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Desinging Single Sampling Plan for Truncated LifeTests Using Minimum Angle Method

2.1 Introduction

Reliability study plays a vital role is quality control analysis. In time truncated acceptance sampling plan, a random sample is chosen from a submitted lot of items and put on the test where the number of failures is observed until the pre-specified time. If the number of failures is higher than the specified acceptance number, then the submitted lot will be rejected. In this chapter Single sampling plan for life test is considered. The Single sampling plan is basic to all sampling situations. The Single sampling plan for life test has become the benchmark against which other sampling plans are judged. Attributes Single sampling plan based on truncated life tests have been proposed for a variety of life distribution by many authors, example Goode and Kao (1961), for the Weibull distribution Gupta and Groll (1961), for the Gamma distribution.

In this chapter a new approach of designing Single sampling plans for truncated life tests using minimum angle method, is proposed when the life time of the items follows different distributions. The distributions considered in this chapter are Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution. The test termination time and mean ratio time are specified. The acceptance number is also specified. The design parameter is obtained such that it satisfies both the producer's risk and consumer's risk simultaneously. The tables of design parameter are provided for easy selection of the plan parameter. The results are analysed with the help of tables and examples.

2.2 Operating Procedure for Single sampling plan for life tests

The Single sampling plan for life test may be described as follows, It is completely specified by three numbers N, n and c where N is the lot size, n is the sample size, and c is the acceptance number,

1. Select a random sample of size n from lot of size N, and put them on test for time t_0.

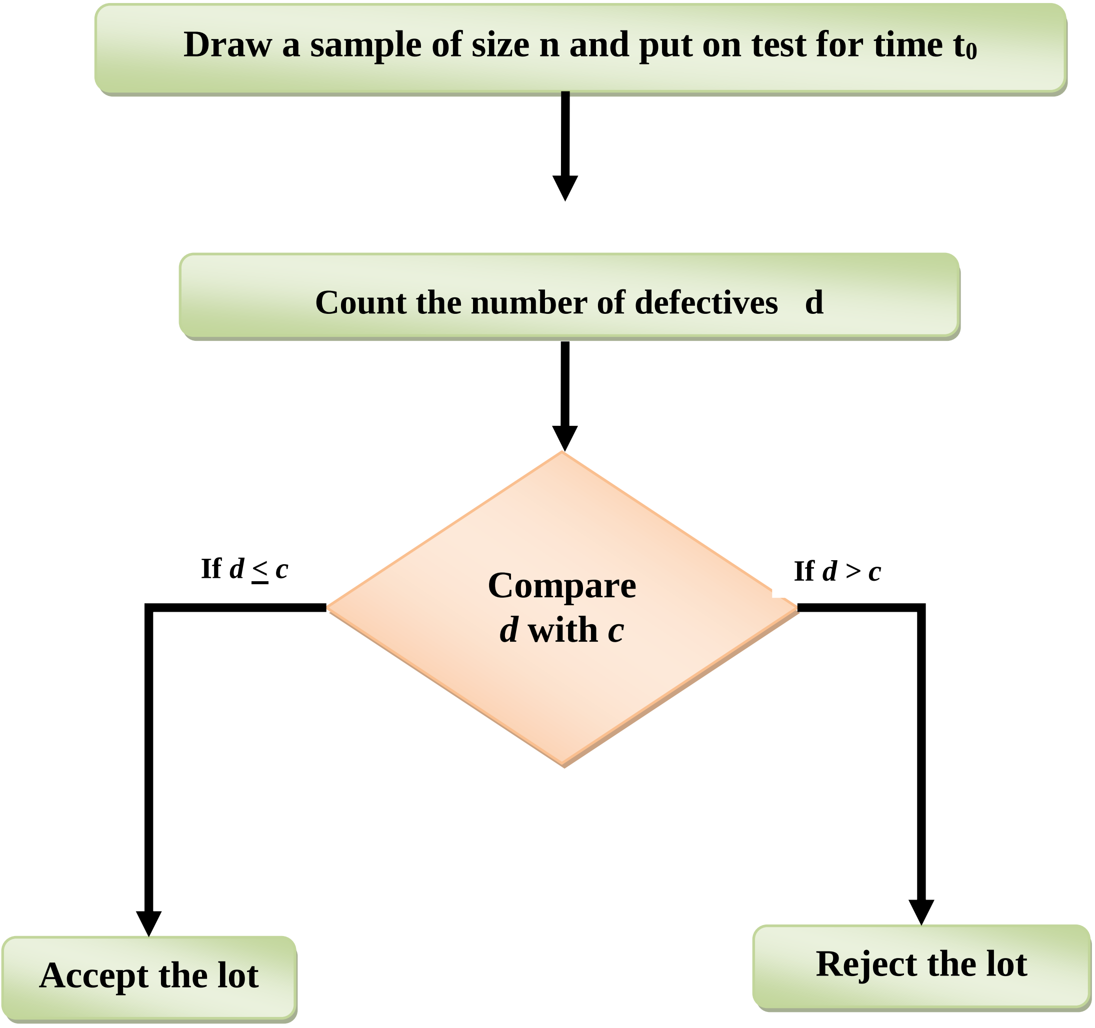
2. Inspect all the items included in the sample. Let d be the number of defectives in the sample.

3. (i) If $d \le c$, accept the lot.

