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β^{**} Generalized Closed Mappings in Intuitionistic Fuzzy Topological Spaces

5.1 Introduction

Seok Jong Lee and Eun Pyo Lee (2000) have introduced intuitionistic fuzzy closed mappings. In this chapter we have introduced intuitionistic fuzzy β^{**} generalized closed mappings, intuitionistic fuzzy β^{**} generalized open mappings, intuitionistic fuzzy M-β^{**} generalized closed mappings and intuitionistic fuzzy contra β^{**} generalized open mappings and provided some interesting propositions.

5.2 Intuitionistic Fuzzy β^{**} Generalized Closed Mappings

In this section we have introduced intuitionistic fuzzy β^{**} generalized closed mappings, intuitionistic fuzzy β^{**} generalized open mappings, intuitionistic fuzzy M-β^{**} generalized closed mappings and investigated some of their properties.

Definition 5.2.1 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an **intuitionistic fuzzy β^{**} generalized** (IFβ^{**}G) **closed mapping** if f(V) is an IFβ^{**}GCS in Y for every IFCS V of X.

Example 5.2.2 : Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTS on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is an IFβ^{**}G closed mapping.

Proposition 5.2.3 : Every IF closed mapping is an IFβ^{**}G closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF closed mapping. Let A be an IFCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since every IFCS is an IFβ^{**}GCS, f(A) is an IFβ^{**}GCS in Y. Hence f is an IFβ^{**}G closed mapping.

Example 5.2.4 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTS on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is an IFβ^{**}G closed mapping but not an IF closed mapping, since $G_2^c = \langle x, (0.2_a, 0.4_b), (0.8_a, 0.6_b)\rangle$ is an IFCS in X and $f(G_2^c)$ is not an IFCS in Y, as $cl(f(G_2^c)) = G_3^c \neq f(G_2^c)$.

Proposition 5.2.5 : Every IF semi closed mapping is an IFβ^{**}G closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF semi closed mapping. Let A be an IFCS in X. Then f(A) is an IFSCS in Y, by hypothesis. Since every IFSCS is an IFβ^{**}GCS, f(A) is an IFβ^{**}GCS in Y. Hence f is an IFβ^{**}G closed mapping.

Example 5.2.6 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTS on X and Y respectively. Here, f is an IFβ^{**}G closed mapping but not an IF semi closed mapping, since $G_2^c = \langle x, (0.2_a, 0.4_b), (0.8_a, 0.6_b)\rangle$ is an IFCS in X but $f(G_2^c)$ is not an IFSCS in Y, as $int(cl(f(G_2^c))) = G_3 \not\subseteq f(G_2^c)$.

Proposition 5.2.7 : Every IF pre closed mapping is an IFβ^{**}G closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF pre closed mapping. Let A be an IFCS in X. Then f(A) is an IFPCS in Y, by hypothesis. Since every IFPCS is an IFβ^{**}GCS, f(A) is an IFβ^{**}GCS in Y. Hence f is an IFβ^{**}G closed mapping.

Example 5.2.8 : Let X = {a, b} and Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFβ^{**}G closed mapping but not an IF pre closed mapping, since $G_2^c = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$ is an IFCS in X, but $f(G_2^c)$is not an IFPCS in Y, as $cl(int(f(G_2^c))) = cl(G_3) = 1_\sim \not\subseteq f(G_2^c)$.

Proposition 5.2.9 : Every IFα closed mapping is an IFβ^{**}G closed mapping but not conversely in general.

Proof : Let f : (X, τ) → (Y, σ) be an IFα closed mapping. Let A be an IFCS in X. Then f(A) is an IFαCS in Y, by hypothesis. Since every IFαCS is an IFβ^{**}GCS, f(A) is an IFβ^{**}GCS in Y. Hence f is an IFβ^{**}G closed mapping.

Example 5.2.10 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTS on X and Y respectively. Here, f is an IFβ^{**}G closed mapping but not an IFα closed mapping, since $G_2^c = \langle x, (0.2_a, 0.4_b), (0.8_a, 0.6_b)\rangle$ is an IFCS in X but $f(G_2^c)$ is not an IFαCS in Y, as $cl(int(cl(f(G_2^c)))) = G_3^c \not\subseteq G_2^c$.

Proposition 5.2.11 : Every IFβ closed mapping is an IFβ^{**}G closed mapping but not conversely in general.

Proof : Let f : (X, τ) → (Y, σ) be an IFβ closed mapping. Let A be an IFCS in X. Then f(A) is an IFβCS in Y, by hypothesis. Since every IFβCS is an IFβ^{**}GCS, f(A) is an IFβ^{**}GCS in Y. Hence f is an IFβ^{**}G closed mapping.

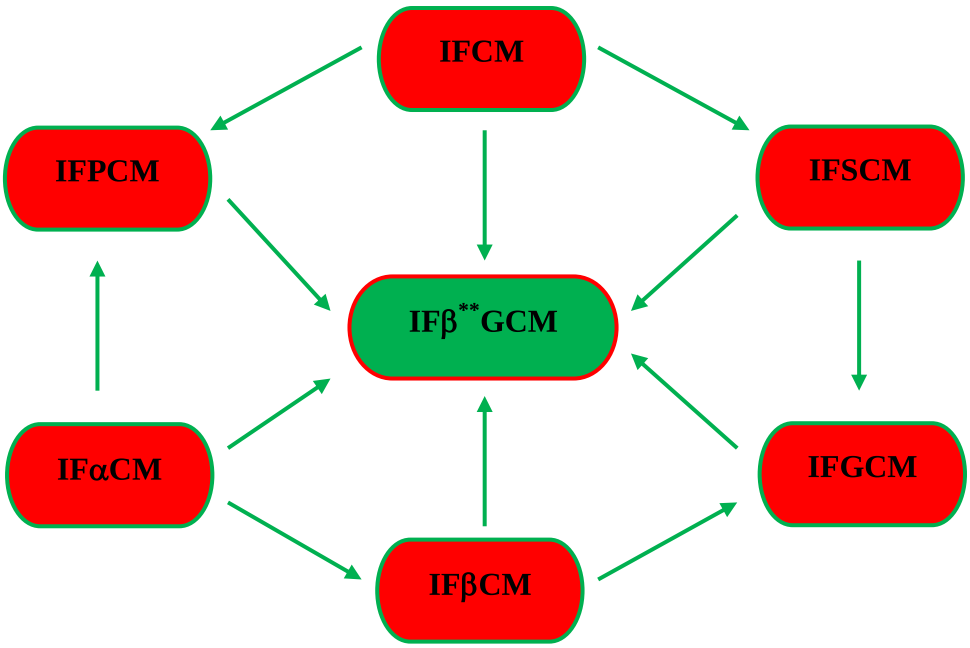
Example 5.2.12 : Let X = {a, b} and Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here, f is an IFβ^{**}G closed mapping but not an IFβ closed mapping, since $G_2^c = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$ is an IFCS in X but $f(G_2^c)$ is not an IFβCS in Y, as $int(cl(int(f(G_2^c)))) = 1_\sim \not\subseteq f(G_2^c)$.

