

ON $G^*B\omega$ - CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT : The aim of this paper is to introduce the concepts of closed sets in bitopological space called (i, j) - generalized star $b\omega$ - closed sets, (i, j) - generalized star $b\omega$ - open sets and study their basic properties.

KEYWORDS : (i, j) - generalized star $b\omega$ - closed sets, (i, j) - generalized star $b\omega$ - open sets.

I. INTRODUCTION

Generalized closed sets form a stronger tool in the characterization of bitopological spaces. The study of bitopological spaces was initiated by Kelly [8] and thereafter a large number of papers have been done to generalize the topological concepts to bitopological setting. Fukutake [5] introduced g - closed sets in bitopological spaces. Abo Khadra and Nasef [1] discussed b - open sets in bitopological spaces. Alswidi et al. [2] introduced a new notions on an $ij - \omega$ - closed sets in bitopological spaces.

In this paper, a new class of sets in bitopological spaces called (i, j) - $g^*b\omega$ - closed sets is introduced. A comparative study has been done with already existing closed sets and (i, j) - $g^*b\omega$ - closed sets.

II. PRELIMINARIES

A triple (X, τ_1, τ_2) where X is a non empty set and τ_1 and τ_2 are topologies on X is called a bitopological space. For a subset A of (X, τ_1, τ_2) , the closure of A and the interior of A with respect to τ_i is denoted by $i - cl(A)$ and $i - int(A)$ respectively for $i = 1, 2$. The intersection of all τ_i - closed sets containing A is called $i - cl(A)$. The union of all τ_i - open sets contained in A is $i - int(A)$.

Definition 2.1 For $i, j = 1, 2$ and $i \neq j$, a subset A of a bitopological space (X, τ_1, τ_2) is called

- (i) **(i, j) - semi closed** (Maheswari et al., 1977 - 78) if $j - int(i - cl(A)) \subseteq A$.
- (ii) **(i, j) - α - closed** [7] if $i - cl(j - int(i - cl(A))) \subseteq A$.
- (iii) **(i, j) - pre closed** [7] if $i - cl(j - int(A)) \subseteq A$.
- (iv) **(i, j) - regular closed** [4] if $i - cl(j - int(A)) \subseteq A$.
- (v) **(i, j) - b - closed** [3] if $(j - int(i - cl(A))) \cup (i - cl(j - int(A))) \subseteq A$.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 5, May 2014

The complements of the above mentioned sets are called (i, j) - semi open, (i, j) - α - open, (i, j) - pre open, (i, j) - regular open and (i, j) - b - open sets respectively.

The intersection of all τ_j - semi closed (resp. τ_j - α - closed, τ_j - pre closed, τ_j - regular closed and τ_j - b - closed) subsets of (X, τ) containing A is called the τ_j - semi closure (resp. τ_j - α - closure, τ_j - pre closure, τ_j - regular closure and τ_j - b - closure) of A and is denoted by τ_j - scl(A) (resp. τ_j - α cl(A), τ_j - pcl(A), τ_j - rcl(A) and τ_j - bcl(A)).

Definition 2.2 For $i, j = 1, 2$ and $i \neq j$, a subset A of a bitopological space (X, τ_1, τ_2) is called

