Acceptance Sampling Plan for Truncated Life Tests at Maximum Allowable Percent Defective

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Abstract:
The paper deals with a reliability acceptance sampling plan developed through maximum allowable percent defective (MAPD) having the single sampling plan as attribute plan. By fixing MAPD, we here obtain the test termination ratios, assuming that the lifetime follows different distributions. Comparisons are made and examples are given to illustrate the procedure.

Keywords: Acceptance sampling, Life test, Burr XII distribution, Weibull distribution, Log – Logistic distribution, Rayleigh distribution, MAPD.

1. Introduction

Sampling plans often used to determine the acceptability of lots of items. Although in recent years more emphasis is placed on process control and off-line quality control methods, acceptance sampling remains as a major tool of many practical quality control system. In acceptance sampling, if the quality variable is the lifetime of an item, the problem of acceptance sampling is known as the reliability sampling, and the test is called the life test. We would like to know whether the lifetimes of items reach our standard or not. When the life test shows that the mean lifetime of items exceeds or equals to the specified one, we treat the lot of items as acceptable; conversely, when the life test shows that the mean lifetime of the lot of items is less than our standard, we treat the lot as unacceptable and reject it. Whenever the quality of the product is related to its lifetime, it is then called as life testing. Here it is common practice to truncate the experiment during the sampling process if no failure occurs within the experimental time period or the number of failure exceeds the specified number. Acceptance sampling plan is an essential tool in the statistical quality control as hundred percent inspections of the products is not possible. This type of sampling plans was first applied by the US military for testing the bullets during World War II. Acceptance sampling plan is a middle path between hundred percent inspections and no inspection at all. Many authors have discussed acceptance sampling based on truncated life tests. Aslam M. (2007) has studied double acceptance sampling based on truncated life tests in Rayleigh distribution. Again Aslam M., with Jun, C.H. (2009) studied a group acceptance sampling plans for truncated life tests based on the inverse Rayleigh and log-logistic distributions. Fertig F.W. and Mann N.R. (1980) developed life-test sampling plans for two-parameter Weibull populations. Goode H.P., and Kao J.H.K. (1961) have studied Sampling plans based on the Weibull distribution. Kantam R.R. L., Rosaiah K. and Srinivasa Rao G. (2001) have studied acceptance sampling based on life tests. Muhammad Hanif, Munir Ahmad and Abdur Rehman (2011) developed economic reliability acceptance sampling plans from truncated life tests based on the Burr Type XII percentiles. MAPD is a key measure assessing to what degree the inflection point empowers the OC curve to discriminate between good and bad lots. Many authors have discussed MAPD, Mayor (1956) introduced the concept of MAPD in a SSP using Poisson model. MAPD locates a point of the OC curve at which the descent is steepest. It is defined as the proportion of defective beyond which consumer won't be willing to accept the lot. Mandelson (1962) has explained the desirability for developing a system of sampling plan indexed by MAPD and suggested a relation $p^* = c/n$. Soundararajan (1975) has indexed SSP through MAPD. Norman Bush (1953) developed a SSP based on tangent at inflection point. This technique states that a straight line will uniquely portray the slope of OC curve and hence the OC is fixed. According to Norman Bush the point of inflection has been chosen because it is most representative point of an OC curve showing the turning point of quality since the maximum tangent occurs at this point. Ramkumar (2009) developed Design of single sampling plan by discriminate at MAPD. We in this paper has developed an acceptance plan by fixing MAPD, which has single sampling plan as attribute plan to obtain the test termination ratios, assuming that the life time of the product follows different distributions.

2. Cumulative Distributive Function

2.1 Burr XII distribution

The cumulative distribution function (cdf) of the Burr XII distribution is given by

$$F(t/\sigma) = 1 - \left(1 + (t/\sigma)^{\gamma}\right)^{-\lambda} \qquad (1)$$

Where σ is a scale parameter and λ and γ are the shape.

2.2 Weibull Distribution
The cumulative distribution function (cdf) of the Log – Logistic distribution is given by

$$F(t,\sigma,\theta) = \left[\frac{(t/\sigma)}{1+(t/\sigma)} \right] \quad (2)$$

where σ is a scale parameter.

2.3 Log – Logistic distribution
The cumulative distribution function (cdf) of the Log – Logistic distribution is given by

$$F(t,\sigma) = \frac{\left(\frac{t}{\sigma}\right)^{\lambda}}{1+\left(\frac{t}{\sigma}\right)^{\lambda}} \quad (3)$$

Where σ is a scale parameter and λ is the shape.