Proposition 5.2.13 : Every IF generalized closed mapping is an IFβ^{**}G closed mapping but not conversely in general.

Proof : Let f : (X, τ) → (Y, σ) be an IF generalized closed mapping. Let A be an IFCS in X. Then f(A) is an IFGCS in Y, by hypothesis. Since every IFGCS is an IFβ^{**}GCS, f(A) is an IFβ^{**}GCS in Y. Hence f is an IFβ^{**}G closed mapping.

Example 5.2.14: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTS on X and Y respectively. Here, f is an IFβ^{**}G closed mapping but not an IF generalized closed mapping, since $G_2^c = \langle x, (0.2_a, 0.4_b), (0.8_a, 0.6_b)\rangle$ is an IFCS in X but $f(G_2^c)$ is not an IFGCS in Y, as $cl(f(G_2^c)) = G_3^c \not\subseteq G_3$ whereas $f(G_2^c) \subseteq G_3$.

The relation between various types of intuitionistic fuzzy closed mappings is given in the following diagram. In this diagram 'CM' means closed mappings.



The reverse implications are not true in general in the above diagram.

Proposition 5.2.15 : A bijective mapping f : (X, τ) → (Y, σ) is an IFβ^{**}G closed mapping if and only if for every IFS B of Y and for every IFOS U containing $f^{-1}(B)$, there is an IFβ^{**}GOS A of Y such that B ⊆ A and $f^{-1}(A) \subseteq U$.

Proof : Necessity : Let B be any IFS in Y. Let U be an IFOS in X such that $f^{-1}(B) \subseteq U$, then U^c is an IFCS in X. By hypothesis $f(U^c)$ is an IFβ^{**}GCS in Y. Let $A = (f(U^c))^c$, then A is an IFβ^{**}GOS in Y and B ⊆ A, as for a bijective mapping $(f(U^c))^c = f(U)$. Now $f^{-1}(A) = f^{-1}(f(U^c))^c = (f^{-1}(f(U^c)))^c \subseteq U$.

Sufficiency : Let A be any IFCS in X, then A^c is an IFOS in X and $f^{-1}(f(A^c)) \subseteq A^c$. By hypothesis there exists an IFβ^{**}GOS B in Y such that $f(A^c) \subseteq B$ and $f^{-1}(B) \subseteq A^c$. Therefore $A \subseteq (f^{-1}(B))^c$. Hence $B^c \subseteq f(A) \subseteq f(f^{-1}(B))^c \subseteq B^c$. This implies that $f(A) = B^c$. Since B^c is an IFβ^{**}GCS in Y, f(A) is an IFβ^{**}GCS in Y. Hence f is an IFβ^{**}G closed mapping.

Proposition 5.2.16 : If f : (X, τ) → (Y, σ) is an IF closed mapping and g : (Y, σ) → (Z, δ) is an IFβ^{**}G closed mapping, then g o f : (X, τ) → (Z, δ) is an IFβ^{**}G closed mapping.

Proof : Let A be an IFCS in X, then f(A) is an IFCS in Y, since f is an IF closed mapping. Since g is an IFβ^{**}G closed mapping, g(f(A)) is an IFβ^{**}GCS in Z. Therefore g o f is an IFβ^{**}G closed mapping.

Proposition 5.2.17 : Let f : (X, τ) → (Y, σ) be an IFβ^{**}G closed mapping. Then for every IFS A of X, f(cl(A)) is an IFβ^{**}GCS in Y.

Proof : Let A be any IFS in X. Then cl(A) is an IFCS in X. By hypothesis f(cl(A)) is an IFβ^{**}GCS in Y.

Proposition 5.2.18 : A bijective mapping f : (X, τ) → (Y, σ) is an IFβ^{**}G closed mapping if cl(int(cl(f(A)))) ⊆ f(cl(A)) for every IFS A in X.

Proof : Let A be an IFCS in X, By hypothesis, cl(int(cl(f(A)))) ⊆ f(cl(A)) = f(A). This implies f(A) is an IFαCS in Y and hence it is an IFβ^{**}GCS in Y. Therefore f is an IFβ^{**}G closed mapping.

Proposition 5.2.19 : If every IFS is an IFCS, then an IFβ^{**}G closed mapping is an IFβ^{**}G continuous mapping.

Proof : Let $f : X \to Y$ be an IFβ^{**}G closed mapping and let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFS in X. Therefore $f^{-1}(A)$ is an IFCS in X, by hypothesis. Since every IFCS is an IFβ^{**}GCS, $f^{-1}(A)$ is an IFβ^{**}GCS in X. This implies that f is an IFβ^{**}G continuous mapping.

Proposition 5.2.20 : Let $f : X \to Y$ be a bijective mapping where Y is an IFβ^{**}p$T_{1/2}$ space. Then the following are equivalent:

(i) f is an IFβ^{**}G closed mapping

(ii) f(B) is an IFβ^{**}GCS in Y for each IFCS B in X

(iii) $int(cl(int(f(B)))) \subseteq f(cl(B))$ for every IFS B in X.

Proof : (i) $\Leftrightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (iii) Let B be an IFS in X, then cl(B) is an IFCS in X. By hypothesis f(cl(B)) is an IFβ^{**}GCS in Y. Since Y is an IFβ^{**}p$T_{1/2}$ space, f(cl(B)) is an IFPCS in Y. Therefore $int(cl(int(f(B)))) \subseteq int(cl(int(f(cl(B))))) \subseteq cl(int(f(cl(B)))) \subseteq f(cl(B))$.

(iii) $\Rightarrow$ (i) Let A be an IFCS in X. By hypothesis, $f(A) = f(cl(A)) \supseteq int(cl(int(f(A))))$. This implies f(A) is an IFβCS in Y and hence an IFβ^{**}GCS in Y. Therefore f is an IFβ^{**}G closed mapping.

Proposition 5.2.21 : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFβ^{**}G closed mapping. Then f is an IFG closed mapping if Y is an IFβ^{**}g$T_{1/2}$ space.

Proof : Let A be an IFCS in X. Then f(A) is an IFβ^{**}GCS in Y, by hypothesis. Since Y is an IFβ^{**}g$T_{1/2}$ space, f(A) is an IFGCS in Y. Hence f is an IFG closed mapping.

Proposition 5.2.22 : Let $f : X \to Y$ be a bijective mapping. Then the following are equivalent if Y is an IFβ^{**}p$T_{1/2}$ space:

(i) f is an IFβ^{**}G closed mapping

(ii) $pcl(f(A)) \subseteq f(cl(A))$ for each IFS A of X

(iii) $f^{-1}(pcl(B)) \subseteq cl(f^{-1}(B))$ for every IFS B of Y

Proof : (i) $\Rightarrow$ (ii) Let A be an IFS in X. then cl(A) is an IFCS in X.

