c}, {a, c}}.

Here the sets {b, c} and {a, c} are not closed in (X, τ).

Section – 2.3

STRONGLY g*s –CONTINUOUS MAPS AND PERFECTLY g*s –CONTINUOUS MAPS

Definition 2.3.1[31]

A map f : (X, τ) → (Y, σ) is said be **strongly g*s –continuous** if the inverse image of every g*s –open set in Y is open in X.

Remark 2.3.2

When Y is T_L, strongly g*s –continuity coincides with continuity.

Theorem 2.3.3

If a map f : (X, τ) → (Y, σ) is strongly g*s –continuous then it is continuous.

Proof:

Assume that f is strongly g*s –continuous. Let G be any open set in Y. Since every open set is g*s –open, G is g*s –open in Y. Since f is strongly g*s –continuous, $f^{-1}(G)$ is open in X. Therefore f is continuous.

Remark 2.3.4

The converse of the above theorem need not be true as seen from the following example.

Example 2.3.5

Let X = Y = {a, b, c} with the topologies τ = {φ, X, {a}} and σ = {φ, Y, {a, b}}. Define a map f : (X, τ) → (Y, σ) by f (a) = a = f (b) and f (c) = b. Then f is continuous. But f is not strongly g*s –continuous. Since $f^{-1}(a)$ = {a, b} is not open in X where {a} is g*s –open in X.

Theorem 2.3.6

A map f : (X, τ) → (Y, σ) from a topological space X into the topological space Y is strongly g*s –continuous if and only if the inverse image of every g*s –closed set in Y is closed in X.

Proof:

Assume that f is strongly g*s –continuous. Let F be any g*s –closed set in Y. Then F^c is g*s –open set in Y. Since f is strongly g*s –continuous, $f^{-1}(F^c)$ is open in X. But $f^{-1}(F^c) = X - f^{-1}(F)$ and so $f^{-1}(F)$ is closed in X.

Conversely assume that the inverse image of every g*s –closed set in Y is closed in X. Let G be any g*s –open set in Y. Then G^c is g*s –closed set in Y.

By assumption, $f^{-1}(G^c)$ is closed in X.But $f^{-1}(G^c) = X - f^{-1}(G)$ and so $f^{-1}(G)$ is open in X. Therefore f is strongly g*s –continuous.

Theorem 2.3.7

If a map f : (X, τ) → (Y, σ) is strongly continuous then it is strongly g*s –continuous.

Proof:

Assume that f is strongly continuous. Let G be any g*s –open set in Y. Since f is g*s –continuous, $f^{-1}(G)$ is open in X. Therefore f is strongly g*s –continuous.

Remark 2.3.8

The converse of the above theorem need not be true as seen from the following example.

Example 2.3.9

Let X = Y = {a, b, c} the topologies τ = {φ, X, {a}, {b}, {a, b}, {b, c}} and σ = {φ, Y, {b}, {a, b}, {b, c}}.Define a map f : (X, τ) → (Y, σ) as the identity map. Then f is strongly g*s –continuous but f is not strongly continuous. For the subset {b} of Y, $f^{-1}(b)$ = {b} is open in X, but not closed in X.

Theorem 2.3.10

If a map f : (X, τ) → (Y, σ) is strongly g*s –continuous and a map g : (Y, σ) → (Z, η) is g*s –continuous, then the composition g ∘ f : (X, τ) → (Z, η) is continuous.

Proof:

Let G be any open set in Z. Since g is g*s –continuous, $g^{-1}(G)$ is g*s –open in Y. Since f is strongly g*s –continuous, $f^{-1}(g^{-1}(G))$ is open in X. But $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$. Therefore g ∘ f is continuous.

Definition 2.3.11[31]

A map f : X → Y is said to be **perfectly g*s –continuous** if the inverse image of every g*s –open set in Y is both open and closed in X.

Theorem 2.3.12

If a map f : (X, τ) → (Y, σ) from a topological space X into the topological space Y is perfectly g*s –continuous then it is strongly g*s – continuous.

Proof :

Assume that f is perfectly g*s – continuous. Let G be any g*s – open set in Y. Since f is perfectly g*s – continuous, $f^{-1}(G)$ is open in X.Therefore f is strongly g*s – continuous.

Remark 2.3.13

The converse of the above theorem need not be true as seen from the following example.

Example 2.3.14

Let X = Y = {a, b, c} with τ = {φ, X, {a}, {b}, {a, b}, {b, c}} and σ = {φ, X, {b}, {a, b}, {b, c}}. Define a map f : (X, τ) → (Y, σ) as the identity map. Then f is strongly g*s – continuous but f is not perfectly g*s – continuous. For the subsets {c} of Y, {a, c} of Y are closed sets in X but not open in X.

Theorem 2.3.15

A map f : (X, τ) → (Y, σ) from a topological space X into the topological space Y is perfectly g*s –continuous if and only if $f^{-1}(G)$ is both open and closed in X for every g*s –closed set G in Y.

Proof:

Assume that f is perfectly g*s –continuous. Let F be any g*s –closed set in Y. Then F^c is g*s –open set in Y. Since f is perfectly g*s –continuous, $f^{-1}(F^c)$ is both open and closed in X.

But $f^{-1}(F^c) = X - f^{-1}(F)$ and so $f^{-1}(F)$ is both open and closed in X.

Conversely assume that the inverse image of every g*s –closed set in Y is both open and closed in X.