2.4 Rayleigh distribution
The cumulative distribution function (cdf) of the Rayleigh distribution is given by

$$F(t/\sigma) = 1 - e^{-\frac{1}{2}\left(\frac{t}{\sigma}\right)^{2}} \quad (4)$$

Where σ is a scale parameter.

If some other parameters are involved, then they are assumed to be known, for an example, if shape parameter of a distribution is unknown it is very difficult to design the acceptance sampling plan. In quality control analysis, the scale parameter is often called the quality parameter or characteristics parameter. Therefore it is assumed that the distribution function depends on time only through the ratio of t/ σ.

3. Design of the proposed sampling plan

3.1 Procedure
We here design a sampling plan which ensures that the true average life is greater than the specified average life. We propose the following plan based on truncated life test with fixed MAPD.

i. Select n items and put them on test
ii. Select the acceptance number c and termination time t_0.
iii. Terminate the experiment if more than c failures are recorded before termination time and reject the lot. Accept if c or fewer failures occurs before termination time.

It would be convenient to determine the termination time t_0 as a multiple of the specified average file σ_0. We know the hypothesis that $H_0{:}\sigma \geq \sigma_0$, we accept the lot and $H_1{:}\sigma < \sigma_0$, we reject the same. When true mean life is equal to specified mean life ($\sigma = \sigma_0$), it is the worst case.

4. Construction of tables Operating Characteristic functions
The MAPD is the quality level that corresponds to the point of inflection of the OC curve. It is the quality level at which the second order derivative of the OC function L(p) with respect to p is zero and third order not equals zero.

$$\frac{d^2L(p)}{dp^2} = 0 \qquad \text{at } p = p^* \qquad (5)$$

$$\frac{d^3L(p)}{dp^3} \neq 0$$

Where L (p) is the probability of acceptance at quality level p fraction defectives. In our proposed plan we equate p to that of the fixed MAPD to obtain the test termination ratio. Soundarajan (1975) has proposed a procedure for designing a single sampling plan with quality standards p* and $k = p_t/p^*$, where p_t is the tangent intercept to the p – axis from the inflection point of the OC curve.

It is well known that points on the OC curve for the sampling plan (n, c) under the conditions for the application of the Poisson model is given by

$$L(p) = e^{-np}\sum_{i=0}^{c}\frac{(np)^r}{r!} \qquad (6)$$

Where p is the failure probability of the lot. Then

$$\frac{dL(p)}{dp} = \frac{-n(np)^c e^{-np}}{c!} \qquad (7)$$

$$\frac{d^2L(p)}{dp^2} = \frac{-n(np)^c e^{-np}(c-np)}{c!\,p} \qquad (8)$$

Equating to zero, the p – coordinates of the inflection point p* of the OC curve is given by

$$p^* = \frac{c}{n} \qquad (9)$$

Here by fixing MAPD and c, the values of n is studied and given in Table 1.

The equation of the tangent to the OC curve at the inflection point is given by

$$f(p) - e^{-c}\sum_{r=0}^{c}\frac{c^r}{r!} = \left(\frac{-nc^c e^{-c}}{c!}\right)(p - \frac{c}{n}) \qquad (10)$$

Which reduces to?

$$c^c np + c!e^c f(p) = c^{c+1} + c!\sum_{r=0}^{c}\frac{c^r}{r!} \qquad (11)$$

Putting f (p) = 0 in (11), the point at which the inflection tangent cuts the p – axis is given by

$$p_t = \left(\frac{c}{n}\right) + \left(\frac{c!}{c^c n}\right)\sum_{r=0}^{c}\frac{c^r}{r!} \qquad (12)$$

And the ratio $R = p_t/p^*$ which is given in Table 1. Table 1 also gives the values of the probability of acceptance for fixed values of the failure probability and the determined values of n.

5. Operating Characteristic Functions

The probability of acceptance can be regarded as a function of the deviation of the specified value μ_0 of the median from its true value μ. This function is called Operating Characteristic (OC) function of the sampling plan. Once the minimum sample size is obtained, one may be interested to find the probability of acceptance of a lot when the quality (or reliability) of the product is sufficiently good. As mentioned earlier, the product is considered to be good if $\mu \geq \mu_0$. The probabilities of acceptance are displayed in Table 3 and 4 for various values of the median ratios μ/μ_0, producer's risks β and time multiplier a.