- (i) **(i, j) - generalized closed** (briefly, **(i, j) - g - closed**) [5] if τ_j - cl(A) \subseteq U whenever $A \subseteq U$ and U is τ_i - open in X .
- (ii) **(i, j) - regular generalized closed** (briefly, **(i, j) - rg - closed**) [3] if τ_j - cl(A) \subseteq U whenever $A \subseteq U$ and U is τ_i - regular open in X .
- (iii) **(i, j) - weakly generalized closed** (briefly, **(i, j) - wg - closed**) [6] if τ_j - cl(A)[τ_i - int(A)] \subseteq U whenever $A \subseteq U$ and U is τ_i - open in X .
- (iv) **(i, j) - generalized star closed** (briefly, **(i, j) - g* - closed**) [13] if τ_j - cl(A) \subseteq U whenever $A \subseteq U$ and U is τ_i - g - open in X .
- (v) **(i, j) - generalized α - closed** (briefly, **(i, j) - g α - closed**) [9] if τ_j - α cl(A) \subseteq U whenever $A \subseteq U$ and U is τ_i - α - open in X .
- (vi) **(i, j) - α - generalized closed** (briefly, **(i, j) - α g - closed**) [12] if τ_j - α cl(A) \subseteq U whenever $A \subseteq U$ and U is τ_i - open in X .
- (vii) **(i, j) - generalized star pre closed** (briefly, **(i, j) - g*p - closed**) [14] if τ_j - pcl(A) \subseteq U whenever $A \subseteq U$ and U is τ_i - g - open in X .
- (viii) **(i, j) - generalized star semi closed** (briefly, **(i, j) - g*s - closed**) [12] if τ_j - scl(A) \subseteq U whenever $A \subseteq U$ and U is τ_i - gs - open in X .
- (ix) **(i, j) - generalized # semi closed** (briefly, **(i, j) - g#s - closed**) [15] if τ_j - scl(A) \subseteq U whenever $A \subseteq U$ and U is τ_i - α g - open in X .

The complement of the above mentioned sets are called their respective open sets.

III. (i, j) - g*b ω - CLOSED SETS

In this section, the concept of (i, j) - g*b ω - closed sets in bitopological spaces is defined and some of their characterizations and properties are studied.

Definition 3.1 A subset A of a bitopological space (X, τ_1, τ_2) is called **(i, j) - generalized star b omega closed** (briefly, **(i, j) - g*b ω - closed**) if τ_j - bcl(A) \subseteq U whenever $A \subseteq U$ and U is τ_i - gs - open in (X, τ_1, τ_2) , where $i, j = 1, 2$ and $i \neq j$.

The set of all (i, j) - g*b ω - closed sets in (X, τ_1, τ_2) is denoted by $G^*b\omega C(i, j)$.

Remark 3.2 By setting $\tau_i = \tau_j$ in definition 3.1, an (i, j) - g*b ω - closed set is a g*b ω - closed set [11].

Example 3.3 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $\emptyset, X, \{b\}, \{c\}, \{b, c\}$ are $(1, 2)$ - g*b ω - closed.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 5, May 2014

Theorem 3.4 Every τ_j - closed (resp. τ_j - semi closed, τ_j - α - closed, τ_j - pre closed, τ_j - regular closed) set in (X, τ_1, τ_2) is (i, j) - $g^*b\omega$ - closed.

Proof: Let A be τ_j - closed (resp. τ_j - semi closed, τ_j - α - closed, τ_j - pre closed, τ_j - regular closed) in (X, τ_1, τ_2) such that $A \subseteq U$, where U is τ_i - g_s - open. Since A is τ_j - closed (resp. τ_j - semi closed, τ_j - α - closed, τ_j - pre closed, τ_j - regular closed), τ_j - $cl(A)$ (resp. τ_j - $scl(A)$, τ_j - $\alpha cl(A)$, τ_j - $pcl(A)$, τ_j - $rcl(A)$) = $A \subseteq U$. But τ_j - $bcl(A) \subseteq \tau_j$ - $cl(A)$ (resp. τ_j - $scl(A)$, τ_j - $\alpha cl(A)$, τ_j - $pcl(A)$, τ_j - $rcl(A)$). Therefore τ_j - $bcl(A) \subseteq U$. Hence A is an (i, j) - $g^*b\omega$ - closed set in (X, τ_1, τ_2) .

The converse of the above theorem is not true in general as can be seen from the following examples:

Example 3.5 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$. The subsets $\{b\}$ is $(1, 2)$ - $g^*b\omega$ - closed but not τ_2 - closed.

Example 3.6 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. The subsets $\{a, c\}$ is $(1, 2)$ - $g^*b\omega$ - closed but not τ_2 - semi closed.

Example 3.7 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. The subsets $\{a\}, \{b\}$ are $(1, 2)$ - $g^*b\omega$ - closed but not τ_2 - α - closed, not τ_2 - pre closed and not τ_2 - regular closed.

