CHAPTER 1

Chapter 1

Preliminaries

Definition 1.1 A subset A of a topological space (X, τ) is called

(1) regular open set (Stone 1937) if A=int(cl(A))

(2) semi-open set (Levine 1963) if A$\subseteq$ cl(int(A))

(3) α-open set (Njastad 1965) if A$\subseteq$int(cl(int(A))

(4) The finite union of regular open sets is said to be π-open (Zaitsav 1968).

(5) Pre-open set (Mashour 1982) if A$\subseteq$ int(cl(A))

(6) semi-pre-open set (Andrjevic 1986) if A$\subseteq$ cl(int(cl(A)))

The complements of the above mentioned sets are called regular closed, semi closed, α-closed, π-closed, pre-closed and semi-pre-closed respectively.

The intersection of all regular-closed (resp. semi closed, α-closed, π closed, pre closed and semi-pre-closed) subsets of (X, τ) containing A is called the regular closure (resp. semi closure, α-closure, π-closure, pre-closure, and semi-pre-closure) of A and is denoted by rcl(A) (resp. scl(A), αcl(A), πcl(A), pcl(A) and spcl(A)). A subset A of (X, τ) is called clopen if it is both open and closed in (X, τ). A subset A of (X, τ) is called nowhere dense if int(cl(A))=φ.

Definition 1.2 A subset A of a topological space (X, τ) is called

(1) generalized closed (briefly g-closed) (Levine 1970) if cl(A)$\subseteq$U whenever A$\subseteq$U and U is open.

(2) semi-generalized closed (briefly sg-closed) (Bhattacharyya(1987) if scl(A)$\subseteq$ U whenever A$\subseteq$ U and U is semi-open.

(3) generalized semi-closed (briefly gs-closed) (Arya 1990) if scl(A) $\subseteq$ U whenever A$\subseteq$U and U is open.

(4) α-generalizead closed (briefly αg-closed) (Maki 1994) if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open

(5) generalized semi-preclosed (briefly gsp-closed) (Dontchev 1995) if spcl(A)$\subseteq$ U whenever A$\subseteq$U and U is open.

(6) generalized pre-closed (briefly gp-closed) (Maki 1996) if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is open.

(7) π g-closed (Dontchev 2000) if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is π -open

(8) πgp-closed (Park 2006) if pcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is π -open

(9) πgs-closed (Aslim 2006) if scl (A) $\subseteq$ U whenever A $\subseteq$ U and U is π -open

Definition 1.3 A function f : (X, τ) $\rightarrow$ (Y, σ) is called

(1) g-continuous (Balachandran 1991) if f^{-1} (V) is g-closed in (X, τ) for every closed set of (Y, σ).

(2) semi-generalized continuous (briefly sg-continuous) (Sundaram 1991)if f^{-1}(V) is sg-closed in (X, τ) for every closed set V of (Y, σ).

(3) generalized semi-continuous (briefly gs-continuous) (Devi 1995) if f^{-1}(V) is gs-closed in (X, τ) for every closed set V of (Y, σ).

(4) α-generalized continuous (briefly αg-continuous) (Maki 1994) if f^{-1}(V) is αg-closed in (X, τ) for every closed set V of (Y, σ).

(5) generalized semi pre-continuous (briefly gsp-continuous) (Dontchev 1995) if f^{-1} (V) is gsp- closed in (X, τ) for every closed set V of (Y, σ).

(6) π-continuous (Dontchev 2000) f^{-1} (V) is π-closed in (X, τ) for every closed set V of (Y, σ).

(7) πg-continuous (Dontchev 2000), if f^{-1} (V) is πg-closed in (X, τ) for every closed set V of (Y, σ).

(8) πgp-continuous (Park 2006) if f^{-1} (V) is πgp-closed in (X, τ) for every closed set V of (Y, σ).

Definition 1.4 A bijection f : (X, τ) → (Y, σ) is called

(1) a g-homeomorphism (Maki 1991) if f is both g-continuous and g-open.