   (ii) If $d > c$, reject the lot.

The following flow chart enumerates the operating procedure for Single sampling plan for life test .

Flow Chart


Draw a sample of size n and put on test for time t_0
Count the number of defectives d
Compare
d with c
If $d \leq c$
If $d > c$
Accept the lot
Reject the lot

2.3 Distributions

The following are the life time distributions used in this chapter.

2.3.1 Rayleigh distribution

The cumulative distribution function (cdf) of the Rayleigh distribution is given by

$$F(t,\lambda) = 1 - e^{-\frac{1}{2}\left(\frac{t}{\lambda}\right)^2}, \qquad t>0, \lambda>0 \qquad (2.1)$$

where λ is the scale parameter

2.3.2 Generalized Exponential distribution

The cumulative distribution function (cdf) of the Generalized Exponential distribution is given by

$$F(t,\lambda) = \left(1 - e^{-\frac{t}{\lambda}}\right)^{\alpha}, \qquad t>0, \lambda>0 \qquad (2.2)$$

where λ is the scale parameter and α is the shape parameter and which is assumed as 2

2.3.3 Weibull distribution

The cumulative distribution function (cdf) of the Weibull distribution is given by

$$F(t,\lambda) = 1 -- e^{-\left(\frac{t}{\lambda}\right)^{m}}, \qquad t>0, \lambda>0 \qquad (2.3)$$

where λ is the scale parameter and m is the shape parameter and which assumed as 2

2.3.4 Gamma distribution

The cumulative distribution function (cdf) of the Gamma distribution is given by

$$F(t,\lambda)=1-e^{-\frac{t}{\lambda}}\sum_{j=0}^{\gamma-1}\left(\frac{t}{\lambda}\right)^{j}/\,j! \quad , \qquad t>0,\ \lambda>0 \qquad (2.4)$$

where λ is the scale parameter and γ is positive integer.

If some other parameters are involved, then they are assumed to be known. The failure probability of an item by time t_0 is given by

$$P=F(t_0,\lambda)\ , \qquad t_0>0,\ \lambda>0 \qquad (2.5)$$

where λ is the scale parameter.

The quality of an item is usually represented by its true mean lifetime. Let us assume that the true mean λ can be represented by the scale parameter. In addition, it is convenient to specify the test time as a multiple of the specified life so that t/λ_0 and the quality of an item as a ratio of the true mean to the specified life **(λ/λ_0)**.

Therefore

$$p=F\,(t/\lambda_0\,,\lambda/\lambda_0)\ , \qquad t>0,\ \lambda>0 \qquad (2.6)$$

when the underlying distribution is the Rayleigh distribution, the failure probability is

$$p=1-e^{-\frac{1}{2}\left(\frac{t/\lambda 0}{\lambda/\lambda 0}\right)^{2}}\ , \qquad t>0,\ \lambda>0 \qquad (2.7)$$

when the underlying distribution is the Generalized Exponential distribution the failure probability is

$$p=\left(1-e^{-\frac{t/\lambda 0}{\lambda/\lambda 0}}\right)^{\alpha}\ , \quad t>0,\ \lambda>0 \qquad (2.8)$$

when the underlying distribution is the Weibull distribution the failure probability is

$$p = 1 - e^{-\left(\frac{t/\lambda 0}{\lambda/\lambda 0}\right)^{m}} , \qquad t > 0, \lambda > 0 \qquad (2.9)$$

when the underlying distribution is the Gamma distribution the failure probability is

$$P = 1 - e^{-\frac{t/\lambda o}{\lambda/\lambda o}} \sum_{j=0}^{\gamma-1} \left(\frac{t/\lambda o}{\lambda/\lambda o}\right)^{j} \Big/ j! , \qquad t > 0, \lambda > 0 \qquad (2.10)$$

2.4 Construction of Tables

It is assumed that the lot size is large enough to use the binomial distribution to find the probability of lot acceptance. The probability of acceptance L(p) for the Single sampling plan is calculated using the following equations,

$$L(p) = \sum_{x=0}^{c} \binom{n}{x} p^{x} q^{n-x} \qquad (2.11)$$

where *p* is the failure probability.

The time termination ratio **t**/λ_0 values are fixed as 0.628, 0.912, 1.257, 1.571, 2.356, 3.141, 3.927 and 4.712, and the mean ratio λ/λ_0 values are fixed as 4,6,8,10 and 12. The failure probability p is obtained such that it satisfies the following inequality at worst case $\lambda = \lambda_0$, $L(p) \leq \beta$ where β is taken as 0.10. The parameter value n is obtained using minimum angle method for predetermined values of acceptance numbers and satisfying the conditions $L(p_1) \geq 0.95$ and $L(p_2) \leq 0.10$ at the same time the sum of risks which is also minimum for Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution and are presented in Table 2.1 , Table 2.2 , Table 2.3 and Table 2.4 respectively. The value of θ and tanθ are also provided in each table. The sample size n is selected corresponding to the minimum value of θ.