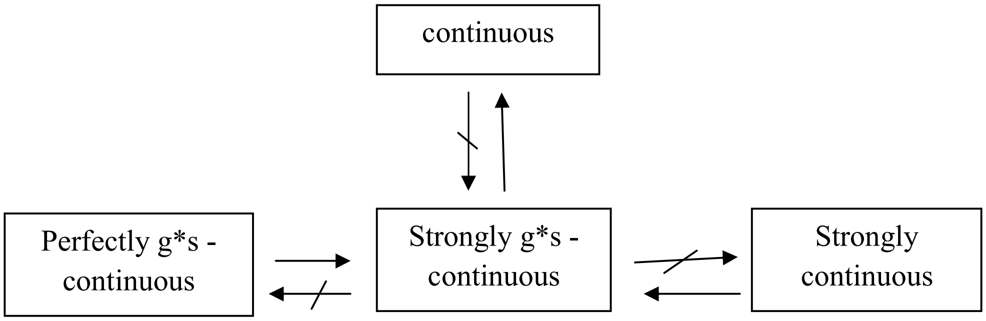
Let G be any g*s –open set in Y. Then G^c is g*s –closed set in Y. By assumption $f^{-1}(G^c)$ is both open and closed in X.

But $f^{-1}(G^c) = X - f^{-1}(G)$ and so $f^{-1}(G)$ is both open and closed in X.

Therefore f is perfectly g*s –continuous.

Remark 2.3.16

From the above observations we have the following implications:



Where A $\longrightarrow$ B (respectively A $\not\longrightarrow$ B) represents that A implies B (respectively A does not always imply B).

Section – 2.4

g*s – COMPACTNESS

Definition 2.4.1[31]

A collection $\{A_i ; i \in I\}$ of g*s – open sets in a topological space X is called a **g*s –open cover** of a subset B in X if $B \subseteq \bigcup_{i\in I} A_i$.

Definition 2.4.2 [31]

A topological space X is **g*s –compact** if every g*s –open cover of X has a finite subcover of X.

Definition 2.4.3 [31]

A subset B of a topological space X is called **g*s –compact relative to X**, if for every collection $\{A_i ; i \in I\}$ of g*s –open subset of X such that $B \subseteq \bigcup_{i\in I} A_i$, there exist a finite subset I_o of I such that $B \subseteq \bigcup_{i \in I_o} A_i$.

Definition 2.4.4 [31]

A subset B of a topological space X is called **g*s –compact** if B is g*s –compact as the subspace of X.

Theorem 2.4.5

A g*s –closed subset of g*s –compact space is g*s –compact relative to X.

Proof:

Let A be a g*s –closed subset of a g*s –compact space X. Then A^c is g*s –open in X. Let S be a g*s –open cover of A in X. Then, S along with A^c form a g*s –open cover of X.

Since X is g*s –compact, it has a finite subcover, say $\{G_1, G_2, \ldots\ldots\ldots, G_n\}$. If this subcover contains A^c , we discard it.

Otherwise leave the subcover as it is. Thus we have obtained a finite subcover of A and so

A is g*s –compact relative to X.

Theorem 2.4.6

A g*s –continuous image of a g*s –compact space is compact.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a g*s –continuous map from a g*s –compact space (X, τ) onto a topological space (Y, σ). Let $\{A_i ; i \in I\}$ be an open cover of (Y, σ). Then $\{f^{-1}(A_i) ; i \in I\}$ is a g*s –open cover of (X, τ).

Since (X, τ) is g*s –compact, it has a finite subcover say $\{ f^{-1}(A_1), f^{-1}(A_2), \ldots\ldots, f^{-1}(A_n) \}$. Since f is onto, $\{A_1, A_2, A_3, \ldots\ldots A_n\}$ is an open cover of (Y, σ) and so (Y, σ) is compact.

Theorem 2.4.7

If f : (X, τ) → (Y, σ) is strongly g*s –continuous map from a compact space (X, τ) onto a topological space (Y, σ) then (Y, σ) is g*s –compact.

Proof:

Let $\{A_i ; i \in I\}$ be a g*s -open cover of (Y, σ). Then $\{f^{-1}(A_i) ; i \in I\}$ is an open cover of (X, τ). Since f is strongly g*s –continuous. Since (X, τ) is compact, it has a finite subcover say $\{f^{-1}(A_1), f^{-1}(A_2), \ldots\ldots\ldots, f^{-1}(A_n)\}$ and since f is onto, $\{A_1, A_2, A_3, \ldots\ldots A_n\}$ is a finite subcover of (Y, σ). Therefore (Y, σ) is g*s - compact.

Theorem 2.4.8

If map f : (X, τ) → (Y, σ) is g*s –continuous from a compact space (X, τ) onto a topological space (Y, σ), then (Y, σ) is g*s –compact.

Proof:

Let $\{A_i ; i \in I\}$ be a g*s -open cover of (Y, σ). Then $\{f^{-1}(A_i) ; i \in I\}$ is an open cover of (X, τ).Since f is g*s –continuous.Since every g*s – continuous map is strongly g*scontinuous and (X,τ) is compact, it has a finite subcover say $\{f^{-1}(A_1), f^{-1}(A_2), \ldots\ldots\ldots, f^{-1}(A_n)\}$ and since f is onto, $\{A_1, A_2, A_3, \ldots\ldots A_n\}$ is a finite subcover of (Y,σ). Therefore (Y, σ) is g*s - compact.

Theorem 2.4.9

If map f : (X, τ) → (Y, σ)is a perfectly g*s –continuous from a compact space X onto a topological space (Y, σ), then (Y, σ) is g*s –compact.

