APPLICATIONS OF OPERATIONS RESEARCH IN BUSINESS AND MEDICINE

By

P.SANGEETHA

(09 PM 14)

A DISSERTATION SUBMITTED TO THE

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COIMBATORE – 641 043

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

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CERTIFIED AS BONAFIDE RESEARCH WORK

Parvathi   L. Sudha

SIGNATURE OF THE HEAD OF THE DEPARTMENT

SIGNATURE OF THE GUIDE

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Introduction

INTRODUCTION

Operations Research (OR), also called Operational Research in the United Kingdom (UK), uses various computational tools for solutions to complex problems within a system. It deals with challenges in planning, scheduling, forecasting, process analysis and decision analysis. It also addresses individual components of the system. Operational Research techniques add speed, efficiency, quality and consistency to the documentation. It also acts as a learning tool which can be audited, and lends itself to research questions.

Operations Research, also has an important role to play in cost-effective management in many sectors. It has its origins in military operations during the Second World War, and began when scientists in Britain were asked to develop procedures for the use of radar in a new and effective air defense system, that is, this was research to improve operations. Operations Research builds mathematical models of decision-making processes, and applies the resulting techniques and algorithms in the fields of engineering, finance, service systems and management. The methodology may also be used in medicine in predictive health, disease modelling, public health, medical preparedness, logistics, quality improvement and informatics. This has applications in reducing waiting times, triage in the emergency department, work-flow development, operation theatre and day surgery management, outsourcing of services, management of information technology, nurse scheduling, bed forecasting, ambulance scheduling, portering operations, stock inventory, waste disposal, catering and in manpower planning.

In recent years, Operations Research has had an increasingly great impact on the management of organizations. Both the number and the variety of its applications continue to grow rapidly, and no slowdown is in sight. The subject is also being used widely in other types of organizations, including business and industry. Many industries including aircraft and missile, automobile, communication, computer, electronics, mining, paper, petroleum and transportation have made widespread use of Operation Research in determining their tactical and strategical decisions more scientifically. In addition to the industries, the financial institutions, governmental agencies and hospitals are also rapidly increasing the use of Operation Research.

There is a large number of papers on the application of OR techniques to finance, mathematics, engineering and other literatures. Equally, OR has played a part in the adoption by the financial markets of the new finance theories.

As part of the increased use of mathematical models in finance (Merton, 1995), investment banks have recruited staff skilled in quantitative techniques, including OR, to devise pricing equations and analyse market data- the so called “quants” or “rocket scientists”.

NATURE AND FEATURES OF OPERATIONS RESEARCH

Some significant features of O.R are highlighted below:

(a) Decision-Making: Primarily, O.R. is addressed to managerial decision-making or problem-solving. Major premise of O.R is that decision-making, irrespective of the situation involved, can be considered as a general systematic process. It consists of the following steps:

1) Define the problem and establish a criterion to be used.
2) Select the alternative course of action for consideration.
3) Determine the model to be used and the values of the parameters involved.
4) Solve the model to choose the best (optimal) alternative.

(b) Scientific Approach: O.R. employs scientific methods for the purpose of solving problems. It is a formalized process of reasoning.

(c) Objective: O.R. attempts to locate the best or optimal solution to the problem under consideration. For this purpose, it is necessary that a measure of effectiveness is defined which is based on the goals of the organization. This measure is then used as the basis to compare the alternative courses of action.

(d) Inter-disciplinary Team Approach: O.R. is inter-disciplinary in nature and requires a team approach to solution of the problem. Managerial problems have economic, physical, psychological, biological, sociological and engineering aspects. This requires a blend of people with expertise in the area of mathematics, statistics, economics, engineering, management, computer science, and so on.

(e) Digital computer: Use of a digital computer has become an integral part of the O.R. approach to decision-making. The computer may be required due to the complexity of the model, volume of data required and the computations to be made.

SCIENTIFIC METHOD IN OPERATIONS RESEARCH.

The scientific method in operation research is its most important feature. It consists of the following three phases:

Judgement phase: This phase includes: (i) identification of the real- life problem, (ii) selection of an appropriate goal and the value of various variables related to the goals, (iii) appropriate scale of measurement, and (iv) formulation of an appropriate model of the problem, abstracting the essential information so that a solution at the decision-maker's goal can be sought.

Research phase: This phase is the largest and longest among the other two. However, the remaining two are also equally important as they provide the basis for a scientific method. This phase utilizes: (i) observation and data collection for a better understanding of what the problem is, (ii) formulation of hypothesis and models, (iii) observation and experimentation to test the hypothesis on the basis of additional data, (iv) analysis of the available information and verification of the hypothesis using pre-established measures of effectiveness, (v) predictions of various results from the hypothesis, and (vi) generalization of the results and consideration of alternative methods.

Action phase: This phase consists of making recommendations for decision process by those who first posed the problem consideration, or by anyone in a position to make a decision influencing the operation in which the problem occurred.

MODELING IN OPERAION RESEARCH

Following are the main characteristics that a good model should have:

i. Capability of taking into account new formulation without having any significant change in its frame.
ii. Assumptions made in the model should be as small as possible.
iii. Simplicity and coherency with less number of variables.
iv. Open to parametric type of treatment.
v. Less time in its construction for any problem.

ADVANTAGES OF MODELS

Models in O.R. are used as an aid for analyzing complex problems. The main advantages of a model are:

i. Through a model, the problems under consideration become controllable.
ii. It provides some logical and systematic approach to the problem.
iii. It indicates the limitations and scope of an activity.
iv. It helps in incorporating useful tools that eliminate duplication of methods applied to solve any specific problems.
v. It helps in finding avenues for new research and improvements in a system.
vi. It provides economic descriptions and explanations of the operations of the system they represent.

Disadvantages of models: Besides the above advantages, a model has the following limitations:

a) Models are only an attempt in understanding operations and should never be considered as absolute in any sense.
b) Validity of any model with regard to corresponding operation can only be verified by carrying out the experiment and observing relevant data characteristics.
c) Constructions of models require experts from various disciplines.

METHODOLOGY OF OPERATIONS RESEACH

The systematic methodology developed for an O.R. study deals with problems involving conflicting multiple objectives, policies and alternatives.

The O.R. approach to problem solving consists of the following seven steps:

1) Formulate the problem.
2) Construct a mathematical model.
3) Acquire the input data.
4) Derive the solution from the model.
5) Validate the model.
6) Establish control over the solution.
7) Implement the final results.

THESIS ORGANIZATION

Among the applications of O.R, this dissertation discusses only the applications relating to production of furniture, shop-floor management and in financial markets and Medicine.

In chapter I, Linear Programming, a method of allocating resources in an optimal way, is applied to production of furniture. Three methods (1) graphical method (2) direct method (3) simplex method are discussed. It is established that the simplex method is superior to the other two in real world problems. The simplex method is applied from an easier problem to more complex ones. The various cases of simplex method are analysed.

In chapter II, we discuss the shop floor management problem .Starting with the explanation of shop floor planning and control, we see the activities included in SFAC (Shop Floor Activity and Control). Then we pass on to Line Balancing Technique "Arranging production line so that there is an even flow of production from one work station to the next, so that there are no delays at any work station that will leave the next work station with idle time." An illustration is explained in one of the line balancing methods namely Heuristic method.

In chapter III, we review the application of OR to financial markets.

After considering reasons for the attractiveness of general finance problems to OR researchers, the main types of financial market problem amenable to OR are identified, and some of the many problems solved using OR are documented. While mathematical programming is the most widely applied technique, Monte Carlo and other simulation methods are increasingly and widely used. OR now plays an important role in the operation of financial markets and this importance is likely to increase, creating the opportunity for OR (and operations researchers) to play an even greater role.

In chapter IV, we see some Operational research methodology in medicine. OR applies scientific methods to provide decision makers with more insights information about their systems. The issue of how best to reduce utilization of diagnostic studies without detracting from patient management necessitates a meeting of managerial science with medicine. Bree et al studied a new utilization management tool, the Consultant Radiologist as a gatekeeper, for its ability to improve inpatient diagnostic imaging. This was an example of the use of Operations Research in medicine. The following concepts (1) Decision-Tree Analysis, (2) Housekeeping and Fuzzy Logic (3) Commissions of Enquiry and Coroner's Inquiries (4) Operational Research in Internal Medicine are explained as the application of operations research in the general medical rounds.

Review of Literature

REVIEW OF LITERATURE

The activity "Operation Research " (abbreviated as O.R.) has become increasingly important in the face of fast moving technology and increasing complexities in business and industry.

No science has ever been born on a specific day. Operation research is no exception. Its roots are as old as science and society. Though the roots of O.R. extend to even early 1800's. It was in 1917, when A.K.Erlang, a Danish mathematician, published his work on the problem of congestion of telephone traffic. The difficulty was that during busy periods, telephone operators were unable to handle the calls the moment they were made, resulting in delayed call. A few years after its appearance, his work was accepted by the British Post Office as the basis for calculating circuit facilities. During the 1930's, H.C. Levinson applied scientific analysis to the problems of merchandising. His work included scientific study of customers' buying habits , response to advertising and relation of environment to the type of article sold.

The term, Operations Research, was first coined in 1940 by McClosky and Trefthen in a small town, Bowdey, of the United Kingdom. This new science came into existence as a result of research on military operations during World War II. During the war there strategic and tactical problems which were greatly complicated, to expect adequate solutions from individuals or specialists was unrealistic. Therefore, the military management called on scientists from various disciplines and organised them into teams to assist in solving strategic and tactical problems,i.e., to discuss , evolve and suggest ways and means to improve the execution of various military projects. By their joint efforts, experience and deliberations, they suggested certain approaches that showed remarkable progress. This new approach to systematic and scientific study of the operations of the system was called the ***Operations Research*** or ***operational Research*** (abbreviated as O.R.).

Following the end of World War II, the success of military teams attracted the attention of industrial managers of U.K. who were seeking solutions to their complex executive type problems. It was becoming apparent that these were basically the same problems but in a different context. Thus, keeping in view the critical economic situation which required drastic increase in production efficiency, O.R. activities were diverted from military to civil government and industries. In this way O.R. began to creep into business and industry. By 1948 it had taken good hold in U.K. and was in the process of achieving the same in the United States . It was only in the early 1950's that the industries in U.S.A. realized the importance of this new science in solving their management problems. Since then, industrial O.R. developed rapidly in U.S.A. as compared to U.K.

The dramatic development and refinement of the various techniques of O.R. and the advent of digital computers are the two prime factors that contributed to the growth and application of O.R. in the post-war period. In the 1950's O.R. achieved recognition as a subject for study in the universities. Since then the subject has gained more and more importance for the students of management, public administration, behavioural sciences, engineering, mathematics, economics and commerce.

In recent years a good amount of research is reported to improve the available methods for solving linear programming models. Many researches have considered the computationally efficient algorithms with less number of steps.

For our thesis, we have reviewed the following articles on Operation Research.

Dantzig (1956) presented simplex method to solve linear programming problems. Dantzig (1963) presented revised simplex algorithm to solve linear programming problems and also its applications.

Miller (1963) provided the introduction of the optimization techniques and simplex methods for solving problems.

Kohler (1973) pointed out that Khachian has modified the algorithm of Shor and designated as Ellipsodial method which is a polynomial algorithm and described a new algorithm for solving linear programming problem.

Shakthivel (1984) presented multiplex algorithm to solve large-scale linear programming problems. This algorithm nips some of the variables at thair bud and prevents the waste of time in computation.

Karmarker also developed an algorithm which is similar to Khachian. Lasserre (1981) showed that Karmarker's method solved problems involving 150,000 design variables and 12,000 constraints in hour.

Williams (1986) showed that Fourier's method based on Kohler rule eliminates redundant constraints and the reduction of CPU time.

Sharma (1989) provided simplex and systematic presentation of different algorithm to demonstrate use of linear programming techniques.

Kanti Swarup et al (1994) provided various optimization techniques and their applications.

Kanniappan and Pandian (1994) studied the types of convex functions and interrelations between them.

Taha (1999) and Singiresu (1996) provided various advanced optimization techniques organized in three parts to solve the problems of optimization in the presence of uncertainity.

Pinheiro (2009) studied the application of operations research in general medical grounds in order to benefit the individual patients.

Corbett et al (1995) emphasized methods to maximise the beneficial environmental effects subject to budget constraint, storage constraint, labour constraint.

Bierman (1977) discussed the Quantitative Analysis for Business Decisions.

Dykstra (1984) Mathematical Programming for Natural Resource Management.

Hillier (1995) Introduction to Operations Research, sixth edition.

Ignizio (1975) Operations Research in Decision Making.

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Chapter I

CHAPTER - I

OPERATIONS RESEARCH IN FURNITURE MANUFACTURING

This chapter presents the application of Operations Research in the manufacturing of wooden tables and chairs with optimum resources.