Theorem 3.8 Every (i, j) - g^*s - closed set in (X, τ_1, τ_2) is (i, j) - $g^*b\omega$ - closed.

Proof: Let $A \subseteq U$ and U be τ_i - g_s - open in (X, τ_1, τ_2) . Since A is (i, j) - g^*s - closed in (X, τ_1, τ_2) , τ_j - $scl(A) \subseteq U$. But τ_j - $bcl(A) \subseteq \tau_j$ - $scl(A) \subseteq U$. Therefore A is (i, j) - $g^*b\omega$ - closed.

The converse of the above theorem is not true in general as can be seen from the following example:

Example 3.9 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b, c\}\}$. The subsets $\{b\}, \{c\}, \{a, b\}$ and $\{a, c\}$ are $(1, 2)$ - $g^*b\omega$ - closed but not $(1, 2)$ - g^*s - closed.

Remark 3.10 The following examples show that (i, j) - $g^*b\omega$ - closed set is independent from (i, j) - semi closed set, (i, j) - α - closed set and (i, j) - pre closed set.

Example 3.11 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. The subset $\{a, c\}$ is $(1, 2)$ - semi closed, $(1, 2)$ - α - closed and $(1, 2)$ - pre closed but not $(1, 2)$ - $g^*b\omega$ - closed.

Example 3.12 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. The subset $\{a, c\}$ is $(1, 2)$ - $g^*b\omega$ - closed but not $(1, 2)$ - semi closed, not $(1, 2)$ - α - closed and not $(1, 2)$ - pre closed.

Remark 3.13 The following examples show that (i, j) - $g^*b\omega$ - closed set is independent from (i, j) - regular closed set and (i, j) - g - closed set, (i, j) - wg - closed.

Example 3.14 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$. The subset $\{a, c\}$ is $(1, 2)$ - regular closed and $(1, 2)$ - g - closed $(1, 2)$ - wg - closed but not $(1, 2)$ - $g^*b\omega$ - closed.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 5, May 2014

Example 3.15 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$. The subset $\{b\}$ is $(1, 2)$ - $g^*b\omega$ -closed but not $(1, 2)$ -regular closed and not $(1, 2)$ - g -closed, not $(1, 2)$ - wg -closed.

Remark 3.16 The following examples show that (i, j) - $g^*b\omega$ -closed set is independent from (i, j) - rg -closed set and (i, j) - g^* -closed set.

Example 3.17 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. The subset $\{a, c\}$ is $(1, 2)$ - rg -closed $(1, 2)$ - g^* -closed but not $(1, 2)$ - $g^*b\omega$ -closed.

Example 3.18 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. The subset $\{b\}$ is $(1, 2)$ - $g^*b\omega$ -closed but not $(1, 2)$ - rg -closed, not $(1, 2)$ - g^* -closed.

Remark 3.19 The following examples show that (i, j) - $g^*b\omega$ -closed set is independent from (i, j) - ga -closed set and (i, j) - g^*p -closed set.

Example 3.20 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. The subset $\{a, c\}$ is $(1, 2)$ - ga -closed and $(1, 2)$ - g^*p -closed but not $(1, 2)$ - $g^*b\omega$ -closed.

Example 3.21 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. The subsets $\{a\}, \{b\}$ are $(1, 2)$ - $g^*b\omega$ -closed but not $(1, 2)$ - ga -closed and not $(1, 2)$ - g^*p -closed.

Remark 3.22 The following examples show that the concepts (i, j) - ag -closed set and (i, j) - $g^*b\omega$ -closed set are independent.

Example 3.23 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$. The subset $\{a, c\}$ is $(1, 2)$ - ag -closed but not $(1, 2)$ - $g^*b\omega$ -closed.

Example 3.24 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. The subsets $\{a\}, \{b\}$ are $(1, 2)$ - $g^*b\omega$ -closed but not $(1, 2)$ - ag -closed.

