CHAPTER - V

CHAPTER V
INTUITIONISTIC FUZZY TOPSIS METHOD

In this chapter the concept of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) for solving multi- criteria decision analysis problems are extended with intutionistic fuzzy data that expressed in the intutionistic fuzzy sets. In many real life situations there may be some hesitation degree between membership and nonmembership. In such cases intutionistic fuzzy sets are appropriate to deal with.

Consider i=1 to m and j=1 to n, the performance measure of i^{th} alternative in terms of j^{th} criterion is represented by $(\mu_{A_i}(x_j), \nu_{A_i}(x_j)\pi_{A_i}(x_j))$

The intutionistic fuzzy decision matrix D is defined as

$$D = \begin{bmatrix} (\mu_{A_1}(x_1), \nu_{A_1}(x_1), \pi_{A_1}(x_1)) & (\mu_{A_1}(x_2), \nu_{A_1}(x_2), \pi_{A_1}(x_2)) & \cdots & (\mu_{A_1}(x_n), \nu_{A_1}(x_n), \pi_{A_1}(x_n)) \\ (\mu_{A_2}(x_1), \nu_{A_2}(x_1), \pi_{A_2}(x_1)) & (\mu_{A_2}(x_2), \nu_{A_2}(x_2), \pi_{A_2}(x_2)) & \cdots & (\mu_{A_2}(x_n), \nu_{A_2}(x_n), \pi_{A_2}(x_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{A_m}(x_1), \nu_{A_m}(x_1), \pi_{A_m}(x_1)) & (\mu_{A_m}(x_2), \nu_{A_m}(x_2), \pi_{A_m}(x_2)) & \cdots & (\mu_{A_v}(x_n), \nu_{A_m}(x_n), \pi_{A_m}(x_n)) \end{bmatrix}$$

Since all criteria cannot be assumed to be of equal importance, a set of grades of importance is received from the decision makers and is represented as an Intutionistic Fuzzy Set (IFS) denoted as $W = \{\langle \mu_w(x), \nu_w(x), \pi_w(x) \rangle | x \in X\}$ where $\mu_w(x): X \to [0,1]$ and $\nu_w(x): X \to [0,1]$ defines the degree of importance and degree of unimportance for an criteria and for each $x \in X$, the hesitancy degree toward the importance of an criteria is $\pi_w(x) = 1 - \mu_w(x) - \nu_w(x)$.

For two Intutionistic Fuzzy Sets A_i and W,

$$A_i.W = \{<x, \mu_{A_i}(x) \cdot \mu_w(x), \nu_{A_i}(x) + \nu_w(x) - \nu_{A_i}(x) \cdot \nu_w(x)> | x \in X\}$$

represents an IFS A_i of the i^{th} alternative characterized as grades of importance

That is $\mu_{A_i.w}(x) = \mu_{A_i}(x) \cdot \mu_w(x)$,

$$\nu_{A_i.w}(x) = \nu_{A_i}(x) + \nu_w(x) - \nu_{A_i}(x)\cdot\nu_w(x),$$

$$\pi_{A_i.w}(x) = 1 - \mu_{A_i}(x)\cdot\mu_w(x) - \nu_{A_i}(x) - \nu_w(x) + \nu_{A_i}(x)\cdot\nu_w(x).$$

The intutionistic fuzzy weighted decision matrix D'

$$D' = \begin{bmatrix} (\mu_{A_1\cdot w}(x_1), \nu_{A_1\cdot w}(x_1), \pi_{A_1\cdot w}(x_1)) & \cdots & (\mu_{A_1\cdot w}(x_n), \nu_{A_1\cdot w}(x_n), \pi_{A_1\cdot w}(x_n)) \\ \vdots & \ddots & \vdots \\ (\mu_{A_m\cdot w}(x_1), \nu_{A_m\cdot w}(x_1), \pi_{A_m\cdot w}(x_1)) & \cdots & (\mu_{A_m\cdot w}(x_n), \nu_{A_m\cdot w}(x_n), \pi_{A_m\cdot w}(x_n)) \end{bmatrix}$$

