CHAPTER V

CHAPTER V

BI-ˇCECH CLOSURE SPACES

In this chapter, the concept of \$-closed set in bi- ˇ Cech closure space and their properties are analysied.

SECTION 5.1

ON \$-CLOSED SETS IN BI-ˇCECH CLOSURE SPACES

In this section, the concept of \$-closed set in bi- ˇ Cech closure space and the properties of \$-closed set in bi- ˇ Cech closure space discussed.

Definition: 5.1.1

Two functions k_1 and k_2 from power set of X to itself are called **bi- ˇ Cech closure operators** (simply biclosure operator) for X if they satisfy the following properties.

(i) $k_1(\phi) = \phi$ and $k_2(\phi) = \phi$

(ii) $A \subset k_1(A)$ and $A \subset k_2(A)$ for any set $A \subset X$

(iii)$k_1(A \cup B)=k_1(A) \cup k_1(B)$ and $k_2(A \cup B)=k_2(A) \cup k_2(B)$ for any $A,B \subset X$

(X, k_1, k_2) is called bi- ˇ Cech closure space.

Example: 5.1.2

Let $X = \{a, b, c\}$ and define a closure operator k_1 on X by $k_1(\{a\}) = \{a\}$, $k_1(\{b\}) = \{b, c\}$, $k_1(\{c\}) = k_1(\{a, c\})=\{a, c\}$, $k_1(\{a, b\}) = k_1(\{b, c\}) = k_1(\{X\}) = X$, $k_1(\phi) =\phi$. Define a closure operator k_2 on X by $k_2(\{a\}) = \{a\}$,

$k_2(\{b\}) = k_2(\{c\}) = k_2(\{b, c\}) = \{b, c\}$, $k_2(\{a, b\}) = k_2(\{a, c\}) = k_2(\{X\}) = X$, $k_2(\phi) = \phi$. Now, (X, k_1, k_2) is a bi- ˇ Cech closure space.

Definition: 5.1.3

A subset A in a **bi- ˇ Cech closure space** (X, k_1, k_2) is said to be

1. k_i-semi open if $A \subseteq k_i(intk_i(A))$, $i = 1, 2$

2. k_i-semi closed if $intk_i(k_i(A)) \subseteq A$, $i = 1, 2$

Definition: 5.1.4

A subset A in bi-cech closure space (X, k_1, k_2) is said to be a **(k_1, k_2)-generalized semi closed set** if $k\text{-}scl_2(A) \subseteq U$ whenever $A \subseteq U$ and U is k_1–open in (X, τ).

Definition: 5.1.5

A subset A in bi-cech closure space (X, k_1, k_2) is said to be a **(k_1, k_2)-\$ closed** if $k\text{-}scl_2(A) \subseteq U$ whenever $A \subseteq U$ and U is k_1-gs open set in X.

Theorem: 5.1.6

If A and B are (k_1, k_2)-\$ closed sets and so is a $A \cup B$.

Proof

Let A and B be the (k_1, k_2) \$-closed sets. Let U be a k_1 gs-open set in X.Let $(A \cup B) \subseteq U$. Therefore, $A \subseteq U$ and $B \subseteq U$.

Then $k\text{-}scl_2(A) \subseteq U$ and $k\text{-}scl_2(B) \subseteq U$ Implies $(k\text{-}cl_2(A) \cup k\text{-}scl_2(B)) \subseteq U$.

Hence $k\text{-}scl_2(A \cup B) \subseteq U$. Thus $A \cup B$ is a (k_1, k_2)-\$ closed set.

Theorem: 5.1.7

If A is a (k_1, k_2)-\$ closed set. Then k-scl_2(A)-A contains no non-empty k_1-gs closed sets.

Proof

Let A be (k_1, k_2)-\$ closed. Let U be k_1-gs closed contained in k-scl_2(A)-A.

