Introduction:

Fuzzy set are sets whose elements have degrees of membership. Fuzzy sets were introduced by Lotfi A. Zadeh and Dieter Klaua in 1965 as an extension of the classical notion of set. Fuzzy set are used in different areas, such as linguistics 9Dw cock, Bodenhofer and Kerre 2000), decision making (Kuzmin 1982), and clustering (Bezdex 1978).

In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition- an element either belong or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval [0, 1].

Example:

Word like young, tall or high are fuzzy.

- There is no single quantitative value which defines the term young.
- For some people, age 25 is young, and for others, age 35 is young.
- The concept young has no clean boundary.
- Age 35 has some possibility of being young and usually depends on the context in which it is being considered

So, we have fuzzy membership

U = {15,20,25,30,35}

= {(15, 0.9) (20, 0.7) (25, 0.8) (30, 0.6) (35, 05)}

Intuitionistic fuzzy sets are developed from the fuzzy set, and the set whose elements have degree of membership and non-membership. Intuitionistic fuzzy set have been introduced by Krassimir Atanassor (1983) as an extension of Lotfi Zadeh's notion of a set. The theory of intuitionistic fuzzy sets futher extends both concepts by allowing the assessment of the elements by two function; μ for membership and ν for the non-membership, which belong to the real unit interval [0, 1].

Intuitionistic fuzzy sets are used in decision making problem, medical problems etc, Recently various applications of intuitionistic fuzzy sets to artificial intelligence have applied-intuitionistic fuzzy expert system, intuitionistic fuzzy neural networks, intuitionistic fuzzy decision making, intuitionistic fuzzy mechine learning ect.

Pythagorean fuzzy sets (PFSs), originally proposed by Yager (Yager, Abbasov. Int J Intell Syst 2013), are a new tool to deal with vagueness considering the membership grades are pairs (μ, V) satisfying the condition $\mu^2 + V^2 \leq 1$. As a generalized set, PFSs have closed relationship with intuitionistic fuzzy sets (IFSs). PFSs can be reduced to IFSs satisfying the condition $\mu + V \leq 1$.

However, the related operations of PFSs do not take different conditions into consideration. To better understand PFSs, we propose two operations: division and subtraction, and discuss their properties in detail. Then, based on Pythagorean fuzzy aggregation operators, their properties such as boundedness, idempotency, and monotonicity are investigated.

Later, we develop a Pythagorean fuzzy superiority and inferiority ranking method to solve uncertainty problem. Finally, an illustrative example for evaluating the internet stocks performance is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Example:

Given that $U = \{u_1, u_2, u_3\}$ is a set of three cars under consideration of a decision maker to purchase is given as follows:

$F_P = \{(u_1, (0.6, 0.3)), (u_2, (0.5, 0.7)), (u_3, (0.6, 0.3))\}$,

In 1999, Molodtsov introduced soft sets and established the fundamental results of the new theory. It is a general mathematical tool for dealing with objects which have been defined using a very loose and hence very general set of characteristics. A soft set is a collection of approximate descriptions of an object. Each approximate description has two parts: a predicate and an approximate value set. In classical mathematics, we construct a mathematical model of an object and define the notion of the exact solution of this model.

Usually the mathematical model is too complicated and we cannot find the exact solution So, in the second step, we introduce the notion of approximate solution and calculate that solution. In the Soft Set Theory (SST), we have the opposite approach to this problem. The initial description of the object has an approximate nature, and we do not need to introduce the notion of exact solution.

The absence of any restrictions on the approximate description in SST makes this theory very convenient and easily applicable in practice. We can use any parametrization we prefer with the help of words and sentences, real numbers, functions, mappings, and so on. Molodtsov also showed how SST is free from parametrization inadequacy syndrom of Fuzzy Set Theory (FST), Rough Set Theory (RST), Probability Theory, and Game Theory. SST is a very general framework. Many of the established paradigms appear as special cases of SST.

Maji et al. initiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union, intersection, complement of a fuzzy set, De Morgen's law etc. Neog and Sut have reintroduced the notion of fuzzy soft sets and redefined the complement of fuzzy soft set accordingly. They have shown that the modified definition of the complement of a fuzzy soft set meets all the requirements that the complement of a set in classical sense really does. Applications of fuzzy soft set theory in many disciplines and real life situations have been studied by many researchers.

Example:

Suppose a fuzzy soft set (F, E) describes attractiveness of the shirts with respect to the given parameter, which the authors are going to wear.

U = $\{x_1,\ x_2, x_3, x_4, x_5\}$ which is the set of all shirts under consideration. Let I^U be the collection of all fuzzy subsets of U. Also let

$$E = \{e_1 = "color\ full, e_2 = "bright", e_3 = "cheap", e_4 = "warm\}$$

Let

$$F\ (e_1) = \{(x_1, 0.3)\ (x_2, 0.5)(x_3, 0.7)(x_4, 0.6)(x_5, 0)\}$$

F (e_2) = $\{(x_1, 0.1)\ (x_2, 0)\ (x_3, 0.3)(x_4, 0.7)(x_5, 0.9)\}$

F (e_3) = $\{(x_1, 0.4)\ (x_2, 0.2)(x_3, 0.7)(x_4, 0.5)(x_5, 0.8)\}$

F (e_4) = $\{(x_1, 0.7)\ (x_2, 0.1)(x_3, 0.3)(x_4, 0.5)(x_5, 0.7)\}$

Then the family {F (e_i), i = 1, 2, 3, 4} of I^U is a fuzzy set (F, E).

Maji and his coworker are introduced the notion of intuitionstic fuzzy soft set which is based on a combination of the intuitionstic fuzzy set and soft set models and studied the properties of intuitionstic fuzzy soft set. Later, a lot of extentions of intuitionstic fuzzy soft are appeared such as the generalized intuitionstic fuzzy soft set, possibility intuitionstic fuzzy soft set, etc.

Example:

(F, A) = {F (e_1) = (a, 0.3,0.2)}

(G, B) = {G (e_1) = (a, 0.4, 0.5)}

(H, C) = {H (e_1) = (a, 0.3, 0.6)}

(F, A) $\cap$ (G, B) $\rightarrow$ (H, C) = [max{max(0.2, min(0.3, 0.4)), 0.3}, min{min (0.3, 0.5), 0.6))} = (0.5,0.3)

(F, A) $\cap$ (G, B) = {(a, 0.3, 0.5)}

A Pythagorean fuzzy soft set (PFSs) is a parameterized family of Pythagorean fuzzy sets and a generalization of intuitionistic fuzzy soft sets. A Pythagorean fuzzy soft set (PFSs) model is an extension of an intuitionistic fuzzy soft sets (IFSs) model to deal with vague knowledge according to different parameters. The PFSs model is a more powerful tool for expressing uncertain information when making decisions and it relaxes the constraint of IFSs. As for the problem, the purpose of this paper is to extend the concept of Pythagorean fuzzy soft set by introducing a possibility of each element in the universe which is attached with the parameterization of Pythagorean fuzzy sets while defining a Pythagorean fuzzy soft set, from which we can obtain a possibility Pythagorean fuzzy soft set model.

Example:

Given that $U = \{u_1, u_2, u_3\}$ is a set of three cars under consideration of a decision maker to purchase, $E = \{e_1 = \textit{low fuel consumption}, e_2 = \textit{high safety}\}$ is a set of parameters. Suppose that F_P: $E \rightarrow PF(U) \times PF(U)$ is given as follows:

F_P (e_1) = {(u_1 /(0.9, 0.2), (0.6, 0.3)), (u_2/(0.8, 0.2),(0.5, 0.7)), (u_3/(0.5, 0.6), (0.6, 0.3))},

F_P (e_2) = {(u_1 /(0.7, 0.3), (0.6, 0.2)), (u_2/(0.9, 0.2)(0.4, 0.5)), (u_3/(0.4, 0.9), (0.8, 0.1))},

In this paper we organized as follows. In this we briefly recall some basic notions. puts forward the concept of possibility Pythagorean fuzzy soft set and explores some of its interesting properties and gives a similarity measure to compare two possibility Pythagorean fuzzy soft sets to deal with decision problems. The utility of the possibility Pythagorean fuzzy soft set is shown by using the similarity measure to solve construction project contractor selection problems.

This paper aims to introduce a new notion called pythagorean soft cubic sets and using cubic sets, Pythagorean cubic sets and soft sets. We also define some new notions such as internal (external) pythagorean soft cubic sets. P-(R-)order, P-(R-)union, P-(R-)intersection are introduced, and related properties are investigated We also investigate some of the core properties of pythagorean soft cubic set.

Review of Literature:

In most real problems, uncertainty can be seen everywhere. In order to cope with the uncertainties, many uncertain theories are put forward such as fuzzy set [1]. In 2006 he analyze a new concept of FM d-value is defined to quantify the divergence of two sets of values. Further he analyze the asymptotic property of FM-test, and then establish the relationship between FM d-valueand p-value [2].

Ismat Beg, Samina Ashraf (2016) they propose a new set of axioms that a value in the interval [0,1] should satisfy to be a degree or a measure of similarity between fuzzy subsets of a given universe. The relevance of the new axioms with previous axioms and categories of similarity measures for fuzzy sets is also studied [3].

In 1983 K. T. Atanassov introduce the concept of Generalized Atanassov's Intuitionistic Fuzzy Index. We characterize it in terms of fuzzy implication operators and we propose a construction method with automorphisms [4], H. Bustince and P. Burillo are recapitulate the definition given by Atanassov (1983) of intuitionistic fuzzy sets as well as the definition of vague sets given by Gau and Buehrer (1993) and see that both definitions coincide [5].

R.K. Verma B.D. Sharma (2012) they characterized the fundamental of interval valued intuitionistic fuzzy sets is that the values of its membership function and non-membership function are intervals rather than exact number [6].

1999, [7] Molodsov initiated the novel concept of soft set theory which is a completely new approach for modeling vagueness and uncertainty [10]. Since the proposal of the theory of soft sets, it has made great achievements both in theory [11 - 15]. and in application [16].

By integrating soft set with interval-valued fuzzy set [17], Yang et al. [18] proposed a multi-fuzzy soft set model. Alkhazaleh et al. [19] defined the concept of possibility fuzzy soft sets where a possibility of each element in the universe is attached with the parameterization of fuzzy sets while defining a fuzzy soft set.

In [20] Maji et al. intiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union , intersection, complement of fuzzy soft set. Moreover in [21] Maji et al extended soft sets to intuitionistic fuzzy soft sets and Neutrosophic soft sets.

[24]Chetia and Das (2012) extended thematrix representation of soft set to fuzzy soft set and intuitionistic fuzzy soft matrix, respectively, and applied it to decision-making problems. Various other researchers have also worked on the concept of soft set and soft matrices which are available in the literature.

In recent years, in order to cope with decision making problems more effectively, Peng et al. [25] has extended fuzzy soft sets to Pythagorean fuzzy environment, and developed the concept of Pythagorean fuzzy soft sets.

Y. B. Jun et al[28]., introduced a new notion, called a cubic set by using a fuzzy set and an interval-valued fuzzy set, and investigated several properties. Nirmala et al.[28] introduced neutrosophic soft cubic set(internal, external). [28] dealt with P-union, P-intersection, R-union and R-intersection of neutrosophic soft cubic sets, and investigated several related properties.

The cubic set was defined by Jun et al. [29] They used the notion of cubic sets in group and initiated the idea of cubic subgroups.The algebraic structures like BCK/BCI-algebra was introduced by Imai et al. Cubic soft set withapplication and subalgebra in BCK/BCI-algebra were studied by Muhiuddin et al. [30].

X. D. Peng,Y. Y. Yang, J. Song and Y. Jiang (2015) are combines the characteristics of Pythagorean fuzzy set with the parameterization of soft set,and constructs Pythagorean fuzzy soft set. Some operation such as complement,union,intersection,and,or,addition,multiplication,necessity,and possibility are defined.Some corresponding results are presented,and the De Morgan's Law of Pythagorean fuzzy soft sets are discussed in detailPenget al [31][. [32]studied some results for PF-sets along with their applications.