Table – 2.1 The sample size and probability of acceptance for Minimum angle method Single sampling plan when the life time of the items follows Rayleigh distribution

t/λ_0	λ/λ_0	c	n	$L(p_1)$	$L(p_2)$	tanθ	θ
0.628	4	2	29	0.994707	0.087405	0.183752	10.41204
	4	1	21	0.972999	0.088719	0.188536	10.67696
	6	2	29	0.999462	0.087410	0.190231	10.77090
	6	1	21	0.994152	0.088719	0.191626	10.84785
	8	1	21	0.998088	0.088719	0.19342	10.94699
	8	2	29	0.9999	0.087405	0.192758	10.91041
	8	0	12	0.963702	0.093827	0.202202	11.43121
	10	1	21	0.999205	0.088719	0.194398	11.00099
	10	2	29	0.999973	0.087405	0.193955	10.97651
	10	0	12	0.976615	0.093827	0.200498	11.33736
	12	0	12	0.983702	0.088719	0.198438	11.22385
	12	1	21	0.999613	0.088719	0.194972	11.03264
	12	2	29	0.999999	0.087405	0.194608	11.01259
0.942	4	1	10	0.970913	0.077916	0.370641	20.33677
	4	2	14	0.994062	0.074621	0.359981	19.79791
	6	1	10	0.993676	0.077916	0.377918	20.70246
	6	2	14	0.999395	0.074621	0.374234	20.51753
	8	1	10	0.99793	0.077916	0.381975	20.90559
	8	2	14	0.999887	0.074621	0.379807	20.7971
	10	0	6	0.97373	0.069802	0.391517	21.3812
	10	1	10	0.999139	0.077916	0.384167	21.01514
	10	2	14	0.99997	0.074621	0.382454	20.92957
	12	0	6	0.981683	0.069802	0.389584	21.28509
	12	1	10	0.999581	0.077916	0.385448	21.07907
	12	2	14	0.99999	0.074621	0.383905	21.00207
1.257	4	1	6	0.969421	0.071827	0.554805	29.02171
	4	2	8	0.994786	0.092103	0.551677	28.88449
	6	1	6	0.993332	0.093474	0.582826	30.23472
	6	2	8	0.999472	0.071827	0.565367	29.4824
	8	1	6	0.997815	0.071827	0.576571	29.96651
	8	2	8	0.999901	0.092103	0.588124	30.4608
	10	0	3	0.976578	0.093474	0.609551	31.36445
	10	1	6	0.99909	0.071827	0.580523	30.13613
	10	2	8	0.999974	0.092103	0.592923	30.66466

t/ λ_0	λ/λ_0	c	n	L(p1)	L(p2)	tanθ	θ
	12	0	3	0.983676	0.093474	0.607385	31.27385
	12	1	6	0.999557	0.071827	0.582815	30.23427
	12	2	8	0.999991	0.092103	0.595552	30.77601
1.571	4	1	4	0.970123	0.077142	0.710713	35.40192
	4	2	6	0.993107	0.063643	0.682816	34.32589
	6	1	4	0.993489	0.063643	0.726123	35.98427
	6	2	6	0.999291	0.063643	0.72162	35.81499
	8	1	4	0.997867	0.077142	0.749174	36.83961
	8	2	6	0.999867	0.063643	0.736772	36.38175
	10	0	3	0.963656	0.024673	0.741883	36.57108
	10	1	4	0.999112	0.077142	0.755573	37.07372
	10	2	6	0.999964	0.063643	0.743993	36.64897
	12	0	3	0.974619	0.024673	0.737249	36.39947
	12	1	4	0.999568	0.077142	0.759245	37.20741
	12	2	6	0.999988	0.063643	0.747959	36.79498
2.356	4	2	4	0.985775	0.021416	0.807193	38.91023
	6	1	3	0.984301	0.01117	0.887317	41.5832
	6	2	5	0.996356	0.0022	0.868552	40.97603
	8	1	3	0.99475	0.01117	0.91018	42.30784
	8	2	5	0.999284	0.0022	0.897854	41.91921
	10	0	1	0.972628	0.062327	0.999999	45.00000
	10	1	3	0.997793	0.01117	0.922643	42.69596
	10	2	5	0.999803	0.0022	0.912489	42.3801
	12	0	1	0.980911	0.062327	0.9999991	45.000000
	12	1	3	0.998921	0.01117	0.929976	42.92208
	12	2	5	0.999932	0.0022	0.920673	42.63492
3.141	4	2	3	0.981325	0.02146	0.757902	37.15857
	6	1	2	0.983603	0.014359	0.892182	41.73878
	6	2	3	0.9979	0.02146	0.885607	41.52833
	8	1	2	0.994497	0.014359	0.937228	43.14409
	8	2	3	0.999592	0.02146	0.939151	43.2027
	10	0	1	0.951868	0.007205	0.9999991	45.00000
	10	1	2	0.997683	0.014359	0.960682	43.85119
	10	2	3	0.999888	0.02146	0.96549	43.9941
	12	0	1	0.966324	0.007205	0.9999991	45.00000
	12	1	2	0.998866	0.014359	0.974212	44.25161
	12	2	3	0.999962	0.02146	0.980191	44.42686
3.972	6	1	2	0.961279	0.00075	0.83584	39.89023
	6	2	3	0.992381	0.001125	0.809931	39.00508

t/λ_0	λ/λ_0	c	n	$L(p_1)$	$L(p_2)$	tanθ	θ
	8	1	2	0.986553	0.00075	0.896388	41.87268
	8	2	3	0.998441	0.001125	0.88604	41.54224
	10	1	2	0.994246	0.00075	0.929819	42.91727
	10	2	3	0.999564	0.001125	0.925217	42.77551
	12	1	2	0.997158	0.00075	0.949729	43.52304
	12	2	3	0.999849	0.001125	0.947527	43.45663
4.712	6	2	3	0.981315	0.000053	0.748648	36.82029
	8	1	2	0.97464	0.000002	0.862638	40.78231
	8	2	3	0.995961	0.000053	0.844183	40.17048
	10	1	2	0.988959	0.00012	0.904929	42.14285
	10	2	3	0.99884	0.000053	0.89599	41.86004
	12	1	2	0.994495	0.000002	0.930941	42.95173
	12	2	3	0.999592	0.000053	0.926208	42.80611

