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Research Article

g_u-Closed Sets in Generalized Topological Spaces

Dhanadeepa P and Jayanthi D

Department of Mathematics, Avinashilingam University, Coimbatore, Tamilnadu, India

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Abstract: In this paper we have introduced a new class of sets in generalized topological spaces called g_u-closed sets. Also we have investigated some of their basic properties and obtained some interesting theorems.

Keywords: Generalized topological spaces, g-closed sets, g_u- closed sets.

INTRODUCTION

The concept of generalized topological spaces was introduced and investigated by A.Csaszar[1]. Many g-closed sets like g-pre closed, g-β closed,etc, in generalized topological spaces were introduced by him. In this paper,we have introduced a new class of sets in generalized topological spaces called g_u- closed sets.Also we have investigated some of their basic properties.

PRELIMINARIES

Definition 2.1[1]: Let X be a non-empty set and g be a collection of subsets of X. Then g is called a generalized topology (GT for short) on X if and only if $\emptyset \in g$ and $G_i \in g$ for $i \in I \neq \emptyset$ implies $G = \bigcup_{i\in I} G_i \in g$. The pair (X, g) is called as a generalized topological space(GTS for short) on X. The elements of g are called g-open sets and their complements are called g-closed sets.

We denote the family of all g-closed sets in X by g(X). The generalized closure of a subset S of X, denoted by $c_g(S)$, is the intersection of generalized closed sets including S. And the interior of S, denoted by $i_g(S)$, is the union of generalized open sets contained in S.

Note that $c_g(S) = X- i_g(X-S)$ and $i_g(S) = X- c_g(X-S)$.

Definition 2.2:[2] Let (X, g) be a generalized topological space and A $\subseteq$ X. Then A is said to be

(i) g-semi closed if $i_g(c_g(A)) \subseteq A$
(ii) g-pre closed if $c_g(i_g(A)) \subseteq A$
(iii) g-α closed if $c_g(i_g(c_g(A))) \subseteq A$
(iv) g-β closed if $i_g(c_g(i_g(A))) \subseteq A$
(v) g- regular closed if $c_g(i_g(A)) = A$

The complement of g-semi closed(resp., g-pre closed, g-α closed, g-β closed, g-regular closed) is said to be g-semi open (resp., g- pre open, g-α open, g-β open, g-regular open).

g_u-CLOSED SETS IN GENERALIZED TOPOLOGICAL SPACES

In this section we have introduced g_u-closed sets in generalized topological spacesand studied some of their basic properties.

Definition 3.1: Let (X, g) be a generalized topological space. Then a non empty subset A is said to be g_u-closed if $c_g(A) \subseteq i_g(c_g(U))$ whenever $A \subseteq U$ and U is g-open. The family of all g_u-closed sets is denoted by $g_u(X)$.

Example 3.2: Let X={a, b, c}and let g ={∅, {b}, {a, b},{b, c}, X} then (X, g) is a generalized topological space. Now let A={a}andU = {a, b} then A $\subseteq$ U and $c_g(A)=c_g(\{a\})$ ={a}and $i_g(c_g(U))$ = $i_g(c_g\{a, b\})$ = X. Therefore $c_g(A) \subseteq i_g(c_g(U))$. Therefore the set A is g_u-closed set in (X, g).

Theorem 3.3: Every g-closed set in (X, g) is a g_u- closed set in (X, g) but not conversely.

Proof: Let A be g-closed in (X, g) then $c_g(A)$ = A. Now let A $\subseteq$ U and U be g-open. Then $c_g(A) = A \subseteq U = i_g(U) \subseteq i_g c_g(U)$,by hypothesis. Therefore $c_g(A) \subseteq i_g c_g(U)$. This implies A is a g_u- closed set in (X, g).

Example 3.4: Let X = {a, b, c} and let g = {∅, {a, b}, {b, c}, X}, then (X, g) is a generalized topological space. Now let A = {b} and U = {a, b}. Then A $\subseteq$ U and $c_g(A) = c_g(\{b\})$ = X and $i_g c_g(U) = i_g c_g(\{a, b\})$=X. Therefore $c_g(A) \subseteq i_g c_g(U)$. Therefore A is a g_u-closed set in (X, g). But $c_g(A) = c_g(\{b\}) = X \neq A$. Hence A is not a g-closed in (X, g).