A key problem faced by managers is how to allocate scarce resources among activities or projects. One of the most widely used operations research (OR) tools is Linear programming, or LP which is a method of allocating resources in an optimal way. It has been used successfully as a decision-making aid in almost all industries, and in financial and service organizations.

Programming refers to a planning process that allocates resources - labor, materials, machines, and capital - in the best possible (optimal) way so that costs are minimized or profits are maximized. These resources are known as decision variables. The criterion for selecting the best values of the decision variables is known as the objective function. The limitations on resource avail- ability form what is known as a constraint set.

Let us consider an example in which a furniture manufacturer produces wooden tables and chairs. Unit profit for tables is \$6, and unit profit for chairs is \$8,say . To simplify our discussion, let's assume the only two resources the company uses to produce tables and chairs are wood (board feet) and labor (hours). Let us consider, it takes 30 bf and 5 hours to make a table, and 20 bf and 10 hours to make a chair.

There are 300 bf of wood available and 110 hours of labor available. The company wishes to maximize profit, so profit maximization becomes the objective function. The resources (wood and labor) are the decision variables. The limitations on resource availability (300 bf of wood and 110 hours of labor) form the constraint set, or operating rules that govern the process. Using LP, the management can decide how to allocate the limited resources to maximize profits.

- The objective function maximization or minimization can be described by a linear function of the decision variables
- The constraint set can be expressed as a set of linear equations.

We shall present here the three methods of solving this L.P. The first method namely the graphical method can be applied only to problems with two or three variables. Most real world problems contain numerous objective criteria and resources, so they are too complicated to represent in graphs. But, this method gives a beginners a better understanding how an LP solution procedures work.

The next method , the direct method has also the same limitations in variables as in the previous method .

The simplex method is the most common way to solve large LP problems. Simplex is a mathematical term. In one dimension, a simplex is a line segment connecting two points. In two dimensions, a simplex is a triangle formed by joining the points. A three- dimensional simplex is a four-sided pyramid having four corners. The underlying concepts are geometrical, but the solution algorithm, developed by George Dantzig in 1947, is an algebraic procedure. As with the graphical method, the simplex method finds the most attractive corner of the feasible region to solve the LP problem.

Simplex usually starts at the corner that represents doing nothing. It moves to the neighboring corner that best improves the solution. It does this over and over again, making the greatest possible improvement each time. When no more improvements can be made, the most attractive corner corresponding to the optimal solution has been found.

A moderately sized LP with 10 products and 10 resource constraints would involve nearly 200,000 corners. An LP problem10 times this size would have more than a trillion corners. Fortunately, the search procedure for the simplex method is efficient enough that only about 20 of the 200,000 corners are searched to find the optimal solution. In the real world, computer software is used to solve LP problems using the simplex method.

(a) The graphical method:

Information about available resources (board feet of wood and hours of labour) and the objective criterion is presented in Table1.

Table 1. Information for the wooden tables and chairs linear programming problem.

Resource	**Table (X1)**	**Chair (X2)**	**Available**
Wood (bf)	30	20	300
Labor (hr)	5	10	110
Unit profit	$6	$8	

In this example, X_1 refers to tables, X_2 refers to chairs, and Z refers to profit.

Maximize: $Z = 6X_1 + 8X_2$ (objective function)

Subject to: $30X_1 + 20X_2 \leq 300$ (wood constraint: 300 bf available)

$5X_1 + 10X_2 \leq 110$ (labor constraint: 110 hours available)

$X_1, X_2 \geq 0$ (nonnegativity conditions)

Based on the above information, it is graphicalled solved in Figure-1. Graph the two constraint equation lines. Then plot two objective function lines by arbitrarily setting Z = 48 and Z = 72 to find the direction to move to determine the most attractive corner. The coordinates for the most attractive corner (where the wood and labor constraint equations intersect) can be found by simultaneously solving the constraint equations with two unknowns.

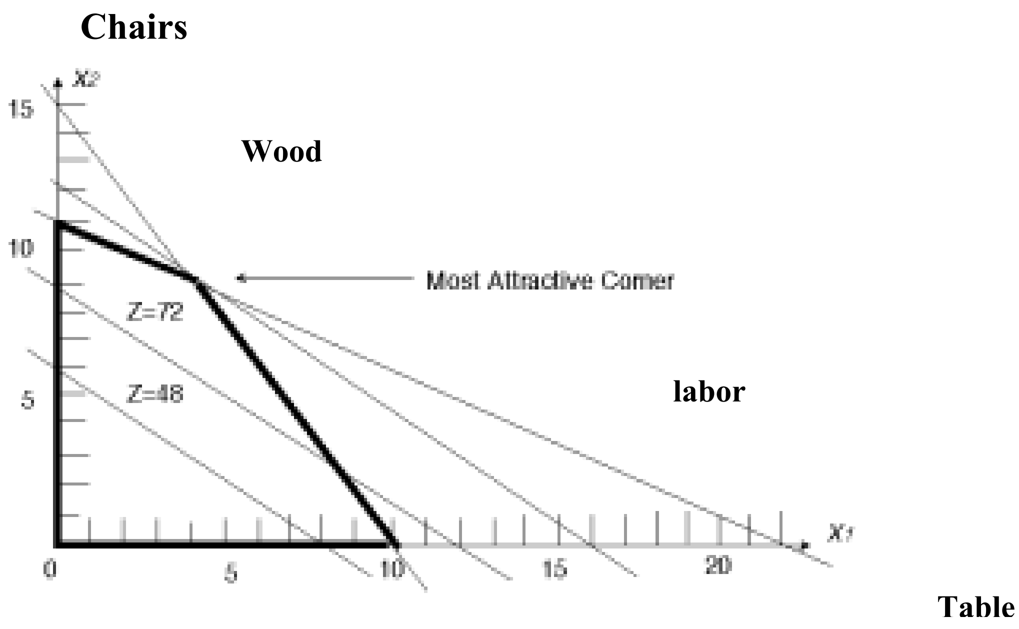


Figure 1. Determining the optimal solution.

(b) Direct Method:

We can also arrive at the optimal solution directly by solving the two constraint equations, first multiply the labor equation by -2, and add it to the wood equation:

$$30X_1 + 20X_2 = 300 \quad \text{(wood)}$$
$$-2\,(5X_1 + 10X_2 = 110) \quad \text{(labor)}$$

$$20X_1 + 0 = 80$$
$$X_1 = 4 \text{ tables}$$

Next, substitute into either of the constraint equations to find the number of chairs. We can substitute into both equations to illustrate that the same value is found.

Wood constraint	**Labor constraint**
$30(4) + 20X_2 = 300$	$5(4) + 10X_2 = 110$
$120 + 20\,X_2 = 300$	$20 + 10\,X_2 = 110$
$20\,X_2 = 300 - 120$	$10\,X_2 = 110 - 20$
$20\,X_2 = 180$	$10\,X_2 = 90$
$X_2 = 180/20$	$X_2 = 90/10$
$X_2 = 9$ chairs	$X_2 = 9$ chairs

Now, determine the value of the objective function for the optimal solution. Substitute into the equation the number of tables and chairs, and solve for Z.

$$Z = \$6(4) + \$8(9) = \$96$$

The optimal solution is to manufacture four tables and nine chairs for a profit of $96.

(c) The Simplex Method

By introducing the idea of slack variables (unused resources) to the tables and chairs problem, one can add two more variables to the problem. With four variables, one can't solve the LP problem graphically. We'll need to use the simplex method to solve this more complex problem.

Brief steps involved in using the simplex method is presented here which gives the general overview of the procedure.

Step 1. Formulation of the LP and construction of a simplex tableau

Add slack variables to represent unused resources, thus eliminating inequality constraints. Construct the simplex tableau—a table that allows us to evaluate various combinations of resources to determine which mix will most improve our solution. Use the slack variables in the starting basic variable mix. Table 2. gives the simplex tableau.

Table 2. Example of a simplex tableau.

Unit profit		**6**	**8**	**0**	**0**		
	Basic mix	**X_1**	**X_2**	**S_W**	**S_L**	**Solution**	
0	S_W	30	20	1	0	300	
0	S_L	5	10	0	1	110	
	Sacrifice	0	0	0	0	0	← **Current profit**
	Improvement	**6**	**8**	**0**	**0**	**-**	

Step 2. Finding the sacrifice and improvement rows

Values in the sacrifice row indicate what will be lost in per-unit profit by making a change in the resource allocation mix. Values in the improvement row indicate what will be gained in per-unit profit by making a change.

Step 3. Applying the entry criteria

Find the entering variable and mark the top of its column with an arrow pointing down. The entering variable is defined as the current non-basic variable that will most improve the objective if its value is increased from 0. If ties occur, arbitrarily choose one as the entering variable. When no improvement can be found, the optimal solution is represented by the current tableau. If no positive number appears in the entering variable's column, this indicates that one or more constraints are unbounded. Since it is impossible to have an unlimited supply of a resource, an unbounded solution indicates that the LP problem was formulated incorrectly.

Step 4. Applying the exit criteria

Using the current tableau's exchange coefficient from the entering variable column, calculate the following exchange ratio for each row as:

Solution value/Exchange coefficient

The exchange ratio tells us which variable is the limiting resource , i.e., the resource that would run out first.

Find the lowest nonzero and nonnegative value. This variable is the limiting resource. The basic variable in this row becomes the exiting variable. In case of identical alternatives, arbitrarily choose one. Mark the exiting variable row with an arrow pointing left.

Step 5. Constructing a new simplex tableau

Constructing a new tableau is a way to evaluate a new corner. One variable will enter the basic mix (entering variable), and one variable will leave the basic mix and become a non-basic variable (exiting variable). The operation of an entering variable and an exiting variable is called a pivot operation. The simplex method is made up of a sequence of such pivots. The pivot identifies the next corner to be evaluated. The new basic mix always differs from the previous basic mix by one variable (exiting variable being replaced by the entering variable).

To construct the new tableau, replace the exiting variable in the basic mix column with the new entering variable. Other basic mix variables remain unchanged. Change the unit profit or unit loss column with the value for the new entering variable. Compute the new row values to obtain a new set of exchange coefficients applicable to each basic variable.

Step 6. Repeating steps 2 through 5 until no longer can improve the solution.

A simplex method example: Production of wooden tables and chairs

Step 1. Formulate the LP and construct a simplex tableau.

From the information in Table 3, we can formulate the LP problem as before.

Table 3.Information for the wooden tables and chairs linear programming problem.

Resource	**Table (X_1)**	**Chair (X_2)**	**Available**
Wood (bf)	30	20	300
Labor (hr)	5	10	110
Unit profit	\$6	\$8	

Maximize: $Z = 6X_1 + 8X_2$ (objective function)

Subject to: $30X_1 + 20X_2 \leq 300$ (wood constraint: 300 bf available)

$5X_1 + 10X_2 \leq 110$ (labor constraint: 110 hours available)

$X_1, X_2 \geq 0$ (nonnegativity conditions)

Slack Variables

Using the simplex method, the first step is to recognize surplus resources, represented in the problem as slack variables. In most real-life problems, it's unlikely that all resources (usually a large mix of many different resources) will be used completely. While some might be used completely, others will have some unused capacity. Also, slack variables allow us to change the inequalities in the constraint equations to equalities, which are easier to solve algebraically. Slack variables represent the unused resources between the left-hand side and right-hand side of each inequality; in other words, they allow us to put the LP problem into the standard form so it can be solved using the simplex method.

The first step is to convert the inequalities into equalities by adding slack variables to the two constraint inequalities. With SW representing surplus wood, and SL representing surplus labor, the constraint equations can be written as:

$$30X_1 + 20X_2 + S_W = 300 \qquad \text{(wood constraint: 300 bf)}$$
$$5X_1 + 10X_2 + S_L = 110 \qquad \text{(labor constraint: 110 hours)}$$

All variables need to be represented in all equations. Add slack variables to the other equations and give them coefficients of 0 in those equations. Rewrite the objective function and constraint equations as:

Maximize:	$Z = 6X_1 + 8X_2 + 0S_W + 0S_L$	(objective function)
Subject to:	$30X_1 + 20X_2 + S_W + 0S_L = 300$	(wood constraint: 300 bf)
	$5X_1 + 10X_2 + 0S_W + S_L = 110$	(labor constraint: 110 hours)
	$X_1, X_2, S_W, S_L \geq 0$	(nonnegativity conditions)

We can think of slack or surplus as unused resources that don't add any value to the objective function. Thus, their coefficients are 0 in the objective function equation.

Basic Variable Mix And Non-Basic Variables

Since there are more unknown variables (four) than equations (two), we can't solve for unique values for the X and S variables using algebraic methods. Whenever the number of variables is greater than the number of equations, the values of the extra variables must be set arbitrarily, and then the other variables can be solved for algebraically.