Chapter-1

Preliminaries

Definition: 1.1

Let E be a universe. Then a fuzzy set μ over E is defined by

$$X = \{ \mu_x (x) / x: x \in E \}$$

where μ_x is called membership function of X and defined by $\mu_x : E \rightarrow [0,1]$. For each x E, the value $\mu_x(x)$ represents the degree of x belonging to the fuzzy set X.

Definition: 1.2

The union of two fuzzy sets A and B is specified in general by a operation on the unit interval function of the form U: $[0,1]\times[0,1] \rightarrow [0,1]$

$$\mu A \cup B(X) = \max[\mu A(X), \mu B(X)] = \mu A(X) \cup \mu B(X), \text{ where } x \in X$$

Definition: 1.3

The intersection of two fuzzy sets A and B is specified in general by a binary operation on the unit interval, a function of the form I: $[0,1] \times [0,1] \rightarrow [0,1]$

$$\mu A \cap B(X) = \min[\mu A(X), \mu B(X)] = \mu A(X) \cap \mu B(X), \text{ where } x \in X$$

Definition: 1.4

Let a set E be fixed. An IFS A in E is an object of the following form :

$$A = \{(x, \mu_A(X), V_A(X)) x \in E\}$$

When $V_A(x) = 1 - \mu_A(x)$ for all $x \in E$ is ordinary fuzzy set.

In addition, for each IFS A in E, if

$$\pi_A(x) = 1 - \mu_X - V_X$$

Then $\pi_A(x)$ is called the degree of indeterminacy of x to A, or called the degree of hesitancy of x to A.

Especially, if $\pi_A(x) = 0$, for all $x \in E$ then the IFS, A is reduced to a fuzzy set.

Definition: 1.5

The union of fuzzy soft set A and B is denoted by AUB and is defined as the smallest fuzzy set that contains both fuzzy soft set A and fuzzy set B. The membership function $\mu_{A\cup B}$ of the union AU B of the fuzzy soft sets A and B is defined as follows:

. $$\mathrm{A} \cup B = \{< x, \min(\mu_A(x), \mu_B(x)), \max(V_A(x)), V_B(x)) > / \, x \in E\}$$

The symbol V is often used instead of the symbol max. The union corresponds to the operation "or".

Definition: 1.6

The intersection of fuzzy sets A and B is denoted by A$\cap$B and defined as the largest fuzzy set contained in both A and B. The intersection corresponds to the operation "and".

$$\mathrm{A} \cap B = \{< x, \max(\mu_A(x), \mu_B(x)), \min(V_A(x), V_B(x)) > / \, x \in E\}$$

Definition: 1.7

Consider U, P as initial universe and parameters sets, respectively. Take ρ(U) as a power set of U. Let X $\subset$ P. Give the mapping m : X $\rightarrow \rho$(U). Therefore, m_X is called a soft set (SS) on U.

Choose set of k objects and set of parameters as U = $\{a_1, a_2, ... a_K\}$, {A(1);A(2); …. ;A(i)}, class of parameters and the elements of A(i) represents a specific property set. Assumed that the property sets can be shown as FSs.

Definition: 1.8

Let $\varphi X, \varphi Y \in \phi(U)$. If the following conditions are hold, then φ_X is φ- soft subset of φ_Y:

i. $\varphi(X)(x) \subseteq_{\varphi} \varphi(X)(x),$

ii. mX (x)$\leq mY(x), nX(x) \geq nY(x)$

for all x $\in$P. ϕ- soft subset is denoted by $\varphi X \subseteq \varphi Y$.

Definition: 1.9

A pair (Λ, Σ) is called a fuzzy soft set over *X*, where Λ : Σ → P(X) is a mapping From Σ *into P(X).*

Definition: 1.10

For two fuzzy soft sets (Λ, Σ) and (Δ,Ω) in a fuzzy soft class (*X*, *E*), we say that (Λ, Σ) is a fuzzy soft subset of (Δ,Ω), if

(i) $\Sigma \subseteq \Omega$,

(ii) For all $\varepsilon \in \Sigma, \Lambda(\varepsilon) \leq \Delta(\varepsilon)$,

and is written as $(\Lambda, \Sigma) \subseteq (\Delta,\Omega)$.

Definition: 1.11

The complement of a fuzzy soft set (Λ, Σ) is denoted by $(\Lambda, \Sigma)^C$ and is defined by $(\Lambda, \Sigma)^C = (\Lambda^C, \Sigma)$, where Λ^C :_Σ → P(X) is a mapping given by $\Lambda^C(\sigma) = (\Lambda(\sigma))^C$, for all $\sigma \in \Sigma$.

Union of two fuzzy soft sets is defined by Maji et al.

Definition: 1.12

Union of two fuzzy soft sets (Λ, Σ) and (Δ,Ω) in a soft class (*X*, *E*) is a fuzzy soft set (Θ,Ξ), where $\Xi = \Sigma \cup \Omega$, and for all $\varepsilon \in \Xi$,

$$\Theta(\varepsilon)=\begin{cases}\Lambda(\varepsilon), if \varepsilon \in \Sigma - \Omega, \\ \Delta(\varepsilon), if \varepsilon \in \Omega - \Sigma, \\ \Lambda(\varepsilon) \vee \Delta(\varepsilon), if \varepsilon \in \Sigma \cap \Omega\end{cases},$$

and is written as $(\Lambda, \Sigma) \vee (\Delta,\Omega) = (\Theta,\Xi)$. For a few basic properties of fuzzy soft union,

Definition: 1.13

Let *U* be a universe of discourse. A Pythagorean fuzzy set (*PFS*) is an object having the following form:

$A = \{(x, \rho_A(x), V_A(x)) | x \in U\}$,

where μ_A: $U \to [0, 1]$ is degree of membership and V_A: $U \to [0, 1]$ is degree of non-membership of the element $x \in U$ to the set A, respectively, and for any $x \in U$, it holds that $0 \leq (\rho_A(x))2 + (V_A(x))2 \leq 1$. The degree of indeterminacy is given as:

$\pi_A(x) = \sqrt{1 - (\rho_A(x))^2 - (V_A(x))^2}$ where $\mu_A \in [0, 1]$, $V_A \in [0, 1]$.

Definition: 1.14

Given that $\beta = A(\rho_{\beta_1}, V_{\beta_1})$, $\beta_1 = A(\rho_{\beta_1}, V_{\beta_2})$, $\beta_2 = A(\rho_{\beta_2}, V_{\beta_2})$ are any three Pythagorean fuzzy numbers (*PFNs*) over (U, E), then the following holds:

(1) $\beta^C = (V_\beta, \rho_\beta)$,

(2) $\beta_1 \cup \beta_2 = (\max(\rho_{\beta_1}, \rho_{\beta_2}), \min(V_{\beta_1}{}' V_{\beta_2}))$,

(3) $\beta_1 \cap \beta_2 = (\min(\rho_{\beta_1}, \mu_{\rho\beta_2}), \max(V_{\beta_1}, V_{\beta_2}))$,

(4) $\beta_1 \geq \beta_2 \, iff \, \rho_{\beta_1} \geq \rho_{\beta_2} \, and \, V_{\beta_1} \leq V_{\beta_2}$,

(5) $\beta_1 = \beta_2 \, iff \, \rho_{\beta_1} = \rho_{\beta_2} \, and \, V_{\beta_2} = V_{\beta_2}$.

Definition: 1.15

Given an initial universe set U and a universe set of parameters E. A pair (F, A) is referred to as a fuzzy soft set on U if $A \subseteq E$ and $F : A \to F(U)$, where $F(U)$ is the set of all fuzzy subsets of U. The fuzzy soft sets to Pythagorean fuzzy environment and developed the concept of Pythagorean fuzzy soft sets as follows:

Definition: 1.16

Given an initial universe set U and a universe set of parameters E. A pair (F,A) is referred to as a Pythagorean fuzzy soft set (*PFSS*) on U if $A \subseteq E$ and $F : A \to PF(U)$, where $PF(U)$ is the family of all Pythagorean fuzzy subsets of U.

Example: 1.1

Given a set of three patients $U = \{u1, u2, u3\}$ and a set of parameters $E = \{e_1 =$ *dizziness* ,e_2 = *cough* e_3, = *stuffed nose*$\}$. Suppose that $F : E \to PF(U)$ is given as follows:

$F(e_1) = \{u1/(0.8, 0.5), u2/(0.6, 0.5), u3/(0.4, 0.8)\}$,

$F(e_2) = \{u1/(0.6, 0.4), u2/(0.4, 0.8), u3/(0.9, 0.2)\}$,

$F(e_3) = \{u1/(0.5, 0.5), u2/(0.8, 0.3), u3/(0.2, 0.8)\}$.

initiated the notion of possibility fuzzy soft set as follows:

Definition: 1.17

Given an initial universe set U and a universe set of parameters E. The pair (U, E) is referred to as a soft universe. Let $F : E \to F(U)$, and μ be a fuzzy subset of E, i.e. $\mu : E \to F(U)$. Let $F_\rho : E \to F(U) \times F(U)$ be a function defined as follows: $F_\rho(e) = (F(e)(x), \rho(e)(x)), \forall x \in U$. Then F_ρ is referred to as a possibility fuzzy soft set (*PFSS*) over the soft universe (U, E).

Definition: 1.18

Let E be a universe. Then a fuzzy set μ over E is defined by $X = \{ \mu_x (x) / x: x \in E \}$ where μ_x is called membership function of X and defined by $\mu_x : E \to [0,1]$. For each x E, the value $\mu_x(x)$ represents the degree of x belonging to the fuzzy set X.

Definition: 1.19

Let E be a universe. Then, an interval valued fuzzy set A over E is defined by

$A = \{ [A^-(x), A^+(x)] / x: x \in E \}$ where $A^-(x)$ and $A^+(x)$ are referred to as the lower and upper degrees of membership $x \in E$ where $0 \le A^-(x) + A^+(x) \le 1$, respectively.

Definition: 1.20

Let X be a non-empty set. By a cubic set, we mean a structure $\Xi = \{\langle x, A(x), \mu(x)\rangle \mid x \in X\}$ in which A is an interval valued fuzzy set (IVF) and μ is a fuzzy set. It is denoted by $\langle A, \mu\rangle$.

Definition: 1.21

Let $\Xi_1 = \langle A_1, \mu_1\rangle$ and $\Xi_2 = \langle A_2, \mu_2\rangle$ be cubic sets in X. Then we define

1. (Equality) $\Xi_1 = \Xi_2$ if and only if $A_1 = A_2$ and $\mu_1 = \mu_2$
2. (P- Order) $\Xi_1 \subseteq_p \Xi_2$ if and only if $A_1 \subseteq A_2$ and $\mu_1 \le \mu_2$
3. (R- Order) $\Xi_1 \subseteq_R \Xi_2$ if and only if $A_1 \subseteq A_2$ and $\mu_1 \ge \mu_2$

Definition: 1.22

Let X be an universe. Then a Pythagorean (PS) set λ is an object having the form

$$\lambda = \{< x : T(x), I(x), F(x) >: x \in X\}$$

where the functions T, I, F : X →]⁻0, 1+[defines respectively the degree of Truth, the degree of indeterminacy, and the degree of Falsehood of the element x ∈ X to the set λ with the condition.

$$^{-}0 \leq T(x) + I(x) + F(x) \leq 3^{+}$$

For two NS, $\lambda_1 = \{< x, T_1(x), I_1(x), F_1(x) > | x \in X\}$ and $\lambda_2 = \{< x, T_2(x), I_2(x), F_2(x) > | x \in X\}$ the operations are defined as follows:

1. $\lambda_1 \subset \lambda_2$ if and only if $T_1(x) \leq T_2(x), I_1(x) \geq I_2(x), F_1(x) \geq F_2(x)$

2. $\lambda_1 = \lambda_1$ if and only if, $T_1(x) = T_2(x), I_1(x) = I_2(x), F_1(x) = F_2(x)$

3. $\lambda_1 \breve{} c = \{x, F_1(x), I_1(x), T_1(x) >: x \in X\}$

4. $\lambda_1 \cap \lambda_2 = \{< x, \min\{T_1(x), T_2(x)\},$
 $\max\{I_1(x), I_2(x)\}, \max\{F_1(x), F_2(x)\} >: x \in X\}$

5. $\lambda_1 \cup \lambda_2 = \{< x, \max\{T_1(x), T_2(x)\},$
 $\max\{I_1(x), I_2(x)\}, \min\{F_1(x), F_2(x)\} >: x \in X\}$

Definition: 1.23

Let X be a non-empty set. An interval Pythagorean set (IPS) A in X is characterized by the truth-membership function A_T, the indeterminacy-membership function A_I and the falsity-membership function A_F. For each point x ∈ X, $A_T(x), A_I(x), A_F(x) \subseteq [0,1]$. For two INS

$$A = \{<x, [A_T^-(x), A_T^+(x)], [A_I^-(x), A_I^+(x)], [A_F^-(x), A_F^+(x)]>: x \in X\}$$

and

$$B = \{<x, [B_T^-(x), B_T^+(x)], [B_I^-(x), B_I^+(x)], [B_F^-(x), B_F^+(x)]>: x \in X\}$$

Then,

1. $A \tilde{\subseteq} B$ if and only if

$A_T^-(x) \leq B_T^-(x), A_T^+(x) \leq B_T^+(x)$

$A_I^-(x) \le B_I^-(x), A_I^+(x) \le B_I^+(x)$

$A_F^-(x) \le B_F^-(x), A_F^+(x) \le B_F^+(x)$ for all x∈ X.