Table – 2.2 The sample size and probability of acceptance for Minimum angle method Single sampling plan when the life time of the items follows Generalized Exponential distribution

t/λ_0	λ/λ_0	c	n	$L(p_1)$	$L(p_2)$	$\tan\theta$	θ
0.628	4	1	17	0.950865	0.088555	0.227718	12.82855
	4	2	23	0.98785	0.095663	0.220093	12.41248
	6	1	17	0.987981	0.088555	0.230813	12.99701
	6	2	23	0.998529	0.095663	0.229934	12.94916
	8	1	17	0.995826	0.088555	0.23342	13.13871
	8	2	23	0.999699	0.095663	0.234255	13.18409
	10	1	17	0.998201	0.088555	0.235003	13.22473
	10	2	23	0.999915	0.095663	0.236405	13.30082
	10	0	10	0.963562	0.086096	0.243622	13.69179
	12	1	17	0.999104	0.088555	0.235984	13.27796
	12	2	23	0.99997	0.095663	0.237613	13.36634
	12	0	10	0.974305	0.086096	0.24192	13.59968
0.942	4	1	9	0.953218	0.095894	0.387408	21.17676
	4	2	13	0.982492	0.084911	0.365716	20.08829
	6	1	9	0.985463	0.095894	0.39477	21.54261
	6	2	13	0.997704	0.084911	0.384726	21.04304
	8	1	9	0.994826	0.095894	0.400416	21.82197
	8	2	13	0.99951	0.084911	0.393557	21.48248
	10	0	6	0.952478	0.061174	0.40862	22.2259
	10	1	9	0.997736	0.095894	0.403845	21.99107
	10	2	13	0.999858	0.084911	0.398061	21.70556
	12	0	6	0.966284	0.061174	0.405019	22.04887
	12	1	9	0.998861	0.095894	0.40598	22.09614
	12	2	13	0.999949	0.084911	0.400624	21.83223
1.257	4	2	9	0.97685	0.07865	0.488992	26.05827
	6	1	6	0.982608	0.098585	0.538681	28.31052
	6	2	9	0.996743	0.07865	0.518691	27.41535
	8	1	6	0.993664	0.098585	0.54832	28.73682
	8	2	9	0.999279	0.07865	0.533103	28.06219
	10	1	6	0.997187	0.098585	0.554171	28.99395
	10	2	9	0.999786	0.07865	0.540614	28.39628
	12	1	6	0.998571	0.098585	0.557832	29.15419

t/ λ_0	λ/λ_0	c	n	L(p1)	L(p2)	tanθ	θ
1.571	4	2	7	0.970359	0.072016	0.581097	30.16076
	6	1	5	0.974711	0.067565	0.633256	32.34427
	6	2	7	0.995551	0.072016	0.622018	31.88236
	8	1	5	0.990522	0.067565	0.645461	32.84066
	8	2	7	0.998979	0.072016	0.642671	32.72768
	10	1	5	0.995719	0.067565	0.653325	33.15759
	10	2	7	0.99969	0.072016	0.653663	33.17115
	12	1	5	0.9978	0.067565	0.658397	33.36079
	12	2	7	0.999886	0.072016	0.660075	33.42782
2.356	6	1	3	0.96898	0.086077	0.808616	38.95956
	6	2	5	0.990048	0.044107	0.75473	37.04293
	8	1	3	0.987847	0.086077	0.836483	39.91192
	8	2	5	0.997506	0.044107	0.791186	38.35054
	10	1	3	0.994347	0.086077	0.853634	40.4852
	10	2	5	0.9992	0.044107	0.811785	39.06918
	12	1	3	0.997035	0.086077	0.864598	40.84664
	12	2	5	0.999694	0.044107	0.824218	39.49598
3.141	6	2	4	0.983953	0.038261	0.792317	38.3904
	8	1	3	0.968991	0.020265	0.853723	40.48814
	8	2	4	0.995682	0.038261	0.845969	40.2302
	10	1	3	0.984929	0.020265	0.873601	41.14048
	10	2	4	0.998549	0.038261	0.877581	41.26958
	12	1	3	0.99186	0.020265	0.887565	41.59115
	12	2	4	0.999427	0.038261	0.897195	41.89831
3.927	6	2	4	0.959391	0.008666	0.768147	37.52956
	8	2	4	0.98791	0.008666	0.827689	39.61417
	10	1	2	0.988875	0.076512	0.937684	43.158
	10	2	4	0.995678	0.008666	0.866766	40.91763
	12	1	2	0.993932	0.076512	0.962577	43.90761
	12	2	4	0.99822	0.008666	0.89241	41.74604
4.712	6	2	3	0.974073	0.052724	0.744708	36.67534
	8	2	4	0.973512	0.001875	0.806862	38.89875
	10	1	2	0.980067	0.035465	0.89024	41.67674
	10	2	3	0.997186	0.052724	0.890371	41.68093
	12	1	2	0.988878	0.035465	0.919483	42.59802
	12	2	3	0.998827	0.052724	0.926587	42.81779

