Studies Relating to Design of AOQL Single Sampling Plans

Vaishnavi, K.N

(13PMA013)

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women,

Coimbatore-641 043

In Partial Fulfilment of the Requirements for the

Degree of Master of Science in Mathematics

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Signature of the Head of the Department

Signature of the Supervisor

Acknowledgement

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Introduction

Introduction

Acceptance sampling is a statistical method which enables us to make the decision of either accepting or rejecting a shipment of items for the lot. In most situations, 100 percent inspection of all items is neither desirable nor economically feasible.

Some advantages of acceptance of sampling plan are:

- It is more economical as against 100 percent inspection in terms of inspection cost.
- It is usually more accurate than 100 percent inspection, since it allows less opportunity for inspection fatigue, which can be responsible for mistakes.
- Less product damage occurs since it requires less handling of the product.
- Rejecting the entire lot on the basis of simple sampling testing can motivate the suppliers of the product to improve their quality control standards and procedures.
- It is the only approach in situation where quality is tested by destroying the item.

Inspection for acceptance purpose is carried out at many stages in manufacturing. There may be inspection of incoming materials and parts, process inspection at various points in the manufacturing operations, final inspection by a manufacturing of own product and ultimately inspection of the finished product by one or more purchasers.

Much of this acceptance inspection is necessarily on a sampling basis. All acceptance tests that are destructive of the item tested much inevitably be done by sampling. In many other situations, sampling inspection is used because the cost of 100 percent inspection is prohibited.

An important advantage of modern acceptance sampling system such as those discussed in the Dodge-Romig system[4] for lot-by-lot acceptance sampling by attributes and acceptance sampling by variables is that they exert more effective pressure for quality improvement than is possible with 100 percent inspection.

The major areas of acceptance sampling according to Dodge [7] are

- ❖ Lot-by-lot sampling by the method of attributes in which each unit in a sample is inspected on a go-not-go basis for one or more characteristics.

- ❖ Lot-by –lot sampling by the method of variables in which each unit in a sample is measured for a single characteristics, such as weight or strength.

- ❖ Continuous sampling of a flow of units by the method of attributes.

- ❖ Special purpose plans including chain sampling, skip-lot sampling, small sampling plans, etc….

Sampling plan, sampling scheme, and sampling system

According to American national standards institute/ American society, for quality control (ANS/ASQC) standard A_2 [1] an acceptance sampling plan is a specific plan that states the sampling rules to be used and the associated acceptance and non-acceptance criteria and an acceptance sampling scheme is a specific set of procedures which usually consists of acceptance sampling plan in which lot sizes, sampling size and acceptance criteria or the amount of 100 percent inspection and sampling are related.

According to Hill[10], a sampling scheme is whole set of sampling plans and operation include in the standard " the over- all strategy specifying the way in which sampling plans are to be used"

The MIL-STD-105 D [19] is a well known sampling scheme. Stephens and Larson [22] have described a sampling system as an assigned grouping of two or more sampling plans and the rules for using these plans for sentencing lots to achieve a blend of the advantage features of each of the sampling plans. Chen , C.H and Chou C.H [2] has described the lot-by-lot AOQL sampling plans which are designed to provide assurance that the long run average of acceptance quality given screening and repairing of rejects lots, will not be worse than the indexed AOQL value. Already Dodge-Romig [7] provided comprehensive table of attribute plans indexed by LTPD and AOQL. Hill [10] has also described the difference between sampling plan and sampling scheme.

Operating Characteristics (OC) curve

Every sampling is associated with an operating characteristic curve, familiarly known as OC curve of the plan. This curve when referred to two axis , the axis of p-proportion nonconforming of the material offered for inspection and the axis of P_a (p) – probability of acceptance of a lot or process, is the locus of (p, P_a (p)). The OC curve gives the practical performance of the sampling plan.

Type A-OC curve

A curve showing for a given sampling plan, the probability of accepting a lot as a function of the lot quality. This curve is for isolated or unique lots or a lot from an isolated sequence.

Type B-OC curve

A curve showing for a given sampling plan, the probability of accepting a lot as a function of the process average. This curve is for continuous stream of lots.

There are three probability distribution that may be used to find the probability of acceptance.

- The hyper geometric distribution
- The binomial distribution
- The Poisson distribution

Although the hyper geometric may be used when the lot sizes are small, the binomial and Poisson are by far the most popular distribution to use when constructing sampling plans.

Hyper geometric distribution

The hyper geometric distribution is used to calculate the probability of acceptance of a sampling when the lot is relatively small. It can be defined as the true basic probability distribution of attribute data but the calculations could become quite cumbersome for large lot sizes. This model exact for the case of nonconforming units under Type A situations and is useful for isolated lots.