Proof:

Let $\{A_i ; i \in I\}$ be a g*s -open cover of (Y, σ). Then $\{f^{-1}(A_i) ; i \in I\}$ is an open cover of (X, τ). Since f is perfectly g*s –continuous. Since every perfectly g*s –continuous function is strongly g*s –continuous and (X, τ) is compact, it has a finite subcover say $\{f^{-1}(A_1), f^{-1}(A_2), \ldots\ldots\ldots, f^{-1}(A_n)\}$ and since f is onto, $\{A_1, A_2, A_3, \ldots\ldots A_n\}$ is a finite subcover of (Y, σ). Therefore (Y, σ) is g*s – compact.

Chapter – 3

CHAPTER - 3

g*s –CONNECTEDNESS AND g*s –LOCALLY CLOSED SETS IN TOPOLOGICAL SPACES

Section 3.1

Preliminaries

Definition 3.1.1 [29]

A subsets of X is called a **locally closed** if S = P ∩ Q. where P is open and Q is closed in X.

Definition 3.1.2 [29]

A subsets of X is called a **generalized locally closed** if S = P ∩ Q. where P is g –open and Q is g –closed in X.

Definition 3.1.3 [29]

A subsets of X is called a **gs - locally closed** if S = P ∩ Q. where P is gs – open and Q is gs – closed in X.

Definition 3.1.4 [29]

A subsets of X is called a **sg - locally closed** if S = P ∩ Q. where P is sg –open and Q is sg – closed in X.

Section 3.2

g*s –CONNECTEDNESS

Definition 3.2.1 [1]

A topological space X is called **g*s –connected** if X cannot be written as a disjoint union of two nonempty g*s –open sets. A subset of X is g*s –connected if it is g*s –connected as subspace of X.

Theorem 3.2.2

For a topological space X. The following are equivalent.

a) X is g*s –connected.
b) The only subsets of X which are both g*s –open and g*s –closed are the empty set φ and X.
c) Each g*s continuous map of X into a discrete space Y with at least two points is a constant map.

Proof:

To prove (a) ⟹ (b) :

Let U be g*s –open and g*s –closed subset of X.

Then X – U is both g*s –open and g*s –closed.

Since X is the disjoint union of the g*s –open sets U and X –U, one of these must be empty (ie.,) U = φ (or) U = X.

To prove (b) ⟹ (c) :

Suppose that X = A ∪ B where A and B are disjoint nonempty g*s –open subset of X. Then A is both g*s –open and g*s –closed.

By assumption, A = φ.

Therefore X is g*s –connected.

To prove (c) ⟹ (b) :

Let U be both g*s –open and g*s –closed in X.

Suppose that U ≠ φ.

Define f : X → Y by f (U) = {y} and f (X - U) = {w} for some distinct points y and w in Y then f is a g*s –continuous map.

By assumption f is constant. Therefore y = w and so U = x.

To prove (b) $\Longrightarrow$ (c) :

Let $f : X \rightarrow Y$ be a g*s –continuous map. Then X is covered by g*s –open and g*s –closed covering $\{f^{-1}(y) : y \in Y\}$.

Suppose that there are two points in Y such that $f^{-1}(y) = \varphi$.

By assumption $f^{-1}(y) =$ or X for each $y \in Y$. If $f^{-1}(y) = \varphi$ for all $y \in Y$ then f fails to be a map. Then there exists only one point $y \in Y$ such that $f^{-1}(y) = \varphi$ and hence $f^{-1}(y) = X$ which shows that f is a continuous map.

Theorem 3.2.3

Every g*s –connected space is connected.

Proof:

Let X be a g*s –connected space. Let X be not connected. Then X can be written as $X = A \cup B$ where A and B are disjoint nonempty open sets in X. Since every open set is g*s –open, $X = A \cup B$ where A and B are disjoint nonempty and g*s –open sets in X.

This contradicts the fact that X is g*s –connected.

Therefore X is connected.

Remark 3.2.4

The converse of the above theorem need not be true as seen from the following example.

Example 3.2.5

Let $X = \{a, b, c\}$ with the topology $\tau = \{\varphi, X\}$ clearly (X, τ) is connected space.

But (X, τ) is not g*s –connected, because X can be written as $X = \{a\} \cup \{b, c\}$,

where $\{a\}$ and $\{b, c\}$ are g*s – open sets in (X, τ).

Theorem 3.2.6

If X is a T_L space and connected, then X is g*s –connected.

Proof:

Let X be T_L –space and connected. Assume that X can be written in the form $X = A \cup B$ where A and B are disjoint nonempty open sets in X.

This contradicts the fact that X is connected.

Therefore X is g*s –connected.

Theorem 3.2.7

If $f : X \rightarrow Y$ is a g*s –continuous surjection and X is g*s –connected, then Y is connected.

Proof:

Suppose that Y is not connected. Let $Y = A \cup B$ where A and B are disjoint nonempty open sets in Y. Since f is g*s –continuous and onto,

$$X = f(f^{-1}(A) \cup f(f^{-1}(B))$$

Where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty g*s - open in X.

This contradicts the fact that X is g*s –connected.

Hence Y is connected.

Theorem 3.2.8

If $f : X \rightarrow Y$ is a g*s –continuous map from a connected space X into a topological space Y, then Y is g*s –connected.

Proof:

Let Y be not g*s connected.

Then Y can be written as $Y = A \cup B$ where A and B are disjoint nonempty g*s –open sets in Y. Since f is g*s continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are open sets in X. Also $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.

This contradicts the fact that X is connected.

Therefore Y is g*s –connected.