2. $A = B$ if and only if

$A_T^-(x) = B_T^-(x), A_T^+(x) = B_T^+(x)$

$A_I^-(x) = B_I^-(x), A_I^+(x) = B_I^+(x)$

$A_F^-(x) = B_F^-(x), A_F^+(x) = B_F^+(x)$ for all x ∈ X.

3. $A^{\tilde{C}} = \{< x, [A_F^-(x), A_F^+(x)], [A_I^-(x), A_I^+(x)], [A_T^-(x), A_T^+(x)] >: x \in X\}$

4. $A \tilde{\cap} B = \{< x, [\min\{A_T^-(x), B_T^-(x)\}, \min\{A_T^+(x), B_T^+(x)\}],$
$[\max\{A_I^-(x), B_I^-(x)\}, \max\{A_I^+(x), B_I^+(x)\}],$
$[\max\{A_F^-(x), B_F^-(x)\}, \max\{A_F^+(x), B_F^+(x)\}] >: x \in X\}$

5. $A \tilde{\bigcup} B = \{< x, [\max\{A_T^-(x), B_T^-(x)\}, \max\{A_T^+(x), B_T^+(x)\}],$
$[\min\{A_I^-(x), B_I^-(x)\}, \min\{A_I^+(x), B_I^+(x)\}],$
$[\min\{A_F^-(x), B_F^-(x)\}, \min\{A_F^+(x), B_F^+(x)\}] >: x \in X\}$

Definition: 1.24

Let U be an initial universe set and E be a set of parameters. Consider A ⊂ E. Let P(U) denotes the set of all Pythagorean sets of U. The collection (F, A) is termed to be the soft Pythagorean set over U, where F is a mapping given by F : A → P(U).

Definition: 1.25

Let (F, A) and (G, B) be two Pythagorean soft sets over the common universe U. (F, A) is said to be Pythagorean soft subset of (G, B) if A⊂ B, and $T_{F(e)}(x) \le T_{G(e)}(x)$, $I_{F(e)}(x) \le I_{G(e)}(x)$, $F_{F(e)}(x) \ge F_{G(e)}(x)$, ∀ e∈A ,x∈U. We denote it by (F, A) ⊆ (G, B).

Definition: 1.26

Complement of A Pythagorean soft set. The complement of a Pythagorean soft set (F, A) denoted by $(F,A)^c$ and is defined as $(F,A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \to P(U)$ is a mapping given by $F^c(\alpha)$ = Pythagorean soft complement with $T_{Fc(x)} = F_{F(x)}$, $I_{Fc(x)} = I_{F(x)}$ and $F_{Fc(x)} = T_{F(x)}$.

Definition: 1.27

The union of two Pythagorean soft sets (F,A)and (G,B) over (U,E) is Pythagorean soft set where C = A∪B, ∀e ∈ C

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

and is written as (F,A) ∪(G,B)= (H,C)

Definition: 1.28

The intersection of two Pythagorean soft sets (F,A) and (G,B) over (U,E) is Pythagorean soft set where C = A∩B ,∀e ∈ C , H(e) = F(e)∩G(e) and is written as (F,A) ∩ (G,B) = (H,C)

Chapter-2

Possibility Pythagorean fuzzy soft set

Definition: 2.1

Given that U is a universal set of elements and E is a set of parameters, then the pair (U, E) is called a soft universe. Suppose that $F: E \to PF(U)$, and p is a Pythagorean fuzzy subset of E, i.e. $p: E \to PF(U)$, where $PF(U)$ denotes the collection of all Pythagorean fuzzy subsets of U. If $F_P \to PF(U) \times PF(U)$ is a function defined as $F_P(e) = (F(e)(x), p(e)(x)), x \in U$, then F_p is referred to as a possibility Pythagorean fuzzy soft set (*PPFSS*) on (U, E).

It's worth noting that for each parameter e, $F_p(e)$ can be written as

$F_P(e) = \{(x, (\rho_F\ (e)(x), V_P(e)(x))\ (\rho_F\ (e)(x), V_P(e)\ (x))), x \in U\}$.

Example: 2.1

Given that $U = \{u1, u2, u3\}$ is a set of three cars under consideration of a decision maker to purchase, $E = \{e1 =$ *low fuel consumption*, $e2 =$ *high safety*, $e3 =$ *cheap*$\}$ is a set of parameters. Suppose that $F_P: E \to PF(U) \times PF(U)$ is given as follows:

$F_P\ (e1) = \{(u1/(0.9, 0.2), (0.6, 0.3)), (u2/(0.8, 0.2),(0.5, 0.7)), (u3/(0.5, 0.6), (0.6, 0.3))\}$,

$F_P\ (e2) = \{(u1/(0.7, 0.3), (0.6, 0.2)), (u2/(0.9, 0.2)(0.4, 0.5)), (u3/(0.4, 0.9), (0.8, 0.1))\}$,

$F_P\ (e3) = \{(u1/(0.5, 0.2), (0.9, 0.4)), (u2/(0.8, 0.4),(0.4, 0.7)), (u3/(0.7, 0.5), (0.6, 0.7))\}$.

In matrix form it can be expressed as

$$F_{p=} \begin{pmatrix} (0.9, 0.2), (0.6, 0.3) & (0.8, 0.2), (0.5, 0.7) & (0.5, 0.6), (0.6, 0.3) \\ (0.7, 0.3), (0.6, 0.2) \vdots & (0.9, 0.2)(0.4, 0.5) & (0.4, 0.9), (0.8, 0.1) \\ (0.5, 0.2), (0.9, 0.4) & (0.8, 0.4), (0.4, 0.7) & (0.7, 0.5), (0.6, 0.7) \end{pmatrix}$$

Definition: 2.2

Given a universal set of elements U and a set of parameters E. suppose that F_P and S_q are two *PPFSS*s over (U, E). Now F_P is referred to as a possibility Pythagorean fuzzy soft subset of S_q if and only if

(1) $q(e)(x) \subseteq p(e)(x)$ if $\rho_P\ (e)(x) \geq \rho_q\ (e)(x), V_p\ (e)(x) \leq V_q(e)(x)$,

(2) $S(e)(x) \subseteq F(e)(x)$ if $\rho_F(e)(x) \geq \rho_S(e)(x), V_F\ (e)(x) \leq V_S(e)(x), \forall\, e \in E$.

This relationship is denoted as $S_q \subseteq F_P$

Example: 2.2

Consider the *PPFSS* F_P over (*U*, *E*) given in Example 3.2. Let S_q be another *PPFSS* over (*U*, *E*) defined as follows:

S_q (*e*1) = {(*u*1/(0.9, 0.4), (0.5, 0.8)), (*u*2/(0.6, 0.5),(0.8, 0.3)), (*u*3/(0.6, 0.7), (0.5, 0.8))},

S_q (*e*2) = {(*u*1/(0.7, 0.3), (0.4, 0.9)), (*u*2/(0.9, 0.2),(0.5, 0.7)), (*u*3/(0.3, 0.8), (0.5, 0.6))},

S_q (*e*3) = {(*u*1/(0.8, 0.2), (0.3, 0.9)), (*u*2/(0.6, 0.3),(0.7, 0.2)), (*u*3/(0.5, 0.5), (0.9, 0.2))}.

Clearly, we have $S_q \subseteq F_P$.

Definition: 2.3

Given a universal set of elements *U* and a set of parameters *E*. suppose that F_P and S_q are two *PPFSS*s over (*U*, *E*). Now F_P and S_q are referred to as a possibility Pythagorean fuzzy soft equal if and only if

(1) $F_P \subseteq S_q$

(2) $F_p \supseteq S_q$, which can be denoted by $F_P = S_q$.

Operations on possibility Pythagorean fuzzy soft sets

Definition: 2.4

Given a universal set of elements *U* and a set of parameters *E*. Let F_P be a *PPFSS over* (*U*, *E*). The complement of F_P denoted by F_P^C, is defined by

$F_P^C = (F^C\ (e)(x), P^C(e)(x),$

Where $F^C(e)(x) = (V_F(\mathrm{e})\ (x), \rho_F\ e)(x)), P^C(e)(x) = (V_P\ (e)(x), \rho_P(e)(x))$.

From the above definition, it is observed that

$(F_P^C)^C = Fp$.

Definition: 2.5

Given a universal set of elements *U* and a set of parameters *E*. Let F_p and S_q be Two *PPFSS*s over (*U*, *E*). The union and intersection operations on two *PPFSSs* F_P and S_q over (*U*, *E*), denoted by "$F_P \cup S_q$" and "$F_P \cap S_P$", is respectively defined by two mappings as follows

$D_d: E \to PF(U) \times PF(U)$,

$J_j: E \to PF(U) \times PF(U)$,

such that for all $x \in U$, $D_d(e)(x) = (D(e)(x), d(e)(x))$, $J_j(e)(x) = (J(e)(x), j(e)(x))$,
where $D(e)(x) = F(e)(x) \cup S(e)(x)$, $d(e)(x) = p(e)(x) \cup q(e)(x)$, $J(e)(x) = F(e)(x) \cap S(e)(x)$, and $j(e)(x) = p(e)(x) \cap q(e)(x)$.

Example: 2.3

Assume that F_P and S_q are two *PPFSS*s over (U, E) defined as follows:

$F_P(e1) = \{(u1/ (0.5, 0.3), (0.9, 0.1)), (u2/(0.6, 0.4), (0.1, 0.7)), (u3/(0.8, 0.2), (0.7, 0.4))\}$,

$F_p(e2) = \{(u1/ (0.7, 0.4), (0.1, 0.8)), (u2/(0.2, 0.9), (0.8, 0.4)), (u3/(0.4, 0.8), (0.3, 0.9))\}$,

$F_p(e3) = \{(u1/ (0.9, 0.4), (0.3, 0.7)), (u2/(0.7, 0.4), (0.2, 0.9)), (u3/(0.6, 0.5), (0.8, 0.4))\}$.

$S_q(e1) = \{(u1/ (0.7, 0.4), (0.9, 0.4)), (u2/(0.8, 0.3), (0.6, 0.5)), (u3/(0.5, 0.8), (0.6, 0.7))\}$,

$S_q(e2) = \{(u1/ (0.4, 0.9), (0.7, 0.3)), (u2/(0.5, 0.7), (0.9, 0.2)), (u3/(0.5, 0.6), (0.3, 0.8))\}$,

$S_q(e3) = \{(u1/ (0.3, 0.9), (0.8, 0.2)), (u2/(0.6, 0.3), (0.7, 0.2)), (u3/(0.9, 0.2), (0.5, 0.5))\}$.