Table – 2.3 The sample size and probability of acceptance for Minimum angle method Single sampling plan when the life time of the items follows Weibull distribution

t/λ_0	λ/λ_0	c	n	$L(p_1)$	$L(p_2)$	tanθ	θ
0.628	4	1	11	0.971826	0.082513	0.33909	18.73129
	4	2	15	0.994728	0.088436	0.332737	18.40421
	6	1	11	0.993884	0.082513	0.345644	19.06738
	6	2	15	0.999467	0.088436	0.345773	19.07398
	8	1	11	0.997999	0.082513	0.349281	19.25332
	8	2	15	0.9999	0.088436	0.350822	19.33198
	10	1	11	0.999168	0.082513	0.351243	19.35349
	10	2	15	0.999973	0.088436	0.353215	19.45399
	12	1	11	0.999595	0.082513	0.352389	19.41191
	12	2	15	0.999991	0.088436	0.354526	19.52073
0.942	4	1	5	0.973909	0.096367	0.608871	31.33604
	4	2	8	0.992839	0.057477	0.571234	29.73646
	6	1	5	0.994355	0.096367	0.627973	32.12773
	6	2	8	0.999263	0.057477	0.59877	30.91189
	8	1	5	0.998156	0.096367	0.637057	32.49947
	8	2	8	0.999861	0.057477	0.609614	31.36708
	10	1	5	0.999233	0.096367	0.641763	32.69085
	10	2	8	0.999963	0.057477	0.614785	31.58257
	12	1	5	0.999627	0.096367	0.644462	32.80026
	12	2	8	0.999987	0.057477	0.617624	31.70047
1.257	4	1	4	0.953363	0.029551	0.757731	37.15233
	4	2	5	0.992814	0.062604	0.75252	36.96218
	6	1	4	0.98956	0.029551	0.782382	38.039
	6	2	5	0.999258	0.062604	0.80189	38.72578
	8	1	4	0.996547	0.029551	0.795917	38.51689
	8	2	5	0.99986	0.062604	0.821172	39.39189
	10	1	4	0.998556	0.029551	0.803255	38.77335
	10	2	5	0.999962	0.062604	0.830374	39.70537
	10	0	3	0.953704	0.008737	0.823689	39.4779
	12	1	4	0.999296	0.029551	0.807555	38.92278
	12	2	5	0.999987	0.062604	0.835434	39.87654
	12	0	3	0.967618	0.008737	0.816705	39.23867

t/λ_0	λ/λ_0	c	n	$L(p_1)$	$L(p_2)$	tanθ	θ
1.571	4	2	4	0.989569	0.020331	0.796816	38.54839
	6	1	3	0.987411	0.020331	0.87789	41.27956
	6	2	4	0.998894	0.038381	0.883892	41.4732
	8	1	3	0.995815	0.020331	0.899471	41.97047
	8	2	4	0.99979	0.038381	0.91264	42.38484
	10	1	3	0.998246	0.020331	0.91099	42.3332
	10	2	4	0.999943	0.038381	0.926483	42.81458
	12	1	3	0.999144	0.020331	0.9177	42.54259
	12	2	4	0.999981	0.038381	0.934127	43.04936
2.356	4	2	3	0.97481	0.011609	0.729833	36.1232
	6	1	2	0.979583	0.007754	0.87796	41.28185
	6	2	3	0.997083	0.011609	0.865804	40.88613
	8	1	2	0.993098	0.007754	0.92662	42.81881
	8	2	3	0.999427	0.011609	0.9243	42.74719
	10	1	2	0.997085	0.007754	0.952281	43.59981
	10	2	3	0.999843	0.011609	0.953337	43.63154
	10	1	2	0.997085	0.007754	0.952281	43.59981
	12	2	3	0.999946	0.011609	0.96961	44.11604
	12	1	2	0.99857	0.007754	0.967185	44.04432
3.141	6	2	3	0.986226	0.000156	0.77098	37.63149
	8	1	2	0.979591	0.000104	0.875037	41.18714
	10	2	3	0.999171	0.000156	0.906893	42.20467
	10	1	2	0.991174	0.000104	0.914163	42.43241
	12	2	3	0.99971	0.000156	0.934146	43.04994
	12	1	2	0.995615	0.000104	0.93794	43.16579
3.927	6	2	3	0.957699	0.000002	0.68035	34.22939
	8	2	3	0.990182	0.000002	0.793667	38.43786
	8	1	2	0.95415	0.000001	0.823638	39.47617
	10	2	3	0.997081	0.000002	0.859598	40.6823
	10	1	2	0.979577	0.000001	0.874959	41.1846
	12	2	3	0.998953	0.000002	0.899385	41.96774
	12	1	2	0.989686	0.000001	0.907806	42.23335
4.712	8	2	3	0.97481	0.000083	0.725127	35.94687
	10	2	3	0.992107	0.000083	0.807264	38.91269
	10	1	2	0.960356	0.000055	0.833953	39.82652
	10	2	3	0.992107	0.000083	0.807264	38.91269
	12	1	2	0.979583	0.000055	0.874976	41.18515
	12	2	3	0.997083	0.000083	0.859619	40.68299