(2) a gc-homeomorphism (Maki 1991) if f and its inverse f^{-1} are gc-irresolute.

Definition 1.5. (Mrˇsevi´c 1986) Let A be a subset of a space (X,τ). Then kernel of A and is denoted by ker (A) and is defined as ∩ {U ∈ τ | A ⊂ U}

Lemma 1.6 (Jafari and Noiri 2000) The following properties hold for subsets A, B of a space X

1). x ∈ ker (A) if and only if A∩F ≠ φ for any closed set F containing x,

2). A ⊆ ker (A) and A = ker (A) if A is open in X,

3). if A ⊆ B, then ker (A) ⊆ ker (B).

Definition 1.7 A topological space (X, τ) is said to be

(a) Ultranormal (Staum 1974) if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

(b) Ultra Hausdorff (Staum 1974) if for each pair of distinct points x and y in X there exist U ∈ CO(X,x) and V ∈ CO(X, y) such that U ∩V = φ.

Lemma 1.8 (Ekici 2004) Let G(f) be the graph of f. For any subset A ⊆ X and B ⊆ Y, f (A)∩B = φ if and only if (A×B) ∩ G(f) = φ.

Definition 1.9 A function f :(X, τ)→(Y, σ) is said to be

1) perfectly continuous (Noiri 1980) if f^{-1}(V) is clopen in X for every open set V of Y.

2) regular set-connected (Dontchev et al 1999) if f^{-1}(V) is clopen in X for every V ∈ RO(Y).

Definition 1.10 (Soundararajan 1971) A space X is said to be weakly Hausdorff if each element of X is an intersection of regular closed sets.

Definition 1.11 A space X is said to be

(1) S-closed (Joseph and Kwack 1980) if every regular closed cover of x has a finite sub-cover

(2) Countably S-closed (Dlaska 1994) if every countable cover of X by regular closed sets has a finite sub-cover

(3) A space X is said to be strongly S-closed (Dontchev 1996) if every closed cover of X has a finite sub-cover.

(4) S-Lindelof (Ekici 2004) if every cover of X by regular closed sets has a countable subcover.

Definition 1.12 A space X is said to be

(1) nearly compact (Singal and Mathur 1969), if every regular open cover of X has a finite sub-cover;

(2) nearly countably compact (Ergun 1980), if every countable cover of X by regular open sets has a finite sub-cover

(3) nearly Lindelof (Ergun 1980), if every cover of X by regular open sets has a countable sub-cover.

Definition 1.13 Two functions k_1 and k_2 from power set X to itself are called bi-ˇCech closure operators (simply biclosure operator) (Chandrasekhara Rao 2006) for X if they satisfy the following properties.

(1) $k_1(\varphi) = \varphi$ and $k_2(\varphi) = \varphi$

(2) $A \subset k_1(A)$ and $A \subset k_2(A)$ for any set $A \subset X$

(3) $k_1(A \cup B) = k_1(A) \cup k_1(B)$ and $k_2(A \cup B) = k_2(A) \cup k_2(B)$ for any $A, B \subset X$

(X, k_1, k_2) is called bi-ˇCech closure space.

Definition 1.14 (Chandrasekhara Rao 2006) A subset A in a bi-ˇCech closure space (X, k_1, k_2) is said to be

(1) k_i-regular open if $A = int_{k_i}(k_i(A))$, $i = 1, 2$

(2) k_i-regular closed if $A = k_i(int_{k_i}(A))$, $i = 1, 2$

(3) k_i-semi open if $A \subseteq k_i(int_{k_i}(A))$, $i = 1, 2$

(4) k_i-semi closed if $int_{k_i}(k_i(A)) \subseteq A$, $i = 1, 2$

(5) The finite union of k_i- regular open sets is said to be k_i π-open.

The complement of a k_i π-open set is said to be k_i π-closed. The smallest k_i π closed set containing A is called k_i-π closure of A and it is denoted by $k_i \pi cl(A)$.