Section 3.3

g*s, g*s*, g*s** -LOCALLY CLOSED SETS

Definition 3.3.1[29]

A subset S of X is called **locally closed set** if S = A ∩ B where A is open in X and B is closed in X.

Definition 3.3.2 [1]

A subset S of X is called **g*s –locally closed set** (g*sl -set) if S = A ∩ B where A is g*s - open in X and B is g*s –closed in X.

The class of all g*s –locally closed sets in X is denoted by g*sLC(X).

Theorem 3.3.3

If a subset S of X is locally closed then it is g*s – locally closed.

Proof:

Let S = P ∩ Q where P is open and Q is closed in X. Since every open set is g*s –open and every closed set is g*s –closed, S is locally g*s –closed in X.

Remark 3.3.4

The converse of the above theorem need not be true as seen from the following example.

Example 3.3.5

Let X = {a, b, c} with $\tau = \{\varphi, X, \{a\}, \{c\}, \{a, c\}\}$.

Then the sets {a} and {c} are g*s –locally closed but are not locally closed.

Theorem 3.3.6

If a subset S of X is g*s –locally closed in X, then S is gs – locally closed.

Proof:

Let S = P ∩ Q where P is g*s -open and Q is g*s -closed in X.

Since open implies gs – open and g*s – closed implies gs –closed,

S is gs -locally closed in X.

Theorem 3.3.7

If A is g*s locally closed in X and B is g*s open in X, then A ∩ B is g*s –locally closed in X.

Proof:

Since A is g*s – locally closed, there exist a g*s –open set P and a g*s –closed set Q such that A = P ∩ Q.

Now A ∩ B = (P ∩ Q) ∩ B = (P ∩ B) ∩ Q.

Since P ∩ B is g*s –open, A ∩ B is g*s –locally closed in X.

Definition 3.3.8 [1]

A subset S of a topological space X is called **g*s* -locally closed set** if S = P ∩ Q where P is g*s-open in X and Q is closed in X.

Definition 3.3.9 [1]

A subset S of a topological space X is called **g*s** -locally closed set** if S = P ∩ Q where P is open in X and Q is g*s closed in X.

Theorem 3.3.10

If A is g*s* -locally closed in X and B is g*s open (or) (g*s -closed), then A ∩ B is g*s* -locally closed set in X.

Proof:

Since A is g*s* -locally closed set, there exist a g*s –open set P and a closed set Q such that A = P ∩ Q. Now A ∩ B = (P ∩ Q) ∩ B = (P ∩ B) ∩ Q.

Since P ∩ Q is g*s –open and Q is closed, A ∩ B is g*s* -locally closed set.

In case of B being a closed set, we have A ∩ B = (P ∩ Q) ∩ B = P ∩ (Q ∪ B).

Since P is g*s –open and Q ∩ B is closed,

A ∩ B is a g*s* - locally closed set.

Theorem 3.3.11

A subset A of a topological space X is g*s* -locally closed set if and only if there exists a g*s –open set P such that A = P ∩ cl(A).

Proof:

Assume that A is g*s* -locally closed set.

There exist a g*s –open set P and a closed set Q such that A = P ∩ Q.

Since A ⊆ Q and Q is closed, A ⊆ cl(A) ⊆ Q.

Then A ⊆ P and A ⊆ cl(A), and hence A ⊆ P ∩ cl(A) …………….. (3.1).

Let x ∊ P ∩ cl(A).

Then x ∊ P and x ∊ cl(A) ∩ Q and so x ∊ P ∩ Q = A.

Hence P ∩ cl(A) ⊆ A ……………… (3.2).

From (3.1) and (3.2), A = P ∩ cl(A).

Conversely, assume that there exist a g*s –open set P such that A = P ∩ cl(A).

Now P is g*s –open set and cl(A) is closed.

Therefore A is g*s* -closed set.

Theorem 3.3.12

If A and B are g*s* -locally closed sets in a topological space X then $A \cap B$ is g*s* -locally closed in X.

Proof:

Since A and B are g*s* - locally closed sets there exist a g*s – open sets P and Q such that $A = P \cap cl(A)$ and $B = Q \cap cl(B)$.

Then $A \cap B = [P \cap cl(A)] \cap [Q \cap cl(B)]$

$= (P \cap Q) \cap [cl(A) \cap cl(B)]$.

Since $P \cap Q$ is g*s –open set and $cl(A) \cap cl(B)$ is closed,

$A \cap B$ is g*s* -locally closed.

Chapter – 4

CHAPTER - 4

g*s –IRRESOLUTE MAPS AND g*s –HOMEOMORPHISMS IN TOPOLOGICAL SPACES

Section – 4.1

g*s –IRRESOLUTE MAPS

Definition 4.1.1 [28]

A map f : (X, τ) → (Y, σ) is called **g*s –irresolute** if the inverse image of every g*s - closed set in Y is g*s –closed in X.

Result 4.1.2 [30]

Every continuous map is g*s –continuous and g*s –continuous map is gs – continuous.

Theorem 4.1.3

A map f : (X, τ) → (Y, σ) is g*s –irresolute if and only if for every g*s –open set A of Y, $f^{-1}(A)$ is g*s –open in X.

Proof:

Necessity:

If f : (X, τ) → (Y, σ) is g*s –irresolute, then for every g*s –closed set B of Y, $f^{-1}(B)$ is g*s –closed in X. If A is any g*s –open subset of Y, then A^c is g*s –closed.

Thus $f^{-1}(A^c) = (f^{-1}(A))^c$ so that $f^{-1}(A)$ is g*s –open in X.