Remark 3.25 The following examples show that the concepts (i, j) - $g^{\#s}$ -closed set and (i, j) - $g^*b\omega$ -closed set are independent.

Example 3.26 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$. The subset $\{a, c\}$ is $(1, 2)$ - $g^{\#s}$ -closed but not $(1, 2)$ - $g^*b\omega$ -closed.

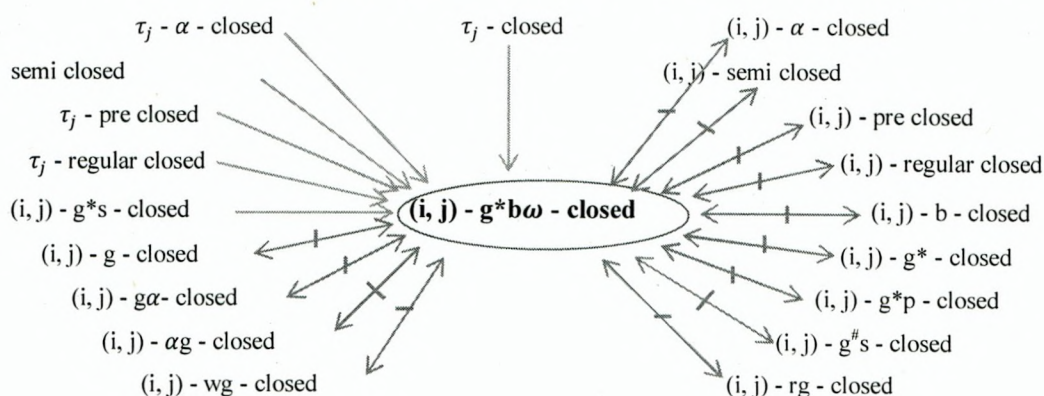
Example 3.27 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b, c\}\}$. The subsets $\{b\}, \{c\}, \{a, b\}$ and $\{a, c\}$ are $(1, 2)$ - $g^*b\omega$ -closed but not $(1, 2)$ - $g^{\#s}$ -closed.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 5, May 2014

The following diagram shows the relationships of (i, j) - $g^*b\omega$ - closed sets with other sets:



where $A \longrightarrow B$ represents A implies B and $A \longleftrightarrow B$ represents A and B are independent.

Remark 3.28 Union of two (i, j) - $g^*b\omega$ - closed sets need not be (i, j) - $g^*b\omega$ - closed as can be seen from the following example:

Example 3.29 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Let $A = \{a\}$ and $B = \{b\}$. Then $A \cup B = \{a, b\}$ is not $(1, 2)$ - $g^*b\omega$ - closed but $A = \{a\}$ and $B = \{b\}$ are $(1, 2)$ - $g^*b\omega$ - closed.

Remark 3.30 Difference of two (i, j) - $g^*b\omega$ - closed sets need not be (i, j) - $g^*b\omega$ - closed set as can be seen from the following example:

Example 3.31 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. Let $A = \{a, c\}$ and $B = \{c\}$. Then A and B are $(1, 2)$ - $g^*b\omega$ - closed but $A \setminus B = \{a\}$ is not $(1, 2)$ - $g^*b\omega$ - closed.

Theorem 3.32 If a subset A of a bitopological space (X, τ_1, τ_2) is (i, j) - $g^*b\omega$ - closed then $\tau_j - \text{bcl}(A) \setminus A$ contains no nonempty τ_i - gs - closed set.

Proof: Let A be an (i, j) - $g^*b\omega$ - closed set and F be a τ_i - gs - closed set such that $F \subseteq \tau_j - \text{bcl}(A) \setminus A$. Therefore $A \subseteq F^c$ and $F \subseteq \tau_j - \text{bcl}(A)$. Since F^c is τ_i - gs - open and A is (i, j) - $g^*b\omega$ - closed, $\tau_j - \text{bcl}(A) \subseteq F^c$. Thus $F \subseteq [\tau_j - \text{bcl}(A)]^c = X \setminus [\tau_j - \text{bcl}(A)]$. Hence $F \subseteq [\tau_j - \text{bcl}(A)] \cap [X \setminus [\tau_j - \text{bcl}(A)]] = \emptyset$. Therefore $F = \emptyset$. Hence $\tau_j - \text{bcl}(A) \setminus A$ contains no nonempty τ_i - gs - closed set.