Table - 2.4 The sample size and probability of acceptance for Minimum angle method Single sampling plan when the life time of the items follows Gamma distribution

t/λ_0	λ/λ_0	c	n	$L(p_1)$	$L(p_2)$	tanθ	θ
0.628	4	2	39	0.990705	0.098666	0.134632	7.66776
	4	1	29	0.958916	0.091065	0.138385	7.87882
	6	2	39	0.998937	0.098666	0.140063	7.973154
	6	1	29	0.990327	0.091065	0.14022	7.981991
	8	1	29	0.996705	0.091065	0.141646	8.062078
	8	2	39	0.999789	0.098666	0.142356	8.101958
	8	0	17	0.951429	0.091537	0.149182	8.484905
	8	1	29	0.996705	0.091065	0.141646	8.062078
	10	1	29	0.998596	0.091065	0.142489	8.109444
	10	2	39	0.999941	0.098666	0.143478	8.164976
	10	0	17	0.96833	0.091537	0.147485	8.389759
	12	0	17	0.977753	0.091065	0.14648	8.333428
	12	1	29	0.999307	0.091065	0.143004	8.138349
	12	2	39	0.99998	0.098666	0.144103	8.200055
0.942	4	1	15	0.951792	0.089443	0.254165	14.2606
	4	2	21	0.987081	0.085063	0.242987	13.65743
	6	1	15	0.988232	0.089443	0.257913	14.46213
	6	2	21	0.998431	0.085063	0.253796	14.24075
	8	1	15	0.995917	0.089443	0.260907	14.62286
	8	2	21	0.999679	0.085063	0.258584	14.49819
	10	0	9	0.963109	0.081714	0.270876	15.15633
	10	1	15	0.998241	0.089443	0.262708	14.71947
	10	2	21	0.999909	0.085063	0.260971	14.62633
	12	0	9	0.973985	0.081714	0.268967	15.05442
	12	1	15	0.999124	0.089443	0.26382	14.77903
	12	2	21	0.999968	0.085063	0.262313	14.6983
1.257	4	2	15	0.9795	0.078344	0.352563	19.42076
	6	1	10	0.985163	0.054616	0.364045	20.0038
	6	2	15	0.997328	0.078344	0.368626	20.2352

	8	1	10	0.994752	0.078344	0.378374	20.72535
	8	2	15	0.999433	0.054616	0.366997	20.15298
	10	0	6	0.957174	0.070104	0.395234	21.56564
	10	1	10	0.997713	0.078344	0.38135	20.87433
	10	2	15	0.999836	0.054616	0.37092	20.35081
	12	0	6	0.969683	0.070104	0.392129	21.4116
	12	1	10	0.998853	0.078344	0.383214	20.96752
	12	2	15	0.999942	0.054616	0.373153	20.46323
1.571	4	2	10	0.981481	0.08329	0.452055	24.32559
	6	1	7	0.984138	0.08329	0.484884	25.86802
	6	2	10	0.997527	0.08329	0.477783	25.53767
	8	1	7	0.99431	0.088316	0.495276	26.3481
	8	2	10	0.999467	0.08329	0.489771	26.09426
	10	0	4	0.956253	0.081527	0.519625	27.45752
	10	1	7	0.997498	0.088316	0.499933	26.56197
	10	2	10	0.999844	0.08329	0.495911	26.37733
	12	0	4	0.96894	0.081527	0.515874	27.28804
	12	1	7	0.998737	0.088316	0.502837	26.69493
	12	2	10	0.999944	0.08329	0.499418	26.53835
2.356	4	2	6	0.974899	0.085816	0.63387	32.36938
	6	1	4	0.980345	0.098071	0.70529	35.19494
	6	2	6	0.996308	0.085816	0.683432	34.34996
	8	1	4	0.992703	0.098071	0.722234	35.83809
	8	2	6	0.99916	0.085816	0.707438	35.27704
	10	1	4	0.99672	0.098071	0.73232	36.21608
	10	2	6	0.999746	0.085816	0.720076	35.75677
	12	0	3	0.950073	0.032201	0.724424	35.92047
	12	1	4	0.998319	0.098071	0.738606	36.44979
	12	2	6	0.999907	0.085816	0.727421	36.03293
3.141	4	2	5	0.952327	0.043092	0.698422	34.93129
	6	1	3	0.973381	0.084701	0.814165	39.15128
	6	2	5	0.992054	0.043092	0.762446	37.32357
	8	1	3	0.989772	0.084701	0.841215	40.07106
	8	2	5	0.998069	0.043092	0.797254	38.56376
	10	1	3	0.9953	0.084701	0.857484	40.61255
	10	2	5	0.999392	0.043092	0.816505	39.23182
	12	1	3	0.997555	0.084701	0.867738	40.94942
	12	2	5	0.999771	0.043092	0.827988	39.62435
3.972	6	1	3	0.954715	0.024669	0.830001	39.69272