Sufficiency:

If for all g*s –open subsets A of Y, $f^{-1}(A)$ is g*s –open in X and if B is any g*s –closed subset of Y, then B^c is g*s –open. Also $f^{-1}(B^c) = (f^{-1}(B))^c$ is g*s –open in X. Thus $f^{-1}(B)$ is closed in X.

Hence f is g*s –irresolute.

Theorem 4.1.4

If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is g*s –irresolute, then it is g*s –continuous.

Proof:

Let A be a closed set in Y. Since every closed set is g*s –closed, A is g*s –closed in Y. Since f is g*s –irresolute, $f^{-1}(A)$ is g*s –closed in X.

Hence f is g*s –continuous.

Remark 4.1.5

The converse of the above theorem need not be true as seen from the following example.

Example 4.1.6

Let $X = Y = \{a, b, c\}$, $\tau = \{\varphi, X, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{\varphi, Y, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = f(c) = b$ and $f(b) = c$. Then f is g*s –continuous.

However, {b} is g*s –closed in Y but $f^{-1}(\{b\}) = \{a, c\}$ is not g*s –closed in X.

Therefore, f is not g*s –irresolute.

Theorem 4.1.7

If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ are both g*s –irresolute, then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is g*s –irresolute.

Proof:

Let A b a g*s –open subset of Z. Since g is g*s –irresolute, $g^{-1}(A)$ is g*s –open in Y. Since f is g*s –irresolute, $f^{-1}(g^{-1}(A))$ is g*s open in X. Thus $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is g*s –open in X.

Hence $g \circ f$ is g*s –irresolute

Theorem 4.1.8

Let (X, τ), (Y, σ) and (Z, η) be any topological spaces. For any g*s –irresolute map $f : (X, \tau) \to (Y, \sigma)$ and any g*s –continuous map $g : (Y, \sigma) \to (Z, \eta)$ the composition $g \circ f : (X, \tau) \to (Z, \eta)$ is g*s –continuous.

Proof:

Let F be a closed set in Z. Since g is g*s –continuous, $g^{-1}(F)$ is g*s –closed in Y. Since f is g*s – irresolute, $f^{-1}(g^{-1}(F))$ is g*s –closed set in X.

Thus $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is g*s –closed in X.

Hence $g \circ f$ is g*s –continuous.

Theorem 4.1.9

If a map $f : (X, \tau) \to (Y, \sigma)$ is g*s –irresolute and a subset B of X is g*s –compact relative to X, then the image f (B) is g*s –compact relative to Y.

Proof:

Let $\{A_i ; i \in I\}$ be any collection of g*s –open subsets of Y such that $f(B) \subseteq \bigcup \{A_i ; i \in I\}$. Then $B \subset \bigcup \{f^{-1}(A_i) : i \in I\}$ holds.

By hypothesis there exists a finite subset I_o of I such that $B \subset \bigcup \{f^{-1}(A_i) : i \in I_o\}$. Therefore we have $f(B) \subset \bigcup \{A_i : i \in I_o\}$

which shows that f (B) is g*s –compact relative to Y.

Theorem 4.1.10

If $f : X \to Y$ is g*s –irresolute surjective and X is g*s –connected, then Y is g*s –connected.

Proof:

Suppose Y is not g*s –connected. Let $Y = A \cup B$ where A and B are disjoint non-empty g*s –open set in Y.

Since f is g*s –irresolute and onto, X = $f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty and g*s –open in X. This contradicts the fact that X is g*s –connected.

Hence Y is g*s –connected.

Section – 4.2

g*s – HAUSDORFF SPACE

Definition 4.2.1 [25]

A space X is said to be Hausdorff if whenever x and y are distinct points of X, there exists disjoint open sets U and V such that $x \in U$ and $y \in V$.

Definition 4.2.2 [28]

A space X is said to be **g*s –Hausdorff** if whenever x and y are distinct points of X, there exists disjoint g*s –open sets U and V such that $x \in U$ and $y \in V$.

Theorem 4.2.3

Let X be a space and Y be a Hausdorff. If $f : X \to Y$ is g*s –continuous injective, then X is g*s – Hausdorff.

Proof:

Let x and y be any two distinct points of X. Then f (x) and f (y) are distinct points of Y, because f is injective. Since Y is Hausdorff, there are disjoint open sets U and V in Y containing f (x) and f (y) respectively. Since f is g*s –continuous and $U \cap V = \varphi$, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint g*s –open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$.

Hence X is g*s – Hausdorff.

Theorem 4.2.4

Let X be a space and Y be a Hausdorff. If $f : X \to Y$ is g*s –irresolute injective, then X is g*s – Hausdorff.

Proof:

Let x and y be any two distinct points of X. Then f (x) and f (y) are distinct points of Y, because f is injective.

Since Y is g*s -Hausdorff, there are disjoint open sets U and V in Y containing f (x) and f (y) respectively. Since f is g*s – irresolute and U ⋂ V = φ, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint g*s –open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$.

Hence X is g*s –Hausdorff.

Theorem 4.2.5

Let X be a space and Y be g*s - Hausdorff. If f : X → Y is strongly g*s –continuous injective, then X is Hausdorff.

Proof:

Let x and y be any two distinct points of X. Then f (x) and f (y) are distinct points of Y, because f is injective. Since Y is g*s -Hausdorff, there are disjoint open sets U and V in Y containing f (x) and f (y), respectively.

Since f is strongly g*s –continuous and U ⋂ V = φ, we have $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint g*s –open sets in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$.