(i) implies that f(cl(A)) is an IFβ^{**}GCS in Y. Since Y is an IFβ^{**}p$T_{1/2}$ space, f(cl(A)) is an IFPCS in Y. Therefore $pcl(f(cl(A))) = f(cl(A))$. Now $pcl(f(A)) \subseteq pcl(f(cl(A))) = f(cl(A))$. Hence $pcl(f(A)) \subseteq f(cl(A))$ for each IFS A of X.

(ii) $\Rightarrow$ (i) Let A be any IFCS in X. Then $cl(A) = A$. (ii) implies that $pcl(f(A)) \subseteq f(cl(A)) = f(A)$. But $f(A) \subseteq pcl(f(A))$. Therefore $pcl(f(A)) = f(A)$. This implies f(A) is an IFPCS in Y. Since every IFPCS is an IFβ^{**}GCS, f(A) is an IFβ^{**}GCS in Y. Hence f is an IFβ^{**}G closed mapping.

(ii) $\Rightarrow$ (iii) Let B be an IFS in Y. Then $f^{-1}(B)$ is an IFS in X. Since f is onto, $pcl(B) = pcl(f(f^{-1}(B)))$ and (ii) implies $pcl(f(f^{-1}(B)) \subseteq f(cl(f^{-1}(B)))$. Therefore $pcl(B) \subseteq f(cl(f^{-1}(B)))$. Now $f^{-1}(pcl(B)) \subseteq f^{-1}(f(cl(f^{-1}(B)))) = cl(f^{-1}(B))$, since f is one to one. Hence $f^{-1}(pcl(B)) \subseteq cl(f^{-1}(B))$.

(iii) $\Rightarrow$ (ii) Let A be any IFS of X. Then f(A) is an IFS of Y. Since f is one to one, (iii) implies that $f^{-1}(pcl(f(A)))) \subseteq cl(f^{-1}(f(A))) = cl(A)$. Therefore $f(f^{-1}(pcl(f(A)))) \subseteq f(cl(A))$. Since f is onto, $pcl(f(A)) = f(f^{-1}(pcl(f(A)))) \subseteq f(cl(A))$.

Proposition 5.2.23 : A bijective mapping $f : (X, \tau) \to (Y, \sigma)$ is an IFβ^{**}G closed mapping if $f(int(B)) \subseteq cl(int(cl(f(B))))$ for every IFS B in X.

Proof : Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis, $f(int(A^c)) = f(A^c) \subseteq cl(int(cl(f(A^c))))$. That is $int(cl(int(f(A)))) \subseteq f(A)$. This implies f(A) is an IFβCS in Y and hence it is an IFβ^{**}GCS in Y. Therefore f is an IFβ^{**}G closed mapping.

Definition 5.2.24 : A mapping $f : X \to Y$ is said to be an **intuitionistic fuzzy β^{**} generalized** (IFβ^{**}G) **open mapping** if f(A) is an IFβ^{**}GOS in Y for each IFOS A in X.

Proposition 5.2.25 : For a bijective mapping $f : (X, \tau) \to (Y, \sigma)$ the following conditions are equivalent:

(i) f is an IFβ^{**}G closed mapping,

(ii) f is an IFβ^{**}G open mapping.

Proof : (i) $\Rightarrow$ (ii) Let A be an IFOS in X. Then A^c is an IFCS in X. By hypothesis, $f(A^c) = (f(A))^c$ is an IFβ^{**}GCS in Y. Hence f(A) is an IFβ^{**}GOS in Y.

(ii) $\Rightarrow$ (i) Let B be an IFCS in X. Then B^c is an IFOS in X. By (ii), $f(B^c) = (f(B))^c$ is an IFβ^{**}GOS in Y. Hence f(B) is an IFβ^{**}GCS in Y. Therefore f is an IFβ^{**}G closed mapping.

Proposition 5.2.26 : Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Then the following are equivalent if Y is an IFβ^{**}p$T_{1/2}$ space:

(i) f is an IFβ^{**}G open mapping

(ii) $f(int(A)) \subseteq pint(f(A))$ for each IFS A of X

(iii) $int(f^{-1}(B)) \subseteq f^{-1}(pint(B))$ for every IFS B of Y.

Proof : This proposition can be easily proved by taking complement in Proposition 5.2.22.

Proposition 5.2.27 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IFβ^{**}G open mapping if $f(\alpha int(A)) \subseteq \alpha int(f(A))$ for every $A \subseteq X$.

Proof : Let A be an IFOS in X. Then int(A) = A. Now $f(A) = f(int(A)) \subseteq f(\alpha int(A)) \subseteq \alpha int(f(A))$, by hypothesis. But $\alpha int(f(A)) \subseteq f(A)$. Hence $\alpha int(f(A)) = f(A)$. Therefore f(A) is an IFαOS in Y. This implies f(A) is an IFβ^{**}GOS in X. Hence f is an IFβ^{**}G open mapping.

Proposition 5.2.28 : A mapping f : (X, τ) → (Y, σ) is an IFβ^{**}G open mapping if and only if $int(f^{-1}(B)) \subseteq f^{-1}(pint(B))$ for every $B \subseteq Y$, where Y is an IFβ^{**}p$T_{1/2}$ space.

Proof : Necessity : Let $B \subseteq Y$. Then $f^{-1}(B) \subseteq X$ and $int(f^{-1}(B))$ is an IFOS in X. By hypothesis, $f(int(f^{-1}(B)))$ is an IFβ^{**}GOS in Y. Since Y is an IFβ^{**}p$T_{1/2}$ space, $f(int(f^{-1}(B)))$ is an IFPOS in Y. Therefore $f(int(f^{-1}(B))) = pint(f(int(f^{-1}(B)))) \subseteq pint(B)$. This implies $int(f^{-1}(B)) \subseteq f^{-1}(f(int(f^{-1}(B)))) \subseteq f^{-1}(pint(B))$.

Sufficiency : Let A be an IFOS in X. Therefore int(A) = A. Then $f(A) \subseteq Y$. By hypothesis $int(f^{-1}(f(A))) \subseteq f^{-1}(pint(f(A)))$. That is $int(A) \subseteq int(f^{-1}(f(A))) \subseteq f^{-1}(pint(f(A)))$. Therefore $A \subseteq f^{-1}(pint(f(A)))$. This implies $f(A) \subseteq f(f^{-1}(pint(f(A)))) \subseteq pint(f(A)) \subseteq f(A)$. Hence f(A) is an IFPOS in Y and hence an IFβ^{**}GOS in Y. Thus f is an IFβ^{**}G open mapping.

Proposition 5.2.29 : For any IFS A in an IFTS (X, τ), where X is an IFβ^{**}p$T_{1/2}$ space, $A \in$ IFβ^{**}GO(X) if and only if for every IFP $p_{(\alpha,\beta)} \in A$, there exists an IFβ^{**}GOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Proof : Necessity : If $A \in$ IFβ^{**}GO(X), then we can take B = A so that $p_{(\alpha,\beta)} \in B \subseteq A$ for every IFP $p_{(\alpha,\beta)} \in A$.