Let J_1 be a collection of benefit criteria (i.e., the larger value of x_j, the greater preference) and J_2 be a collection of cost criteria (i.e., the smaller value of x_j, the greater preference). The intutionistic positive-ideal solution is denoted as A^+ in which the weighted benefit attributes use max-min method while the weighted cost attributes use min-max method. The intutionistic negative-ideal solution is denoted as A^- in which the weighted benefit attributes use min-max method while the weighted cost attributes use max-min method. The max-min method takes the maximum of membership degree while takes the minimum of non-membership degree among the comparative IFSs. The min-max method takes the minimum of membership degree while takes the maximum of non-membership degree among the comparative IFSs. The definitions are as follows:

$$A^+ = \Big\{\Big\langle x_j, \Big((\max_i \mu_{A_i\cdot w}(x_j) | j \in J_1), (\min_i \mu_{A_i\cdot w}(x_j) | j \in J_2) \Big),$$

$$\Big((\min_i \mu_{A_i\cdot w}(x_j) | j \in J_1), (\max_i \mu_{A_i\cdot w}(x_j) | j \in J_2) \Big)\Big\rangle \Big| i = 1,2,\ldots,m \Big\}$$

$$= \Big\{\Big\langle x_1, \mu_{A_i^+\cdot w}(x_1), \nu_{A_i^+\cdot w}(x_1) \Big\rangle, \Big\langle x_2, \mu_{A_i^+\cdot w}(x_2), \nu_{A_i^+\cdot w}(x_2) \Big\rangle$$

$$,\ldots, \Big\langle x_n, \mu_{A_i^+\cdot w}(x_n), \nu_{A_i^+\cdot w}(x_n) \Big\rangle$$

$$A^- = \left\{\left\langle x_j, \left((\min_i \mu_{A_i \cdot w}(x_j) | j \in J_1), (\max_i \mu_{A_i \cdot w}(x_j) | j \in J_2) \right), \right.\right.$$

$$\left.\left. \left((\max_i \mu_{A_i \cdot w}(x_j) | j \in J_1), (\min_i \mu_{A_i \cdot w}(x_j) | j \in J_2) \right) \right\rangle \middle| i = 1,2,\ldots,m \right\}$$

$$= \left\{ \left\langle x_1, \mu_{A_i^- \cdot w}(x_1), \nu_{A_{\bar{i}} \cdot w}(x_1) \right\rangle, \left\langle x_2, \mu_{A_i^- \cdot w}(x_2), \nu_{A_i^- \cdot w}(x_2) \right\rangle \right.$$

$$\left. ,\ldots, \left\langle x_n, \mu_{A_i^- \cdot w}(x_n), \nu_{A_{\bar{i}} \cdot w}(x_n) \right\rangle \right.$$

The intutionistic separation between alternatives can be measured by distance measures and similarity measures, which are a dual concept. The "distance" is a compliment of "similarity".

(A) Intutionistic separation measures characterized by the square root

i) The definition according to Atanassov(1999)

$$S^1 = \left\{ \frac{1}{2} \sum_{i=1}^{n} [(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2] \right\}^{\frac{1}{2}} \tag{1}$$

ii) The definition according to Atanassov(1999)

$$S^2 = \left\{ \frac{1}{2n} \sum_{i=1}^{n} [(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2] \right\}^{\frac{1}{2}} \tag{2}$$

iii) The definition according to Szmidt and Kacprzyk(2000)

$$S^3 = \left\{ \frac{1}{2} \sum_{i=1}^{n} [(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_A(x_i))^2] \right\}^{\frac{1}{2}} \tag{3}$$

iv) The definition according to Szmidt and Kacprzyk(2000)

$$S^4 = \left\{ \frac{1}{2n} \sum_{i=1}^{n} [(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_A(x_i))^2] \right\}^{\frac{1}{2}} \tag{4}$$

(B) Intutionistic separation measures characterized by the maximum

i) The definition according to Grzegorzewski(2004)

$$S^5 = \sum_{i=1}^{n} \max\{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\} \tag{5}$$

ii) The definition according to Grzegorzewski(2004)

$$S^6 = \frac{1}{n} \sum_{i=1}^{n} \max\{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\} \tag{6}$$

iii) The definition according to Wang and Xing (2005)

$$S^7 = \frac{1}{n} \sum_{i=1}^{n} \times \left[\frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{4} + \frac{\max\{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\}}{2} \right] \tag{7}$$

iv) The definition according to Hung and Yang (2004)

$$S^8 = 1 - \frac{1 - \frac{1}{n} \sum_{i=1}^{n} \max\{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\}}{1 + \frac{1}{n} \sum_{i=1}^{n} \max\{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\}} \tag{8}$$