In matrix notation, then we have

$$F_p \cup S_q = \begin{pmatrix} (0.7,0.3)(0.9,0.1) & (0.8,0.3)(0.6,0.5) & (0.8,0.2)(0.7,0.4) \\ (0.7,0.4)(0.7,0.3) & (0.5,0.7)(0.9,0.2) & (0.5,0.6)(0.3,0.8) \\ (0.9,0.4)(0.8,0.2) & (0.7,0.3)(0.7,0.2) & (0.9,0.2)(0.8,0.4) \end{pmatrix}$$

$$,F_p \cap S_q = \begin{pmatrix} (0.5,0.4)(0.9,0.4) & (0.6,0.4)(0.1,0.7) & (0.5,0.8)(0.6,0.7) \\ (0.4,0.9)(0.1,0.8) & (0.2,0.9)(0.8,0.4) & (0.4,0.8)(0.3,0.9) \\ (0.3,0.9)(0.3,07) & (0.6,0.4)(0.2,0.9) & (0.6,0.5)(0.8,0.5) \end{pmatrix}$$

Definition: 2.6

A *PPFSS* $\emptyset_\theta(e)(x) = (\emptyset(e)(x), \theta(e)(x)$ is said to a possibility null Pythagorean fuzzy Soft set $\emptyset_\theta : E \to PF(U) \times PF(U)$, where $\emptyset(e)(x) = (0, 1)$ and $\theta(e)(x) = (0, 1)$, $\forall x \in U$.

Definition: 2.7

A *PPFSS* $\Omega_\Lambda(e)(x) = (\Omega(e)(x), \Lambda(e)(x))$ is said to a possibility absolute Pythagorean fuzzy soft set

$$\Omega_\Lambda : E \to PF(U) \times PF(U),$$

where $\Omega(e)(x) = (1, 0)$ and $\Lambda(e)(x) = (1, 0)$, $\forall x \in U$.

Theorem: 2.1

Given that F_P is a *PPFSS* over (U, E), then the following holds:

(1) $F_P = F_P \cup F_P$, $F_P = F_P \cap F_P$,

(2) $F_P \subseteq F_P \cup F_P$, $F_P \subseteq F_P \cap F_P$,

(3) $F_P \cup \emptyset_\theta = F_P$, $F_P \cap \emptyset_\theta = \emptyset_\theta$,

(4) $F_P \cup \Omega_\Lambda = \Omega_\Lambda$, $F_P \cap \Omega_\Lambda = F_P$

Proof. Straightforward.

Remark: 2.1

Let F_P be a *PPFSS* over (U, E). If $F_P \neq \Omega_\Lambda$ *or* $F_P \neq \emptyset_\theta$ then $F_P \cup F_P^C \neq \Omega_\Lambda$ and $F_P \cap F_P^C \neq \emptyset_\theta$.

Theorem: 2.2

Given that F_P, s_q and R_r are any three PPFSSs over (U, E), then the commutative and associative properties hold

(1) $F_P \cup S_q = S_q \cup F_P$,

(2) $F_P \cap S_q = S_q \cap F_P$,

(3) $F_P \cup (S_q \cup R_r) = (F_P \cup S_q) \cup R_r$,

(4) $F_P \cap (S_q \cap R_r) = (F_P \cap S_q) \cap R_r$.

Theorem: 2.3

Given that F_P, S_q and R_r are any three PPFSSs on (U, E), then the following results hold:

(1) $(F_P \cup S_q)^C = F_P^C \cap S_q^C$,

(2) $(F_P \cap S_q)^C = F_P^C \cup S_q^C$,

(3) $(F_P \cup S_q) \cap F_P = F_P$,

(4) $(F_P \cap S_q) \cup F_P = F_P$,

(5) $F_P \cup (S_q \cap R_r) = (F_P \cup S_q) \cap (F_P \cup R_r)$,

(6) $F_P \cap (s_q \cup R_r) = (F_P \cap s_q) \cup (F_P \cap R_r)$.

Proof:

The properties follow from Definitions 4 and 5.In what follows, AND and OR operations over two *PPFSS*s are introduced as follows. _

Definition: 2.8

Given that (F_P, M) and (S_q, N) are two *PPFSS*s on (U, E), then the operation "(F_P, M) *AND* (F_P, N)", denoted by (F_P, M) $\wedge$ (S_q, N), is defined by

$$(F_P, M) \wedge (G_q, N) = (H_h, M \times N)$$

Where R_r (γ, δ) = (R(γ, δ)(x), r(γ, δ)(x)), for all (α, β) $\in$ $M \times N$, such that R(γ, δ) = F(γ) $\cap$ G(δ),and r(γ, δ) = p(γ) $\cap$ q(δ).

Definition: 2.9

Given that (F_P, M) *and* (S_q, N) are two *PPFSS*s on (U, E), then the operation "(F_P, M) *OR* (S_q, N)", denoted by (F_P, M) $\vee$ (S_q, N), is *defined as*

$$(F_P,\ M) \vee (S_q\ , M \times N) = (R_r, M\ \times N)$$

Where $R_r(\gamma, \delta) =\ R_r\ (\gamma, \delta)(x), r(\gamma, \delta)(x))$, *for all* $(\gamma, \delta) \epsilon M\ \times N$, *such that* $R\ (\gamma, \delta) =\ F(\gamma) \cup S(\delta), and\ r(\gamma, \delta) = p(\gamma) \cup q(\delta)$.

Remark: 2.2

Given that ($F_{P,}$ M) and (S_q, N) are two PPFSSs over (U, E). for all $(\gamma, \delta) \in M\ \times N$, if $\gamma\ \neq\ \delta$, then

$$(S_q, N) \wedge (F_P, M) \neq (F_P, M) \wedge (S_q, N), \text{ and } (s_q, N) \vee (F_P, M) \neq (F_P, M) \vee (S_q, N).$$

Theorem: 2.4

Given that F_P and S_q are two *PPFSS*s over (U, E), then

(1) $((F_{P,}\ M) \wedge (S_{q,}\ N)^C)^C =\ (F_{P,}\ M)^C \vee (S_{q,}\ N)^C$,

(2) $((F_{P,} M) \vee (S_{q,} N)^C)^C\ =\ (F_P, M)^C \wedge (S_{q,} N)^C$.

Proof.

(1) Suppose that $(F_{P,}M)\wedge(S_q,N) = (R_r, M \times N)$.

Here $R_r^c(\gamma,\delta) = R^c(\gamma,\delta)(x), r^c(\gamma,\delta)(x))$,, forall $(\gamma, \delta) \in M \times N$. By Definition 8, for all $(\gamma, \delta) \in M \times N$, we have $R^c(\gamma, \delta) = (F(\gamma) \cap S(\delta))c = F^c(\gamma) \cup (\delta), S^c$ and $r^c(\gamma, \delta) = (p(\gamma) \cap q(\delta))c = p^c(\gamma) \cup q^c(\delta)$.

On the other hand, given that $(F_P, M)^c \vee (S_{q,}N)^c = (\wedge_0, M \times N)$, where $\wedge_0(\gamma, \delta) = (\Lambda(\gamma, \delta)(x), o(\gamma, \delta)(x))$, for all $(\gamma, \delta) \in M \times N$, such that $\Lambda(\gamma, \delta) = F^c(\gamma) \cup S^c(\delta)$, and $o(\gamma, \delta) = P^c(\gamma) \cup q^c(\delta)$.

Hence $R_r^c = \wedge_0$

Likewise, the proof of (2) can be made similarly.

Chapter-3

Similarity between two possibility Pythagorean fuzzy soft sets:

The concept of similarity measure between two *PPFSS*s is introduced as follows.

Definition: 3.10

Given a universal set of elements *U* and a set of parameters *E*. Suppose that F_P and S_q are two *PPFSS*s over (*U*, *E*). The similarity between two *PPFSS*s F_P and S_q, denoted by *Sim* (F_P, S_q), is defined as follows:

$$Sim\,(F_P, S_q) = \phi\,(F, S) \cdot \psi\,(p, q),$$

such that

$$\varphi(F,S) = \frac{A\big(F(e)(x), S(e)(x)\big) + B(F(e)(x),\ S(e)(x))}{2}$$

$$\psi(p, q) = 1 - \frac{\sum|\gamma i - \delta i|}{\sum|\gamma i + \delta i|}$$

where

$$A\,(F(e)(x), S(e)(x)) = \frac{\sum_{i=1}^{n}(\mu_{F(e_i)}(x).\mu_{S(e_i)}(x))}{\sum_{i=1}^{n}(1-\sqrt{(1-\mu^2_{F(e_i)}(x).(1-\mu^2_{S(e_i)}(x)))}},$$

$$B\,(F(e)(x), S(e)(x)) = \sqrt{1 - \frac{\sum_{i=1}^{n}|v^2_{F(e_i)}(x) - v^2_{S(e_i)}(x)|}{\sum_{i=1}^{n}(1+v^2_{F(e_i)}(x).v^2_{S(e_i)}(x))}},$$

$$\gamma_i = \frac{\mu^2_{p(e_i)}(x)}{\mu^2_{p(e_i)}(x) + v^2_{p(e_i)}(x)},$$

$$\delta_i = \frac{\mu^2_{q(e_i)}(x)}{\mu^2_{q(e_i)}(x) + v^2_{q(e_i)}(x)}.$$

Theorem: 2.5

Given that F_P, S_q *and* R_r are any three *PPFSS*s over (U, E), then

(1) $Sim\ (F_P, S_q) = Sim\ (S_q, F_P)$,

(2) $0 \leq Sim\ (F_P, S_q) \leq 1$,

(3) $F_P = S_q \Rightarrow Sim\ (F_P, S_q) = 1$,

(4) $F_P \subseteq S_q \subseteq R_r \Rightarrow Sim\ (F_P, R_r) \leq Sim\ (S_q, R_r)$,

(5) $F_P \cap S_q = \emptyset \Leftrightarrow Sim\ (F_P, S_q) = 0$.

Proof.

The proof follows from Definition 10.

Example: 2.4

Find the similarity between the two *PPFSS*s, F_P *and* S_q as specified in Example 3. The *PPFSS*s, F_P *and* S_q of the first sample $x1$ and $E = \{e1, e2, e3\}$ can be respectively, defined as follows:

$$F_P = \begin{pmatrix} (0.5, 0.3), (0.9, 0.1) \\ (0.7, 0.4), (0.1, 0.8) \\ (0.9, 0.4), (0.3,0.7) \end{pmatrix},$$

$$S_q = \begin{pmatrix} (0.7, 0.4), (0.9, 0.4) \\ (0.4, 0.9), (0.7, 0.3) \\ (0.3, 0.9), (0.8, 0.2) \end{pmatrix}$$

Using Definition 3.10, we obtained

$$\text{A(F(e)(x), S(e)(x)} = \frac{0.350 + 0.280 + 0.270}{0.5583 + 0.4001 + 0.6033}$$

$$=\frac{0.9000}{1.5617}$$

$$= 0.5763$$

$$B(F(e)(x), S(e)(x) = \sqrt{1 - \frac{0.07+0.65+0.65}{1.0144+1.1296+1.9700}}$$

$$= \sqrt{1 - \frac{1.3700}{4.1140}}$$

$$= 0.8167$$

$$\varphi\ (F, S) = \frac{0.5763+0.8167}{2}$$

$$= 0.6965$$

Here,

$$\psi(F, S) = 1 - \frac{0.1528+0.2165+0.1667}{1.8229+1.0635+1.0137}$$

$$= 1 - \frac{0.5360}{3.9001}$$

$$= 0.8626$$

So

$$\text{Sim}\ (F_P, S_q) = 0.6965 \times 0.8626$$

$$=0.6008$$

Application of similarity measure in decision making

Decisions are ubiquitous in life. So we need to make a decision in most real-life problems, such as politics, economy, management, technology and daily life. In economy, we know that decisions have a major impact on customer cost, manufacturing, service and efficiency. The same is true for construction units bid for the project. It is the best result for construction units bid to choose the best construction contractors. For a building unit, the importance of selecting the best construction project contractor for the businesses is also testified by the numerous studies in the literatures. In the selection of construction project contractor, the evaluation of construction contractors is carried out according to various standards of experts. Our goal is to select the optimal one out of a great number of alternatives
based on the assessment of experts against the criteria.