Sufficiency : Let A be an IFS in X and assume that there exists $B \in$ IFβ^{**}GO(X) such that $p_{(\alpha,\beta)} \in B \subseteq A$. Since X is an IF$\beta^{**}pT_{1/2}$ space, B is an IFPOS of X. Then $A = \bigcup_{p_{(\alpha,\beta)} \in A} \{p_{(\alpha,\beta)}\} \subseteq \bigcup_{p_{(\alpha,\beta)} \in A} B \subseteq A$. Therefore A is an IFPOS and hence an IFβ^{**}GOS in X. Thus $A \in$ IFβ^{**}GO(X).

Proposition 5.2.30 : Let f : (X, τ) → (Y, σ) be a mapping that satisfies $f(int(B)) \subseteq cl(int(cl(f(B))))$ for every IFS B in X. Then f is an IFβ^{**}G open mapping.

Proof : Let B be an IFOS in X. Then int(B) = B. By hypothesis $f(B) \subseteq cl(int(cl(f(B))))$. This implies f(B) is an IFβOS in Y. Therefore it is an IFβ^{**}GOS in Y and hence f is an IFβ^{**}G open mapping.

Proposition 5.2.31 : Let f : (X, τ) → (Y, σ) be a bijective mapping. Then the following conditions are equivalent if X and Y are $IF\beta^{**}pT_{1/2}$ spaces:

(i) f is an $IF\beta^{**}G$ closed mapping,

(ii) f(B) is an $IF\beta^{**}GOS$ in Y for each IFOS B in X,

(iii) for each IFP $p_{(\alpha,\beta)}$ in Y and for every IFOS B in X such that $f^{-1}(p_{(\alpha,\beta)}) \in B$, there exists an IFPOS A in Y such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$.

Proof : (i) ⇔ (ii) is obvious as for a bijective mapping $f(A^c) = (f(A))^c$.

(ii) ⇒ (iii) Let B be any IFOS in X and let $p_{(\alpha,\beta)} \in Y$. Given $f^{-1}(p_{(\alpha,\beta)}) \in B$. By hypothesis f(B) is an $IF\beta^{**}GOS$ in Y. As Y is an $IF\beta^{**}pT_{1/2}$ space, f(B) is an IFPOS in Y. Take A = f(B). Then $p_{(\alpha,\beta)} \in f(B) = A$ and $f^{-1}(A) = f^{-1}(f(B)) = B$.

(iii) ⇒ (i) Let A be an IFCS in X. Then its complement, say B is an IFOS in X. Let $p_{(\alpha,\beta)} \in Y$ and $f^{-1}(p_{(\alpha,\beta)}) \in B$. By hypothesis there exists an IFPOS C in Y such that $p_{(\alpha,\beta)} \in C$ and $f^{-1}(C) \subseteq B$. This implies $p_{(\alpha,\beta)} \in C \subseteq f(f^{-1}(C)) \subseteq f(B)$. That is $p_{(\alpha,\beta)} \in f(B)$. Since C is an IFPOS, C = pint(C) ⊆ pint(f(B)). Therefore $p_{(\alpha,\beta)} \in$ pint(f(B)). But $f(B) = \bigcup_{p_{(\alpha,\beta)} \in f(B)} \{p_{(\alpha,\beta)}\} \subseteq p\,int(f(B)) \subseteq f(B)$. Hence f(B) is an IFPOS in Y and is an $IF\beta^{**}GOS$ in Y. Thus f(A) is an $IF\beta^{**}GCS$ in Y and f is an $IF\beta^{**}G$ closed mapping.

Definition 5.2.32 : A mapping f : (X, τ) → (Y, σ) is said to be an **intuitionistic fuzzy M-β^{**} generalized** (IFM-β^{**}G) **closed mapping** if f(A) is an $IF\beta^{**}GCS$ in Y for every $IF\beta^{**}GCS$ A in X.

Example 5.2.33 : Let X = {a, b} and Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFM-β^{**}G closed mapping.

Proposition 5.2.34 : Every IFM-β^{**}G closed mapping is an IFβ^{**}G closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFM-β^{**}G closed mapping. Let A be an IFCS in X. Then A is an IFβ^{**}GCS in X. By hypothesis f(A) is an IFβ^{**}GCS in Y. Hence f is an IFβ^{**}G closed mapping.

Example 5.2.35 : Let $X = \{a, b\}$ and $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.8_v), (0.2_u, 0.1_v)\rangle$ and $G_3 = \langle y, (0.3_u, 0.3_v), (0.2_u, 0.2_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is an IFβ^{**}G closed mapping but not an IFM-β^{**}G closed mapping, since $A = \langle x, (0.3_a, 0.3_b), (0.2_a, 0.2_b)\rangle$ is an IFβ^{**}GCS in X but f(A) is not an IFβ^{**}GCS in Y, as $int(cl(int(f(A)))) \cap cl(int(cl(f(A)))) = 1_\sim \not\subseteq G_3$ whereas $f(A) \subseteq G_3$.

Proposition 5.2.36 : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFβ^{**}G closed mapping and $g : (Y, \sigma) \to (Z, \delta)$ be an IFM-β^{**}G closed mapping then $g \circ f : (X, \tau) \to (Z, \delta)$ is an IFβ^{**}G closed mapping.

Proof : Let V be an IFCS in X. Then f(V) is an IFβ^{**}GCS in Y. Since g is an IFM-β^{**}G closed mapping, g(f(V)) is an IFβ^{**}GCS in Z. Hence $g \circ f$ is an IFβ^{**}G closed mapping.

Proposition 5.2.37 : The composition of two IFM-β^{**}G closed mappings is an IFM-β^{**}G closed mapping.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two IFM-β^{**}G closed mappings. Let V be an IFβ^{**}GCS in X. Then f(V) is an IFβ^{**}GCS in Y by hypothesis. Since g is an IFM-β^{**}G closed mapping, g(f(V)) is an IFM-β^{**}GCS in Z. Hence $g \circ f$ is an IFM-β^{**}G closed mapping.

Proposition 5.2.38 : Let f : X $\rightarrow$ Y be a bijective mapping then the following are equivalent:

(i) f is an IFM-β^{**}G closed mapping,

(ii) f(A) is an IFβ^{**}GCS in Y for each IFβ^{**}GCS A in X,

(iii) f(A) is an IFβ^{**}GOS in Y for every IFβ^{**}GOS A in X.

Proof : (i) $\Leftrightarrow$ (ii) is obviously true.

(ii) $\Leftrightarrow$ (iii) is obvious as $f(A^c) = (f(A))^c$ for a bijective mapping.

(iii) $\Rightarrow$ (i) Let A be an IFβ^{**}GCS in X. Then A^c is an IFβ^{**}GOS in X. By hypothesis, $f(A^c)$ is an IFβ^{**}GOS in Y. That is $f(A)^c$ is an IFβ^{**}GOS in Y. Hence f(A) is an IFβ^{**}GCS in Y. Thus f is an IFM-β^{**}G closed mapping.