Theorem 3.33 Let A be an (i, j) - $g^*b\omega$ - closed set in (X, τ_1, τ_2) . Then A is τ_j - b - closed if and only if $\tau_j - \text{bcl}(A) \setminus A$ is τ_i - gs - closed in (X, τ_1, τ_2) .

Proof: Suppose that A is (i, j) - $g^*b\omega$ - closed. Let A be τ_j - b - closed. Then $\tau_j - \text{bcl}(A) = A$. Therefore $\tau_j - \text{bcl}(A) \setminus A = \emptyset$ is τ_i - gs - closed in (X, τ_1, τ_2) .

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 5, May 2014

Conversely, suppose that A is (i, j) - $g^*b\omega$ -closed and $\tau_j - \text{bcl}(A) \setminus A$ is τ_i -gs-closed. Since A is (i, j) - $g^*b\omega$ -closed, $\tau_j - \text{bcl}(A) \setminus A$ contains no nonempty τ_i -gs-closed set (by Theorem 3.32). Since $\tau_j - \text{bcl}(A) \setminus A$ is τ_i -gs-closed, $\tau_j - \text{bcl}(A) \setminus A = \emptyset$. Then $\tau_j - \text{bcl}(A) = A$. Hence A is τ_j -b-closed.

Theorem 3.34 Let A and B be subsets of (X, τ_1, τ_2) such that $A \subseteq B \subseteq \tau_j - \text{bcl}(A)$. If A is (i, j) - $g^*b\omega$ -closed then B is (i, j) - $g^*b\omega$ -closed.

Proof: Let A and B be subsets such that $A \subseteq B \subseteq \tau_j - \text{bcl}(A)$. Suppose that A is (i, j) - $g^*b\omega$ -closed. Let $B \subseteq U$ and U be τ_i -gs-open in (X, τ_1, τ_2) . Then $A \subseteq U$. Since A is (i, j) - $g^*b\omega$ -closed, $\tau_j - \text{bcl}(A) \subseteq U$. Since $B \subseteq \tau_j - \text{bcl}(A)$, $\tau_j - \text{bcl}(B) \subseteq \tau_j - \text{bcl}[\tau_j - \text{bcl}(A)] = \tau_j - \text{bcl}(A) \subseteq U$. Therefore B is (i, j) - $g^*b\omega$ -closed.

Remark 3.35 In general an (i, j) - $g^*b\omega$ -closed set need not be equal to an (j, i) - $g^*b\omega$ -closed set.

Example 3.36 consider $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then the subsets $\{a\}$ and $\{a, c\}$ are $(1, 2)$ - $g^*b\omega$ -closed but not $(2, 1)$ - $g^*b\omega$ -closed.

Theorem 3.37 If $\tau_1 \subseteq \tau_2$ in (X, τ_1, τ_2) then $G^*b\omega C(2, 1) \subseteq G^*b\omega C(1, 2)$.

Proof: Let $A \in G^*b\omega C(2, 1)$. Let $U \in \text{GSO}(X, \tau_1)$ such that $A \subseteq U$. Since $\text{GSO}(X, \tau_1) \subseteq \text{GSO}(X, \tau_2)$, $U \in \text{GSO}(X, \tau_2)$. Since A is $(2, 1)$ - $g^*b\omega$ -closed, $\tau_1 - \text{bcl}(A) \subseteq U$. Since $\tau_1 \subseteq \tau_2$, $\tau_2 - \text{bcl}(A) \subseteq \tau_1 - \text{bcl}(A)$. Thus $\tau_2 - \text{bcl}(A) \subseteq U$. Hence A is $(1, 2)$ - $g^*b\omega$ -closed. That is, $A \in G^*b\omega C(1, 2)$.