Case-study:

A construction company intends to choose the most appropriate construction project contractor to build a new construction. After the first round of bids, three bidder businesses are nominated. The score of the bidder business evaluated by the experts is represented by $E = \{e1$: *domain experience,* $e2$: *project plan and description*, $e3$: *expected time cost*, $e4$: *brand value*$\}$. The ideal construction project contractor is given in Table 1. Suppose that decision makers in the construction company can

Table 1

PPFSS for the ideal construction project contractor

$L_\delta(e)$	e_1	e_2	e_3	e_4
L(e) (x)	(09, 0.2)	(0.5, 0.6)	(0.8, 0.3)	(0.7, 0.5)
$\delta(e)(x)$	(0.4, 0.3)	(0.6, 0.5)	(0.7, 0.6)	(0.5, 0.5)

Table 2

PPFSS for the first construction project contractor

$X_{\alpha}(e)$	e_1	e_2	e_3	e_4
X(e)(x)	(0.5, 0.6)	(0.8, 0.2)	(0.7, 0.6)	(0.6, 0.5)
$\alpha(e)(x)$	(0.3, 0.6)	(0.4, 0.6)	(0.5, 0.4)	(0.9, 0.3)

Table 3

PPFSS for the second construction project contractor

$Y_{\beta}(e)$	e_1	e_2	e_3	e_4
Y(e)(x)	(0.6, 0.4)	(0.7, 0.5)	(0.5, 0.4)	(0.9, 0.2)
β(e)(x)	(0.7, 0.3)	(0.8, 0.4)	(0.9, 0.3)	(1, 0)

provide the *PFN* values for the ideal construction project contractor, which reflect the pursuit of the ideal qualities of the construction project contractor. The evaluations of the construction project contractors as per *PPFSS* are shown as Tables 1–3. The *PFN*s values in Tables 1-3 are provided by the experts, depending on their assessment of the alternatives against the criteria under consideration.

In this example, in order to find the construction project contractor which is closest to the ideal construction project contractor, we should calculate the similarity measure of *PPFSS*s in Tables 1-3 with the one in Table 1 based on Definition3.10. The threshold of the similarity should rely on the company. Generally speaking, among all the available construction project contractors, the construction project contractor with the similarity measure above this threshold is the most appropriate choice. Calculating the similarity measure for the first construction project contractor as follows, we have

$$\varphi(L,X) = \frac{A\big(L(e)(x), X(e)(x)\big) + B(L(e)(x),\ X(e)(x))}{2}$$

By the definition 3.10, then

$$A\big(L(e)(x), X(e)(x)\big) = \frac{1.8300}{2.1031} = 0.\ 8701$$

$$B(L(e)(x), X(e)(x)) = \sqrt{1 - \frac{0.9100}{4.013}}$$

$$= \sqrt{1 - 0.2207} = 0.8828$$

So we get,

$$\varphi(L, X) = \frac{0.8701 + 0.8828}{2}$$

$$= 0.8765$$

From definition 3.10, we get

$$\psi(\delta, \alpha) = 1 - \frac{\sum |\gamma i - \delta i|}{\sum |\gamma i + \delta i|}$$

Therefore,

$$\psi(\delta, \alpha) = 1 - \frac{1.1568}{4.3242}$$

$$= 0.7325$$

From the definition 3.10, we get

$$\text{Sim } (L_\delta, X_\sigma) = 0.8765 \times 0.7325 = 0.6420$$

Likewise, the similarity of the second construction project contractor with the ideal construction project contractor is computed as,

$$\varphi(L, Y) = \frac{A\big(L(e)(x), Y(e)(x)\big) + B(L(e)(x), Y(e)(x))}{2}$$

By the definition 3.10, then

$$A\big(L(e)(x), Y(e)(x)\big) = \frac{1.9200}{2.2019}$$

$$= 0.\ 8720$$

$$B(\text{L(e) (x), Y(e) (x)}) = \sqrt{1 - \frac{0.5100}{4.1208}}$$

$$= \sqrt{1 - 0.1238}$$

$$= 0.9361$$

So we get,

$$\varphi(L, Y) = \frac{0.8720 + 0.9361}{2} \quad = 0.9040$$

From definition 3.10, we get

$$\psi(\delta, \beta) = 1 - \frac{\sum |\gamma i - \delta i|}{\sum |\gamma i + \delta i|}$$

Therefore,

$$\psi(\delta, \beta) \ = 1 - \frac{1.2381}{5.8515} = 0.7884$$

From the definition 3.10, we get

$$\text{Sim } (L_{\delta}, Y_{\beta}) = 0.9040 \times 0.7884$$

$$\text{Sim } (L_{\delta}, Y_{\beta}) = 0.7127$$

From the above results, we find that the first construction project contractor is closest to the ideal construction project contractor with the highest value of the similarity measure as

0.830. The second construction project contractor with the values of similarity measure as 0.7127 follow the first construction project contractor.

Comparison of PPFSS approach with PFSS approach without the generalization parameter

In this subsection, we shall again investigate the above mentioned case-study using the *PFSS* approach to consider the effect of the possibility parameter. Calculating the similarity measure for the first construction project contractor as follows, we have from

$$\varphi(L,X) = \frac{A\big(L(e)(x),X(e)(x)\big) + B(L(e)(x),\ X(e)(x))}{2}$$

By the definition 3.10, then

$$A\big(L(e)(x),X(e)(x)\big) = \frac{1.8300}{2.1031}$$

$$= 0.\ 8701$$

$$B(L(e)(x),X(e)(x)) = \sqrt{1-\frac{0.9100}{4.013}}$$

$$= \sqrt{1-0.2207}$$

$$= 0.8828$$

So

$$\text{Sim}\ (L_\delta, X_\sigma) = \frac{0.8701+0.8828}{2}$$

$$= 0.8765$$

Likewise, the similarity of the second construction project contractor with the ideal construction project contractor is computed as,

$$\varphi(L,Y) = \frac{A\big(L(e)(x), Y(e)(x)\big) + B(L(e)(x),\ Y(e)(x))}{2}$$

By the definition 3.10, then

$$A\big(L(e)(x), Y(e)(x)\big) = \frac{1.9200}{2.2019}$$

$$= 0.\ 8720$$

$$B(\mathrm{L(e)\ (x),\ Y(e)\ (x)}) = \sqrt{1 - \frac{0.5100}{4.1208}}$$

$$= \sqrt{1 - 0.1238}$$

$$= 0.9361$$

So

$$\mathrm{Sim}\ (L_\delta, Y_\beta) = \frac{0.8720 + 0.9361}{2} = 0.9040.$$

As can be seen from the results above, the likelihood parameter has a significant impact on the calculation of the similarity measure of *PPFSS*s. It is observed that the second and third construction project contractors from the perspective of similarity are quite away from the ideal construction project contractor. If the building unit chooses the threshold 0.7, we should choose the first construction project contractor as a potential candidate. On the contrary, when using *PFSS* approach without the generalization parameter, we cannot distinguish which the construction project contractor is the best one. So the possibility parameter has an important influence to the similarity of the two construction project contractor. Therefore, *PPFSS* approach is more scientific and reasonable than *PFSS* approach without the generalization parameter in the process of decision-making.

Chapter-4

Pythagorean soft cubic sets

Definition: 4.1

Let U be an initial universe set. Let PC(U) denote the set of all Pythagorean cubic sets and E be the set of parameters. Let $A \subset E$ then $(P, A) = \{ P(e_i\ ) =$ $\{< x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in U\} e_i \in A\}$, where $A_{e_i}(x) = \{< x,\ A_{e_i}^T(x), A_{e_i}^I(x), A_{e_i}^F(x) > / \, x \in U\}$, is an interval Pythagorean set , $\lambda_{e_i}(x) = \{< x, (\lambda_{e_i}^T(x), \lambda_{e_i}^I(x), \lambda_{e_i}^T(x) > / \, x \in U\}$ is a Pythagorean set. The pair (P, A) is termed to be the Pythagorean soft cubic set over U where P is a mapping given by $P : \mathrm{A} \rightarrow \mathrm{PC(U)}$. The sets of all Pythagorean soft cubic sets over U will be denoted by C_N^U

Example:4.2

Let U = {x_1, x_2, x_3, x_4} be the set of cricket players under consideration and

E = {e_1, e_2, e_3, e_4} be the set of parameters, where e_1, e_2, e_3, e_4 represent fitness, good current form, good domestic cricket record and good moral characters respectively.

Let $A = \{e_1, e_2, e_3\} \subseteq E$. Then, the Pythagorean soft cubic set

$(P, A) = \{ P(e_i\ ) =\ \{< x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in U\} e_i \in A\}$ i=1,2,3 in U is

U	P(e_1)		P(e_2)		P(e_3)	
	Ae_1	λe_1	Ae_2	λe_2	Ae_3	λe_3
x_1	[0.2, 0.4][0.4, 0.7][0.1, 0.3]	[0.2, 0.5, 0.55]	[0.3,0.7][0.3,0.5][0.5, 0.7]	[0.3,0.4, 0.5]	[0.4,0.5][0.5,0.6] [0.3, 0.4]	[0.5,055,0.65]
x_2	[0.4, 0.7][0.3, 0.5][0.5, 0.6]	[0.3, 0.6, 0.7]	[0.5,0.7][0.4,0.6][0.3, 0.4]	[0.6, 0.65, 0.66]	[0.3,0.6][0.5,0.7] [0.2, 0.5]	[0.5, 0.6, 0.7]
x_3	[0.5,0.7][0.2, 0.4][0.4,0.7]	[0.5,0.6,0.75]	[0.6,0.7][0.1,0.4][0.2,0.5]	[0.15,0.2,0.4]	[0.4,0.5][0.3,0.6] [0.2,0.5]	[0.3,0.7,0.9]
x_4	[0.6,0.7][0.3,0.6][0.2,0.5]	[0.1,0.4,0.2]	[0.4,0.7][0.3,0.5][0.5,0.6]	[0.1,0.5,0.65]	[0.5,0.7][0.2,0.6] [0.4,0.6]	[0.2,0.3,0.5]
x_5	[0.4,0.5][0.5,0.6][0.5,0.7]	[0.1,0.55, 0.65]	[0.5,0.7][0.4,0.6][0.4,0.6]	[0.55,0.6,0.65]	[0.3,0.5][0.1,0.4] [0.4,0.7]	[0.45,0.35, 0.55]

Definition:4.3

Let U be a non-empty set. A Pythagorean soft cubic set (P, A) in U is said to be

- truth-internal (briefly, T-internal) if the following inequality is valid

$$(\forall\, x \in U, e_i \in E)\ \ (A_{e_i}^{-T}(x) \le \lambda_{e_i}^{T}(x) \le A_{e_i}^{+T}(x)), \qquad (4.1)$$

indeterminacy-internal (briefly, I-internal) if the following inequality is valid

$$(\forall\, x \in U, e_i \in E)\ \ (A_{e_i}^{-I}(x) \le \lambda_{e_i}^{I}(x) \le A_{e_i}^{+I}(x)), \qquad (4.2)$$

- falsity-internal (briefly, F-internal) if the following inequality is valid

$$(\forall\, x \in U, e_i \in E)(A_{e_i}^{-F}(x) \le \lambda_{e_i}^{F}(x) \le A_{e_i}^{+F}(x)). \qquad (4.3)$$

If a Pythagorean soft cubic set in U satisfies (4.1), (4.2) and (4.3) we say that (P, A) is an internal Pythagorean soft cubic in U.