Remark 3.5: Every g_u-closed set and g-semi closed set in (X, g) are independent to each other.

Example 3.6: Let X = {a, b, c} and let g = {∅, {a, b},{b, c}, X}.Then (X, g) is a generalized topological space. Let A = {b} and U= {a, b}. Then A $\subseteq$ U and $c_g(A) = c_g(\{b\})$ = X and $i_g c_g(U) = i_g c_g(\{a, b\})$ = X. Therefore $c_g(A) \subseteq i_g c_g(U)$. Therefore A is a$g_u$-closed set in (X, g). But $i_g(c_g(A)) = i_g(c_g(\{b\}) = X \not\subseteq A$. Therefore A is not a g-semi closed in (X, g).

Example 3.7: Let X = {a, b, c, d} and let g = {∅,{a}, {c},{a, b},{a, c},{a, b, c},X}.Then (X, g) isa generalized topological space. Let A = {a, b} and U= {a, b}. Then A $\subseteq$ U and$i_g(c_g(A)) = i_g(c_g(\{a, b\})$ = {a, b} $\subseteq$ A.Therefore A is a g-semi closed set in (X, g). But$c_g(A) = c_g(\{a, b\})$ = {a, b, d} and $i_g c_g(U) = i_g c_g(\{a, b\})$ = {a, b}.Therefore $c_g(A) \not\subseteq i_g c_g(U)$. Therefore A is not a$g_u$-closed set in (X, g).

Remark 3.8: Every g_u-closed set and g-pre closed set in (X, g) are independent to each other.

Example 3.9: Let X = {a, b, c, d} and let g = {∅,{a},{a, b},{c, d},{a, b, d},{a, b, c},{a, c, d}, X}.Then (X,g) is a generalized topological space. Let A = {a} and U = {a, b}. Then A ⊆ U and $c_g(A) = c_g(\{a\}) = \{a, b\}$ and $i_gc_g(U) = i_gc_g(\{a, b\}) = \{a, b\}$. Therefore $c_g(A) \subseteq i_gc_g(U)$. Therefore A is a g_u- closed set in (X, g). But $c_g(i_g(A)) = c_g(i_g(\{a\}) = \{a, b\} \not\subseteq A$. Therefore A is not a g-pre closed set in (X, g).

Example 3.10: Let X = {a, b, c, d} and let g = {∅, {a}, {c},{a, b},{a, c},{a, b, c}, X}.Then (X, g) is a generalized topological space. Let A = {b} and U = {a, b}. Then A ⊆ U and $c_g(i_g(A)) = c_g(i_g(\{b\}) = \emptyset \subseteq$ A. Therefore A is a g-pre closed set in (X, g). But $c_g(A) = c_g(\{b\}) = \{b, d\}$ and $i_gc_g(U) = i_gc_g(\{a, b\}) = \{a, b\}$. Therefore $c_g(A) \not\subseteq i_gc_g(U)$. Therefore A is not a g_u- closed set in (X, g).

Remark 3.11: Every g_u-closed set and g-α closed set in (X, g) are independent to each other.

Example 3.12: Let X = {a, b, c} and let g = {∅,{a, b},{b, c}, X }.Then (X, g) is a generalized topological space. Let A = {b} and U = {a, b}. Then A ⊆ U and $c_g(A) = c_g(\{b\}) = X$ and $i_gc_g(U) = i_gc_g(\{a, b\}) = X$. Therefore $c_g(A) \subseteq i_gc_g(U)$. Therefore A is a$g_u$- closed set in (X, g). But $c_g(i_g(c_g(A))) = c_g(i_g(c_g(\{b\})) = X \not\subseteq A$. Therefore A is not a g-α closed set in (X, g).

Example 3.13: Let X = {a, b, c, d} and let g = {∅, {a}, {c},{a, b},{a, c},{a, b, c}, X}.Then (X, g) is a generalized topological space. Let A = {b} and U = {a, b}. Then A ⊆ U and $c_g(i_g(c_g(A))) = c_g(i_g(c_g(\{b\})) = \emptyset \subseteq A$. Therefore A is a g-α closed set in (X, g). But $c_g(A) = c_g(\{b\}) = \{b, d\}$ and $i_gc_g(U) = i_gc_g(\{a, b\}) = \{a, b\}$. Therefore $c_g(A) \not\subseteq i_gc_g(U)$. Therefore A is not a g_u- closed set in (X, g).