First we'll choose which variables to solve for algebraically. These variables are defined to be in the basic variable mix . We can solve for these variables after we fix the other variables at some arbitrary level.

The fixed-value variables are identified as not being in the basic mix and are called non-basic variables . We'll arbitrarily give the non-basic variables the value of 0. The algebraic solution of the constraint equations, with non-basic variables set to 0, represents a corner.

For any given set of variables, there are several possible combinations of basic variables and non-basic variables. For illustration, Table 4 contains the six basic mix pairs and the corresponding non-basic variables for the tables and chairs LP problem. Figure 2 illustrates where each corner (A through F in Table 4) lies on a graph.

Table 4. Basic variable mix combinations and algebraic solutions.

Basic Variable Mix	**Non-basic variables**	**Algebraic solution**					**Corners (Figure2)**
		X_1	X_2	S_W	S_L	Z($)	
S_W S_L	X_1 X_2	0	0	300	110	0	A
S_W X_2	X_1 S_L	0	11	80	0	88	B
S_L X_1	X_2 S_W	10	0	0	60	60	C
X_1 X_2	S_W S_L	4	9	0	0	96	D
S_L X_2	S_W X_1	0	15	0	-40	infeasible	E
S_W X_1	S_L X_2	22	0	-360	0	infeasible	F

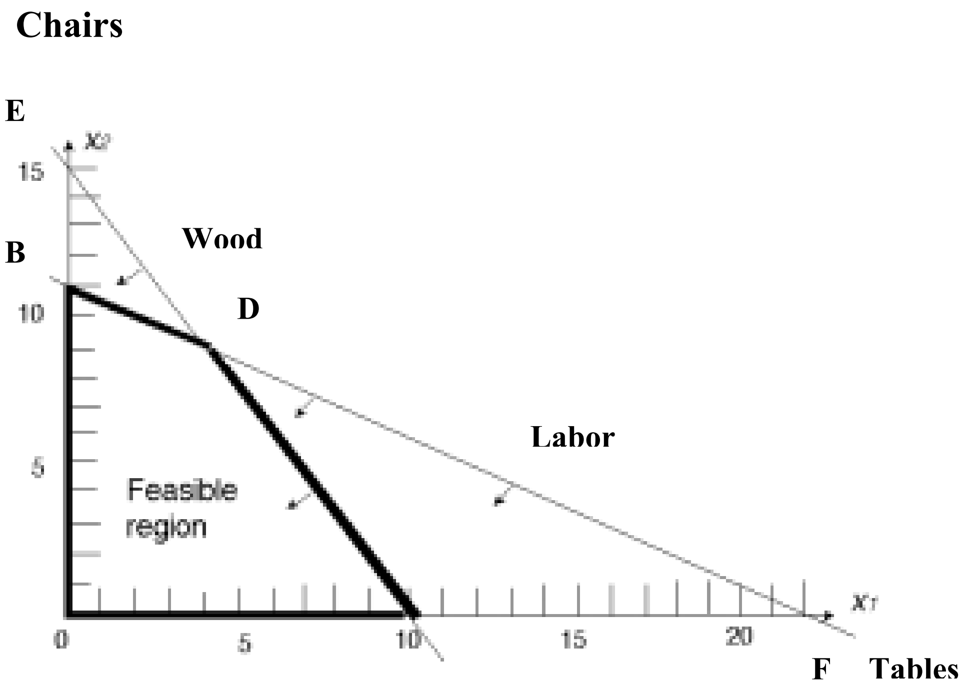


Figure2. Corners corresponding to Table 4 data

One can evaluate each corner to find the values of the basic variables and of *Z*:

Corner A:

Set $X_1 = 0$ and $X_2 = 0$

$30(0) + 20(0) + S_W + 0S_L = 300$

$5(0) + 10(0) + 0S_W + S_L = 110$

Therefore, $S_W = 300$

and $S_L = 110$

Solution: $X_1 = 0$, $X_2 = 0$, $S_W = 300$, $S_L = 110$

Profit: $Z = \$6(0) + \$8(0) + 0(300) + 0(110) = 0$

Corner B:

Set $X_1 = 0$ and $S_L = 0$

$30(0) + 20X_2 + S_W + 0(0) = 300$

$5(0) + 10X_2 + 0S_W + 0 = 110$

Therefore, $20X_2 + S_W = 300$

and $10X_2 = 110$

Solution: $X_1 = 0$, $X_2 = 11$, $S_W = 80$, $S_L = 0$

Profit: $Z = \$6(0) + \$8(11) + 0(80) + 0(0) = \$88$

Corner C:

Set $X_2 = 0$ and $S_W = 0$

$30X_1 + 20(0) + 0 + 0S_L = 300$

$5X_1 + 10(0) + 0(0) + S_L = 110$

Therefore, $30X_1 = 300$

and $5X_1 + S_L = 110$

Solution: $X_1 = 10$, $X_2 = 0$, $S_W = 0$, $S_L = 60$

Profit: $Z = \$6(10) + \$8(0) + 0(0) + 0(60) = \$60$

Corner D:

Set $S_W = 0$ and $S_L = 0$

$30X_1 + 20X_2 + 0 + 0(0) = 300$

$5X_1 + 10X_2 + 0(0) + 0 = 110$

Therefore, $30X_1 + 20X_2 = 300$

and $5X_1 + 10X_2 = 110$

Solution: $X_1 = 4$, $X_2 = 9$, $S_W = 0$, $S_L = 0$

Profit: $Z = \$6(4) + \$8(9) + 0(0) + 0(0) = \$96$

Point E is infeasible because it violates the labor constraint, and point F is infeasible because it violates the wood constraint. The simplex algorithm never evaluates infeasible corners.

With slack variables added, the tables and chairs LP is now four-dimensional and is not represented by Figure 2. Points on the constraint lines in Figure 2 represent 0 slack for both wood and labor resources. A feasible point off a constraint line represents positive slack and cannot be read off the two-dimensional graph. In Table 4, all corners in the LP were identified, and all feasible corners were algebraically evaluated to find the optimum solution. You can see that a graph wasn't necessary to list all variable mixes and that each variable-mix pair corresponded to a corner solution.

One reason we can't use this procedure to solve most LP problems is that the number of corners for real-life LP problems usually is very large. Another reason is that each corner evaluation requires a lengthy algebraic solution. To obtain each corner solution for a 10-constraint linear program, 10 equations with 10 unknowns must be solved, which is not a simple arithmetic task. Many LP problems are formulated with many more than 10 constraints.

Simplex tableau

Formulates of production the tables and chairs LP in standard form as:

Maximize: $Z = 6X_1 + 8X_2 + 0S_W + 0S_L$ (objective function)

Subject to: $30X_1 + 20X_2 + S_W + 0S_L = 300$ (wood constraint: 300 bf)

$5X_1 + 10X_2 + 0S_W + S_L = 110$ (labor constraint: 110 hours)

$X_1, X_2, S_W, S_L \geq 0$ (nonnegativity conditions)

The information for the tables and chairs example can be incorporated into a simplex tableau (Table 5). A tableau is a table that allows you to evaluate the values of variables at a given corner to determine which variable should be changed to most improve the solution.

Table 5. Tables and chairs simplex tableau.

Unit profit		6	8	0	0	
	Basic mix	**X_1**	**X_2**	**S_W**	**S_L**	**Solution**
	S_W	**30**	**20**	**1**	**0**	**300**
	S_L	**5**	**10**	**0**	**1**	**110**

The top of the tableau lists the per-unit profit for the objective function. The rows in the body of the tableau indicate the basic variable mix for the corner point being evaluated. The first row in the body of the tableau lists the coefficients of the first constraint equation (the wood constraint) in their original order. The second row lists the coefficients of the second constraint equation (the labor constraint) in their original order.

The basic mix column lists the slack variables. All variables not listed in this column are designated as non-basic variables and will be arbitrarily fixed at a value of 0 when we plug them into the constraint equations.

The solution column lists the values of the basic variables, S_W= 300 and S_L = 110. Thus, the solution mix shows that all of the resources (wood and labor) remain unused. The solutions to the two constraint equations after zeroing out X_1 and X_2 are as follows:

$$30(0) + 20(0) + S_W + 0S_L = 300 \quad \text{or} \quad S_W = 300$$
$$5(0) + 10(0) + 0S_W + S_L = 110 \quad \text{or} \quad S_L = 110$$

The original constraint coefficients, highlighted in the simplex tableau, are called exchange coefficients. They indicate how many units of the variable listed on the left (basic mix column) must be given up to achieve a unit increase in the variable listed at the top of the tableau. The 30 indicates that 30 board feet of unused wood can be exchanged for one table, and the 20 indicates that 20 board feet of wood can be exchanged for one chair. Likewise, 5 hours of labor can be exchanged for one table, and 10 hours of labor can be exchanged for one chair.

The exchange coefficients are 0 or 1 for the basic mix variables. These numbers are not very meaningful. For example, the 1 in the first row indicates that 1 board foot of wood can be exchanged for 1 board foot of wood. The 0 in row one indicates that no unused wood is required to accommodate more unused labor.

Step 2. Finding the sacrifice and improvement rows.

The next step is to expand the simplex tableau as in Table 6.

Table 6. Expanded simplex tableau.

Unit profit		**6**	**8**	**0**	**0**		
	Basic mix	**X_1**	**X_2**	**S_W**	**S_L**	**Solution**	
0	S_W	30	20	1	0	300	
0	S_L	5	10	0	1	110	
	Sacrifice	0	0	0	0	0 ←	**Current profit**
	Improvement	**6**	**8**	**0**	**0**	**-**	

In the expanded tableau, list the per-unit profit for the basic variables in the far left-hand column (Unit profit). The per-unit profit for both slack variables is 0.

Values in the sacrifice row indicate what will be lost in per-unit profit by making a change in resource allocation. Values in the improvement row indicate what will be gained in per-unit profit by making a change in resource allocation. The sacrifice and improvement rows help you decide what corner to move to next.

Sacrifice row Identification

Values for the sacrifice row are determined by:

Unit sacrifice = Unit profit column * Exchange coefficient column

To obtain the first sacrifice row value, calculations are:

	(Unit profit column value)	*	**(X_1 column value)**		
	0	*	30	= 0	
+	0	*	5	= 0	Add the products
				0	First value in the sacrifice row

The first product is the unit profit of unused wood multiplied by the amount needed to make one table. The second product is the unit profit of unused labor multiplied by the amount needed to make one table. Together, these products are the profit that is sacrificed by the basic mix variables for producing one more table. Since both basic mix variables are slack variables, and slack refers to unused resources, zero profit is sacrificed by producing another table.

The next sacrifice row value is calculated in the same manner:

(Unit profit column value) * (X_2 column value)

	0	*	20 = 0	
+	0	*	10 = 0	Add the products
			0	Second value in the sacrifice row

The other sacrifice row values are calculated in the same manner. In the case of our example, the unit profit values are 0, so the sacrifice row values are all 0.

The values in the solution column of the sacrifice row are calculated as:

(Unit profit column value) * (Solution column value)

	0	*	300	= 0
+	0	*	110	= 0 Add the products
				0

The sum of these products represents the current profit (Z).

Improvement row identification

Improvement row values are calculated by subtracting each value in the sacrifice row from the value found above it in the unit profit row. Therefore:

Unit improvement = Unit profit - Unit sacrifice

For example, the improvement for X_1 is calculated as:

Unit profit = 6

-Unit sacrifice = 0

$6 (first value in the improvement row)

Since all of the sacrifice values are 0 in this example, all of the improvement row values are the same as those found in the unit profit row.

Step 3. Applying the entry criterion.

The next step is to apply the entry criterion, that is, to determine the entering variable. The entering variable is defined as the current non-basic variable that will most improve the objective if it is increased from 0. It's called the entering variable because it will enter the basic mix when you construct your next tableau to evaluate a new corner. For profit maximization problems, you determine the entering variable by finding the largest value in the improvement row.

In our example, the largest value in the improvement row is 8. Thus, we can increase profit (improve the current solution) by $8 per unit for each chair made. Increasing the value of X_2 from 0 to $8 is the best improvement that can be made. If we increase the value of X_1, our solution improves by only $6. Therefore, X_2 is the entering variable. The entering variable is marked by placing a downward facing arrow in the X_2 (chair) column (Table7).

Table 7. Entering variable, exchange ratios, exiting variable, and pivot element.

Unit profit		6	8	0	0		
	Basic mix	**X_1**	**X_2**	**S_W**	**S_L**	**Solution**	**Exchange ratios:**
0	S_W	30	20	1	0	300	
0	S_L ←	5	(10)	0	1	110	
	Sacrifice	0	0	0	0	0 ←	**Current profit**
	Improvement	**6**	**8**	**0**	**0**	**-**	

Limiting resource. . ."The resource that would run out first."