Definition 1.15 (Noiri 2009) Let X be a nonempty set and P(X) be the power set of X. A subfamily m_X of P(X) is called a minimal structure (briefly m-structure) if $\emptyset \in m_X$ and $X \in m_X$.

By (X, m_X), we denote a nonempty set X with a m-structure on X and it is called a m-space. Each member of m_X is said to be a m_X-open set and the complement of a m_X-open set is said to be m_X-closed.

Definition 1.16 (Noiri 2009) Let X be a nonempty set and m_X be m-structure on X.

For a subset A of X the m_X-closure of A and the m_X-interior of A are defined as follows :

(1) m_X-Cl(A) = $\cap\{F \mid A \subseteq F, X - F \in m_X\}$;

(2) m_X-Int(A) = $\cup\{U \mid U \subseteq A, U \in m_X\}$.

(3) m_X-Cl(m_X-Cl(A)) = m_X-Cl(A) and m_X-Int(m_X-Int(A)) = m_X-Int(A).

Definition 1.17 (Maki 1999) A m-structure m_X on a nonempty set X is said to have property B if the union of any family of subsets belonging to m_X belongs to m_X.

Definition 1.18 (Boonpok 2010) Let X be a nonempty set and let be minimal structures on X.

A triple (X, m_X^1, m_X^2) is called a biminimal structure space (briefly bim-space).

Throughout the present paper, (X, m_X^1, m_X^2) denote a biminimal structure space and A is a subset of X. The m_X-closure and m_X-interior of A with respect to m_X^i are denoted by m_X^i- Cl(A) and m_X^i- Int(A), respectively, for i = 1, 2.

Definition 1.19 (Boonpok 2010) Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces.

i) A function f : $(X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is said to be (i, j)-M continuous at a point x ∈ X if for each V ∈ m_X^j containing f(x), there exists U ∈ m_X^i containing x such that f(U) ⊆ V, where i, j = 1, 2 and i ≠ j.

ii) A function f : $(X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is said to be (i, j)-M-continuous if it has this property at each point x ∈ X.

Definition 1.20 (Boonpok 2010) A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be:

(1) m_X^i-regular open if A = m_X^i-Int(m_X^i-Cl(A)), for i = 1, 2;

(2) m_X^i-semi-open if A ⊆ m_X^i-Cl(m_X^i-Int(A)), for i = 1, 2;

(3) m_X^i-preopen if A ⊆ m_X^i-Int(m_X^i-Cl(A)), for i = 1, 2;

(4) m_X^i-α-open if A ⊆ m_X^i-Int(m_X^i-Cl(m_X^i-Int(A))), for i = 1, 2;

(5) m_X^i-β-open if A ⊆ m_X^i-Cl(m_X^i-Int(m_X^i-Cl(A))), for i = 1, 2.

The complement of a m_X^i-regular open (resp. m_X^i-semi open, m_X^i-preopen, m_X^i-α-open, m_X^i-β-open) set is called a m_X^i-regular closed (resp. m_X^i-semiclosed, m_X^i-preclosed, m_X^i-α-closed, m_X^i-β-closed) set. The finite union of m_X^i regular open sets is said to be m_X^i π -open.The complement of a m_X^iπ-open set is said to be m_X^iπ-closed.

Definition 1.21. (Boonpok 2010) A subset A of biminimal structure space (X, m_X^1, m_X^2) is said to be:

(1) (i, j)-m_X-regular open if A = m_X^i-Int(m_X^j-Cl(A)), where i, j = 1, 2 and i ≠ j

(2) (i, j)-m_X-semi-open if A ⊆ m_X^i-Cl(m_X^j-Int(A)), where i, j = 1, 2 and i ≠ j

(3) (i, j)-m_X-preopen if A ⊆ m_X^i-Int(m_X^j-Cl(A)), where i, j = 1, 2 and i ≠ j;

(4) (i, j)-m_X-α-open if A ⊆ m_X^i-Int(m_X^j-Cl(m_X^i-Int(A))), where i, j = 1, 2 and i ≠ j;

(5) (i, j)-m_X-β-open if A ⊆ m_X^i-Cl(m_X^j-Int(m_X^i-Cl(A))), where i, j = 1, 2 and i ≠ j.