The probability of exactly X defective parts in a sample n:

$$P(X) = \frac{\binom{n}{X}\binom{N-n}{n-X}}{\binom{N}{n}}$$

Binomial distribution

The binomial distribution is used when the lot is very large. For large lots, the non replacement of the sampled product does not a affect the probabilities. The hyper geometric takes into consideration that each sample taken affects the probability associated with the next sample. This is called sampling without replacement. The binomial assumes that the probabilities associated with all samples are equal. This is sometimes referred to as sampling with replacement although the parts are not physically replaced. The binomial is used extensively in the construction of sampling plan.

The probability of exactly X defective parts in a sample n:

$$P(X) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

The symbol p represents the value of incoming quality expressed as a decimal.(1%=0.1, 2%=0.02, etc).

This model is exact for the case of nonconforming under type A and type B situations. Under situation of type A, for the case of nonconforming units whenever n/N $\leq$ 0.10, where n and N are the sample and lot sizes respectively.

Poisson distribution

The Poisson distribution is used for sampling plans involving the number of defects per unit rather than the number of defective parts. It is also used to approximate the binomial probabilities involving the number of defective parts when the sample (n) is large and p is very small. When n is large and p is small, the Poisson distribution formula may be used to approximate the binomial. Using the Poisson to calculate probabilities associated with various sampling plans is relatively simple because the Poisson tables can be used.

The probability of exact X defects or defectives parts in a sample n:

$$P(X) = \frac{e^{-np}\ (np)^{x}}{x!}$$

where e represents the value of the base of the natural logarithm system. It is constant value (e=2.71828)

This model is exact for the case of nonconforming under type A and type B situations. Under situation of type A, for the case of nonconforming units, this model can be used whenever $n/N \leq 0.01$, n is large and p is small such that $np<5$. Under situation of type B, for the case of nonconforming units, this model can be used whenever n is large and p is small such that $np<5$.

Designing sampling plan

In designing a sampling plan one has to accomplish a number of different purposes,

- To strike a proper balance between the consumer’s requirements, the producer’s capabilities, inspector’s capacity.
- To separate bad lots from good.
- Simplicity of procedures and administration.

- Economy in number of observations.

- To reduce the risk of wrong decisions with increasing lot sizes.

- To use accumulated sample data as a valuable source of information.

- To exert pressure on the producer or supplier when the quality of lots received is unreliable or not up to standard and

- To reduce sampling when the quality is reliable and satisfactory.

Some of the major types of designing the plans, which are classified according to types of production:

1. The plan is specified by requiring the OC curve to pass through (or nearly through) two fixed points.
2. The plan is specified by fixing the one point only, through which the OC curve is required to pass and setting up one or more conditions, not explicitly in terms of the OC curve.
3. The plan is specified by imposing upon the OC curve of two or more independent conditions none of which explicitly involves the OC curve.

Acceptance Sampling Plan

A specific plan that states the sample size or sizes to be used and the associated acceptance and non-acceptance criteria.

Probability of Acceptance

The probability that the lot will be accepted under the given sampling plan.

Probability of Rejection

The probability that the lot will not be accepted under the given sampling plan.

Quality

Quality means fitness for use and it is inversely proportional to variability.

Acceptance Quality Level (AQL)

The acceptable quality level (AQL) is the maximum percentage or proportion of nonconforming items or number of nonconformities in a lot or batch that can be considered satisfactory as a process average. The AQL concept has been troublesome to many who consider it to condone an acceptance of less-than-perfect quality. To statisticians, AQL simply denotes an economic decision that is associated with producer's risk.

Average Outgoing Quality

The expected quality of outgoing product following the use of an acceptance sampling plan for a given value of incoming product quality.

Average Outgoing Quality Level (AOQL)

For a given acceptance sampling plan, the maximum average outgoing quality over all possible levels of incoming quality.

Lot Tolerance Percent Defective

Lot tolerance percent defective is defined as a maximum percentage of defective item in a lot beyond which the lot should be rejected.

Consumer risk

For a given sampling plan, the probability of acceptance of a lot, the quality of which has a designed numerical value representing a level which it is seldom desired to accept.

Producer risk

For a given sampling plan, the probability of not accepting a lot, the quality of which has designated numerical value representing a level which it is generally desired to accept.

Limiting Quality Level (LQL)

The percentage or proportion of variant units in a batch or a lot for which, for the purpose of acceptance sampling the consumer wishes the probability of acceptance to be restricted to a specified low value.