The converse of the above theorem need not be true as seen from the following example:

Example 3.38 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $G^*b\omega C(2, 1) \subseteq G^*b\omega C(1, 2)$ but $\tau_1 \not\subseteq \tau_2$.

IV. (i, j) - $g^*b\omega$ -OPEN SETS

In this section, (i, j) - $g^*b\omega$ -open sets in bitopological spaces is introduced and their properties are studied.

Definition 4.1 A set A of a bitopological space (X, τ_1, τ_2) is called (i, j) -generalized star b omega open (briefly, (i, j) - $g^*b\omega$ -open) if its complement is (i, j) - $g^*b\omega$ -closed.

The set of all (i, j) - $g^*b\omega$ -open sets in (X, τ_1, τ_2) is denoted by $G^*b\omega O(i, j)$.

Theorem 4.2 A subset A of a bitopological space (X, τ_1, τ_2) is (i, j) - $g^*b\omega$ -open if and only if $F \subseteq \tau_j - \text{bint}(A)$ whenever $F \subseteq A$ and F is τ_i -gs-closed in (X, τ_1, τ_2) .

Proof: Suppose that A is (i, j) - $g^*b\omega$ -open. Let $F \subseteq A$ and F be τ_i -gs-closed. Then $A^c \subseteq F^c$ and F^c is τ_i -gs-open. Since A^c is (i, j) - $g^*b\omega$ -closed, $\tau_j - \text{bcl}(A^c) \subseteq F^c$. Since $\tau_j - \text{bcl}(A^c) = [\tau_j - \text{bint}(A)]^c$, $[\tau_j - \text{bint}(A)]^c \subseteq F^c$. Hence $F \subseteq \tau_j - \text{bint}(A)$.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 5, May 2014

Conversely, suppose that $F \subseteq \tau_j$ -bint(A) whenever $F \subseteq A$ and F is τ_i -gs-closed in (X, τ_1, τ_2) . Let U be τ_i -gs-open in (X, τ_1, τ_2) and $A^c \subseteq U$. Then U^c is τ_i -gs-closed and $U^c \subseteq A$. Hence by assumption $U^c \subseteq \tau_j$ -bint(A). That is τ_j -bcl(A^c) $\subseteq U$. Therefore A^c is (i, j) - $g^*b\omega$ -closed. Hence A is (i, j) - $g^*b\omega$ -open.

Theorem 4.3 If a subset A is (i, j) - $g^*b\omega$ -closed in (X, τ_1, τ_2) then τ_j -bcl(A) \setminus A is (i, j) - $g^*b\omega$ -open.

Proof: Suppose that A is (i, j) - $g^*b\omega$ -closed in (X, τ_1, τ_2) . Let $F \subseteq \tau_j$ -bcl(A) \setminus A and F be τ_i -gs-closed. Since A is (i, j) - $g^*b\omega$ -closed, τ_j -bcl(A) \setminus A does not contain nonempty τ_i -gs-closed sets (by Theorem 3.32). Hence $F = \emptyset$. Thus $F \subseteq \tau_j$ -bint[τ_j -bcl(A) \setminus A]. Hence τ_j -bcl(A) \setminus A is (i, j) - $g^*b\omega$ -open.

Theorem 4.4 If a set A is (i, j) - $g^*b\omega$ -open in (X, τ_1, τ_2) then $G = X$ whenever G is τ_i -gs-open and τ_j -bint(A) $\cup A^c \subseteq G$.

Proof: Suppose that A is (i, j) - $g^*b\omega$ -open in (X, τ_1, τ_2) , G is τ_i -gs-open and τ_j -bint(A) $\cup A^c \subseteq G$. Then $G^c \subseteq \{\tau_j$ -bint(A) $\cup A^c\}^c = \tau_j$ -bcl(A^c) \setminus A^c. Since A^c is (i, j) - $g^*b\omega$ -closed, τ_j -bcl(A^c) \setminus A^c contains no nonempty τ_i -gs-closed set in (X, τ_1, τ_2) (by Theorem 3.32). Therefore $G^c = \emptyset$. Hence $G = X$.