Hence X is Hausdorff.

Section – 4.3

g*s – HOMEOMORPHISMS AND g*sc - HOMEOMORPHISMS

The notation of homeomorphism plays a very important role in topology. By defnition, a homeomorphism between two topological spaces X and Y is a bijective map f : X → Y when both f and f^{-1} are continuous.

Definition 4.3.1 [28]

A bijection f : (X, τ) → (Y, σ) is called a **g*s -homeomorphism** if f is both g*s -continuous and g*s –open.

We denote the family of all g*s –homeomorphisms of a topological space (X, τ) onto itself by g*s –h(X, τ).

Theorem 4.3.2

Every homeomorphism is a g*s –homeomorphism.

Proof:

Let f : (X, τ) → (Y, σ) be a homeomorphism.Since f is homeomorphism, f is bijection and also f is both open and continuous. Since every open map is g*s –open and every continuous map is g*s –continuous, f is bijection, g*s –open and g*s –continuous. Hence f is g*s –homeomorphism.

Remark 4.3.3

The converse of the above theorem need not be true as seen from the following example.

Example 4.3.4

Let X = Y = {a, b, c}, τ = {φ, X, {a, b}} and σ = {φ, Y, {a}}.

Let f : (X, τ) → (Y, σ) be an identity map. Then f is g*s –homeomorphism but not a homeomorphism. Since {a, b} is open in (X, τ) but the image f({a, b}) = {a, b} is not open in (Y, σ).

Theorem 4.3.5

Every g*s -homeomorphism is a gs –homeomorphism.

Proof:

Let f : (X, τ) → (Y, σ) be a g*s -homeomorphism. Since f is g*s -homeomorphism, f is bijection and also f is both g*s - open and g*s -continuous. Since every g*s -open map is gs –open and every g*s -continuous map is gs –continuous, we have f is bijection, gs –open and gs –continuous. Hence f is gs –homeomorphism.

Remark 4.3.6

The converse of the above theorem need not be true as seen from the following example.

Example 4.4.7

Let $X = Y = \{a, b, c\}$, $\tau = \{\varphi, X, \{a\}\}$ and $\sigma = \{\varphi, Y, \{a\}, \{c\}, \{a, c\}\}$.

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an identity map. Then f is gs –homeomorphism but not a g*s -homeomorphism. Since $\{a, b\}$ is closed in (Y, σ) but $f^{-1}(\{a, b\}) = \{a, b\}$ is not g*s -closed in (X, τ).

Theorem 4.3.8

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective g*s –continuous map, then the following are equivalent:

a) f is g*s –open map.
b) f is g*s –homeomorphism.
c) f is g*s –closed map.

Proof:

To prove: (a) ⟹ (b) :

Suppose f is g*s –open map. To prove that f is g*s –homeomorphism.

By hypothesis, f is bijective and g*s –continuous map.

By definition of g*s –homeomorphism, f is g*s –homeomorphism.

To prove :(b) ⟹ (c) :

Suppose that f is g*s –homeomorphism. To prove that f is g*s –closed map.

Since f is g*s –homeomorphism, f is bijective and also f is g*s –open and g*s –continuous. Let F be a closed set of (X, τ). Then F^c is open set in (X,τ).

Since f is g*s –open map, $f(F^c)$ is g*s –open in (Y, σ). $f(F^c) = (f(F))^c$ is g*s –open set in (Y,σ). Thus $f(F)$ is g*s –closed set in (Y, σ).

Hence f is g*s –closed map.

To prove :(c) $\Longrightarrow$ (a) :

Suppose that f is g*s –closed map. To prove that f is g*s –open map.

Let A be a closed set in (X, τ). Since f is g*s –closed map, f (A) is g*s –closed set in (Y, σ). f (A) = $(f^{-1})^{-1}$(A) is g*s –closed set in (Y, σ), which implies that f^{-1} is g*s –continuous on (Y, σ).

By **theorem 1.4.15,** f is g*s –open map.

Remark 4.3.9

The composition of two g*s –homeomorphism need not be g*s –homeomorphism in general as seen from the follwing example.

Example 4.3.10

Consider X = Y = Z = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}}, σ = {φ, Y, {a, b}} and η = {φ, Z, {a}}**.** Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) be the identity maps.

Then both f and g are g*s –homeomorphism but their composition g ∘ f : f : (X, τ) → (Z, η) is not g*s –homeomorphism. Because for the open set {b} of (X, τ), [g ∘ f] ({b}) = g (f ({b})) = g ({b}) = {b} which is not g*s –open in (Z, η).

Definition 4.3.11 [28]

A bijective f : (X, τ) → (Y, σ) is said to be **g*sc –homeomorphism** if both f and f^{-1} are g*s –irresolute.

We denote the family of all g*sc –homeomorphism of a topological space (X, τ) onto itself by g*sc –h (X, τ).

Theorem 4.3.12

Every g*sc -homeomorphism is a g*s –homeomorphism.

Proof:

Let f : (X, τ) → (Y, σ) be a g*sc –homeomorphism. To prove f is g*s –homeomorphism. That is to prove f is bijective and also f is both g*s –open and g*s –continuous. Since f is g*sc –homeomorphism, f and f^{-1} are g*s –irresolute and f is bijective.

Let V be a closed set in (Y, σ). Since f is g*s –irresolute, $(f^{-1})^{-1}(V)$ is g*s –closed in (X, τ). Therefore f^{-1} is g*s-continuous. **By theorem 1.4.15 ,** f is g*s open. Hence f is g*s –homeomorphism.