Table 2. Tabular representation of (P, A)

U	P(e_1)		P(e_2)		P(e_3)	
	A$e_1(x)$	λe_1(x)	A$e_2(x)$	λe_2(x)	Ae_3(x)	λe_3(x)
x_1	[0.4,0.6][0.3,0.6] [0.2,0.6]	[0.5,0.55 ,0.6]	[0.2,0.4][0.4,0.7] [0.3,0.6]	[0.3,0.5, 0.55]	[0.3,0.6][0.3,0.5] [0.5,0.7]	[0.5,0.6,0. 7]
x_2	[0.1,0.4][0.2,0.4] [0.1,0.5]	[0.3,0.35 ,0.4]	[0.4,0.7][0.3,0.6] [0.5,0.7]	[0.5,0.55 ,0.7]	[0.3,0.6][0.4,0.7] [0.3,0.7]	[0.3,0.5,0. 65]
x_3	[0.3,0.6][0.2,0.5] [0.4,0.8]	[0.4,0.45 ,0.7]	[0.5,0.6][0.2,0.7] [0.4,0.6]	[0.5,0.55 ,0.6]	[0.4,0.5][0.1,0.7] [0.2,0.6]	[0.5,0.6,0. 6]
x_4	[0.2,0.5][0.1,0.4] [0.3,0.7]	[0.3,0.35 ,0.5]	[0.6,0.7][0.3,0.6] [0.2,0.5]	[0.15,0.5 ,0.4]	[0.2,0.6][0.3,0.6] [0.4,0.7]	[0.55,0.45, 0.55]
x_5	[0.3,0.7][0.1,0.3] [0.2,0.5]	[0.25,0.3 ,0.45]	[0.4,0.7][0.5,0.5 5][0.5,0.6]	[0.1,0.5, 0.55]	[0.3,0.6][0.2,0.5] [0.4,0.7]	[0.4,0.5,0. 7]

Definition: 4.4

Let U be a non-empty set. A Pythagorean soft cubic set (P, A) in U is said to be

- truth-external (briefly, T -external) if the following inequality is valid

$$(\forall x \in U, e_i \in E) \ (\lambda_{e_i}^{T}(x) \notin (A_{e_i}^{-T}(x), A_{e_i}^{+T}(x))), \qquad (4.4)$$

- indeterminacy-external (briefly, I -external) if the following inequality is valid

$$(\forall x \in U, e_i \in E) \ (\lambda_{e_i}^{I}(x) \notin (A_{e_i}^{-I}(x), A_{e_i}^{+I}(x))), \qquad (4.5)$$

- falsity-external (briefly, F -external) if the following inequality is valid

$$(\forall x \in U, e_i \in E) \ (\lambda_{e_i}^{F}(x) \notin (A_{e_i}^{-F}(x), A_{e_i}^{+F}(x))). \qquad (4.6)$$

If a Pythagorean soft cubic set (P, A)) in *X* satisfies (4.4), (4.5) and (4.6), we say that (P, A) is an external Pythagorean soft cubic in *U*.

Example: 4.5

Let U = {p_1, p_2, p_3, p_4} be the set of cricket players under consideration and E = {e_1, e_2, e_3, e_4} be the set of parameters, where e_1, e_2, e_3, e_4 represent fitness, good current form, good domestic cricket record and good moral character, resp. Let $I = \{e_1, e_2, e_3\} \subseteq E$. Then the PSCSS $(\tilde{P}, I) = \{ P(e_i) = \{ < p, A_{e_i}(p), \lambda_{e_i}(p) > : p \in X \} \ e_i \in I, i = 1,2,3 \}$ in U is external Pythagorean soft cubic set (EPSCS) in U.

Table 3. Tabular representation of (P, A)

U	$P(e_1)$		$P(e_2)$		$P(e_3)$	
	$Ae_1(x)$	$\lambda e_1(x)$	$Ae_2(x)$	$\lambda e_2(x)$	$Ae_3(x)$	$\lambda e_3(x)$
x_1	[0.4,0.6][0.3,0.3][0.5,0.6]	[0.1,0.2,0.3]	[0.2,0.4][0.4,0.5][0.5,0.6]	[0.1,0.3,0.4]	[0.4,0.6][0.5,0.5][0.5,0.7]	[0.3,0.4,0.4]
x_2	[0.4,0.4][0.5,0.6][0.6,0.7]	[0.3,0.4,0.4]	[0.4,0.7][0.3,0.6][0.5,0.6]	[0.2,0.2,0.4]	[0.5,0.6][0.6,0.6][0.3,0.3]	[0.4,0.5,0.2]
x_3	[0.3,0.6][0.7,0.7][0.4,0.6]	[0.2,0.6,0.2]	[0.5,0.6][0.6,0.7][0.4,0.6]	[0.4,0.5,0.35]	[0.6,0.7][0.6,0.6][0.5,0.6]	[0.1,0.5,0.4]
x_4	[0.5,0.6][0.6,0.7][0.3,0.4]	[0.4,0.5,0.2]	[0.6,0.7][0.4,0.6][0.6,0.6]	[0.2,0.3,0.5]	[0.5,0.6][0.3,0.4][0.4,0.4]	[0.4,0.25,0.35]
x_5	[0.5,0.7][0.5,0.6][0.6,0.6]	[0.45,0.4,0.5]	[0.4,0.5][0.5,0.6][0.5,0.7]	[0.3,0.45,0.4]	[0.3,0.6][0.6,0.7][0.4,0.4]	[0.2,0.5,0.3]

Theorem: 4.6

Let $(P, A) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in U\}\ e_i \in I\}$ be a Pythagorean soft cubic set in U which is not an EPSCS. Then, there exists at least one $e_i \in I$ for which there exists some $x \in U$ such that $\lambda_{e_i}^{T}(x) \in (A_{e_i}^{-T}(x), A_{e_i}^{+T}(x))$, $\lambda_{e_i}^{I}(x) \in (A_{e_i}^{-I}(x), A_{e_i}^{+I}(x))$, $\lambda_{e_i}^{T}(x) \in (A_{e_i}^{-F}(x), A_{e_i}^{+F}(x))$

Proof: Straightforward

Theorem: 4.7

Let (P, A) be a Pythagorean soft cubic set in U. If (P, A) is both T-internal and T-external in U, then

$$(\forall\, x \in U, e_i \in E)\ \ (\lambda_{e_i}^{T}(x) \in \{A_{e_i}^{-T}(x) / x \in U, e_i \in E\} \cup \{A_{e_i}^{+T}(x) / x \in U, e_i \in E\})$$

Proof.

Consider the conditions (3.1) and (3.4) which implies that $A_{e_i}^{-T}(x) \le \lambda_{e_i}^{T}(x) \le A_{e_i}^{+T}(x)$ and $\lambda_{e_i}^{T}(x) \notin (A_{e_i}^{-T}(x), A_{e_i}^{+T}(x)$ for all $x \in U,\ e_i \in E$.

Then it follows that $\lambda_{e_i}^{T}(x) = A_{e_i}^{-T}(x)$ *or* $\lambda_{e_i}^{T}(x) = A_{e_i}^{+T}(x)$,

And hence $\lambda_{e_i}^{T}(x) \in \{A_{e_i}^{-T}(x) / x \in U, e_i \in E\} \cup \{A_{e_i}^{+T}(x) / x \in U, e_i \in E\}$.

Hence Proved.

Similarly, the following propositions hold for the indeterminate and falsity values.

Theorem: 4.8

Let (P, A) be a Pythagorean soft cubic set in a non-empty set *U*. If (P, A) is both I-internal and I-external, then $(\forall x \in U, e_i \in E)\ (\lambda_{e_i}^{I}(x) \in \{A_{e_i}^{-I}(x) / x \in U, e_i \in E\} \cup \{A_{e_i}^{+I}(x) / x \in U, e_i \in E\})$

Proposition Theorem be a Pythagorean soft cubic set in a non-empty set *X*. If (P, A) is both F-internal and F-external, then $(\forall x \in U, e_i \in E)\ (\lambda_{e_i}^{F}(x) \in \{A_{e_i}^{-F}(x) / x \in U, e_i \in E\} \cup \{A_{e_i}^{+F}(x) / x \in U, e_i \in E\})$.

Definition: 4.10

Let $\Im = (P, A) \in C_N^X$. If

$A_{e_i}^{-T}(x) \le \lambda_{e_i}^{T}(x) \le A_{e_i}^{+T}(x)$, $A_{e_i}^{-I}(x) \le \lambda_{e_i}^{I}(x) \le A_{e_i}^{+I}(x)$ and $\lambda_{e_i}^{F}(x) \notin (A_{e_i}^{-F}(x), A_{e_i}^{+F}(x)$ or $A_{e_i}^{-T}(x) \le \lambda_{e_i}^{T}(x) \le A_{e_i}^{+T}(x)$, $A_{e_i}^{-F}(x) \le \lambda_{e_i}^{F}(x) \le A_{e_i}^{+F}(x)$ and $\lambda_{e_i}^{I}(x) \notin (A_{e_i}^{-I}(x), A_{e_i}^{+I}(x)$ or $A_{e_i}^{-F}(x) \le \lambda_{e_i}^{F}(x) \le A_{e_i}^{+F}(x)$, $A_{e_i}^{-I}(x) \le \lambda_{e_i}^{I}(x) \le A_{e_i}^{+I}(x)$ and $\lambda_{e_i}^{T}(x) \notin (A_{e_i}^{-T}(x), A_{e_i}^{+T}(x)$ for all $x \in U$ corresponding to each $e_i \in I$. Then $\Im$ is called an external Pythagorean soft cubic set $\frac{2}{3}$ IPSCS or $\frac{1}{3}$ EPSCS.

Example: 4.11

Let $\Im = (P, A) \in C_N^X$. If $(\mathrm{P, A}) = \mathrm{P(e_1)} = \{< \mathrm{x}, ([0.2,0.5],[0.5,0.7],[0.3,0.5]), (0.3,0.4,0.4) >\}$ all $x \in U$ corresponding to each $e_i \in I$ .Then $\Im = (P, A)$ is a $\frac{2}{3}$ IPSCS.

Definition: 4.12

Let $\Im = (P, A) \in C_N^X$. If

$A_{e_i}^{-T}(x) \le \lambda_{e_i}^{T}(x) \le A_{e_i}^{+T}(x)$, $\lambda_{e_i}^{I}(x) \notin (A_{e_i}^{-I}(x), A_{e_i}^{+I}(x)$ and $\lambda_{e_i}^{F}(x) \notin (A_{e_i}^{-F}(x), A_{e_i}^{+F}(x)$ or , $A_{e_i}^{-F}(x) \le \lambda_{e_i}^{F}(x) \le A_{e_i}^{+F}(x)$, $\lambda_{e_i}^{T}(x) \notin (A_{e_i}^{-T}(x), A_{e_i}^{+T}(x)$ and $\lambda_{e_i}^{I}(x) \notin (A_{e_i}^{-I}(x), A_{e_i}^{+I}(x)$ or $A_{e_i}^{-I}(x) \le \lambda_{e_i}^{I}(x) \le A_{e_i}^{+I}(x)$, $\lambda_{e_i}^{F}(x) \notin (A_{e_i}^{-F}(x), A_{e_i}^{+F}(x)$ and $\lambda_{e_i}^{T}(x) \notin (A_{e_i}^{-T}(x), A_{e_i}^{+T}(x)$ for all $x \in X$ corresponding to each $e_i \in I$. Then $\Im$ is called an external Pythagorean soft cubic set $\frac{1}{3}$ IPSCS or $\frac{2}{3}$ EPSCS.

Example: 4.13

Let $\Im = (P, A) \in C_N^X$. If $(\mathrm{P, A}) = \mathrm{P(e_1)} = \{< \mathrm{x}, ([0.2,0.5],[0.5,0.7],[0.3,0.5]), (0.3,0.4,0.6) >\}$ where all $x \in U$ corresponding to each $e_i \in I$ .Then $\Im = (P, A)$ is a $\frac{1}{3}$ IPSCS.

Theorem: 4.14

Let $\Im = (P, A) \in C_N^X$ .Then

i. *Every IPSCS is a generalization of the ICS*

ii. *Every EPSCS is a generalization of the ECS.*

iii. *Every PSCS is the generalization of cubic set.*

Proof. The proof is direct from the above definitions.

Definition 4.15.

Let $(P, I) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in U\}\ e_i \in I \}$ and $(Q, J) = \{ Q(e_i) = B_i = \{< x, B_{e_i}(x), \mu_{e_i}(x) >: x \in U\}\ e_i \in I \}$ be two Pythagorean soft cubic sets in U. Let I and J be any two subsets of E (set of parameters), then we have the following

1. $(P, I) = (Q, J)$ if and only if the following conditions are satisfied
   a) I = J and
   b) $P(e_i) = Q(e_i)$ for all $e_i \in I$ if and only if $A_{e_i}(x) = B_{e_i}(x)$ and $\lambda_{e_i}(x) = \mu_{e_i}(x)$ for all $x \in U$ corresponding to each $e_i \in I$.
2. (P, I) and (Q, J) are two Pythagorean soft cubic set then we define and denote P-order as $(P, I) \subseteq_P (Q, J)$ if and only if the following conditions are satisfied
   c) $I \subseteq J$ and
   d) $P(e_i) \leq_P Q(e_i)$ for all $e_i \in I$ if and only if $A_{e_i}(x) \subseteq_P B_{e_i}(x)$ and $\lambda_{e_i}(x) \leq_P \mu_{e_i}(x)$ for all $x \in U$ corresponding to each $e_i \in I$.
3. (P, I) and (Q, J) are two Pythagorean soft cubic set then we define and denote P- order as $(P, I) \subseteq_R (Q, J)$ if and only if the following conditions are satisfied
   e) $I \subseteq_R J$ and
   f) $P(e_i) \leq_R Q(e_i)$ for all $e_i \in I$ if and only if $A_{e_i}(x) \subseteq_R B_{e_i}(x)$ and $\lambda_{e_i}(x) \geq_R \mu_{e_i}(x)$ for all $x \in U$ corresponding to each $e_i \in I$.