Proposition 5.2.39 : If f : (X, τ) $\rightarrow$ (Y, σ) is a bijective mapping, where X and Y are IFβ^{**}p$T_{1/2}$spaces, then the following are equivalent :

(i) f is an IFM-β^{**}G closed mapping,

(ii) f(A) is an IFβ^{**}GOS in Y for every IFβ^{**}GOS A in X,

(iii) $f(pint(B)) \subseteq pint(f(B))$ for every IFS B in X,

(iv) $pcl(f(B)) \subseteq f(pcl(B))$ for every IFS B in X.

Proof : (i) $\Leftrightarrow$ (ii) is obvious.

(i) $\Rightarrow$ (iii) Let B be any IFS in X. Since pint(B) is an IFPOS, it is an IFβ^{**}GOS in X. Then by hypothesis, f(pint(B)) is an IFβ^{**}GOS in Y. Since Y is an IFβ^{**}p$T_{1/2}$ space, f(pint(B)) is an IFPOS in Y. Therefore $f(pint(B)) = pint(f(pint(B))) \subseteq pint(f(B))$.

(iii) $\Rightarrow$ (iv) can be easily proved by taking complement in (iii).

(iv) $\Rightarrow$ (i) Let A be an IFβ^{**}GCS in X. By hypothesis, $pcl(f(A)) \subseteq f(pcl(A))$. Since X is an IF$\beta^{**}pT_{1/2}$ space, A is an IFPCS in X. Therefore, $pcl(f(A)) \subseteq f(pcl(A)) = f(A) \subseteq pcl(f(A))$. Hence f(A) is an IFPCS in Y and hence an IFβ^{**}GCS in Y. Thus f is an IFM-β^{**}G closed mapping.

Proposition 5.2.40 : If f : (X, τ) → (Y, σ) is a bijective mapping, then the following are equivalent :

(i) f is an IFM-β^{**}G closed mapping,

(ii) f(A) is an IFβ^{**}GOS in Y for every IFβ^{**}GOS A in X,

(iii) for every IFP $p_{(\alpha,\beta)} \in$ Y and for every IFβ^{**}GOS B in X such that $f^{-1}(p_{(\alpha,\beta)}) \in$ B, there exists an IFβ^{**}GOS A in Y such that $p_{(\alpha,\beta)} \in$ A and f^{-1}(A) = B.

Proof : (i) ⇒ (ii) is obvious by Proposition 5.2.38.

(ii) ⇒ (iii) Let $p_{(\alpha,\beta)} \in$ Y and let B be an IFβ^{**}GOS in X such that $f^{-1}(p_{(\alpha,\beta)}) \in$ B. This implies $p_{(\alpha,\beta)} \in$ f(B). By hypothesis, f(B) is an IFβ^{**}GOS in Y. Let A = f(B). Therefore $p_{(\alpha,\beta)} \in$ f(B) = A and $f^{-1}(A) = f^{-1}(f(B))$ = B.

(iii) ⇒ (i) Let B be an IFβ^{**}GCS in X. Then B^c is an IFβ^{**}GOS in X. Let $p_{(\alpha,\beta)} \in$ Y and $f^{-1}(p_{(\alpha,\beta)}) \in B^c$. This implies $p_{(\alpha,\beta)} \in f(B^c)$. By hypothesis there exists an IFβ^{**}GOS A in Y such that $p_{(\alpha,\beta)} \in$ A and $f^{-1}(A) = B^c$. Then $A = f(f^{-1}(A)) = f(B^c)$. Hence by proposition 5.2.29, $f(B^c)$ is an IFβ^{**}GOS in Y. As f is a bijective mapping, $f(B^c) = (f(B))^c$. Therefore f(B) is an IFβ^{**}GCS in Y. Thus f is an IFM-β^{**}G closed mapping.

5.3 Intuitionistic Fuzzy Almost β^{**} Generalized Closed Mappings

In this section we have introduced intuitionistic fuzzy almost β^{**} generalized closed mappings and investigated some of their properties.

Definition 5.3.1 : A mapping f : (X, τ) → (Y, σ) is called an **intuitionistic fuzzy almost β^{**} generalized** (IF almost β^{**}G) **closed mapping** if f(V) is an IFβ^{**}GCS in Y for every IFRCS V of X.

Example 5.3.2 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$, $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF almost β^{**}G closed mapping.

Proposition 5.3.3 : Every IF closed mapping is an IF almost β^{**}G closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF closed mapping. Let V be an IFRCS in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFCS in Y by hypothesis. Since every IFCS is an IFβ^{**}GCS, f(V) is an IFβ^{**}GCS in Y. Hence f is an IF almost β^{**}G closed mapping.

Example 5.3.4 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$, $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF almost β^{**}G closed mapping but not an IF closed mapping, since $G_1^c = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$ is an IFCS in X but $f(G_1^c)$ is not an IFCS in Y, as $cl(f(G_1^c)) = 1_\sim \neq f(G_1^c)$.

Proposition 5.3.5 : Every IF semi closed mapping is an IF almost β^{**}G closed mapping in (X, τ) but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF semi closed mapping. Let V be an IFRCS in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFSCS in Y by hypothesis. Since every IFSCS is an IFβ^{**}GCS, f(V) is an IFβ^{**}GCS in Y. Hence f is an IF almost β^{**}G closed mapping.

Example 5.3.6 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$, $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$.

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF almost β^{**}G closed mapping but not an IF semi closed mapping, since $G_1^c = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$ is an IFCS in X but $f(G_1^c)$ is not an IFSCS in Y, as $int(cl(f(G_1^c))) = 1_\sim \not\subseteq f(G_1^c)$.

Proposition 5.3.7 : Every IF pre closed mapping is an IF almost β^{**}G closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF pre closed mapping. Let V be an IFRCS in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFPCS in Y by hypothesis. Since every IFPCS is an IFβ^{**}GCS, f(V) is an IFβ^{**}GCS in Y. Hence f is an IF almost β^{**}G closed mapping.

Example 5.3.8 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF almost β^{**}G closed mapping, but not an IF pre closed mapping since G_2^c is an IFCS in X but $f(G_2^c)$ is not an IFPCS in Y, as $cl(int(f(G_2^c))) = cl(G_3) = 1_\sim \not\subseteq f(G_2^c)$.

Proposition 5.3.9 : Every IFα closed mapping is an IF almost β^{**}G closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFα closed mapping. Let V be an IFRCS in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFαCS in Y by hypothesis. Since every IFαCS is an IFβ^{**}GCS, f(V) is an IFβ^{**}GCS in Y. Hence f is an IF almost β^{**}G closed mapping.

Example 5.3.10 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$, $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$.

Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF almost β^{**}G closed mapping but not an IFα closed mapping, since $G_1^c = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$ is an IFCS in X but $f(G_1^c)$ is not an IFαCS in Y, as $cl(int(cl(f(G_1^c)))) = 1_\sim \not\subseteq f(G_1^c)$.

Proposition 5.3.11 : Every IFβ closed mapping is an IF almost β^{**}G closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFβ closed mapping. Let V be an IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFβCS in Y by hypothesis. Since every IFβCS is an IFβ^{**}GCS, f(V) is an IFβ^{**}GCS in Y. Hence f is an IF almost β^{**}G closed mapping.