Remark 3.14: Every g_u-closed set and g-β closed set in(X, g) are independent to each other.

Example 3.15: Let X={a, b, c, d} and$g$ ={∅,{c},{a, b},{a, b, c},{a, b, d}, X}. Then(X, g) is a generalized topological space. Now let A= {a, b} and U={a, b, d}. Then A ⊆ U and$c_g(A) = c_g\{(a, b)\} = \{a, b, d\}$ and $i_g(c_g(U)) = i_g(c_g(a, b, d)) = \{a, b, d\}$.Therefore $c_g(A) \subseteq i_gc_g(U)$.Therefore A is a g_u closed set in (X, g).But $i_g(c_g(i_g(A))) = \{a, b, d\} \not\subseteq A$. Therefore A is not a g-β closed set in (X, g).

Example 3.16: Let X = {a, b, c, d} and let g = {∅, {a}, {c},{a, b},{a, c},{a, b, c}, X}.Then (X,g) is a generalized topological space. Let A={b} and U={a, b}.Then A ⊆ U and $i_g(c_g(i_g(A)) = i_g(c_g(i_g\{b\})) = \emptyset \subseteq$ A. Therefore A is a g-β closed set in (X, g). But $c_g(A) = c_g(\{b\}) = \{b, d\}$ and $i_g(c_g(U)) = i_g(c_g\{a, b)\} = \{a, b\}$. Therefore $c_g(A) \not\subseteq i_g(c_g(U))$. Therefore A is not a g_u-closed set in (X, g).

Theorem 3.17: If A and B are g_u-closed in (X, g) then A∩B is also g_u-closed in (X, g).

Proof: Let Aand B be any two g_u-closed sets in X and U be any g-open set containing A and B, that is A ⊆ U and B ⊆ U. Then A∩B ⊆ U. We have $c_g(A) \subseteq i_g(c_g(U))$and $c_g(B) \subseteq i_g(c_g(U))$ by hypothesis. Now $c_g(A \cap B) \subseteq c_g(A) \cap c_g(B) \subseteq i_g(c_g(U)) \cap i_g(c_g(U)) = i_g(c_g(U))$. Thus $c_g(A \cap B) \subseteq i_g(c_g(U))$. Hence A∩B is g_u-closed in (X, g)

Theorem 3.18: If a set A is g_u-closed in (X, g) then $c_g(A)$-A contains no non empty g-closed set in (X, g).

Proof: Suppose that A is a g_u- closed set in (X, g). Let F be a g-closed subset of $c_g(A)$-A. Then $F \subseteq A^c$ and hence $A \subseteq F^c$. Since F^c is g-open and A is a g_u-closed set. $c_g(A) \subseteq F^c$. This implies $F \subseteq (c_g(A))^c$. But $F \subseteq c_g(A)$ and $c_g(A) \cap (c_g(A))^c = \emptyset$. Therefore F = ∅. Hence the theorem is proved.

Theorem 3.19: In a generalized topological space X, for each $x \in X$, $\{x\}$ is g-closed or its complement X-$\{x\}$ is g_u-closed in (X, g).

Proof: Suppose that $\{x\}$ is not g-closed in (X, g).Then X-$\{x\}$ is not g-open and the only g-open set containing X-$\{x\}$ is X. Therefore $c_g(X-\{x\}) \subseteq X \subseteq i_g(c_g(X))$. Therefore X-$\{x\}$ is g_u-closed in (X, g).

Theorem 3.20: If A is g-open and g_u-closed in (X, g) then A isg-semi open in (X, g).

Proof: Since A is g-open and g_u-closed, we have $A \subseteq A$ and$c_g(A) \subseteq i_g c_g(A)$.Now $A \subseteq c_g(A) \subseteq i_g c_g(A) \subseteq c_g(A)=c_g(i_g(A))$. Then $A \subseteq c_g(i_g(A))$. Thus A is g-semi open in (X, g).

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Corresponding author: Dhanadeepa P

Department of Mathematics, Avinashilingam University,
Coimbatore, Tamilnadu, India