We now define the P-union, P-intersection, R-union and R-intersection of Pythagorean cubic soft sets as follows:

Definition: 4.16

Let (F, I) and (G, J) be two Pythagorean soft cubic sets (PSCS) in U where I and J are any two subsets of the parametric set E. Then we define **P-union** as $(F, I) \cup_p (G, J) = (H, C)$ where $C = I \cup J$

$$H(e_i) = \begin{cases} F(e_i) & If\ e_i \in I - J \\ G(e_i) & If\ e_i \in J - I \\ F(e_i) \vee_P G(e_i) & If\ e_i \in I \cap J \end{cases}$$

where $F(e_i) \vee_P G(e_i)$ is defined as

$$F(e_i) \vee_P G(e_i) = \{<\mathrm{x}, \max\{\mathrm{A}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda_{\mathrm{e_i}} \vee \mu_{\mathrm{e_i}})(\mathrm{x}) >: \mathrm{x} \in \mathrm{U}\}\ \mathrm{e_i} \in \mathrm{I} \cap \mathrm{J}$$

where $\mathrm{A}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}_{\mathrm{e_i}}(\mathrm{x})$ represent interval Pythagorean sets. Hence

$$F^T(e_i) \vee_P G^T(e_i) = \{<\mathrm{x}, \max\{\mathrm{A}^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda_{\mathrm{e_i}} \vee \mu_{\mathrm{e_i}})^{\mathrm{T}}(\mathrm{x}) >: \mathrm{x} \in \mathrm{U}\}\ \mathrm{e_i} \in \mathrm{I} \cap \mathrm{J}\ ,$$

$$F^I(e_i) \vee_P G^{\mathrm{I}}(e_i) = \{<\mathrm{x}, \max\{\mathrm{A}^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda_{\mathrm{e_i}} \vee \mu_{\mathrm{e_i}})^{\mathrm{I}}(\mathrm{x}) >: \mathrm{x} \in \mathrm{U}\}\ \mathrm{e_i} \in \mathrm{I} \cap \mathrm{J}\ ,$$

$$F^F(e_i) \vee_P G^F(e_i) = \{<\mathrm{x}, \max\{\mathrm{A}^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda_{\mathrm{e_i}} \vee \mu_{\mathrm{e_i}})^{\mathrm{F}}(\mathrm{x}) >: \mathrm{x} \in \mathrm{U}\}\ \mathrm{e_i} \in \mathrm{I} \cap \mathrm{J}\ .$$

Definition: 4.17

Let (F, I) and (G, J) be two Pythagorean soft cubic sets (PSCS) in U where I and J are any subsets of parameter's set E.

Then we define **P-intersection** as $(F, I) \cap_p (G, J) = (H, C)$ where $C = I \cap J$, $H(e_i) = F(e_i) \wedge_P G(e_i)$ *and* $e_i \in I \cap J$. Here $F(e_i) \wedge_P G(e_i)$ is defined as

$$F(e_i) \wedge_P G(e_i) = H(e_i) = \{<\mathrm{x}, \min\{\mathrm{A}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda_{\mathrm{e_i}} \wedge \mu_{\mathrm{e_i}})(\mathrm{x}) >: \mathrm{x} \in \mathrm{U}\}\ \mathrm{e_i} \in \mathrm{I} \cap \mathrm{J}\ .$$

where $\mathrm{A}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}_{\mathrm{e_i}}(\mathrm{x})$ represent interval Pythagorean sets. Hence

$$F^T(e_i) \wedge_P G^T(e_i) = \{<\mathrm{x}, \min\{\mathrm{A}^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda_{\mathrm{e_i}} \wedge \mu_{\mathrm{e_i}})^{\mathrm{T}}(\mathrm{x}) >: \mathrm{x} \in \mathrm{U}\}\ \mathrm{e_i} \in \mathrm{I} \cap \mathrm{J}\ ,$$

$$F^I(e_i) \wedge_P G^{\mathrm{I}}(e_i) = \{<\mathrm{x}, \min\{\mathrm{A}^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}^{\mathrm{I}}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda_{\mathrm{e_i}} \wedge \mu_{\mathrm{e_i}})^{\mathrm{I}}(\mathrm{x}) >: \mathrm{x} \in \mathrm{U}\}\ \mathrm{e_i} \in \mathrm{I} \cap \mathrm{J}\ ,$$

$$F^F(e_i) \wedge_P G^F(e_i) = \{<\mathrm{x}, \min\{\mathrm{A}^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}^{\mathrm{F}}_{\mathrm{e_i}}(\mathrm{x})\}, (\lambda_{\mathrm{e_i}} \wedge \mu_{\mathrm{e_i}})^{\mathrm{F}}(\mathrm{x}) >: \mathrm{x} \in \mathrm{U}\}\ \mathrm{e_i} \in \mathrm{I} \cap \mathrm{J}\ .$$

Example: 4.18 (P-ORDER)

Let U={x_1,x_2,x_3,x_4,x_5} be initial universe, I = J= {e_1,e_2} are any subsets of parameter's set E={e_1,e_2,e_3}.

Let (F,I) be PSCS defined as $(F, I) = \{ F(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in U\}\ e_i \in I \}$ is

U	F(e_1)		F(e_2)	
	A$e_1(x)$	$\lambda e_1(x)$	A$e_2(x)$	$\lambda e_2(x)$
x_1	[0.5,0.6][0.3,0.5][0.3,0.4]	[0.4,0.5,0.5]	[0.2,0.3][0.1,0.3][0.2,0.8]	[0.1,0.5,0.6]
x_2	[0.2,0.2][0.4,0.4][0.5,0.6]	[0.3,0.5,0.6]	[0.4,0.5][0.3,0.5][0.2,0.4]	[0.6,0.4,0.2]
x_3	[0.3,0.4][0.7,0.7][0.1,0.2]	[0.5,0.6,0.6]	[0.2,0.3][0.1,0.3][0.4,0.5]	[0.5,0.5,0.6]
x_4	[0.1,0.1][0.2,0.4][0.6,0.7]	[0.2,0.3,0.4]	[0.5,0.6][0.4,0.5][0.3,0.5]	[0.2,0.5,0.5]
x_5	[0.5,0.7][0.3,0.5][0.2,0.4]	[0.1,0.4,0.5]	[0.3,0.6][0.2,0.3][0.5,0.7]	[0.4,0.5,0.7]

Let (G,J) be PSCS defined as $(G, J) = \{ G(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) >: x \in U\}\ e_i \in J \}$ is

U	G(e_1)		G(e_2)	
	A$e_1(x)$	$\lambda e_1(x)$	A$e_2(x)$	$\lambda e_2(x)$
x_1	[0.3,0.4][0.7,0.9][0.1,0.1]	[0.4,0.6,0.7]	[0.4,0.6][0.7,0.7][0.1,0.4]	[0.2,0.6,0.9]
x_2	[0.6,0.6][0.3,0.4][0.1,0.7]	[0.4,0.6,0.6]	[0.1,0.5][0.4,0.7][0.5,0.6]	[0.5,0.6,0.7]
x_3	[0.3,0.6][0.4,0.5][0.3,0.4]	[0.3,0.4,0.6]	[0.4,0.7][0.1,0.3][0.2,0.4]	[0.5,0.6,0.6]
x_4	[0.6,0.7][0.3,0.3][0.2,0.4]	[0.1,0.5,0.5]	[0.3,0.4][0.7,0.9][0.1,0.2]	[0.4,0.4,0.5]
x_5	[0.2,0.6][0.2,0.4][0.3,0.4]	[0.3,0.5,0.7]	[0.5,0.7][0.6,0.7][0.3,0.4]	[0.2,0.5,0.7]

Then P- union is denoted by $(F, I) \cup_p (G, J)$ and defined as

U	F∪ $G(e_1)$		F∪ $G(e_2)$	
	< A∪ $Be_1(x)$	λ∪ $\mu e_1(x)$ >	< A∪ $Be_2(x)$	λ∪ $\mu e_2(x)$ >
x_1	[0.5,0.6][0.7,0.9][0.3,0.4]	[0.4,0.6,0.7]	[0.4,0.6][0.7,0.7][0.2,0.8]	[0.2,0.6,0.9]
x_2	[0.6,0.6][0.4,0.4][0.5,0.7]	[0.4,0.6,0.6]	[0.4,0.5][0.4,0.7][0.5,0.6]	[0.6,0.6,0.7]
x_3	[0.3,0.6][0.7,0.7][0.3,0.4]	[0.5,0.6,0.6]	[0.4,0.7][0.1,0.3][0.4,0.5]	[0.5,0.6,0.6]

x_4	[0.6,0.7][0.3,0.4][0.6,0.7]	[0.2,0.5,0.5]	[0.5,0.6][0.7,0.9][0.3,0.5]	[0.4,0.5,0.5]
x_5	[0.5,0.7][0.3,0.5][0.3,0.4]	[0.3,0.5,0.7]	[0.5,0.7][0.6,0.7][0.5,0.7]	[0.4,0.5,0.7]

Then P- intersection denoted by $(F,I) \cap_p (G,J)$ and defined as

U	F∩ $G(e_1)$		F∩ $G(e_2)$	
	< A∩ $Be_1(x)$	λ∩ $\mu e_1(x)$ >	< A∩ $Be_2(x)$	λ∩ $\mu e_2(x)$ >
x_1	[0.3,0.4][0.3,0.5][0.1,0.1]	[0.4,0.5,0.5]	[0.2,0.3][0,.1,0.3][0.1,0.4]	[0.1,0.5,0.6]
x_2	[0.2,0.2][0.3,0.4][0.1,0.6]	[0.3,0.5,0.6]	[0.1,0.5][0.3,0.5][0.2,0.4]	[0.5,0.4,0.2]
x_3	[0.3,0.4][0.4,0.5][0.1,0.2]	[0.3,0.4,0.6]	[0.2,0.3][0.1,0.3][0.4,0.4]	[0.5,0.5,0.6]
x_4	[0.1,0.1][0.2,0.3][0.2,0.4]	[0.1,0.3,0.4]	[0.3,0.4][0.4,0.5][0.1,0.2]	[0.2,0.4,0.5]
x_5	[0.2,0.6][0.2,0.4][0.2,0.4]	[0.1,0.4,0.5]	[0.3,0.6][0.2,0.3][0.3,0.4]	[0.2,0.5,0.7]

Definition: 4.18

Let (F,I) and (G,J) be two Pythagorean soft cubic sets (PSCS) in U where I and J are any subsets of parameter's set E.

Then we define **R-union** as $(F,I) \cup_R (G,J) = (H,C)$ where $C = I \cup J$

$$H(e_i) = \begin{cases} F(e_i) & if\ e_i \in I - J \\ G(e_i) & if\ e_i \in J - I \\ F(e_i) \vee_R G(e_i) & if\ e_i \in I \cap J \end{cases}$$

Where $F(e_i) \vee_R G(e_i)$ is defined as

$$F(e_i) \vee_R G(e_i) = \{< \mathrm{x}, \max\{ \mathrm{A}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}_{\mathrm{e_i}}(\mathrm{x}) \}, (\lambda_{\mathrm{e_i}} \wedge \mu_{\mathrm{e_i}})(\mathrm{x}) >: \mathrm{x} \in \mathrm{U}\}\ \mathrm{e_i} \in \mathrm{I} \cap \mathrm{J}$$

Where $\mathrm{A}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}_{\mathrm{e_i}}(\mathrm{x})$ represent interval Pythagorean sets.