Example 5.3.12 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$, $G_3 = \langle y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)\rangle$ and $G_4 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF almost β^{**}G closed mapping but not an IFβ closed mapping, since $G_1^c = \langle x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)\rangle$ is an IFCS in X but $f(G_1^c)$ is not an IFβCS in Y, as $int(cl(int(f(G_1^c))) = 1_\sim \not\subseteq f(G_1^c)$.

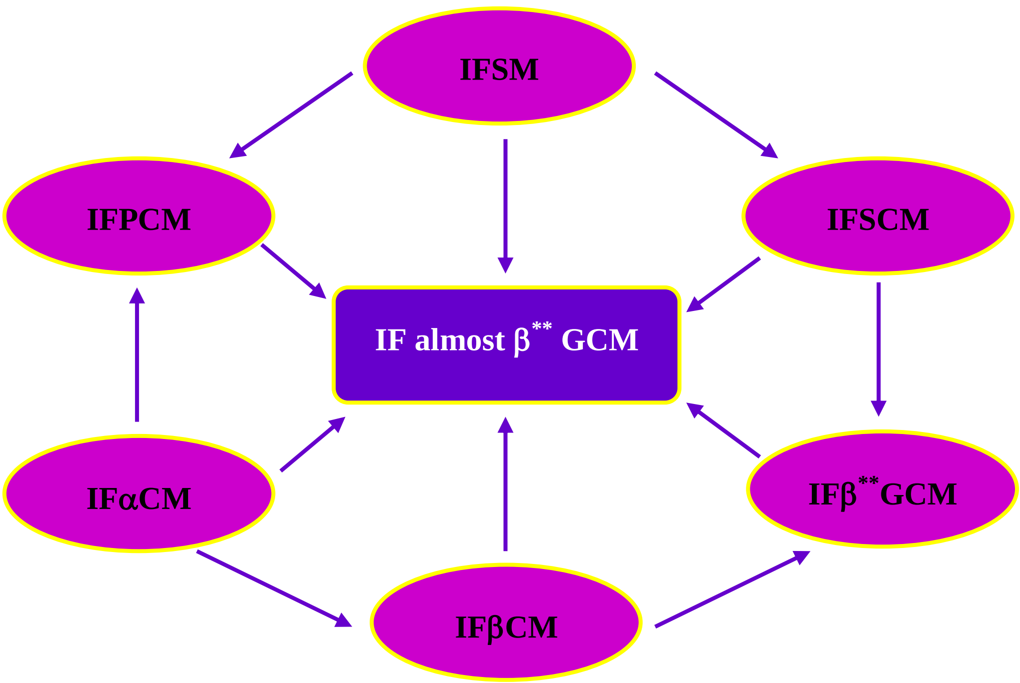
Proposition 5.3.13 : Every IFβ^{**}G closed mapping is an IF almost β^{**}G closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFβ^{**}G closed mapping. Let V be an IFRCS is in X. Since every IFRCS is an IFCS in X, V is an IFCS in X. Then f(V) is an IFβ^{**}GCS in Y by hypothesis. Hence f is an IF almost β^{**}G closed mapping.

Example 5.3.14 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.2_a, 0.2_b), (0.5_a, 0.8_b)\rangle$, $G_2 = \langle x, (0.2_u, 0.2_v), (0.5_u, 0.6_v)\rangle$ and $G_3 = \langle y, (0.5_u, 0.8_v), (0.2_u, 0.2_v)\rangle$. Then

$\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF almost β^{**}G closed mapping but not an IFβ^{**}G closed mapping, since $G_1^c = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)\rangle$ is an IFCS in X but $f(G_1^c)$ is not an IFβ^{**}GCS in Y, as $f(G_1^c) \subseteq G_3$ whereas $int(cl(int(f(G_1^c)))) \cap cl(int(cl(f(G_1^c)))) = 1_\sim \not\subseteq G_3$.

The relation between various types of intuitionistic fuzzy closed mappings is given in the following diagram. In this diagram 'CM' means closed mappings. The reverse implications are not true in general in the below diagram.



The reverse implications are not true in general in the above equation.

Definition 5.3.15 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an **intuitionistic fuzzy almost β^{**}generalized** (IF almost β^{**}G) **open mapping** if f(A) is an IFβ^{**}GOS in Y for each IFROS A in X.

Proposition 5.3.16 : A bijective mapping f : (X, τ) → (Y, σ) is an IF almost β^{**}G closed mapping if and only if the inverse image of each IFROS in X is an IFβ^{**}GOS in Y.

Proof : Necessity : Let A be an IFROS in X. This implies A^c is an IFRCS in X. Then $f(A^c)$ is an IFβ^{**}GCS in Y, by hypothesis. Since $f(A^c) = (f(A))^c$, for a bijective mapping f(A) is an IFβ^{**}GOS in Y.

Sufficiency : Let A be an IFRCS in X. Then A^c is an IFROS in X. By hypothesis $f(A^c)$ is IFβ^{**}GOS in Y. Therefore f(A) is an IFβ^{**}GCS in Y. Hence f is an IF almost β^{**}G closed mapping.

Proposition 5.3.17 : Let $p_{(\alpha,\beta)}$ be an IFP in X. A mapping f : X → Y is an IF almost β^{**}G open mapping if for every IFOS A in X with $f^{-1}(p_{(\alpha,\beta)}) \in A$, there exists an IFOS B in Y with $p_{(\alpha,\beta)} \in B$ such that f(A) is IFD in B.

Proof : Let A be an IFROS in X. Then A is an IFOS in X. Let $f^{-1}(p_{(\alpha,\beta)}) \in A$, then there exists an IFOS B in Y such that $p_{(\alpha,\beta)} \in B$ and cl(f(A)) = B. Since B is an IFOS, cl(f(A)) = B is also an IFOS in Y. Therefore int(cl(f(A))) = cl(f(A)). Now f(A) ⊆ cl(f(A)) = int(cl(f(A))) ⊆ cl(int(cl(f(A)))). This implies f(A) is an IFβOS in Y and hence an IFβ^{**}GOS in Y. Thus f is an IF almost β^{**}G open mapping.

Proposition 5.3.18 : Let f : X → Y be a mapping. If f is an IF almost β^{**}G closed mapping, then β^{**}gcl(f(A)) ⊆ f(cl(A)) for every IFβOS A in X.

Proof : Let A be an IFβOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(cl(A)) is an IFβ^{**}GCS in Y. Then β^{**}gcl(f(cl(A))) = f(cl(A)). Now β^{**}gcl(f(A)) ⊆ β^{**}gcl(f(cl(A))) = f(cl(A)). That is β^{**}gcl(f(A)) ⊆ f(cl(A)).

Corollary 5.3.19 : Let f : X $\rightarrow$ Y be a mapping. If f is an IF almost β^{**}G closed mapping, then β^{**}gcl(f(A)) $\subseteq$ f(cl(A)) for every IFSOS A in X.