Exchange ratio. . ."Tells you which variable is the limiting resource."

The lowest nonzero and nonnegative exchange ratio denotes the limiting resource. The basic variable in this row becomes the exiting variable.

Step 4. Applying the exit criterion.

The next step is to determine the exiting variable. The exiting variable is the variable that will exit the basic mix when you construct your next simplex tableau.

We'll find the exiting variable by calculating the exchange ratio for each basic variable. The exchange ratio tells us how many tables or chairs can be made by using all of the resource for the current respective basic variable. To find the exchange ratio, divide the solution value by the corresponding exchange coefficient in the entering variable column. The exchange ratios are:

$300/20 = 15$ (S_W basic mix row)

and $110/10 = 11$ (S_L basic mix row)

By using all 300 board feet of wood, we can make 15 chairs because it takes 20 board feet of wood to make a chair. By using all 110 hours of labor, we can make 11 chairs because it takes 10 hours of labor per chair. Thus, it's easy to see the plant can't manufacture 15 chairs. We have enough wood for 15 chairs but only enough labor for 11. In this case, labor is the limiting resource. If all the labor were used, there would be leftover wood.

The exit criterion requires that the limiting resource (the basic mix variable with the smallest exchange ratio) exit the basic mix. In this case, the exiting variable is SL. Because of this, wood (S_W) remains in the basic mix. Indicate the exiting variable by placing a small arrow pointing toward the S_L (Table 7).

Next circle the pivot element —the value found at the intersection of the entering variable column and the exiting variable row. In this case, the value 10 (X_2 column and S_L row) is the pivot element. We'll use this value to evaluate the next corner point represented by exchanging X_2 and S_L.

Constructing a new tableau is a way to evaluate a new corner point. One variable will enter the basic mix (entering variable), and one variable will leave the basic mix and become a non-basic variable (exiting variable). The operation. The simplex method is made up of a sequence of such pivots.

Step 5. Construct a new simplex tableau.

The next step is to create a new simplex tableau. First, let's look at the old constraint equations that represented the X_1 and X_2 rows in our original tableau:

$30X_1 + 20X_2 + S_W + 0S_L = 300$ (wood constraint)

$5X_1 + 10X_2 + 0S_W + S_L = 110$ (labor constraint)

Since X_2 is to replace S_L , we need to transform the second equation so that X_2 will have a coefficient of 1. This requires some algebraic manipulation. Although the resulting equations will look different, they will be equivalent to the original constraints of the LP problem.

First, we'll multiply the labor constraint equation by 1/10 (the same as dividing each of the variables by 10). We get the following equivalent equation:

$$½ X_1 + X_2 + 0S_W + 1/10\ S_L = 11$$

This now becomes the new X_2 row. By setting the non-basic variables,

X_1 and S_L, both to 0, we get:

$$½ (0) + X_2 + 0S_W + 1/10\ (0) = 11$$

$$X_2 = 11$$

We want the solution, $X_2 = 11$, to satisfy both constraint equations. We can do this by zeroing the X_2 term in the first equation (the wood constraint). We'll do this by multiplying the second equation by -20 and adding it to the first equation:

Multiply times -20:

$$-20\left(\tfrac{1}{2}X_1 + X_2 + 0S_W + 1/10\, S_L\right) = 11$$

$$-10X_1 - 20X_2 - 0S_W - 2S_L = -220$$

Add to the first equation:

$$30X_1 + 20X_2 + S_W + 0S_L = 300$$

$$-10X_1 - 20X_2 - 0S_W - 2S_L = -220$$

$$20X_1 + 0X_2 + S_W - 2S_L = 80$$

This equation becomes the new S_W row. Thus, our new constraint equations are:

$$20X_1 + 0X_2 + S_W - 2S_L = 80 \quad \text{(wood)}$$

$$\tfrac{1}{2}X_1 + X_2 + 0S_W + 1/10\, S_L = 11 \quad \text{(labor)}$$

When the non-basic variables X_1 and S_L are set to 0, the solution becomes:

$$20(0) + 0 + S_W - 2(0) = 80$$

$$S_W = 80$$

and $\tfrac{1}{2}(0) + X_2 + 0 + 1/10\,(0) = 11$

$$X_2 = 11$$

These two new equations give us some information. Remember, S_W represents the amount of surplus or slack wood, that is, the amount of wood not used. When 11 chairs are manufactured, 80 board feet of surplus or slack wood will remain.

The new simplex tableau is shown in Table 8c.

An Easier Method

It's easier to find a solution by using the simplex tableau than by doing the algebra.

The above equations can be calculated much more easily directly from the original simplex tableau than by doing the algebra. Refer to Tables 8a through 8c as we work through the example.

1. Fill in the new X_2 row.

Referring to Table 8a, divide all values in the exiting variable row, S_L, by the pivot element, 10. The calculations are 5/10, 10/10 (pivot element divided by itself), 0/10, and 1/10. Place the new values in the same location in the new tableau (Table 8b). Place the unit profit row value for X_2, the new entering variable (8), into the unit profit column.

Table 8a. Original simplex tableau.

Unit profit		**6**	**8**	**0**	**0**		
	Basic mix	**X_1**	**X_2 ↓**	**S_W**	**S_L**	**Solution**	**Exchange**
0	S_W	30	20	1	0	300	**ratios:**
0	S_L ←	5	(10)	0	1	110	**300/20=15**
	Sacrifice	0	0	0	0	0	**110/10=11**
	Improvement	**6**	**8**	**0**	**0**	**-**	**Current profit**

Table 8b. Second simplex tableau – X_2 row.

Unit profit		6	8	0	0	
	Basic mix	X_1	X_2	S_W	S_L	**Solution**
0	S_W					
8	X_2	0.5	1	0	0.1	11
	Sacrifice					
	Improvement					

2. Fill in the new S_W row.

Now we'll find the values for the S_W row. Referring to Table 8a, find the value in the S_W row in the old tableau in the pivot element column (20). Multiply it times the first value in the new X_2 row (0.5 from Table 8b). Subtract your answer from the value in the first position of the old S_W row.

Thus, for the first value (to replace the 30 in the first tableau):

(20 * 0.5) = 10

30 - 10 = 20

For the second value (to replace 20 in the first tableau):

(20 * 1) = 20

20 – 20 = 0

For the third value in this row:

20 * 0 = 0

1 – 0 = 1 (Stays the same in the new tableau.)

For the fourth value in this row:

$20 * 0.1 \quad = 2$

$0 - 2 \quad = -2$

For the solution column value for this row:

$20 * 11 \quad = 220$

$300 – 220 = 80$

The new S_W row is shown in Table 8c.

Table 8c. Second simplex tableau—S_W row.

Unit profit		6	8	0	0	
	Basic mix	**X_1**	**X_2**	**S_W**	**S_L**	**Solution**
0	S_W	20	0	1	-2	80
8	X_2	0.5	1	0	0.1	11
	Sacrifice					
	Improvement					

3. Find the sacrifice and improvement rows.

Find the sacrifice and improvement rows using the same method as in the first tableau. See Table 8d.

Table 8d.—Second simplex tableau - sacrifice and improvement rows.

Unit profit		6	8	0	0		
	Basic mix	**X_1**	**X_2**	**S_W**	**S_L**	**Solution**	
0	S_W	20	0	1	-2	80	
8	X_2	0.5	1	0	0.1	11	
	Sacrifice	4	8	0	0.8	88	← **Current profit**
	Improvement	**2**	**0**	**0**	**-0.8**	**-**	

We now see that profit has been improved from 0 to \$88.

4. Complete the pivot operation (entering and exiting variables).

Recall that the pivot operation results in new entering and exiting variables. The greatest per- unit improvement is 2 (X_1 column). The others offer no improvement (either 0 or a negative number). X_1 becomes the new entering variable. Mark the top of its column with an arrow (Table 8e). Remember, when no improvement can be found at this step, the current tableau represents the optimal solution.

Now determine the exiting variable. To do so, first determine the exchange ratios:

$$80/20 = 4$$

$$\text{and } 11/0.5 = 22$$

Now choose the smallest nonnegative exchange ratio (4 versus 22). SW becomes the exiting variable. Mark that row with an arrow. Draw a circle around the pivot element, 20. (Table 8e).

Table 8e. Second simplex tableau—pivot operation.

Unit profit		**6**	**8**	**0**	**0**		
	Basic mix	X_1	X_2 ↓	S_W	S_L	**Solution**	**Exchange ratios:**
0	S_W ←	(20)	0	1	-2	80	**80/20=4**
8	X_2	0.5	1	0	0.1	11	**110/0.5=22**
	Sacrifice	4	8	0	0.8	88	
	Improvement	**2**	**0**	**0**	**-0.8**	**-**	

5. Construct the third tableau from the second tableau.

Replace the entering variable in the basic mix where the exiting variable left. Bring over the unit profit from the top row of the old table to the new table. Fill in the pivot element row by dividing through by the pivot element (Table 8f).

Table 8f. Third simplex tableau - X_1 row

Unit profit		6	8	0	0	
	Basic mix	**X_1**	**X_2**	**S_W**	**S_L**	**Solution**
6	**X_1**	1	0	0.05	-0.1	4
8	**X_2**					
	Sacrifice					
	Improvement					

Fill in the first value in the X2 row as follows. First, multiply the previous tableau's X2 pivot value (0.5) times the first value in the new tableau's X1 row (1):

$$0.5 * 1 = 0.5$$

Now subtract this number from the first value in the previous tableau's X2 row (0.5):

$$0.5 - 0.5 = 0$$

Place this value in the first position of the new tableau's X2 row. Repeat this process to fill in the remaining values in the new X2 row (Table8g).

Table 8g.—Third simplex tableau - X

Unit profit		6	8	0	0	
	Basic mix	**X_1**	**X_2**	**S_W**	**S_L**	**Solution**
6	**X_1**	1	0	0.05	-0.1	4
8	**X_2**	0	1	-0.025	0.15	9
	Sacrifice					
	Improvement					

Fill in the sacrifice row (Table 8h). The first value is (6 * 1) + (8 * 0) = 6.

Fill in the improvement row. The first value is 6 - 6 = 0.

Table 8h.—Third simplex tableau—sacrifice and improvement rows.

Unit profit		6	8	0	0	
	Basic mix	**X_1**	**X_2**	**S_W**	**S_L**	**Solution**
6	**X_1**	1	0	0.05	-0.1	4
8	**X_2**	0	1	-0.025	0.15	9
	Sacrifice	6	8	0.1	0.6	96
	Improvement	**0**	**0**	**-0.1**	**-0.6**	**-**

There are no positive numbers in the new improvement row. Thus, we no longer can improve the solution to the problem. This simplex tableau represents the optimal solution to the LP problem and is interpreted as:

X1= 4, X2= 9, SW= 0, SL= 0, and profit or Z = $96

The optimal solution (maximum profit to be made) is to manufacture four tables and nine chairs for a profit of $96.

Shortcuts

Several shortcuts can make the construction of simplex tableaus easier.

• In the new tableau, only the columns for the non-basic and exiting variables change. Move the values for all other basic variables directly into the new tableau.

• When 0 is found in the pivot column, that row always is the same in the new tableau. When 0 is found in the pivot row, that column always is the same in the new tableau.

•With two exceptions, the newly entered basic variable's column will contain a zero in all locations. A 1 will go in the same location as the pivot element in the preceding tableau, and the per-unit profit or loss for this variable will appear in the sacrifice row.

Chapter II

CHAPTER- II

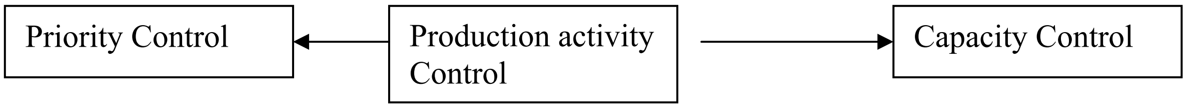
SHOP FLOOR PLANNING AND CONTROL

Shop floor planning and control include the principles and techniques the are necessary to plan , schedule, control and evaluate the effectiveness of production operations. Shop floor activity control integrates the activities o the factors or production of a manufacturing facility such as workers, machines, materials and material handling systems. Shop floor activity control helps in efficient implementation of master production schedule; control of priorities in processing and ensuring minimum work-in-progress and finished goods inventories. Ultimately it minimises the manufacturing cycle time and helps in improving customer service by meeting the promised delivery dates.

Management of shop floor planning and control activities differs widely in make-to-order and make-to-stock manufacturing firms. In make-to-order situations ,due dates (or promised delivery schedules) are important and sequencing of customer order at various work centres becomes an important function. Make-to-stock products are generally high volume consumer products. Manufacture of standardized high volume products involves flow shop or continuous production systems requiring a shop floor control system.