6. Notations

n	-	Sample size
c	-	Acceptance number
t	-	Termination time
p	-	Failure probability
p_t	-	Tangent intercept
p*	-	MAPD
σ	-	Mean life
σ_0	-	Specified life

6. Description of tables and examples

Suppose that an experimenter wants to run an experiment at t = 1500 hours ensuring that the specified average life is at least 1000 hours. This leads to the termination ratio of 1.5. For the producers risk of 0.05, the sampling plan (n = 4, c = 1, t/σ_0 = 1.500), by Tsai and Wu (2006) states that, if during 1500 hours no more than 1 failure out of 4 is recorded, the lot is accepted, otherwise rejected. For the same sampling plan Mohammad Aslam (2008) with the proposed plan (n = 4, c = 1, t/σ_0 = 0.362), states that, we reject the product if more than one failure occur during 362 hours, otherwise we accept it. Our proposed plan is to reduce the time to the minimum by introducing maximum allowable percent defective. Suppose that n = 100, c = 1 and p* = 0.01 then from the Table 2, t/σ_0 = 0.070976 for that the life time that follows Burr XII distribution, t/σ_0 = 0.100251 for that the life time that follows Weibull distribution, t/σ_0 = 0.100504 for that the life time that follows Log – Logistic distribution, t/σ_0 = 0.141777 for that the life time that follows Rayleigh distribution. This states that, we reject the product if more than one failure occurs within 71 hours for Burr XII distributions,100 hours for Weibull, 101 hours for Log – Logistic and 142 hours for Rayleigh distribution. Among these distributions Burr XII using MAPD is better than all other distributions.

Table (2) and Figure 1 represents the termination time according to different distributions with fixed MAPD. It is clear from the table that as MAPD increases the termination time also increases. Thus for a lesser MAPD, the average life time is very high as MAPD decreases the life time increases this is clearly shown in Figure 1.

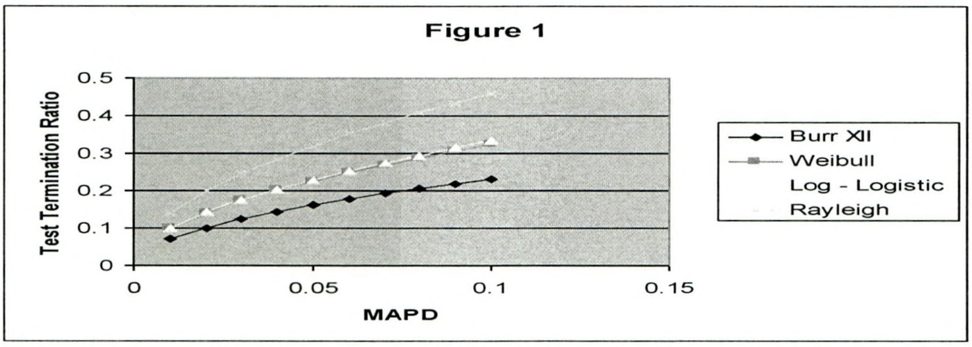

Figure 1. Test termination ratio against MAPD

7. Conclusion

The paper determines the termination time according to different distributions with fixed MAPD. We have developed an acceptance plan by fixing MAPD, which has single sampling plan as attribute plan to obtain the test termination ratios, assuming that the life time of the product follows different distributions. This plan has reduced the test termination ratios to the maximum reducing the time of inspection and increasing the mean life of the product. The paper can be extended by introducing more distributions which can be compared with the distributions present in the paper to reduce the termination ratios even more. The paper can further be extended by fixing the existing time termination ratios one can try to find the mean ratios.

Table 1:
Probability of acceptance for fixed MAPD and acceptance number c

p*	c	n	L(p)	p_t	R = p_t/p^*
0.01	1	100	0.735759	0.03	3
0.02	2	200	0.676676	0.06	3
0.03	3	300	0.647232	0.09	3
0.04	4	400	0.628837	0.12	3
0.05	5	500	0.615961	0.15	3
0.06	6	600	0.606303	0.18	3
0.07	7	700	0.598714	0.21	3
0.08	8	800	0.592547	0.24	3
0.09	9	900	0.587408	0.27	3
0.1	10	1000	0.583040	0.3	3

Table 2:
Test termination ratios with fixed MAPD for different distributions

p*	t/σ_0			
	Burr XII	Weibull	Log - Logestic	Rayleigh
0.01	0.070976	0.100251	0.100504	0.141777
0.02	0.100760	0.142136	0.142857	0.201011
0.03	0.123880	0.174526	0.175863	0.246817
0.04	0.143599	0.202045	0.204124	0.285734
0.05	0.161178	0.226480	0.229416	0.320291
0.06	0.177260	0.248748	0.252646	0.351782
0.07	0.192228	0.269389	0.274352	0.380974
0.08	0.206330	0.288759	0.294884	0.408367
0.09	0.219738	0.307100	0.314485	0.434306
0.10	0.232578	0.324593	0.333333	0.459044

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