Proof : Since every IFSOS is an IFβOS, the proof directly follows from the Proposition 5.3.18.

Corollary 5.3.20 : Let f : X $\rightarrow$ Y be a mapping. If f is an IF almost β^{**}G closed mapping, then β^{**}gcl(f(A)) $\subseteq$ f(cl(A)) for every IFPOS A in X.

Proof : Since every IFPOS is an IFβOS, the proof directly follows from the Proposition 5.3.18.

Proposition 5.3.21 : Let f : (X, τ) $\rightarrow$ (Y, σ) be a mapping. If f is an IF almost β^{**}G closed mapping, then β^{**}gcl(f(A)) $\subseteq$ f(cl(βint(A))) for every IFβOS A in X.

Proof : Let A be an IFβOS in X. Therefore βint(A) = A and cl(A) is an IFRCS in X. By hypothesis, f(cl(A)) is an IFβ^{**}GCS in Y. Then β^{**}gcl(f(A)) $\subseteq$ β^{**}gcl(f(cl(A))) = f(cl(A)) = f(cl(βin(A))).

Corollary 5.3.22 : Let f : X $\rightarrow$ Y be a mapping. If f is an IF almost β^{**}G closed mapping, then β^{**}gcl(f(A)) $\subseteq$ f(cl(βint(A))) for every IFSOS A in X.

Proof : Since every IFSOS is an IFβOS, the proof directly follows from the Theorem 5.3.21.

Corollary 5.3.23 : Let f : X $\rightarrow$ Y be a mapping, If f is an IF almost β^{**}G closed mapping, then β^{**}gcl(f(A)) $\subseteq$ f(cl(βint(A))) for every IFPOS A in X.

Proof : Since every IFPOS is an IFβOS, the proof directly follows from the Theorem 5.3.21.

Proposition 5.3.24 : Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Then the following are equivalent.

(i) f is an IF almost β^{**}G open mapping,

(ii) f is an IF almost β^{**}G closed mapping,

(iii) f^{-1} is an IF almost β^{**}G continuous mapping.

Proof : (i) $\Leftrightarrow$ (ii) is obvious as for a bijective mapping, $f(A^c) = f(A)^c$.

(ii) $\Rightarrow$ (iii) Let $A \subseteq X$ be an IFRCS. Then by hypothesis, f(A) is an IFβ^{**}GCS in Y. That is $(f^{-1})^{-1}$ (A) is an IFβ^{**}GCS in Y. This implies f^{-1} is an IF almost β^{**}G continuous mapping.

(iii) $\Rightarrow$ (ii) Let $A \subseteq X$ be an IFRCS. Then by hypothesis, $(f^{-1})^{-1}$ (A) is an IFβ^{**}GCS in Y. That is f(A) is an IFβ^{**}GCS in Y. Hence f is an IF almost β^{**}G closed mapping.

5.4 Intuitionistic Fuzzy Contra β^{**} Generalized Open Mappings

In this section we have introduced intuitionistic fuzzy contra β^{**} generalized open mappings and intuitionistic fuzzy contra β^{**} generalized closed mappings and analyzed some of their properties.

Definition 5.4.1 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an **intuitionistic fuzzy contra β^{**} generalized** (IF contra β^{**}G) **open mapping** if f(V) is an IFβ^{**}GCS in (Y, σ) for every IFOS V of (X, τ).

For the sake of simplicity, we shall use the notation $A = \langle x, (\mu_a, \mu_b), (\nu_a, \nu_b)\rangle$ instead of $A = \langle x, (a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b) \rangle$ in the following examples. Similarly we shall use the notation $B = \langle y, (\mu_u, \mu_v)\ (\nu_u, \nu_v)\rangle$ instead of $B = \langle y, (u/\mu_u, v/\mu_v), (u/\nu_u, v/\nu_v)\rangle$ in the following examples.

Example 5.4.2 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is an IF contra β^{**}G open mapping.

Definition 5.4.3 : A mapping f : (X, τ) → (Y, σ) is called an **intuitionistic fuzzy contra β^{**} generalized** (IF contra β^{**}G) **closed mapping** if f(V) is an IFβ^{**}GOS in (Y, σ) for every IFCS V of (X, τ).

Proposition 5.4.4 : A bijective mapping f : (X, τ) → (Y, σ) is an IF contra β^{**}G open mapping if and only if the image of each IFCS in X is an IFβ^{**}GOS in Y.

Proof : As $f(A^c) = (f(A))^c$ for a bijective mapping, the proposition is obviously true.

Proposition 5.4.5 : Let f : X → Y be a bijective mapping, suppose that one of the following properties hold:

(i) $f(cl(B)) \subseteq int(\beta cl(f(B)))$ for each IFS B in Y,

(ii) $cl(\beta int(f(B))) \subseteq f(int(B))$ for each IFS B in Y,

(iii) $f^{-1}(cl(\beta int(A))) \subseteq int(f^{-1}(A))$ for each IFS A in Y,

(iv) $f^{-1}(cl(A)) \subseteq int(f^{-1}(A))$ for each IFβOS A in Y.

Then f is an IF contra β^{**}G open mapping.

Proof : (i) ⇒ (ii) is obvious by taking the complement in (i).

(ii) ⇒ (iii) Let A ⊆ Y. Put $B = f^{-1}(A)$ in X. This implies A = f(B) in Y. Now $cl(\beta int(A)) = cl(\beta int(f(B))) \subseteq f(int(B))$ by (ii). Therefore $f^{-1}(cl(\beta(int(A)))) \subseteq f^{-1}(f(int(B))) = int(B) = int(f^{-1}(A))$.

(iii) $\Rightarrow$ (iv) Let $A \subseteq Y$ be an IFβOS. Then βint(A) = A. By hypothesis, f^{-1}(cl(βint(A))) $\subseteq$ int(f^{-1}(A)). Therefore f^{-1}(cl(A)) = f^{-1}(cl(βint((A))) $\subseteq$ int(f^{-1}(A)).

Suppose (iv) holds : Let A be an IFOS in X. Then f(A) is an IFS in Y and βint(f(A)) is an IFβOS in Y. Hence by hypothesis, f^{-1}(cl(βint(f(A)))) $\subseteq$ int(f^{-1}(βint(f(A)))) $\subseteq$ int(f^{-1}(f(A))) = int(A) = A. Therefore cl(βint(f(A))) = f(f^{-1}(cl(βintf(A)))) $\subseteq$ f(A). Now cl(int(f(A))) $\subseteq$ cl(βint(f(A))) $\subseteq$ f(A). This implies f(A) is an IFPCS inY and hence an IFβ^{**}GCS in Y. Thus f is an IF contra β^{**}G open mapping.

Proposition 5.4.6 : If f : X $\rightarrow$ Y is an IF contra β^{**}G open mapping, where Y is an IFβ^{**}p$T_{1/2}$ space, then the following conditions hold :

(i) pcl(f(B)) $\subseteq$ f(int(pcl(B))) for every IFOS B in X,

(ii) f(cl(pint(B))) $\subseteq$ pint(f(B)) for every IFCS B in X.