	6	2	4	0.989619	0.046283	0.80951	38.99051
	8	1	3	0.977577	0.024669	0.85759	40.61607
	8	2	4	0.997356	0.046283	0.859245	40.67065
	10	1	3	0.989359	0.024669	0.876495	41.23441
	10	2	4	0.999142	0.046283	0.887378	41.58515
	12	1	3	0.994345	0.024669	0.889245	41.64494
	12	2	4	0.999669	0.046283	0.90444	42.12744
4.712	6	2	4	0.97787	0.014749	0.791928	38.37667
	8	1	3	0.961331	0.007635	0.870691	41.04579
	8	2	4	0.993966	0.014749	0.847998	40.29789
	10	1	3	0.981111	0.007635	0.890692	41.6912
	10	2	4	0.99796	0.014749	0.881874	41.40823
	12	1	3	0.989768	0.007635	0.905244	42.15279
	12	2	4	0.999191	0.014749	0.903121	42.08587

2.5 Example

Assume that an experimenter wants to establish that the lifetime of the electrical devices produced in the factory ensures that the true unknown mean life is at least 1000 hours when the ratio of the unknown average life is $t/\lambda_0 = 0.628$ and specified life is $\lambda/\lambda_0 = 4$ and acceptance number $c = 2$. Following are the results obtained when the lifetime of the test items follow the Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution respectively :

2.5.1 Rayleigh distribution

Suppose that the life time of a product follows the Rayleigh distribution. For the above example, from Table 2.1, the sample size required is obtained as n = 29 and one can observe that the minimum angle is θ = 10.41204° and also α = 0.0053, and β = 0.0874 which is very much less than the specified risk. The lot is accepted if during 628 hours no more than 2 failures are observed in the sample of size 29 which satisfies the condition of the producer's risk and consumer's risk $\alpha \leq 0.05$, $\beta \leq 0.10$. Thus the required Single sampling plan has parameters (29,2). For the same conditions when the time of experiment is 3141 hours, the probability of acceptance is 0.981325, the producer's risk is 0.018675 and consumer's risk 0.02146. The sample size is 3 which is very much less. Thus it is clear that as the time of experiment increases, the sample size decreases. When the ratio of unknown average life is specified as 12, there is no change in the sample size but there is increase in the probability of acceptance is 0.99999 which is almost equal to 1 and the consumer's risk is 0.087405 which shows that there is a reduction in consumer's risk.

2.5.2 Generalized Exponential distribution

Suppose that the lifetime of a product follows the Generalized Exponential distribution. For the above example from Table 2.2, the sample size required is obtained as n = 23 and one can observe that the minimum angle is $\theta = 12.41248°$ and also $\alpha = 0.0122$, and $\beta = 0.0956$ which is very much less than the specified risk. The lot is accepted if during 628 hours no more than 2 failures are observed in the sample of size 23 which satisfies the condition of the producer's risk and consumer's risk $\alpha \leq 0.05$, $\beta \leq 0.10$. Thus the required Single sampling plan has parameters (23,2). For the same conditions when the time of experiment is 1571 hours, the probability of acceptance is 0.97035, the producer's risk is 0.029641 and consumer's risk 0.072016. The sample size is 7 which is very much less. Thus it is clear that as the time of experiment increases the sample size decreases. When the ratio of unknown average life is specified life is 12, there is slight change in the sample size but there is increase in the probability of acceptance is 0.9991 which is almost equal to 1 and the consumer's risk is 0.0885 which shows that there is a reduction in consumer's risk.

2.5.3 Weibull distribution

Suppose that the lifetime of a product follows the Weibull distribution. For the above example from Table 2.3, the sample size required is obtained as n = 15 and one can observe that the minimum angle is $\theta = 18.40421°$ and also $\alpha = 0.0053$, and $\beta = 0.0884$ which is very much less than the specified risk. The lot is accepted if during 628 hours no more than 2 failures are observed in the sample of size 15 which satisfies the condition of the producer's risk and consumer's risk $\alpha \leq 0.05$, $\beta \leq 0.10$, Thus the required Single sampling plan has parameters (15,2). For the same conditions when the time of experiment is 2356 hours, the probability of acceptance is 0.97481, the producer's risk is 0.02519 and consumer's risk 0.011609. The sample size is 3 which is very much less. Thus it is clear that as the time of experiment increases, the sample size decreases. When the ratio of unknown average life is specified life is 12, there is slight change in the sample size but there is increase in the probability of acceptance is 0.9995 which is almost equal to 1 and the consumer's risk is 0.082513 which shows that there is a reduction in consumer's risk.

2.5.4 Gamma distribution

Suppose that the lifetime of a product follows the Gamma distribution ,for the above example from Table 2.4, the sample size required is obtained as $n = 39$ and one can observe that the minimum angle is $\theta = 7.66776°$ and also $\alpha = 0.0093$, and $\beta = 0.0986$ which is very much less than the specified risk. The lot is accepted if during 628 hours no more than 2 failures are observed in the sample of size 39 which satisfies the condition of producer's risk and consumer's risk $\alpha \leq 0.05$, $\beta \leq 0.10$, Thus the required Single sampling plan has parameters (39,2). For the same conditions when the time of experiment is 3141 hours, the probability of acceptance is 0.952327, the producer's risk is 0.047673 and consumer's risk 0.043092. The sample size is 5 which is very much less. Thus it is clear that as the time of experiment increases, the sample size decreases. When the ratio of unknown average life is specified life is 12, there is slight change in the sample size but there is increase in the probability of acceptance is 0.9993 which is almost equal to 1 and the consumer's risk is 0.0910 which shows that there is a reduction in consumer's risk.