Hence $F^T(e_i) \vee_R G^T(e_i) = \{< \mathrm{x}, \min\{ \mathrm{A}^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{x}), \mathrm{B}^{\mathrm{T}}_{\mathrm{e_i}}(\mathrm{x}) \}, (\lambda_{\mathrm{e_i}} \wedge \mu_{\mathrm{e_i}})^{\mathrm{T}}(\mathrm{x}) >: \mathrm{x} \in \mathrm{U}\}\ \mathrm{e_i} \in \mathrm{I} \cap \mathrm{J}$,

$F^I(e_i) \vee_R G^I(e_i) = \{< x, \min\{A^I_{e_i}(x), B^I_{e_i}(x)\}, (\lambda_{e_i} \wedge \mu_{e_i})^I(x) >: x \in U\}\ e_i \in I \cap J$,

$F^F(e_i) \vee_R G^F(e_i) = \{< x, \min\{A^F_{e_i}(x), B^F_{e_i}(x)\}, (\lambda_{e_i} \vee \mu_{e_i})^F(x) >: x \in U\}\ e_i \in I \cap J$

Definition: 4.19

Let (F,I) and (G,J) be two Pythagorean soft cubic sets (PSCS) in U where I and J are any subsets of parameter's set E.

Then we define **R-intersection** as $(F,I) \cap_R (G,J) = (H,C)$ where $C = I \cap J$,

$H(e_i) = F(e_i) \wedge_R G(e_i)$ *and* $e_i \in I \cap J$. Here $F(e_i) \wedge_R G(e_i)$ is defined as

$F(e_i) \wedge_R G(e_i) = H(e_i) = \{< x, \min\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i} \vee \mu_{e_i})(x) >: x \in U\}\ e_i \in I \cap J\}$.

Example 4.21: (R –ORDER)

Let U={x_1,x_2,x_3,x_4,x_5} be initial universe, I = J= {e_1,e_2} are any subsets of parameter's set E={e_1,e_2,e_3}.

Let (F,I) be PSCS defined as $(F, I) = \{F(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in U\}\ e_i \in I\}$ i

U	P(e_1)		P(e_2)	
	A$e_1(x)$	$\lambda e_1(x)$	A$e_2(x)$	$\lambda e_2(x)$
x_1	[0.2,0.3][0.3,0.5][0.3,0.5]	[0.35,0.35,0.4]	[0.3,0.2][0.3,0.5][0.1,0.5]	[0.25,.025,0.6]
x_2	[0.4,0.7][0.1,0.5][0.2,0.5]	[0.35,0.5,0.6]	[0.5,0.5][0.5,0.6][0.2,0.5]	[0.25,0.3,0.4]
x_3	[0.6,0.9][0.0,0.2][0.3,0.5]	[0.5,0.5,0.55]	[0.4,0.6][0.4,0.7][0.3,0.5]	[0.15,0.2,0.45]

Let (G,J) be PSCS defined as $(G, J) = \{G(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) >: x \in U\}\ e_i \in J\}$ is

U	G(e_1)		G(e_2)	
	B$e_1(x)$	$\mu e_1(x)$	B$e_2(x)$	$\mu e_2(x)$
x_1	[0.3,0.5][0.4,0.7][0.4,0.5]	[0.25,0.3,0.5]	[0.1,0.3][0.5,0.7][0.3,0.5]	[0.4,0.6,0.7]
x_2	[0.2,0.5][0.5,0.2][0.3,0.5]	[0.3,0.4,0.6]	[0.4,0.7][0.4,0.6][0.5,0.6]	[0.5,0.6,0.6]
x_3	[0.6,0.7][0.4,0.7][0.3,0.5]	[0.2,0.3,0.4]	[0.7,0.8][0.4,0.5][0.1,0.4]	[0.5,0.55,0.7]

Then R-union denoted by $(F, I) \cup_R (G, J)$ and defined as

U	F∪ $G(e_1)$		F∪ $G(e_2)$	
	A∪ $Be_1(x)$	λ∩ $\mu e_1(x)$	A∪ $Be_2(x)$	λ∩ $\mu e_2(x)$
x_1	[0.3,0.5][0.4,0.7][0.4,0.5]	[0.25,0.3,0.4]	[0.3,0.3][0.5,0.7][0.3,0.5]	[0.25,0.25.0.6]
x_2	[0.4,0.7][0.5,0.5][0.3,0.5]	[0.3,0.4,0.6]	[0.5,0.7][0.5,0.6][0.5,0.6]	[0.25,0.3,0.4]
x_3	[0.6,0.9][0.4,0.7][0.3,0.5]	[0.2,0.3,0.4]	[0.7,08][0.4,0.7][0.3,0.5]	[0.15,0.2,0.45]

Then R- intersection is denoted by $(F, I) \cap_R (G, J)$ and defined as

U	F∩ $G(e_1)$		F∩ $G(e_2)$	
	A∩ $Be_1(x)$	λ∪ $\mu e_1(x)$	A∩ $Be_2(x)$	λ∪ $\mu e_2(x)$
x_1	[0.2,0.3][0.3,0.5][0.3,0.5]	[0.35,0.35,0.5]	[0.1,0.2][0.3,0.5][0.1,0.5]	[0.4,0.6,0.7]
x_2	[0.2,0.5][0.1,0.2][0.2,0.5]	[0.35,0.5,0.6]	[0.4,0.5][0.4,0.6][0.2,0.5]	[0.5,0.6,0.6]
x_3	[0.6,0.7][0.0,0.2][0.3,0.5]	[0.5,0.5,0.65]	[0.4,0.6][0.4,0.5][0.1,0.4]	[0.5,0.55,0.7]

Conclusion:

The main goal of this work is to present a possibility Pythagorean fuzzy soft set to solve the phenomena related to decision-making in which the sum of membership and non-membership is greater than 1 by considering the possibility of belongingness of the elements in the universe. Moreover, we have discussed some operational properties such as p-union and p-intersection of Pythagorean soft cubic set. In addition, a similarity measure is developed to compare two possibility Pythagorean fuzzy soft sets to deal with decision problems. Finally, in order to illustrate the validity of this similarity measure, possibility Pythagorean fuzzy soft sets are applied to decision making problems, Pythagorean soft cubic sets are applied to interval Pythagorean sets. In this paper, we have defined the notion of possibility Pythagorean fuzzy soft set theory and Pythagorean soft cubic sets P-order, P-union, P-intersection as well as R-order, R-union, R-intersection of Pythagorean soft cubic sets.

Reference:

[1] L. A. Zadeh (1965) "Fuzzy sets" Archived 2015-08-13 at the Wayback Machine. *Information and Control* 8 (3) 338–353.

[2] *Liang, Lily R.; Lu, Shiyong; Wang, Xuena; Lu, Yi; Mandal, Vinay; Patacsil, Dorrelyn; Kumar, Deepak (2006). "FM-test: A fuzzy-set-theory-based approach to differential gene expression data analysis". BMC Bioinformatics.* ***7****: S7. doi:10.1186/1471-2105-7-S4-S7. PMC 1780132. PMID 17217525.*

[3] Ismat Beg, Samina Ashraf: Similarity measures for fuzzy sets, at: Applied and Computational Mathematics, March 2009, available on Research Gate since November 23rd, 2016.

[4] K. T. Atanassov. Intuitionistic fuzzy sets, VII ITKR's session, 1983. De-posed in Central Science Technical Library of Bulgarian Academy of Sci-ence (in Bulgarian).

[5] H. Bustince and P. Burillo. Vague sets are intuitionistic fuzzy sets.FuzzySets and Systems, 79(3):403–405, 1996

[6] R.K. Verma B.D. Sharma, Some new equalities connected with intuitionistic fuzzy set, Note on intuitionistic Fuzzy Sets, 2012.

[7] *Yager, Ronald R. (June 2013). "Pythagorean fuzzy subsets". 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS). IEEE: 57–61. doi:10.1109/IFSA-NAFIPS.2013.6608375. ISBN 978-1-4799-0348-1.*

[8] *Yager, Ronald R. (December 2015). Properties and applications of Pythagorean fuzzy sets. Springer, Cham. pp. 119–136. ISBN 978-3-319-26302-1.*

[9] Yager, R.R.: Pythagorean membership grades in multi-criteria decision making. IEEE Trans. Fuzzy Syst. **22**, 958–965 (2014)

[10] D. Molodtsov, Soft set theory-First results, *Computers and Mathematics with Applications* **37** (1999), 19–31.

[11] Y.B. Jun, Soft BCK/BCI-algebras, *Computers and Mathematics with Applications* **56** (2008), 1408–1413.

[12] Y.B. Jun and C.H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, *Information Sciences* **178** (2008), 2466–2475.

[13] M.I. Ali, F. Feng, X. Liu, W.K. Min, and M. Shabir, On some new operations in soft set theory, *Computers and Mathematics with Applications* **57** (2009), 1547–1553.

[14] K.Y. Qin and Z.Y. Hong, On soft equality, *Journal of Computational and Applied Mathematics* **234** (2010), 1347–1355.

[15] J.H. Park, O.H. Kima, and Y.C. Kwun, Some properties of equivalence soft set relations, *Computers and Mathematics with Applications* **63** (2012), 1079–1088.

[16] X.L. Ma et al, A survey of decision making methods based on two classes of hybrid soft set models, *Artificial Intelligence Review* **49**(4) (2018), 511–529.

[17] G. Deschrijver and E.E.Kerre, Implicators based on binary aggregation operators in interval-valued fuzzy set theory, *Fuzzy Sets and Systems* **153** (2005), 229–248.

[18] Y. Yang, X. Tan, and C.C. Meng, The multi-fuzzy soft set and its application in decision making, *Applied Mathematical Modelling* **37** (2013), 4915–4923.

[19] S. Alkhazaleh, A.R. Salleh, and N. Hassan, Possibility fuzzy soft set, Advances in Decision Sciences, Volume 2011, Article ID 479756, 18 pages.

[20] P.K.Maji , R. Biswas ans A.R.Roy, "Fuzzy soft sets", Journal of Fuzzy Mathematics, Vol 9, no.3, 2001 pp – 589-602,

[21] P.K.Maji, R. Biswas ans A.R.Roy, "Intuitionistic Fuzzy soft sets", The journal of fuzzy Mathematics, Vol 9, (3)(2001), 677 – 692.

[22] Chetia B, Das PK (2012) Some results of Intuitionistic fuzzy soft matrix theory. Adv Appl Sci Res 3:412–423

[23] Chen SM, Chang CH (2015) A novel similarity measure between Atanssovs intuitionistic fuzzy sets based on transformation techniques with applications to pattern recognition. Inf Sci 291:96–114

[24] Chetia B, Das PK(2012) Some results of Intuitionistic fuzzy soft matrix theory. Adv Appl Sci Res 3:412–423

[25] X.D. Peng, Y. Yang, J.P. Song et al, Pythagorean fuzzy soft set and its application, *Computer Engineering* **41**(7) (2015), 224–229.

[26] Peng X, Yuan H, Yang Y (2017) Pythagorean fuzzy information measures and their applications. Int J Intell Syst 32(10):991–1029

[27] Yager RR (2013) Pythagorean fuzzy subsets. In: Proceedings of joint IFSA world congress and NAFIPS annual meeting, Edmonton, Canada, pp 57–61

[28] Y.B. Jun, C.S. Kim and K.O. Yang, Cubic sets, Annals of Fuzzy Mathematics and Informatics 4(3) (2012), 83–98.

[29] Jun, Y. B. Kim, C. S. Yang, K. O. Cubic sets, Annuals of Fuzzy Mathematics and Informatics, 2012 4, 83-98.10.Jun, Y. B. Jung, S. T. Kim, M. S. Cubic subgroup, Annals of Fuzzy Mathematics and Infirmatics, 2011, 2, 9-15.

[30] Muhiuddin, G. Al-roqi, A. M. Cubic soft sets with applications in BCK=BCI-algebras, Ann. Fuzzy Math. Inform. 8(2) (2014),291-304..

[31] X. D. Peng,Y. Y. Yang, J. Song and Y. Jiang,Pythagorean Fuzzy Soft Set and ItsApplication, Computer Engineering,41(7)(2015), 224-229.

[32]X. D. Peng,New similarity measure and distance measure for Pythagorean fuzzy set,Complex & Intelligent Systems, (2018), DOI.org/10.1007/s40747-018-0084-x.