The production activity control (PAC) or shop floor activity control (SFAC) directs how, when and where the products/ components should be made in order to ensure the delivery of goods as per schedules or due dates. The figure shows the major concerns of production activity control.

Fig : Concerns of Production Activity Control



Priority control ensures that the production or shop floor activities are carried out as per a predetermined priority plan. This involves of control orders to vendors /sub contractors and in-house production shops.

Capacity control ensures that the amount of equipment and labour hours necessary at various work centres to carry out the scheduled work are provided.

The concept of Priority control and capacity control can be applied both 9in production systems and service systems even though the problems associated vary in nature and difficulty.

The various activities included in shop floor planning and control are:

1) Assigning a priority to each order which help in setting the sequence of processing orders at work centres.
2) Issuing dispatching lists to each work centre. These lists indicate which orders due to be produced at a work centre, their priorities and completion dates/ times.
3) Updating the work-in-progress inventory. Informations such as number of good parts coming out of each processing step (operation) ,amount of scrap, amount of rework required and number of units short on each order.
4) Providing input-output control on all work centres.

5) Measuring the efficiency, utilization and productivity of workers and machines at each work centre.

The technique used in such production situation where it necessary to nearly equally divide the work to be done among the workers so that the total number of employees required on the assembly line is minimized is known as line balancing technique or assembly line balancing.

LINE BALANCING

Line Balancing is arranging production line so that there is an even flow of production from one work station yo the next, i.e., so that there are no delays at any work station that will leave the next work station with idle time.

Line Balancing is also defined as "the apportionment of sequential work activities into work stations inorder to gain a high utilization of labour and equipment and therefore minimize idle time ". Balancing may be achieved by rearrangement of the work stations or by adding machines and or workers at some of the stations so that all operations take about the same amount of time.

Line Balancing Procedure in Assembly Layouts

Step1: Determine what tasks must be performed to complete the one unit of a finished product and the sequence in which the tasks must be performed. Draw the precedence diagram.

Step2: Estimate the task time (amount of time it takes a worker o perform each task).

Step3: Determine the cycle time (amount of time that would elapse between products coming off the end of the assembly line if the desired hourly production were being produced).

Step 4: Assign each task to a worker and balance the assembly line. This process results in determining the scope of each workers job or which tasks that he or she will perform.

Steps involved in combining the Tasks into Worker's jobs

1. Starting at the beginning of the precedence diagram ,combine tasks into a work station in the order of the sequence of tasks so that the combined tasks times approach but do not exceed the cycle time or multiples of the cycle time.

2. When tasks are combined into a work station, the number of multiples of the cycle time is the number of workers required at the work station, all performing the same job.

Analysis of Line Balancing Problems

The procedure involve the following steps

1. Determine the no. of workstations and time available at each workstation.
2. Group the individual tasks into amounts of work at each work station.
3. Evaluate the efficiency of grouping.

When the available work time at any station exceeds that which can be done by one worker, additional workers must be added at that work station.

The key to efficient line balancing is to group activities or tasks in such a way that the work times at the work station are at or slightly less than the cycle time or a multiple of cycle time if more than one worker is required in any work station.

Determination of cycle time (CT): When the amount of output units required per period (period may be hour, shift ,day or week etc) is specified and the available time per period is given(i.e., the no. of working hours per shift , the no. of shifts per day, the no. of working days per week etc.) then,

$$\text{Cycle time (CT)} = \frac{\text{Available time per period}}{\text{Output units required per period}}$$

Cycle time is the time interval at which completed products leave the production line.

Determination of the ideal or theoretical minimum number of workers required in the line

$$\left.\begin{array}{l}\text{Ideal or theoretical minimum}\\ \text{no. opf workers required in the assy.}\\ \text{line/ production line}\end{array}\right\} = \frac{\begin{pmatrix}\text{Total operation}\\ \text{or task time}\end{pmatrix} \times \begin{pmatrix}\text{Output units}\\ \text{required per period}\end{pmatrix}}{\text{Available time per period per worker}}$$

$$\mathbf{N=} \sum t \times \left(\frac{1}{CT}\right) = \frac{\sum t}{CT}$$

Balancing efficiency:

An efficient line balancing will minimize the amount of idle time . The balance efficiency can be calculated as :

$$\text{(i) } E_{ffB} = \frac{\text{Output of task time}}{\text{Input by work station times}} = \frac{\sum t}{CT \times N}$$

Where , $\sum t$ = sum of the actual worker times or task times to complete one unit.

CT = cycle time;

N = No. of workers or work stations

$$\text{(ii) } E_{ffB} = \frac{\text{Theoretical minimum number of workers}}{\text{Actual number of workers}}$$

The grouping of tasks is done with the aid of a precedence diagram. The precedence diagram is divided into work zones or stations and the appropriate activities are granted under each workstation until the cycle time is as fully utilized as possible.

Line Balancing Procedure:

Steps:

1. Calculate the cycle time and determine the theoretical minimum number of workstations

$$N_t = \frac{\sum t}{CT} = \frac{\text{Sum of all task time}}{\text{Cycle time}}$$

$$\text{Cycle time (CT)} = \frac{\text{Available time}}{\text{Output required}}$$

2. Compute the actual number of workstation (N) required by rounding up the theoretical number of workstations to the next higher integer value.

3. Assign the tasks to the workstations beginning with station 1.Tasks are assigned to work stations moving from left to right through the precedence diagram.

4. Before assigning each task to a workstation, use the following criteria to determine which tasks are eligible to be assigned to a workstation.

   a) All preceeding tasks in the sequence have been assigned already.
   b) The task time does not exceed the time remaining at the workstation .If no tasks are eligiblie to be assigned to a particular workstation, move to the next workstation.

5. After each task assignment, determine the time remaining at the current work station by subtracting the sum of times for tasks already assigned to the work station from the cycle time.

6. When there is a tie between two tasks (parallel tasks) to be assigned, use one of these rules:

(a)Assign the task with the longest task time.

(b) Assign the task with greatest number of followers.

If there is still a tie, choose one task arbitrarily.

7. Continue assignment of tasks until all tasks have been assigned to workstations.

8. Calculate the idle time (or balance delay),percent idle time and efficiency of balancing the line.

The various line balancing methods or techniques used are:

1. Heuristic methods
2. Linear Programming
3. Dynamics Programming
4. Computerised line-balancing

Heuristic and computer based technique are most widely used for solving large scale line balancing problems and use of linear programming and dynamic programming is limited. Here, we see an illustration in Heuristic Method.

Heuristic Method

It is a thumb rule method which gives a satisfactory rather optimal solution to the line balancing problem. Heuristic methods are acceptable when optimizing solutions are not feasible or are too costly to obtain. Thetrial and error technique used in line balancing is a heuristic method in which work elements are grouped such that the cycle time is not violated and the precedence diagram is made use of to group the activities as per the sequence of operations.

Illustration:

The table below shows the number of work stations (N),cycle time (C) and daily production for a product.

Table 15.6		
Number of workstations(N)	**Cycle (C) (seconds)**	**Daily production (in 8 hours shift)**
1	120	240
2	60	480
3	40	720
4	30	960
5	24	1200
6	20	1440

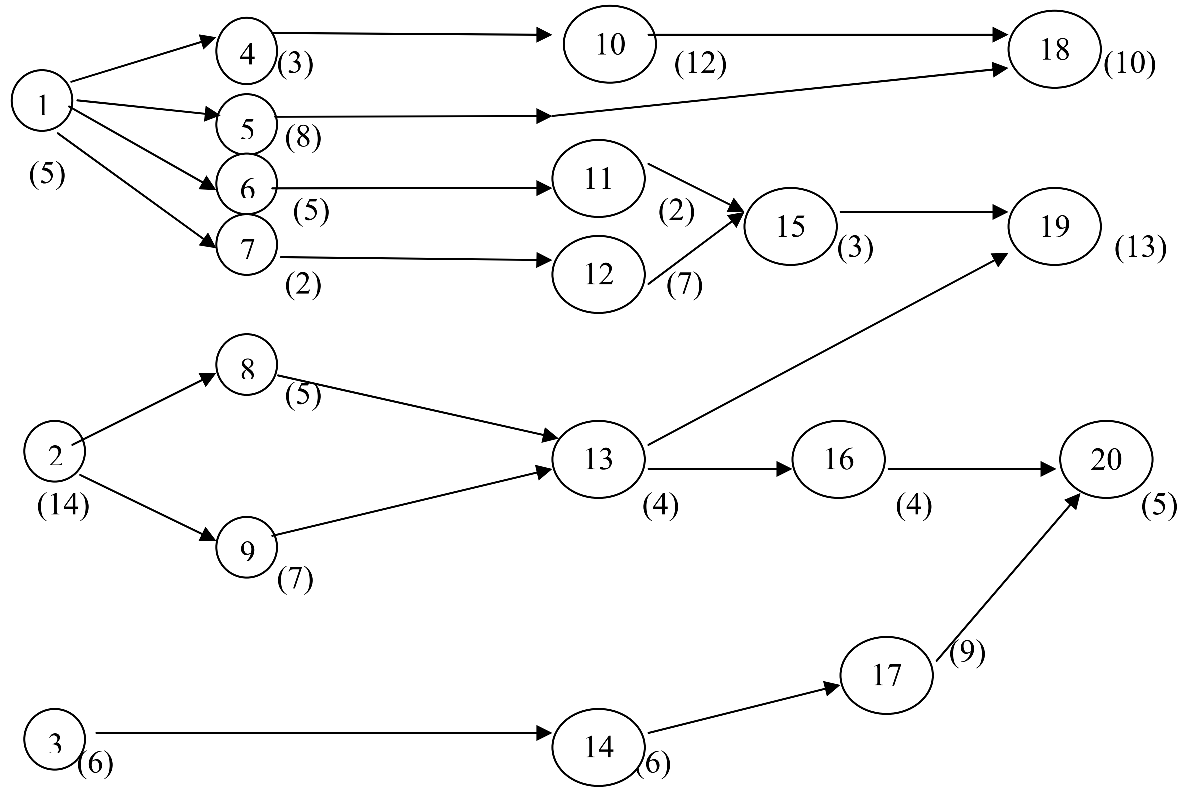
If it is desired to have assy lines each producing 720 units per day, the cycle time will be 40 seconds and there will be 3 work stations in each assembly line. The precedence diagram for the activities is shown below. The activity time in seconds are given in bracket for each of the twenty operations involved.

Assuming that activities may be combined within a " given Zone " without violating the precedence relationship, assign the activities into three work stations. This can be done on trial and error basis by adding activity times upto or nearly equal to the cycle time (I,e.,40 seconds). One such solution is given in the table:

Table 15.7		
	Activities	**Total time for activities (secs)**
Work station 1	1,6,7,2,8,9,11	5+5+2+14+5+7+2=40
Work station 2	4,5,10,12,13,3	3+8+12+7+4+6=40
Work station 3	14,15,16,17,18,19,20	6+3+4+9+10+3+5=40

A perfect is obtained since all the work stations have exactly the same work load of 40 seconds. The balance delay is Nil.

Exhibit 15.20



Chapter III

CHAPTER - III

THE APPLICATION OF OPERATIONS RESEARCH TECHNIQUES TO FINANCIAL MARKETS

3.1 INTRODUCTION

Operations Research, OR, has been applied to problems in finance for at least the last half century. The INFORMS data base of academic papers in OR journals since 1982 classifies nearly 3% of the entries as being concerned with finance. For the journal Management Science over the same period this proportion is over 10%. There is an even larger number of papers on the application of OR techniques to finance in the finance, mathematics, engineering and other literatures, so that, in total, there are several thousand papers which apply OR techniques to finance in academic journals. Equally, OR has played a part in the adoption by the financial markets of the new finance theories. For example, in the 1960s and 1970s the Management Science group at Wells Fargo Bank in San Francisco pioneered the used of new finance theories, and introduced the first index tracking fund in July 1971 (Bernstein, 1992). As part of the increased use of mathematical models in finance (Merton, 1995), investment banks have recruited staff skilled in quantitative techniques, including OR, to devise pricing equations and analyse market data - the so called "quants" or "rocket scientists". This chapter considers the application of OR techniques to financial markets. This covers decisions concerning trading by decision makers in financial markets (e.g. the debt, equity and foreign exchange markets and the corresponding derivatives markets), and represents a more recent and still growing area for the application of OR techniques to finance. This chapter does not consider the more traditional applications of OR to the management of the firm's finances: working capital management (which can be subdivided into the management of cash, receivables and liabilities), capital investment (including the appraisal and implementation of sets of large interdependent investments), multinational taxation, and financial planning models (such as those developed for banks); which have been reviewed by Ashford, Berry and Dyson (1988).