Remark 4.3.13

The converse of the above theorem need not be true as seen from the following example.

Example 4.3.14

Let X = Y = {a, b, c}, τ = {φ, X, {a}}, σ = {φ, Y, {a, b}}**.**

Let f : (X, τ) → (Y, σ) be an identity map. Then f is g*s –homeomorphism but not g*sc –homeomorphism. Since f is not g*s –irresolute, the inverse image of {a, c} is not g*s –closed set in (X, τ).

Theorem 4.3.15

Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are g*sc -homeomorphism. Then their composition g ◦ f : (X, τ) → (Z, η) is also g*sc –homeomorphism.

Proof:

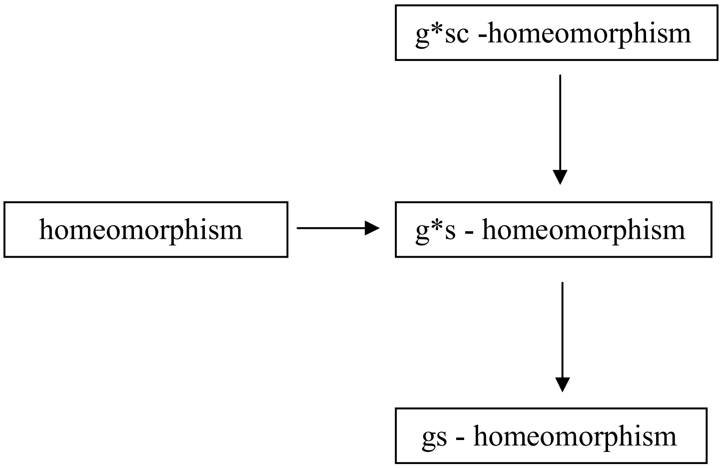
Let U be a g*s –open set in (Z, η). Since g is g*s –irresolute, $g^{-1}(U)$ is g*s –open in (Y, σ). Since f is g*s –irresolute, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is g*s –open in (X, τ). Therefore, g ◦ f is g*s –irresolute.

Also for a g*s –open set G in (X, τ). We have (g ◦ f) (G) = g (f (G)) = g (W), where W = f (G). By hypothesis, f(G) is g*s –open in (Y, σ), g (f (G)) is g*s –open set in (Z, η). (g ◦ f) (G) is g*s –open set in (Z, η). $(g \circ f)^{-1}$ is g*s –irresolute and also g ◦ f is bijective.

Hence g ∘ f is g*sc –homeomrphism.

Remark 4.3.16

The following diagram shows that the relationship between g*s - homeomorphism and other homeomorphisms.



Where A ⟶ B represents that A implies B and A does not imply B.

Note 4.3.18

Let Γ be a collection of all topological spaces. We introduce a relation , say "g*sc", into the family Γ as follows: for two elements (X, τ) and (Y, σ) of Γ, (X, τ) is g*sc –homeomorphic to (Y, σ) say $(X, \tau) \equiv_{g*sc} (Y, \sigma)$, if there exists a g*sc –homeomorphism f : (X, τ) → (Y, σ).

Then we have the following theorem on the relation " $\equiv_{g*sc}$ ".

Theorem 4.3.19

The relation $\equiv_{g*sc}$ is an equivalence relation in the collection of all topological spaces Γ.

Proof:

a) **Reflexive :**

For any element (X,τ) ∈ Γ, $(X, \tau) \equiv_{g*sc} (X, \tau)$ holds. Indeed the identity function $I_x : (X, \tau) \rightarrow (X, \tau)$ is a g*sc –homeomorphism.

Hence the relation $\equiv_{g^*sc}$ is reflexive.

b) **Symmetric :**

Suppose $(X, \tau) \equiv_{g^*sc} (Y, \sigma)$. where (X, τ) and $(Y, \sigma) \in \Gamma$. Then, there exist a g*sc –homeomorphism $f : (X, \tau) \to (Y, \sigma)$. By definition it is seen that $f^{-1} : (Y, \sigma) \to (X, \tau)$ is a g*sc –homeomorphism and hence $(Y, \sigma) \equiv_{g^*sc} (X, \tau)$.

Therefore the relation $\equiv_{g^*sc}$ is symmetric.

c) Transitive :

Suppose that $(X, \tau) \equiv_{g^*sc} (Y, \sigma)$ and $(Y, \sigma) \equiv_{g^*sc} (Z, \eta)$, where (X, τ), (Y, σ) and $(Z, \eta) \in \Gamma$. **By theorem 4.3.15 ,** we get that $(X, \tau) \equiv_{g^*sc} (Z, \eta)$.

Therefore the relation $\equiv_{g^*sc}$ is transitive.

Therefore the relation $\equiv_{g^*sc}$ is an equivalence relation in the collection of all topological spaces Γ.

Theorem 4.3.20

The set g*sc –h(X, τ) is a group under the composition of maps.

Proof:

Define a binary operation $*$: g*sc –h $(X, \tau) \times$ g*sc –h(X, τ) by $f * g = g \circ f$ for all $f ,g \in$ g*sc –h(X, τ) and $\circ$ is the usual operation of composition of maps. Then **By theorem 4.3.15,** $g \circ f \in$ g*sc –h(X, τ). We know that the composition of maps is associative and the identity map $I : (X, \tau) \to (X, \tau)$ belonging to g*sc –h(X, τ) serves as the identity element.

If $f \in$ g*sc –h(X, τ), then $f^{-1} \in$ g*sc –h(X, τ) such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of g*sc –h(X, τ).