The complement of a (i, j)-m_X-regular open (resp. (i, j)-m_X-semi open, (i, j)-m_X preopen, (i, j)-m_X-α-open, (i, j)-m_X-β-open) set is called a (i, j)-m_X-regular closed (resp. (i, j)-m_X-semi-closed, (i, j)-m_X-preclosed, (i, j)-m_X-α-closed, (i, j)-m_X-β closed) set. The finite union of (i, j)-m_X regular open sets is said to be (i, j)-m_X π open.The complement of a(i, j)-m_X π-open set is said to be (i, j)-m_X π-closed.

Definition 1.22 (Boonpok 2011) A subset A of biminimal structure space (X, m_X^1, m_X^2) is said to be $m_X^{(i,j)}$-closed if m_X^i-Cl(m_X^j-Cl(A)) = A, where i, j = 1, 2 and i ≠ j.The complement of a $m_X^{(i,j)}$-closed set is said to be $m_X^{(i,j)}$-open.

Definition 1.15 A subset A of biminimal structure space(X, m_X^1, m_X^2) is said to be $m_X^{(i,j)}$– regular open if A = m_X^i-Int(m_X^i-Cl(m_X^j-Int(m_X^j-Cl(A)))), for i = 1, 2

A finite union of $m_X^{(i,j)}$- regular open sets is said to be $m_X^{(i,j)}$ π -open. The complement of a $m_X^{(i,j)}$π-open set is said to be $m_X^{(i,j)}$π-closed.

Definition 1.23 (Choquet 1947) A non-null collection $\mathcal{G}$ of subsets of a topological spaces X is said to be a grill on X if

(i) $\varphi \notin \mathcal{G}$,

(ii) A ∈ $\mathcal{G}$ and A ⊆ B ⊆ X ⇒ B ∈ $\mathcal{G}$, and

(iii) A,B ⊆ X and A∪B ∈ $\mathcal{G}$ ⇒ A ∈ $\mathcal{G}$ or B ∈ $\mathcal{G}$.

Definition 1.24 (Roy 2007) Let (X, τ, $\mathcal{G}$) be a grill topological space. An operator $\Phi_{\mathcal{G}}$ from the power set $\mathcal{P}$(X) of X to $\mathcal{P}$(X) was defined by $\Phi_{\mathcal{G}}$(A) = {x ∈ X : U ∩ A ∈ $\mathcal{G}$, for each open neighbourhood U of x} for each A ∈ $\mathcal{P}$(X). The map $\Psi_{\mathcal{G}}$: $\mathcal{P}$(X) → $\mathcal{P}$(X), given by $\Psi_{\mathcal{G}}$(A) = A∪ $\Phi_{\mathcal{G}}$(A) (for A∈ $\mathcal{P}$(X)) is a Kuratowski closure operator. Hence, induces a topology $\tau_{\mathcal{G}}$ on X, strictly finer than τ, in general.

Definition 1.25 (Mukherjee 2012) Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X. Then a subset A of X is said to be g-closed with respect to the grill $\mathcal{G}$ ($\mathcal{G}$-g-closed, for short) if $\Phi_{\mathcal{G}}$(A) ⊆ U whenever A ⊆ U and U is open in X.

1. 2 CONTRIBUTIONS OF THE AUTHOR

In the light of the above work, the author has obtained some interesting generalizations on the following topics.

(1) Generalized π-closed sets in topological spaces.

(2) Generalized π-compactness and generalized π-connectedness

(3) Quasi generalized π-closed function

(4) Generalized π-closed sets in Bi- ˇ Cech closure spaces

(5) Generalized π-closed sets in Biminimal structure spaces

(6) Generalized π -closed sets and grill