Remark 4.5 The converse of the above theorem is not true in general as can be seen from the following example:

Example 4.6 Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Let $A = \{c\}$ and $G = X$. Then G is τ_1 -gs-open, τ_2 -bint(A) $\cup A^c = \emptyset \cup \{a, b\} = \{a, b\} \subseteq G$, but $A = \{c\}$ is not $(1, 2)$ - $g^*b\omega$ -open.

Theorem 4.7 Let (X, τ_1, τ_2) be a bitopological space. If $x \in X$ then singleton $\{x\}$ is either τ_i -gs-closed or (i, j) - $g^*b\omega$ -open.

Proof: Let $x \in X$ and suppose that $\{x\}$ is not τ_i -gs-closed. Then $X \setminus \{x\}$ is not τ_i -gs-open. Consequently, X is the only τ_i -gs-open set containing the set $X \setminus \{x\}$. Therefore $X \setminus \{x\}$ is (i, j) - $g^*b\omega$ -closed. Hence $\{x\}$ is (i, j) - $g^*b\omega$ -open.

V. CONCLUSION

In this research, we introduce the concept of $g^*b\omega$ -continuous, closed maps in these spaces and present some results.

REFERENCES

1. Abo Khadra, A.A. and Nasef, A.A., On extension of certain concepts from a topological space to a bitopological space, *Pro. Math. Phys. Soc. Egypt*, 79, 91 - 102, 2003.
2. Alswidi, L.A. and Alhosaini, A.M.A., Weak forms of ω -open sets in Bitopological spaces and connectedness, *European Journal of Scientific Research*, 52(2), 204 - 212, 2011.
3. Arockiarani, I., Balachandran, K. and Ganster, M., On regular generalized locally closed sets and RGL-continuous functions, *Indian J. Pure. Appl. Math.*, 28, 661 - 669, 1997.
4. Bose, S. and Sinha, D., Almost open, almost closed, θ -continuous and almost compact mappings in bitopological spaces, *Bull. Calcutta Math. Soc.*, 73, 345 - 354, 1981.
5. Fukutake, T., On generalized closed sets in bitopological spaces, *Bull. Fukuoka Univ. Ed. Part III*, 35, 19 - 28, 1986.
6. Fukutake, T., Sundaram, P. and Nagaveni, N., On Weakly generalized closed sets, Weakly generalized continuous maps and T_{wg} -spaces in bitopological spaces, *Bull. Fukuoka Univ. Ed. Part III*, 48, 33 - 40, 1999.
7. Jelic, M., A decomposition of pairwise continuity, *J. Inst. Math. Comp. Sci.*, 3, 25 - 29, 1990.
8. Kelly, J.C., Bitopological spaces, *Proc. London Math. Soc.*, 13, 71 - 89, 1963.
9. Khedr, F.H. and Hanan S. Al Saddi., On pairwise semi generalized closed sets, *JKAU: Sci.*, 21, (2), 269 - 295, 2009 A.D/1430 A.H.



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10. Maheswari, S.N. and Prasad, R., Semi open sets and semi continuous functions in bitopological spaces, *Math Notae*, 26, 29 - 37, 1977 - 78.
11. Parvathi, A., Priyadharsini, P. and Chandrika, G.K., On $g^*b\omega$ - closed sets in Topological spaces, *Int. J. adv. Sci. tech. research*, 2(6), 318 - 329, 2012.
12. Selvanayaki, N., On g^*s - closed sets in Bitopological Spaces, *Int. J. Math. Archive*, 3(5), 1991 - 1995, 2012.
13. Sheik John, M. and Sundaram, P., g^* - closed sets in bitopological spaces, *Indian J. Pure. Appl. Math.*, 35(1), 71 - 80, 2004.
14. Vadivel, A. and Swaminathan, A., g^*p - closed sets in bitopological spaces, *J. Advance Studies in topology*, 3(1), 81 - 8, 2012.
15. Veronica, V. and Reena, K., g^*s - closed sets in Bitopological Spaces, *Int. J. Math. Archive*, 3(2), 556 - 565, 2012.