Now, $U \subseteq$ k-$scl_2(A)$ and $U \subseteq A^c$ (1)

Now,

$U \subseteq A^c$ then $A \subseteq U^c$.

Since U is k_1-gs closed, U^c is k_1-gs open. Thus, k-$scl_2(A) \subseteq U^c$.

Consequently,

$U \subseteq [\text{k-}scl_2(A)]^c$ (2)

From (1) and (2)

$U \subseteq \text{k-}scl_2(A) \cap [\text{k-}scl_2(A)]^c = \phi$

Therefore, $U=\phi$.

Hence, k-scl_2(A)-A contain no non-empty k_1-gs closed sets.

Theorem: 5.1.8

If A is a (k_1, k_2) -\$ closed set, then k-$scl_1(x) \cap A \neq \phi$ holds for each $x \in$ k-scl_2(A)

Proof

Let A be a (k_1, k_2) -\$ closed set.

Suppose $k\text{-}scl_1(x) \cap A = \phi$, for some $x \in k\text{-}scl_2(A)$,

Hence, $A \subseteq [k\text{-}scl_1(x)]^c$.

Now $k\text{-}scl_1(x)$ is k_1-semi closed. Therefore $[k\text{-}scl_1(x)]^c$ is k_1-semi open.

Thus $[k\text{-}scl_1(x)]^c$ is k_1-gs open.

Since A is a (k_1, k_2) \$-closed set, we have $k\text{-}scl_2(A) \subseteq [k\text{-}scl_1(x)]^c$

Implies $k\text{-}scl_2(A) \cap k\text{-}scl_1(x) = \phi$.

Then $x \notin k\text{-}scl_2(A)$ is a contradiction.

Hence $k\text{-}scl_2(x) \cap A \neq \phi$ holds for each $x \in k\text{-}scl_2(A)$.

Theorem: 5.1.9

Let (X, k_1, k_2) be bi-cech closure space. For each x in X, $\{x\}$ is k_1-gs closed or $\{x\}^c$ is (k_1, k_2) -\$ closed set.

Proof

Let (X, k_1, k_2) be bi-cech closure space.

Suppose that $\{x\}$ is not k_1-gs closed, $\{x\}^c$ is not k_1-gs open.

Therefore, the only k_1-gs open set containing $\{x\}^c$ is X.

Thus $\{x\}^c \subset X$. Now, $k\text{-}scl_2[\{x\}^c] \subseteq k\text{-}scl_2(X) = X$.

Hence $\{x\}^c$ is a (k_1, k_2) -\$ closed set.

Theorem: 5.1.10

Let A be a (k_1, k_2) -$ closed subset and if A is k_1-gs open then A= k-scl_2(A).

Proof

Let A be a (k_1, k_2) -$ closed subset of a bi-cech closure space (X, k_1, k_2) and let A be a k_1-gs open set.

Then k-scl_2(A) $\subseteq$ U, whenever A$\subseteq$ U and U is a k_1-gs open set in X.

Since A is k_1-gs open and A $\subseteq$A, We have k-scl_2(A)$\subseteq$A but always, A $\subseteq$ k-scl_2(A)

Thus, A= k-scl_2(A).

Theorem: 5.1.11

Let A $\subseteq$ Y$\subseteq$ X and suppose that A is (k_1, k_2) -$ closed in (X, k_1, k_2). Then A is (k_1, k_2)-$ closed relative to Y.

Proof

Let S be any k_1-gs open set in Y such that A $\subseteq$S.

Then S=U $\cap$ Y for some U is k_1-gs open in X.

Therefore A$\subset$ U $\cap$ Y implies A$\subseteq$ U. Since A is a (k_1, k_2) -$ closed set in X,

We have k-scl_2(A) $\subseteq$ A. Hence Y $\cap$ k-scl_2(A) $\subseteq$ Y $\cap$ U =S.

Thus A is a (k_1, k_2)-$ closed set relative to Y.