(C) Intutionistic separation measures characterized by the parameter

i) The definition according to Wang and Xin (2005)

$$S^9 = \frac{1}{n^{\frac{1}{p}}}\left[\sum_{i=1}^{n}\left(\frac{|\mu_A(x_i)-\mu_B(x_i)|}{2}+\frac{|\nu_A(x_i)-\nu_B(x_i)|}{2}\right)^p\right]^{\frac{1}{p}} \tag{9}$$

where p is a positive integer and $1 \leq p < \infty$.

ii) The definition according to Liang and Shi (2003)

$$S^{10} = \frac{1}{n^{\frac{1}{p}}}\left[\sum_{i=1}^{n}\left(\frac{|\mu_A(x_i)-\mu_B(x_i)|}{2}+\left|\frac{(1-\nu_A(x_i))}{2}-\frac{(1--\nu_B(x_i))}{2}\right|\right)^p\right]^{\frac{1}{p}} \tag{10}$$

where p is a positive integer and $1 \leq p < \infty$.

iii) The definition according to Liang and Shi (2003)

$$S^{11} = \frac{1}{n^{\frac{1}{p}}}\left\{\sum_{i=1}^{n}\left[\frac{1}{8}\left(|3\mu_A(x_i)-3\mu_B(x_i)-\nu_A(x_i)+\nu_B(x_i)|\right.\right.\right.$$
$$\left.\left.\left.+|\mu_A(x_i)-\mu_B(x_i)-3\nu_A(x_i)+3\nu_B(x_i)|\right)\right]^p\right\}^{\frac{1}{p}} \tag{11}$$

where p is a positive integer and $1 \leq p < \infty$.

iv) The definition according to Mitchell (2003)

$$S^{10} = 1-\frac{1}{2}\left\{1-\frac{1}{n^{\frac{1}{p}}}\left[\sum_{i=1}^{n}|\mu_A(x_i)-\mu_B(x_i)|\right]+1\right.$$
$$\left.-\frac{1}{n^{\frac{1}{p}}}\left[\sum_{i=1}^{n}|\mu_A(x_i)-\mu_B(x_i)|^p\right]^{\frac{1}{p}}\right\} \tag{12}$$

where p is a positive integer and $1 \leq p < \infty$.

(D) Intutionistic separation measures without the characteristics of (A), (B) and (C)

i) The definition according to Atanassov (1999)

$$S^{13} = \frac{1}{2}\sum_{i=1}^{n}\left(\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\nu_A(x_i) - \nu_B(x_i)\right|\right) \tag{13}$$

ii) The definition according to Atanassov (1999)

$$S^{14} = \frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\nu_A(x_i) - \nu_B(x_i)\right|\right) \tag{14}$$

iii) The definition according to Szmidt and Kacprzyk(2000)

$$S^{15} = \frac{1}{2}\sum_{i=1}^{n}\left(\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\nu_A(x_i) - \nu_B(x_i)\right| + \left|\pi_A(x_i) - \pi_B(x_i)\right|\right) \tag{15}$$

iv) The definition according to Szmidt and Kacprzyk(2000)

$$S^{15} = \frac{1}{2n}\sum_{i=1}^{n}\left(\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\nu_A(x_i) - \nu_B(x_i)\right| + \left|\pi_A(x_i) - \pi_B(x_i)\right|\right) \tag{16}$$

The separation measure S_{i^+} of each alternative from the intutionistic positive-ideal solution are derived from substituting $A=A_i$ and $B=A^+$ in any one of the above equations. And the separation measure S_{i^-} of each alternative from the intutionistic negative-ideal solution are derived from substituting $A=A_i$ and $B=A^-$ in the same equation used for S_{i^+}.

The relative closeness of an alternative A_i with respect to the intutionistic positive-ideal solution A^+ is defined as

$$C_{i^*} = \frac{S_{i^-}}{S_{i^+} + S_{i^-}} \text{ where } 0 \le C_{i^*} \le 1 \text{ and } i=1,2,...,m.$$

Then, the preference order of alternatives is ranked according to the descending order of C_{i^*} 's.

ALGORITHM:

Step 1: Construct the intutionistic fuzzy decision matrix D.

Step 2: Construct the intutionistic fuzzy weighted decision matrix D'.

Step 3: Determine the intutionistic positive-ideal and negative-ideal solutions.

Step 4: Calculate the intutionistic separation measures.

Step 5: Calculate the relative closeness to the intutionistic ideal solution.

Step 6: Rank the preference order.