Proof : (i) Let $B \subseteq X$ be an IFOS. Then int(B) = B. By hypothesis f(B) is an IFβ^{**}GCS in Y. Since Y is an IFβ^{**}p$T_{1/2}$ space, f(B) is an IFPCS in Y. This implies pcl(f(B)) = f(B) = f(int(B)) $\subseteq$ f(int(pcl(B))).

(ii) can be proved by taking complement in (i).

Proposition 5.4.7 : If f : (X, τ) $\rightarrow$ (Y, σ) is an IF open mapping and g : (Y, σ) $\rightarrow$ (Z, δ) is an IF contra β^{**}G open mapping then g o f : (X, τ) $\rightarrow$ (Z, δ) is an IF contra β^{**}G open mapping.

Proof : Let V be an IFOS in X. Then f(V) is an IFOS in Y, since f is an IF open mapping. Since g is an IF contra β^{**}G open mapping, g(f(V)) is an IFβ^{**}GCS in Z. Therefore g o f is an IF contra β^{**}G open mapping.

Proposition 5.4.8 : If f : (X, τ) → (Y, σ) is an IF contra β^{**}G open mapping and g : (Y, σ) → (Z, δ) is an IFM-β^{**}G closed mapping then g o f : (X, τ) → (Z, δ) is an IF contra β^{**}G open mapping.

Proof : Let V be an IFOS in X. Then f(V) is an IFβ^{**}GCS in Y, since f is an IF contra β^{**}G open mapping. Since g is an IFM-β^{**}G closed mapping, g(f(V)) is an IFβ^{**}GCS in Z. Therefore g o f is an IF contra β^{**}G open mapping.

Remark 5.4.9 : The composition of two IF contra β^{**}G open mapping is not an IF contra β^{**}G open mapping in general as seen in the following example.

Example 5.4.10 : Let X = {a, b}, Y = {u, v}, Z = {p, q}, $G_1 = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $G_2 = \langle y, (0.2_u, 0.2_v), (0.5_u, 0.7_v)\rangle$, $G_3 = \langle z, (0.6_p, 0.8_q), (0.2_p, 0.2_q)\rangle$ and $G_4 = \langle z, (0.5_p, 0.6_q), (0.5_p, 0.4_q)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$, $\sigma = \{0_\sim, G_2, 1_\sim\}$ and $\delta = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFTs on X, Y and Z respectively. Now define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v and g : (Y, σ) → (Z, δ) by g(u) = p and g(v) = q. Here f and g are IF contra β^{**}G open mappings but their composition g o f : (X, τ) → (Z, δ) defined by g(f(a)) = p and g(f(b)) = q is not an IF contra β^{**}G open mapping, since $G_1 = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)\rangle$ is an IFOS in X but $g(f(G_1)) = \langle z, (0.5_p, 0.8_q), (0.2_p, 0.2_q)\rangle$ is not an IFβ^{**}GCS in Z, as $cl(int(cl(g(f(G_1))))) \cap int(cl(int(g(f(G_1))))) = 1_\sim \not\subseteq G_3$ whereas $g(f(G_1)) \subseteq G_3$.

Proposition 5.4.11 : A mapping f : (X, τ) → (Y, σ) is an IF contra β^{**}G closed mapping if f(βcl(B)) ⊆ int(f(B)) for every IFS B in X.

Proof : Let B ⊆ X be an IFCS. Then cl(B) = B. Since every IFCS is an IFβCS, βcl(B) = B. Now by hypothesis, f(B) = f(βcl(B)) ⊆ int(f(B)) ⊆ f(B). This implies f(B) = int(f(B)). Therefore f(B) is an IFOS in Y and hence is an IFβ^{**}GOS. Thus f is an IF contra β^{**}G closed mapping.

Proposition 5.4.12 : A mapping $f : X \to Y$ is an IF contra β^{**}G closed mapping, where Y is an IFβ^{**}p$T_{1/2}$ space if and only if $f(pcl(B)) \subseteq pint(f(cl(B))$ for every IFS B in X.

Proof : Necessity : Let $B \subseteq X$ be an IFS. Then cl(B) is an IFCS in X. By hypothesis, f(cl(B)) is an IFβ^{**}GOS in Y. Since Y is an IFβ^{**}p$T_{1/2}$ space, f(cl(B)) is an IFPOS in Y. Therefore $f(pcl(B)) \subseteq f(cl(B)) = pint(f(cl(B)))$.

Sufficiency : Let $B \subseteq X$ be an IFCS. Then cl(B) = B. By hypothesis, $f(pcl(B)) \subseteq pint(f(cl(B))) = pint(f(B))$. But pcl(B) = B. Therefore $f(B) = f(pcl(B)) \subseteq pint(f(B)) \subseteq f(B)$. This implies f(B) is an IFPOS in Y and hence an IFβ^{**}GOS in Y. Hence f is an IF contra β^{**}G closed mapping.

Proposition 5.4.13 : Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Suppose that one of the following properties hold :

(i) $f^{-1}(\beta cl(A)) \subseteq int(f^{-1}(A))$ for each IFS A in Y,

(ii) $\beta cl(f(B)) \subseteq f(int(B))$ for each IFS B in X,

(iii) $f(cl(B)) \subseteq \beta int(f(B))$ for each IFS B in X.

Then f is an IF contra β^{**}G closed mapping.

Proof : (i) $\Rightarrow$ (ii) Let $B \subseteq X$. Then f(B) is an IFS in Y. By hypothesis, $f^{-1}(\beta cl(f(B))) \subseteq int(f^{-1}(f(B))) = int(B)$. Now $\beta cl(f(B)) = f(f^{-1}(\beta cl(f(B)))) \subseteq f(int(B))$.

(ii) $\Rightarrow$ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds. Let A be an IFCS in X. Then cl(A) = A and f(A) is an IFS in Y. Now $f(A) = f(cl(A)) \subseteq \beta int(f(A)) \subseteq f(A)$, by hypothesis. This implies f(A) is an IFβOS in Y and hence an IFβ^{**}GOS in Y. Thus f is an IF contra β^{**}G closed mapping.

Proposition 5.4.14 : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. Then f is an IF contra β^{**}G closed mapping if $cl(f^{-1}(A)) \subseteq f^{-1}(\beta int(A))$ for every IFS A in Y.

Proof : Let A be an IFCS in X. Then $cl(A) = A$ and $f(A)$ is an IFS in Y. By hypothesis $cl(f^{-1}(f(A))) \subseteq f^{-1}(\beta int(f(A)))$. Since f is bijective, $f^{-1}(f(A)) = A$. Therefore $A = cl(A) = cl(f^{-1}(f(A))) \subseteq f^{-1}(\beta int(f(A)))$. Now $f(A) \subseteq f(f^{-1}(\beta int(f(A)))) = \beta int(f(A)) \subseteq f(A)$. Hence f(A) is an IFβOS in Y and hence an IFβ^{**}GOS in Y. Thus f is an IF contra β^{**}G closed mapping.