Therefore (g*sc –h(X, τ), $\circ$) is a group under the operation of composition of maps.

Theorem 4.3.21

Let $f : (X, \tau) \to (Y, \sigma)$ be a g*sc –homeomorphism. Then f induces an isomorphism from the group g*sc –h(X, τ) onto the group g*sc –h(Y, σ).

Proof:

Let $f \in$ g*sc –h(X, τ). Then we define a map ψ_f : g*sc –h$(X, \tau) \rightarrow$ g*sc-h(Y,σ) by $\psi_f(h) = f \circ h \circ f^{-1}$ for every $h \in$ g*sc –h(X, τ). **By theorem 4.3.15,** ψ_f is well defined in general, because $f \circ h \circ f^{-1}$ is a g*sc –homeomorphism for every g*sc –homeomorphism $h : (X, \tau) \rightarrow (Y, \sigma)$.

Let $h_1, h_2 \in$ g*sc –h(X, τ). Then $\psi_f (h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1})$ o $(f \circ h_2 \circ f^{-1})$. Therefore $\psi_f (h_1 \circ h_2) = \psi_f (h_1) \circ \psi_f (h_2)$. Since $\psi_f (f^{-1} \circ h \circ f) = h$, ψ_f is onto. Now, $\psi_f (h) = I$. That implies $h = I$. This proves that ψ_f is one – one.

This shows that ψ_f is an isomorphism induced by f.

Theorem 4.3.22

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is g*sc –homeomorphism, then

g*s - cl$(f^{-1}(B)) = f^{-1}($g*s –cl$(B))$ for every $B \subseteq Y$.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a g*sc –homeomorphism.

By definition 4.3.11, both f and f^{-1} are g*s –irresolute and f is bijective.

Let $B \subseteq Y$. Since g*s –cl (B) is a g*s –closed set in (Y, σ), using the definition of g*s –irresolute, $f^{-1}($g*s –cl $(B))$ is g*s –closed in (X, τ).

But g*s –cl $(f^{-1}(B))$ is the smallest g*s –closed set containing $f^{-1}(B)$.

Therefore g*s –cl $(f^{-1}(B)) \subseteq f^{-1}($g*s –cl $(B))$ …………………(4.1)

g*s –cl $(f^{-1}(B))$ is g*s –closed set in (X, τ). Since f^{-1} is g*s –irresolute, f (g*s –cl$(f^{-1}(B)))$ is g*s –closed set in (Y, σ).

Now, $B = f(f^{-1}(B)) \subseteq f($g*s –cl $(f^{-1}(B)))$.

Since f (g*s –cl $(f^{-1}(B)))$ is g*s –closed and g*s- cl (B) is the smallest g*s –closed set containing B, g*s –cl $(B) \subseteq f($g*s –cl $(f^{-1}(B)))$

$\Rightarrow f^{-1}($g*s –cl $(B)) \subseteq$ g*s –cl $(f^{-1}(B))$ ………………… (4.2)

From (4.1) and (4.2), g*s –cl $(f^{-1}(B)) = f^{-1}($g*s –cl $(B))$.

Corollary 4.3.23

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is g*sc –homeomorphism, then g*s –cl $(f(B)) = f($g*s –cl $(B))$ for every $B \subseteq Y$.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a g*sc –homeomorphism. Since f is g*sc –homeomorphism, f^{-1} is also a g*sc –homeomorphism. Therefore **By theorem 4.3.22,** it follows that g*s –cl $(f(B)) = f($g*s –cl $(B))$ for every $B \subseteq Y$.

Corollary 4.3.24

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is g*sc –homeomorphism, then $f($g*s -int$(B))$ = g*s –int$(f(B))$ for every $B \subseteq Y$.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a g*sc –homeomorphism. For any set $B \subseteq Y$, g*s –int $(B) = ($g*s –cl $(B^c))^c$.

$$\Rightarrow f(\text{g*s –int }(B)) = f(\text{g*s –cl }(B^c))^c = (f(\text{g*s –cl }(B^c)))^c.$$

Then using **corollary 4.3.23**, we get that,

$$f(\text{g*s –int}(B)) = (\text{g*s –cl}(f(B^c)))^c = \text{g*s –int }(f(B)).$$

Corollary 4.3.25

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is g*sc –homeomorphism, then for every $B \subseteq Y$, $f^{-1}($g*s –int $(B))$ = g*s –int$(f^{-1}(B))$.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a g*sc –homeomorphism.

Since f is g*sc –homeomorphism. f^{-1} is also a g*sc –homeomorphism. Therefore by **corollary 4.3.24**,

$f^{-1}($g*s –int$(B))$ = g*s –int$(f^{-1}(B))$ for every $B \subseteq Y$.

Summary and Conclusion

SUMMARY AND CONCLUSION

In Chapter 1, Preliminary definitions, g*s – closed sets, g*s – open sets, g*s – continuous functions, g*s – closed maps and g*s – open maps in topological spaces are studied and their properties are discussed.

In Chapter 2, g*s – separation axioms, strongly g*s – continuous maps, perfectly g*s – continuous maps and g*s – compactness in topological spaces are discussed and its properties, characterizations are analyzed.

Chapter 3 deals with g*s – connectedness, g*s – locally closed sets in topological spaces due to Anitha and Pushpalatha **[1]**. Properties and characterizations of these sets are discussed.

In Chapter 4, g*s – irresolute maps, g*s – Hausdorff spaces, g*s – homeomorphisms and g*sc - homeomorphisms, their properties and characterization theorems due to Perachi Sundari and Latha Martin **[28]** are studied.

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