After considering some of the reasons for the attractiveness of finance problems for the application of OR techniques, we identify the main types of problem that are amenable to OR analysis, and documents some of the many problems in financial markets which have been addressed using OR techniques.

3.2 PROBLEMS IN FINANCIAL MARKETS AND OR TECHNIQUES USED TO ANALYSE THE PROBLEMS

1. Attractiveness of Finance Problems

An important distinguishing feature of problems in financial markets is that they are generally separable and well defined. The objective is usually to maximise profit or minimise risk, and the relevant variables are amenable to quantification, almost always in monetary terms. In contrast to some other OR applications, the investigator has few worries about ensuring that they have identified the correct question (i.e. there is no need to consider whether the problem is to reschedule the company's vehicle fleet to meet customer needs, rather than the broader question of whether it needs to operate a fleet of vehicles at all). In finance problems, the relationships between the variables are usually well defined, so that, for example, the way in which an increase in the proportion of a portfolio invested in a particular asset affects the mean and variance of the portfolio is clear. Thus the resulting OR model is a good representation of reality, particularly as the role of non-quantitative factors is often small. Finance problems also have the advantage that any solution produced by the analysis can probably be implemented, while in other areas there may be unspecified restrictions concerned with human behaviour and preferences that prevent the implementation of some solutions. Furthermore, finance practitioners are accustomed to the quantitative analysis of problems.

The investigator is likely to find that much of the requisite historical data has already been collected and is available from company records or recorded market transactions, and that large amounts of real time data are available on prices (traded and quoted) in financial markets which can readily be used in OR models. In addition, non-quantitative factors are generally absent from formulations of finance problems.

The availability of real time data means that solutions can often be implemented very quickly (e.g. a few seconds) and, as trading in financial markets often involves very large sums of money, even a very small improvement in the quality of the solution (under 0.5%) is beneficial. Furthermore, such problems tend to recur, possibly many times per day, spreading the costs of developing an OR solution over a large number of transactions. This scale and repetition makes the development of an OR model more attractive than for small or one-off decisions.

Thus, because finance applications (especially applications to financial markets) are largely numerical problems with well defined boundaries and objectives, clear relationships between the variables, large benefits from very small improvements in the quality of decision making and excellent data, they are well suited to OR analysis. The application of OR techniques to financial markets in more detail by considering some of the major types of problem in financial markets, and the OR techniques that have been used to analyse them are dicussed.

2. Portfolio Theory

A seminal application of OR techniques to finance was by Harry Markowitz (1952, 1987) when he specified portfolio theory as a quadratic programming problem (for a survey of this theory, see Board, Sutcliffe and Ziemba, 1999). Participants in financial markets usually wish to construct diversified portfolios because this has the substantial advantage of reducing risk, while leaving expected returns unchanged. The objective function for the portfolio problem is generally specified as minimising risk for a given level of expected return, or maximizing expected return for a given level of risk. While returns produce a linear objective function, risk is modeled using the variance, leading to an objective function with quadratic variance and covariance terms. The Markowitz model also includes non-negativity constraints on the decision variables to rule out short selling of the asset concerned. As well as specifying the portfolio problem within a mean-variance framework, Markowitz also developed solution algorithms for more general quadratic programming problems. This provides an example of the interaction between OR techniques and finance, with the former sometimes being tailored to meet the needs of the latter. Subsequently the idea of using the variance to model risk has been extensively used in finance, and hence in applications of OR to finance.

Although the most obvious application of portfolio theory is to the choice of equity portfolios, and empirical papers (e.g. Board and Sutcliffe, 1994; and Perold, 1984) have used quadratic programming to compute efficient equity portfolios, the technique can be applied to a much wider range of problems. Konno and Kobayashi (1997) proposed using quadratic programming to form portfolios of both equities and bonds. Other authors have been concerned with managing bond portfolios to maximize their expected value, and have used stochastic linear programming to allow for interest rate risk (e.g. Bradley and Crane, 1972). Golub et al. (1995), Zenios (1991, 1993b) and Zenios et al (1998) employed stochastic programming to select a portfolio of fixed interest securities (Mortgage Backed Securities, MBS) that maximised the expected utility of terminal wealth, after using Monte Carlo simulation to generate the various scenarios, while Ben-Dov, Hayre and Pica (1992, used stochastic programming to form portfolios of MBS and other assets for clients that were expected to outperform some pre-specified target return.

Pension funds hold portfolios of both assets and liabilities, making investments in shares, bonds, and other financial assets; to fund their obligations to existing and future pensioners. The problem of selecting an investment policy for a pension fund can be analysed using asset-liability management models that allow for the non-zero correlations between the values of the assets and liabilities (e.g. rapid inflation increases the value of both equities and the liabilities of a pension scheme based on final salaries) which, if positive, reduces risk. While these problems may be formulated using quadratic programming, they have usually been solved in other ways (see Ziemba and Mulvey, 1998). Mulvey (1994) assumed that the objective was to maximise the expected value of a non-linear utility of wealth function, and specified the problem as a nonlinear network problem, with the simulation of future pension fund liabilities. Similar assetliability problems are also faced by insurance companies, for example Cariño et al (1994, 1998a, 1998b) formulated this problem for a Japanese insurance company. Their model maximises the expected market value of the company, with risk measured as the underachievement of the specified goals. It allows for various scenarios and finds solutions using stochastic linear programming. Hillier and Eckstein (1993) applied stochastic linear programming to generate the risk-return efficient frontier for a pension fund, while Holmer (1994) devised a utility maximizing approach using Monte Carlo simulation to manage the assets and liabilities of the Federal National Mortgage Association (Fannie Mae). Klaassen (1998) criticised the use of Monte Carlo simulation, as this can bias the results by including arbitrage opportunities in the sampled scenarios. To avoid this, he aggregated an arbitrage-free event tree before its inclusion in a multi-stage stochastic programming model of the asset-liability problem.

Quadratic programming has been used to form portfolios of currencies (Levy, 1981), and commercial loan portfolios (Gollinger and Morgan, 1993), in which the objective was to lend money to industrial sectors to minimise variations in industry credit quality for a given level of return (spread over LIBOR, plus fees).

Another application of quadratic programming is generalised hedging, in which the objective is usually to minimise the variance of a portfolio of a given set of assets and the chosen hedging instruments (Levy, 1987). Rudolf and Zimmermann (1998) widened this approach, and applied quadratic programming to selecting a portfolio of domestic equities, foreign equities and foreign exchange forward contracts. If the hedging instruments include options, this introduces a nonlinearity into the hedging decision. Murtagh (1989), devised a non-linear programming model to hedge foreign currency exposure using a mixture of currency forward and options contracts. The aim was to minimise the expected cost, subject to a chance constraint that the probability of the cost of the chosen hedging strategy does not exceed the cost of using the forward market alone. Clewlow, Hodges and Pascoa (1998) show how linear, goal and dynamic programming can be used to hedge options in the presence of transactions costs. Nielsen and Zenios, 1996, used stochastic programming with recourse to hedge the risks of single premium deferred annuities with mortgage backed securities (MBS).

Quadratic programming has also been used for constructing index tracking portfolios, where the purpose is to select a portfolio of assets (e.g. equities or bonds) which, when combined with a matching short position in the index to be tracked, has minimum risk (Meade and Salkin, 1989, 1990; Rudd, 1980; Seix and Akhoury, 1986). Multi-stage stochastic programming with recourse, in conjunction with Monte Carlo simulation to generate the scenarios, has been used by Vassiadou-Zeniou and Zenios (1996) and Zenios et al (1998) to track an index of MBS.

In some applications of portfolio theory, the decision variables must be integer. While it is acceptable to round the number of shares traded to the nearest integer, this may not be the case for futures contracts, which are generally worth over £100,000 per contract. For this reason, Peterson and Leuthold (1987) and Shanker (1993) used quadratic integer programming to compute hedging strategies involving futures. Similarly, because of indivisibilities in the quantities of short term foreign currency securities, Cotner and Levary, 1987, imposed an integer requirement when forming portfolios of currencies, making the solution procedure zero-one quadratic programming. Shapiro, 1988, used stochastic integer programming with recourse to construct bond portfolios that allow for some of the bonds being called (or redeemed early) if interest rates are low. Zero-one variables indicated whether a particular bond was included in the portfolio, while non-integer variables gave the scale of investment in each included bond.

Some authors have argued that formulation and solving quadratic programming portfolio problems is too onerous, and proposed simplified solution techniques. Sharpe (1963) proposed a single index model which simplifies the variance-covariance matrix required by the Markowitz model by assuming that assets are related to each other only through their correlation with a single common factor. This simplification removes the need for large numbers of covariance terms in the objective function, enabling the use of special purpose quadratic programming algorithms. When each asset represents only a small proportion of the portfolio, Sharpe (1967) shows that his single index model can be treated as having a linear objective function. In essence, well diversified portfolios have only systematic risk, and this is measured by asset betas, which then gives a linear objective function. In 1971, Sharpe suggested using a piecewise linear approximation to the quadratic objective function, enabling the application of linear programming to solve portfolio problems.

Another proposal is to minimize the mean absolute deviation (MAD), which can be solved using linear programming, rather than quadratic programming. Konno and Yamazaki (1991 and 1997) applied MAD to forming portfolios of Japanese equities and also equities and bonds. Yawitz, Hempel and Marshall (1976) proposed viewing bond portfolios as a risk-return problem, but to avoid quadratic programming used the mean absolute price change to measure risk. The formation of portfolios of MBSs was modeled by Zenios and Kang (1993) who used both simulation and linear programming. They used Monte Carlo simulation to generate rates of return on each MBS during the holding period, and then applied linear programming to minimize the MAD, subject to the expected return exceeding some specified amount. Worzel, Vassiadou- Zeniou and Zenios (1994) suggested using both simulation and linear programming for tracking fixed-interest indices. First, the holding period returns for the securities in the index are simulated; then linear programming, with risk measured by the MAD, is used to select a portfolio which maximises the expected return, subject to the risk of underperforming the index not exceeding some specified upper bound. This bound is then minimised by iteratively solving the linear programming problem. Seix and Akhoury (1986) proposed using linear programming to devise a portfolio to track a bond index by maximising the value of the portfolio, while matching the duration, quality, sectors, and coupons of the bond index. Another approach to removing the need to solve a quadratic programming problem is to specify the problem as choosing between a range of pre-specified equity portfolios using data envelopment analysis (DEA) (Premachandra, Powell and Shi, 1998).

A different approach to removing the need for quadratic programming is to reformulate the portfolio problem as a non-linear generalised network model for which efficient solution algorithms exist (Mulvey, 1987). In this case the objective function is non-linear, and risk can be measured in a wide variety of ways; including the variance.

Glover and Jones (1988) proposed using a network model, in conjunction with the Fourier transform, to give a linear model.

Portfolio problems, with the twin objectives of maximising returns and minimising risk, can also be viewed as goal programming problems with two goals. Additional goals can be introduced, and a number of authors have solved portfolio problems using goal programming. For example, Kumar, Philippatos and Ezzell (1978), Kumar and Philippatos (1979) and Lee and Lerro (1973) have specified models with five or six goals for forming equity portfolios. Measuring risk using systematic (beta) and unsystematic risk, rather than the variance, enables the use of linear rather than quadratic programming. Similarly, Lee, Lerro and McGinnis (1971) constructed a linear programming model with six goals for managing bond portfolios. Sharda and Musser (1986) specified a model with four goals for hedging the risk of Treasury bonds. Because they included zero-one variables to specify the week in which the hedging instrument is traded, the problem is a mixed integer goal programming problem.

Cheng (1962), analysed the problem of maintaining a bond portfolio over time where there is a choice between a short maturity bond with a high yield, and a longer maturity bond with a lower yield, and there is uncertainty over the available bond yields when the existing bonds mature. Cheng suggested quadratic programming by essentially treating the situation as a single period problem. Multi-period portfolio problems have been specified as dynamic programming problems (Elton and Gruber, 1971), while Mulvey and Vladimirou (1992), used a stochastic generalised network model.

In portfolio immunization the aim is to construct a portfolio of interest rate dependent securities whose value is the same as some target asset (usually another interest rate dependent asset). By matching the duration of the portfolio with that of the target asset, the portfolio is immunized against small parallel shifts in the yield curve. Fong and Vasicek (1983) proposed a measure of the risk from general interest rate movements (e.g. non-parallel shifts in the yield curve) and proposed devising a bond portfolio to minimise this, subject to achieving a specified duration, by linear programming. They also suggest that the investor could compute an efficient frontier of portfolios that minimise the standard deviation of the return on the immunized portfolio for a given level of expected return, again using linear programming. Using higher moments of a generalised duration measure, Kornbluth and Salkin (1987) show it is possible to immunize against changes in the shape of the yield curve, as well as parallel shifts, using linear fractional goal programming. Nawalkha and Chambers (1996) used a modified duration measure to quantify the risk of an immunized portfolio, and minimised this using linear programming. Alexander and Resnick (1985) who incorporated default risk, also specified immunization as a linear goal programming problem. What all these immunization studies have in common is that the chosen risk measure does not involve squares or cross products of the decision variables, so that linear programming, not quadratic programming, is the solution technique.