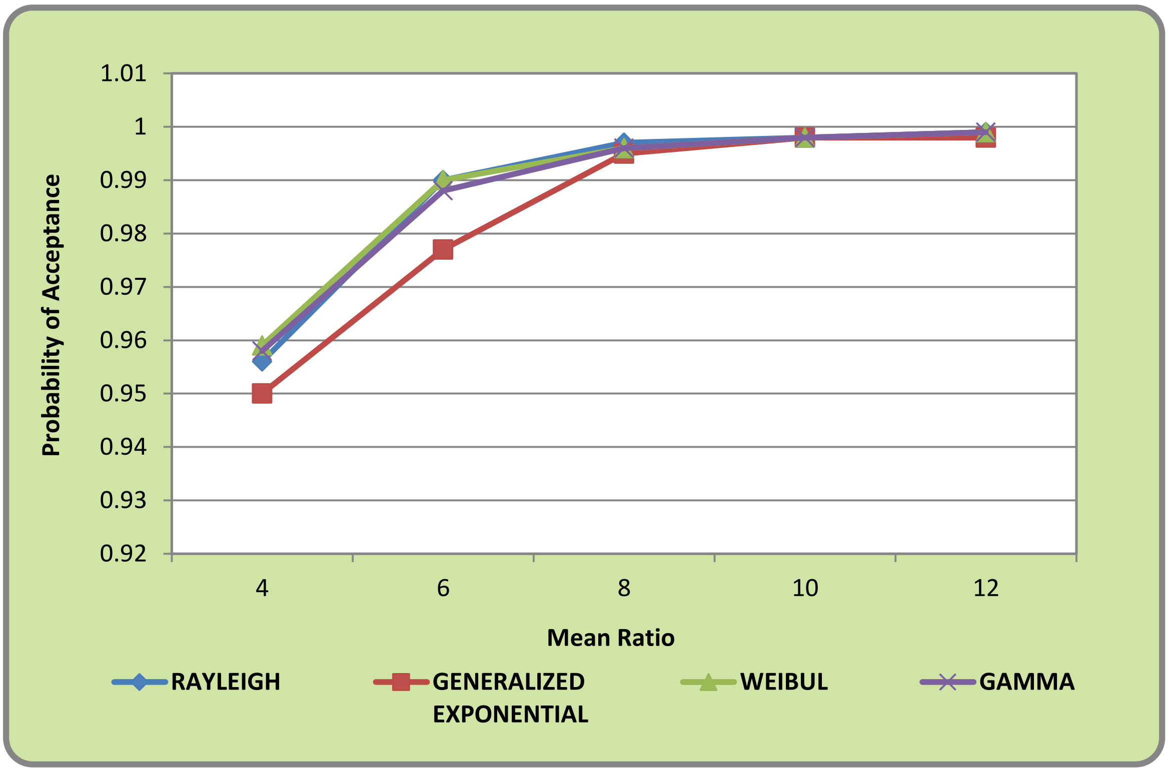
Comparison of the results of Producer's risk, Consumer's risk and sample size for Single sampling plan when the life time of the items follow different distributions Is provided in Table 2.5.

Table – 2.5 Comparison of Producer's risk, Consumer's risk and Sample size for Single sampling plan when the life time of the items follows different distributions (t/λ_0=0.628)

S.No.	λ/λ_0	Distribution	Producer's risk	Consumer's risk	c	n
1	4	Rayleigh	0.0053	0.0874	2	29
		Generalized Exponential	0.0121	0.0956	2	23
		Weibull	0.0052	0.0884	2	15
		Gamma	0.0092	0.0986	2	39
2	6	Rayleigh	0.0005	0.0874	2	29
		Generalized Exponential	0.0014	0.0956	2	23
		Weibull	0.0061	0.0825	1	11
		Gamma	0.0010	0.0986	2	39
3	8	Rayliegh	0.0001	0.0874	2	29
		Generalized Exponential	0.0041	0.0885	1	17
		Weibull	0.0020	0.0825	1	11
		Gamma	0.0032	0.0910	1	29
4	10	Rayleigh	0.0001	0.0874	2	29
		Generalized Exponential	0.0018	0.0885	1	17
		Weibull	0.0008	0.0825	1	11
		Gamma	0.0014	0.0910	1	29
5	12	Rayleigh	0.0001	0.0874	2	29
		Generalized Exponential	0.0008	0.0885	1	17
		Weibull	0.0004	0.0825	1	11

	Gamma	0.0006	0.0910	1	29

Figure - 2.1 OC curve of Single sampling plan when the life time of the items follows different distributions with (t/λ_0=0.628)



From Table 2.5 one can conclude that when the Weibull distribution is followed, the sample size is very much less than the sample size of all other distributions. At the same time producer's risk and consumer's risk also less and the sum of the risks is also very much less for Weibull distribution. Figure 2.1 shows the OC curves of all four distributions. From the figure, one can observe that the probability of acceptance is more for Rayleigh distribution and Weibull distribution than any other distribution. It can be seen that by applying minimum angle method one can obtain parameters which satisfy both the conditions on producer's risk as well as consumer's risk and at the same time the sum of risk is also minimum.