Portfolio theory (and quadratic programming) has also been applied to problems that do not directly involve traded financial assets. For example, Freund (1956) examined farming, and Board and Sutcliffe (1991) applied the approach to tourism.

3. The Valuation of Financial Instruments

It is very important when trading in financial markets to have a good model for valuing the asset being traded, and OR techniques have made a substantial contribution in this area.

Although European style call and put options can be valued using the Black-Scholes model, which provides a good closed form solution, OR techniques have made a substantial contribution to the pricing of more complex derivatives. In 1977, Boyle proposed the use of Monte Carlo simulation as an alternative to the binomial model for pricing options for which a closed form solution is not readily available. Monte Carlo simulation has the advantage over the binomial model that its convergence rate is independent of the number of state variables (e.g. the number of underlying asset prices and interest rates), while that of the binomial model is exponential in the number of state variables.

Monte Carlo simulation is used to generate paths for the price of the underlying asset until maturity. The cash flows from the option for each path, weighted by their risk neutral probabilities, can then be discounted back to the present using the risk free rate, allowing the average present value across all the sample paths to be computed to give the current price of the option (Boyle, Broadie and Glasserman, 1997). A range of variance reduction methods have been used in the Monte Carlo pricing of options (e.g. control variates, antithetic variates, stratified sampling, Latin hypercube sampling, importance sampling, moment matching and conditional Monte Carlo). In addition, quasi-Monte Carlo methods have been applied to finance problems to speed up the simulation (Joy, Boyle and Tan, 1996). As well as generating option prices, Monte Carlo simulation can be used to compute the various sensitivities - "the Greeks" - including the hedge ratio, which are essential for many trading strategies (Broadie and Glasserman, 1996).

There are no closed form solutions for American style options, and until recently it was thought that Monte Carlo simulation could not be used to price such options. This is a major problem, as the majority of options are American style. However, progress is being made in developing Monte Carlo simulation techniques for pricing American style options (Broadie and Glasserman, 1997; Grant, Vora and Weeks, 1997). Options have also been priced using finite difference approximations, and Dempster and Hutton (1996) and Dempster, Hutton and Richards (1998) have proposed the use of linear programming to solve the finite difference approximations to the price of American style put options. In addition, American style options can be priced using dynamic programming, Dixit and Pindyck (1994).

If a closed form pricing equation cannot be derived for an option or other derivative; provided a price history is available, a neural network can be trained to produce prices using a specified set of inputs, which can then be used for out-of-sample pricing (Hutchinson, Lo and Poggio, 1994). This approach was able to outperform the Black-Scholes formula when pricing options on S&P500 futures, and has considerable potential for generating prices for "hard to price" derivatives that are already traded on competitive markets.

Empirical research has found that, although the Black-Scholes pricing model provides accurate prices for at-the-money options, there are some unexpected patterns in options prices, such as the "volatility smile". Modeling this effect, and given a contemporaneous set of prices for

European style put and call options on the same underlying asset, Rubinstein (1994) has shown how the implied risk neutral probability distribution can be computed using quadratic programming. This procedure selects a set of risk neutral probabilities that minimise the sum of the squared difference between themselves and the risk neutral probabilities generated by some prior guess. These probabilities can be used to infer a recombining binomial tree that is consistent with the observed options prices, which is then used in hedging or valuing European style options on the underlying asset over the period until maturity in a way that allows for the presence of the “smile”. Jackwerth and Rubinstein (1996) generalised this approach using nonlinear programming to minimise four other objective functions.

Municipal authorities in the USA who wish to borrow money by issuing bonds usually invite bids from underwriting syndicates. These bids must specify a schedule of bond coupons (i.e. interest payments), subject to various restrictions imposed by the municipality, and by the need for the underwriting syndicate to market the bonds to the public. The winning bid is generally that with the lowest net interest cost to the municipality. The underwriting syndicate typically have only 15 to 30 minutes to prepare a bid, and so a computerized solution procedure is needed. This decision was formulated as a linear programming problem by Percus and Quinto (1956) and Cohen and Hammer (1965, 1966), while Weingartner (1972) respecified it as a dynamic programming problem. If the municipality places an upper bound on the number of different coupon rates, it becomes an integer programming problem, that can also be solved as a zero-one dynamic programming problem (Weingartner, 1972; Friemer, Rao and Weingartner, 1972). Nauss and Keeler (1981) added the constraint that the coupon rates be set to integers times a specified multiplier, and proposed an integer programming formulation. The municipality can specify the true interest cost (which is the internal rate of return (IRR) on the bond), rather then the net interest cost, as the selection criterion to be used. The use of the IRR as the objective to be minimised makes the problem non-linear. Bierwag, 1976, proposed a linear programming algorithm for solving this problem. Nauss, 1986, added some additional restrictions which make the problem integer, and suggested an approximate solution using integer linear programming.

Mortgage backed securities (MBS) are created by the securitisation of a pool of mortgages. For any specific mortgage, the borrower has the right to repay the loan early - the prepayment option, or may default on the payments of capital and interest. These risks feed through to the owners of MBS, in addition to the risks of fluctuations in the rate of interest payable on flexible rate mortgages (Zipkin, 1993). Thus MBS are hybrid securities, as they are variable interest rate securities with an early exercise option. Monte Carlo simulation can be used to generate interest rate paths for future years. Forecasts of the mortgage prepayment rates then permit the computation of the cash flows from each interest rate path, and these sequences of cash flows are used to value the MBS (Zenios, 1993a; Ben-Dov, Hayre and Pica ,1992; Boyle, 1989). This procedure, which can be used to identify mispriced MBS in real time, is computationally demanding and parallel (and massively parallel) and distributed processing have been used in the solution of the problem. Simulation has also been used to price collateralised mortgage obligations or CMOs (Paskov, 1997). Other hybrid securities, such as callable and putable bonds and convertible bonds face similar valuation problems to MBSs and require similarly intensive solution methods.

There is an active secondary market in loan portfolios which may carry a significant default risk. Del Angel el al (1998) used a Markov chain analysis with 14 loan performance states and Monte Carlo simulation to generate the probability distribution of the present value of loan portfolios.

4. Imperfections in Financial Markets

As well as accurately pricing financial securities, traders are interested in finding imperfections in financial markets which can be exploited to make profits (Keim and Ziemba, 1999; Ziemba, 1994). One aspect of this is the search for weak form inefficiency (i.e. that an asset's past prices can be used as the basis of a profitable trading rule). Among the early attempts to find such exploitable regularities in stock prices were Dryden's (1968, 1969) use of Markov chains.

A fundamental feature of financial markets is the existence of no-arbitrage relationships between prices, and small price discrepancies can be exploited by arbitrage trades to give large riskless profits. Network models have been used to find arbitrage opportunities between sets of currencies (Christofides, Hewins and Salkin, 1979; Kornbluth and Salkin, 1987; Mulvey, 1987; Mulvey and Vladimirou, 1992). This problem can be specified as a maximal flow network, where the aim is to maximise the flow of funds out of the network, or as a shortest path network. While some network formulations are linear and could be formulated and solved as linear programming models, interpretation of the problem as a network enables the use of computationally faster algorithms.

Chandy and Kharabe (1986) developed a model for identifying underpriced bonds. They suggested solving a linear programming model to form a bond portfolio with maximum yield. This solution then gives the break even yield, which is the minimum bond yield necessary for inclusion in the portfolio. Hodges and Schaefer (1977) devised a linear programming model which minimises the cost of a given pattern of cash flows, enabling underpriced bonds to be traded.

There has been a growing interest in using artificial intelligence based techniques (expert systems, neural networks, genetic algorithms, fuzzy logic and inductive learning) to develop trading strategies for financial markets (e.g. Trippi and Turban, 1993; Refenes, 1995; Goonatilake and Treleaven, 1995; Wong and Selvi, 1998). Such approaches have the advantage that they can pick up non-linear dynamics, and require little prior specification of the relationships involved.

Firer, Sandler and Ward, 1992, simulated the returns from a stock market timing strategy for a range of levels of forecasting skill, so quantifying the likely benefits from various levels of forecasting ability. Taylor (1989) used Monte Carlo simulation to generate a long time series of data for use in back-testing the performance of trading rules for a variety of financial assets.

5. Funding Decisions

OR techniques have also been used to help firms to determine the most appropriate method by which to raise capital from the financial markets to finance their activities. Brick, Mellon, Surkis and Mohl (1983) put forward a chance constrained linear programming model to compute the values of the debt-equity ratio each period that maximize the value of the firm. Other studies have specified the choice between various types of funding as a linear goal programming problem (Hong, 1981; Lee and Eom, 1989). Ness (1972) used linear programming to find the least cost financing decision for an investment project by a multi-national company. Kornbluth and Vinso (1982) modeled the financing decision of a multi-national corporation as involving two goals - minimizing the overall cost of capital and achieving target debt/equity ratios in each country. Since the debt/equity goals involve ratios of the decision variables, the model becomes a fractional linear goal programming problem.

A different approach to the debt problem is to assume that the firm has found its desired debtequity ratio, and is purely concerned with raising the requisite debt as cheaply as possible. In this case, debt can be treated like any other input to the productive process, and inventory models used to determine the optimal “reorder” times and quantities (Bierman, 1966; Litzenberger and Rutenberg, 1972). An additional aspect of the problem is that, bonds’ maturity must be chosen by the borrower to reflect the different current interest rates payable on alternative maturities, the uncertain costs of future borrowing and the marketability of alternative maturities. Crane, Knoopand Pettigrew (1977) formulated this as a linear programming problem to minimise costs, which they solved for three different interest rate scenarios.

Firms, governmental organizations and others may choose to issue callable bonds in which the issuer has the option to repay the bond at a time of their choosing before the maturity date of the bond. The issuer must choose various parameters of the callable bond, and Consiglio and Zenios (1997a, 1997b) have used nonlinear programming to design such securities in a way that is most beneficial to the issuer, while Holmer, Yang and Zenios (1998) used a simulated annealing algorithm.

Firms which have issued callable debt must decide when to call (repay) the existing debt and refinance it with a new issue, presumably at a lower cost - the bond scheduling problem. This is a dynamic programming problem and has been modelled as such by Weingartner (1967), Elton and Gruber (1971) and Kraus (1973). Baker and Van Der Weide (1982) extended this model to cover a multi-subsidiary company with debt requirements for each subsidiary. Dempster and Ireland (1988) developed a model which applies a range of OR techniques in a complementary fashion to the bond scheduling problem. The model begins by using stochastic linear programming to devise a multi-period plan for both issuing and calling bonds. The plan is refined using heuristics, possibly leading to multiple plans, and the probability distributions of these revised plans are derived using simulation. Finally, an expert system is used to help in deciding between alternative plans.

An important question when appraising investment projects is determining the appropriate cost of capital, i.e. the price which must be paid in the financial markets to finance the project. Boquist and Moore (1983) proposed the use of linear goal programming to estimate the cost of capital for divisions by incorporating corporate prior beliefs concerning betas.

Certificates of deposit (CDs) are issued by banks and indicate that a specified sum has been deposited at the issuing depository institution. As such, CDs represent a source of funding for banks. Russell and Hickle (1986) developed a simulation model to predict the impact of various interest rate scenarios on the cost of this funding source.

Finally, the problem facing borrowers of choosing between alternative mortgage contracts (e.g. fixed rate, variable rate and adjustable rate mortgages) has been modeled using decision trees (Heian and Gale, 1988; Luna and Reid, 1986).

6. Strategic Problems

In recent years, some of the decisions facing traders and market makers in financial markets have been analysed using game theory (O'Hara, 1995; Dutta and Madhavan, 1997). These models typically involve one or more market makers, and traders who may be informed or uninformed, and discretionary or non-discretionary. Traders in stock markets seek to trade at the most attractive prices and large trades are often broken up into a sequence of smaller trades in an effort to minimise the price impact. This can be viewed as a strategic problem with the aim of devising a strategy for trading the block of shares. The initial trades influence the price of subsequent trades, and so executing the large trade at the lowest cost is a dynamic problem. Bertsimas and Lo (1998) use stochastic dynamic programming to define "best execution" and to compute an optimal trading strategy.

Powers (1987) applied game theory to the situation where a company has two major shareholders, and a large number of very small shareholders. This can be modeled as an oceanic game, in which the two large players behave strategically while the many small shareholders (the ocean) do not. This approach can be used to derive the highest price a large shareholder will pay in the market for corporate control.

7. Regulatory and Legal Problems

Financial regulators have become increasingly concerned about financial markets with their very large and rapid international financial flows. OR techniques have proved useful in regulating the capital reserves held by banks and other financial institutions to cover their risk exposure. OR techniques have also been used to ensure compliance with various legal requirements by designing appropriate strategies and to solve other legal problems relating to financial markets.

A key regulatory issue is determining the capital required by financial institutions to underpin their activities in financial markets. An increasingly popular approach to this problem is to quantify the value at risk (VAR). If the specified period and probability are 1 day and 1% respectively, then the VAR is the largest loss that will occur due to market risk 99% of the time. Thus, VAR involves quantification of the lower tail of the probability distribution of outcomes from the firm's portfolio. A particular problem with measuring risk exposure is that portfolios usually include options (or financial securities with option-like characteristics), and options have highly asymmetric payoffs. For such securities, analytical solutions to finding the probabilities in the lower tail of the payoff distribution are unreliable. Riskmetrics™ uses approximations based on "the Greeks" for options that are at or near the money, and Monte Carlo simulation for other options positions (Morgan and Reuters, 1996). Monte Carlo simulation can either make distributional assumptions, or use the distribution of historical realizations, i.e. bootstrapping (Pritsker, 1997).

While Riskmetrics™ quantifies market risk, some securities are also subject to credit risk. Although the market risk of financial instruments (apart from options) tends to produce returns with an approximately normal distribution, credit risk produces returns that are highly nonnormal for all instruments. Usually there is no default, while occasionally there is a substantial or total default. Therefore, Monte Carlo simulation is relevant to modeling the credit risk of portfolios of financial instruments (e.g. loans, letters of credit, bonds, trade credit, swaps, forwards) as in CreditMetrics™ (Morgan, 1997).

Data envelopment analysis (DEA) has been used to assist in bank regulation by measuring bank efficiency, which is then used to predict bank failure (Barr, Seiford and Siems, 1993; Bauer, Berger, Ferrier and Humphrey, 1998).

Traders are required to put up margin when they trade options, and there are complicated rules for determining the total margin required on a portfolio of options and shares. Traders wish to minimise their margin payments, and Rudd and Schroeder (1982) have developed a linear programming model in which the problem was modeled as a transportation problem for determining the minimum required margin.

Some MBS are traded on a "to-be-announced" basis with forward delivery. In these cases the originators have mortgages that have not yet been pooled, and this gives them some flexibility in structuring the securitisation in a manner beneficial to themselves. An extensive set of rules governs the way in which a "to-be-announced" MBS can be structured, leading to a complex problem in devising a feasible solution. This can be specified as a complicated integer programming problem (with the objective of maximising the originator's profit). Collateralised mortgage obligations (CMOs) also involve the securitisation of a mortgage pool, but in this case the pool is structured into a series of bonds (or tranches), each with a different maturity and risks. In the USA various legal restrictions apply to how CMOs can be structured, and it may be difficult to find a feasible solution. Dahl, Meeraus and Zenios (1993) have proposed a complex zero-one programming model for solving this problem, with the objective of maximizing the proceeds from the issue.

To receive the tax benefits, leveraged leases in the USA are designed to satisfy the Internal Revenue Service rules. Capettini and Toole (1981) proposed an integer programming model to structure leveraged leases to meet the IRS rules, with the objective of maximising the net present value of the lessor's cash flow. Litty (1994) developed an approach to this problem using linear programming heuristics that provided fast solutions for untrained users.

Sharda (1987) proposed a linear programming formulation to establish the maximum loss that investors could have sustained from trading in a company's shares. This figure can then be used by the company's lawyers when fighting a lawsuit claiming damages from a misleading statement by the company.

In August 1982 the Kuwait Stock Market collapsed leaving $94 billion of debt to be resolved. This led to the problem of devising a fair method for distributing the assets seized from insolvent brokers among the other brokers and private investors. This problem was solved using linear programming, which reduced the total unresolved debt to $20 billion, saving an estimated $10.34 billion in lawyer's fees (Taha, 1991, Elimam, Girgis and Kotob, 1996, 1997).

8. Economic Understanding

In addition to its traditional role of improving the quality of decision making, OR can also help in trying to understand the economic forces shaping the finance sector. Financial innovation may occur when there is an exogenous change in the constraints or in the costs of meeting existing constraints. Using a linear programming model of a bank, Ben-Horim and Silber (1977) employed annual data to compute movements in the shadow prices of the various constraints. They suggested that a rise in the shadow price of the deposits constraint led to the financial innovation of negotiable CDs.

Arbitrage Pricing Theory (APT), which can be viewed as a generalization of the Capital Asset Pricing Model (CAPM), seeks to identify the factors which affect asset returns. Most tests of the APT use factor analysis, and have difficulty in determining the number and definition of the factors that influence asset returns. To overcome these problems Ahmadi (1993) suggested using a neural network to test the APT. This also has the advantage that the results are distribution free.

Chapter IV

CHAPTER - IV
OPERATIONAL RESEARCH METHODOLOGY IN THE GENERAL MEDICAL ROUNDS

4.1 INTRODUCTION

Operations Research has a vital role in emergency planning for natural disasters, during which there may be periods of mass utilization of men and materials. The discipline of managerial science, also known as Operations Research, applies scientific methods to provide decision makers with more insights information about their systems. The issue of how best to reduce utilization of diagnostic studies without detracting from patient management necessitates a meeting of managerial science with medicine. Bree et al studied a new utilization management tool, the Consultant Radiologist as a gatekeeper, for its ability to improve inpatient diagnostic imaging. This was an example of the use of Operations Research in medicine. The concepts such as decision-tree analysis, housekeeping, fuzzy logic, commissions of enquiry and coroner's inquiries are discussed in general medicine.

4.2 DECISION-TREE ANALYSIS

Decision Analysis is the methodology used in Operations Research to design formal processes upon which decisions are based. It is an approach to selecting a choice from among several variable alternatives. The graphical demonstration of decision analyses is the decision tree. Every clinical examination finding or laboratory investigation can be considered a "test", which must necessarily have a high likelihood-ratio to justify its cost-effective use. Uncertainties are addressed using probability tests. An example is the use of Bayesian analyses of pre-test probability and post test probability. The rapid analysis of the input is part of the neural network used for clinical reasoning. This cognitive function depends upon the clinician's experience, qualifications, training, religious and cultural orientation, language and communication skills, interpretation of body language, innate wisdom, and understanding of the human condition. Though Operations Research methodology uses science to make decisions, the human element cannot be discounted, for the art of medicine is intuitive. The "red herrings" in the data must be accurately identified. The cognitive, analytical, subjective, physical and emotional environment plays a major role in the output, and impairment of any of these parameters will mar the quality of the output. When this process is sub-optimal, there is no new plan for the patient, and the daily entry continues to be the same, or the words "status quo" is entered into the case notes. When the process of clinical reasoning is simplistic, the broad catch-all terms such as "sepsis", "nil acute" or "nil hyperacute" are used. Courage and experience is needed to say if something is normal.

4.3 HOUSEKEEPING AND FUZZY LOGIC

Sometimes clinical charts are mixed up, which leads to wrong entries, wrong administration of medication, missed drug allergies, wrong patients being sent for tests, and duplication of tests on the same day, thus increasing the rate of adverse events and contribute to unnecessary costs and wastage within the system. Repair of damaged case notes is as much the responsibility of the doctors, as that of the ward clerks and nurses. This due care for the patient and the case notes, along with precautions taken with standard drugs, dangerous drugs and controlled drugs is what distinguishes quality care from sub-standard care. Certificates of accreditation for the institution must be translated into the care of the individual patient, rather than to portray broad statistics. This is dependant upon individual doctors and nurses, assisted by the team of pharmacists, physiotherapists, ward clerks, porters, occupational therapists, radiographers, electrocardiogram (ECG) technicians, speech and swallowing therapists, dieticians and podiatrists (i.e. the medical team).

Fuzzy Logic

Individual human input is subject to the usual human frailties. This is manifest in simple things such as handwriting, and what is documented in the notes reflects clarity of the thought processes. Human beings cannot be compared with machines, for there is fuzzy logic used in their thinking. Alignment of thought may not always be present, and sometimes the conclusions may defy logic. Encouraging the process of lateral thinking or thinking "outside the box" contributes to the wide variation in output. This output is also subject to personal feelings. All documentation must nonetheless, be medico-legally defensible.

4.4 COMMISSIONS OF ENQUIRY AND CORONER'S INQUIRIES

The judgements of many commissions of enquiry and other public enquiries like those of the Coroner often cite the causes of error as being inadequate or improper documentation, flaws in the seamless delivery of care and poor communication. Correcting these operational factors avoids unnecessary harm to patients, and prevents ill-will and costly and time-consuming litigation. A court case is enough to result in disillusionment, cynicism, resignations and feelings of abandonment. Public ire is also provoked when the system fails or does not seem to work. Operational research methodology can address all these flaws in the system. The concepts of operational research mentioned here are, however, not meant to be comprehensive. Logistics is often the key to provision of good service, rather than clinical acumen, for the clinician works within the system, and Operations Research improves the logistics within the system. A strong infrastructure, adequate equipment and funding is possible only with proper utilisation of resources. Proper utilisation is an all-encompassing term, and is the sum total of clinical treatment, materials management and cost savings. The benefit to society is obvious. It should be to this aspect that the search for new talent in healthcare be directed. Clinical service champions, known as "blackbelts" in those institutions where lean healthcare is practiced, are the key drivers of the concept of proper utilisation. They also require institutional support. The traditionalists among these leaders will drive the "train" along the same track. The innovators will drive the train onto a new track, but must first construct and test a track. A turtle can move forward only by sticking its neck out!

4.5 OPERATIONAL RESEARCH IN INTERNAL MEDICINE

Healthcare management science is a speciality. Although this is relevant for those studying management, this article is to emphasise that operational techniques of management science can also be used by practising clinicians in Internal Medicine. The relevant books may have to be consulted for the mathematical principles of operational research. The process is often binary, and thus offers a choice of alternatives for the practising clinician. Variations in medical practice are inherent in the system. In operational terms, this translates into a stochastic model in which the outcome depends upon several random events. Although there are several possible outcomes, some are more probable than others. Thus, probability theory and forecasting is used to make a decision, and is based upon the prevalence of the clinical condition. In simple terms, the process is driven by inputs over a period of time, for example, during a period of in-patient stay. Stochastic (random) processes in the medical field may be speech, blood glucose control, such as a patient's ECG, Electroencephalography (EEG), blood pressure or temperature. The deterministic model of Operations Research is one that is used in planning rosters. The use of Operational Research methodology is cost-effective, efficient and reduces wastage. Operations Research when adapted for use in the general medical rounds can be used to formulate a template for ward rounds. This is especially useful when different teams of doctors look after 1 patient. Handover would be more streamlined. The variance lies in the interpretation of the findings elicited, and the different permutations and combinations possible would allow flexibility in management plans. Thus, the template will not promote "cookbook" medicine. The general pattern of following "SOAP" (Subjective, Objective, Assessment and Plan) may not be consistently practiced. In the UK, General Internal Medicine is an integral part of the specialist's work. In many other countries, subspecialty physicians may be called upon to take care of general medical patients. A template for carrying out ward rounds would standardise the documentation and ensure that particular findings are not overlooked. It would be of a sufficient standard to pass a rigorous audit. The purpose of this article is to introduce a template that would add to the value stream of medical care. The template (an example is given in the appendix) describes the condition of the patient at the time of admission and the daily progress, and acts as a basis for in house case discussions and for mortality rounds. It enables the clinicians from differing specialties doing internal medicine to assess the risks posed to patients with multiple general medical conditions, and the interactions with the medications and fluid balance, on a daily basis. For example, an inordinately high urine output raises the possibility of the polyuric phase of acute tubular necrosis or secondary diabetes insipidus. Complex clinical situations can be analysed in all its permutations and combinations for a cost-effective result, be it in the investigations, medications or any other relevant intervention. Alternatives could be generated, and this is the inherent strength of Operational Research. In this globalised world, doctors from overseas would also find a template useful, since they may initially have problems adjusting to a new medical and cultural (including language) system. It would also facilitate clinical governance.

Summary and Conclusion

SUMMARY AND CONCLUSION

In this thesis we have given some applications of Operations Research.

In chapter I, linear programming is applied to production of furniture. In this chapter, it is discussed how L.P is applied gradually from graphical method to simplex method and from easier example to complex problems.

In chapter II, the shop floor management problem is discussed. Starting with the explanation of shop floor planning and control, an illustration is explained in one of the line balancing method namely Heuristic method.

In chapter III, the application of OR to financial markets is reviewed. Here the main types of problems that are amenable to OR analysis is identified and documented some problems in financial markets which have been addressed using OR techniques.

In chapter IV, the application of OR in general medical rounds is discussed.

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