Average Sample Number (ASN)

The average number of sample units per lot for making decisions (acceptance or non-acceptance).

Average Total Inspection (ATI)

The average number of units inspected per lot based on the sample size for accepted lots and all inspected units in not-accepted units.

Average Fraction Inspected (AFI)

The fraction of product that will be inspected over the long run if the process average is a particular value.

Inspection

Inspection is the process of measures examining, testing or otherwise comparing the units of the product with the requirements.

Sampling Inspection

Sampling inspection means the inspection for the defects concerned where the units selected for inspection are selected by sampling.

100 Percent Inspection

100 percent inspection means the inspection of every unit of product for the defects concerned listed for an inspection station. The two terms screening and 100 percent inspection are used inter changeably.

Single – sampling plan

The single-sampling plan is a decision rule to accept or reject a lot based on the results of one random sample from the lot. The procedure is to take a random sample of size (n) and inspect each item in the sample. If the number of defects does not exceed a specified acceptance number (c), the consumer accepts the entire lot. Any defects found in the sample are either repaired or returned to the producer. If the number of defects in the sample is greater than c the consumer subjects the entire lot to 100% inspection or rejects the entire lot and returns it to the producer. The single sampling plan is easy to use but usually results in a larger ANI than the other plans.

Double Sampling

Sampling inspection in which the inspection of the first sample of size n_1 leads to a decision to accept a lot, or not to accept it; or to take a second sample of size n_2 and the inspection of second sample then leads to a decision to accept or not to accept the lot.

Percent defective

Percent defective is a measure of quality in terms of percentage. It can vary from 0% to 100%. It is also termed as fraction defective. It represents the number of defective items per 100 items present in a lot of size N.

Dodge and Romig system

Dodge –Romig sampling inspection tables are designed to minimize average total inspection for given average outgoing quality level.

Glossary of symbols:

AOQL	- Average outgoing quality limit
N	- Number of items in the lot
$\overline{p}$	- Process average fraction defective
p_L	- Average outgoing quality limit
p_t	- Lot tolerance fraction defective
n	- Number if items in the sample
c	- Acceptance number
L(p; n, c)	- Operating characteristic
LTPD	- Lot tolerance percent defective
U	- Upper specification limit
L	- Lower specification limit
μ	- Mean
σ	- Standard deviation
s	- Sample standard deviation
k	- Critical value
C_s^*	- The cost of inspection of one item by attributes
C_m^*	- The cost of inspection of one item by variables
C_{ms}	- The mean inspection cost per lot of process average quality
α	- Producer's risk
β	- Consumer's risk

e	- Dodge-Romig AOQL plans for inspection by attributes from economical point of view
I_s	- Minimize the mean number of items inspected per lot of process average quality
$p_a(p)$	- Probability of acceptance for given p
P	- Lot or process quality
AQL	- Acceptance quality level
ASN	- Average Sample Number
SSP	- Single Sampling Plan
AOQ	- Average Outgoing Quality
QLF	- Quality Loss Function
QC	- Quality Control
OC	- Operating Characteristic
TC_0	- The expected total cost unit for no inspection
TC_1	- The expected total cost unit for 100% inspection
TC_s	- The expected total cost per unit
C_i	- Inspection cost per unit
C_r	- The scrap cost per unit (or) Replacement cost per unit
y	- Quality Characterics
z	- Specification limit

K - The co-efficient of quality loss

f(y) - The probability density function

$z\sigma$ - The lower specification limit for screening product

$\mu + z\sigma$ - The upper specification limit for screening product

X_m - Mean of assumed distribution based on given AQL

P - Fraction defective of the submitted lot

TC_f - The total expected quality loss per unit

Φ (.) - Cumulative distribution function of standard normal random variable

k - The quality loss coefficient

Y - normally distributed with known process mean μ_0 and process standard deviation σ_0

y_0 - Target value of product

μ_T - Target value of process mean

σ_T - Target value of process standard deviation

μ_I - Improved process mean

σ_I - Improved process standard deviation

This thesis is devoted to the study of designing Average Outgoing Quality Limit [AOQL] single sampling plans.

The first chapter deals with the design of acceptance sampling plans when the remainder of rejected lots is inspected. Two types of AOQL plans are considered – for inspection by variables and for inspection by variables and attributes. These plans are compared with the corresponding Dodge – Romig AOQL plans for inspection by attributes. An algorithm allowing the calculation of these plans with software mathematica is used. From the results of numerical investigations it follows that under the same protection of consumer the AOQL plans for inspection by variables are in many situations more economical than the corresponding Dodge-Romig attribute sampling plans.

The second chapter deals with the design of integrating Dodge-Romig average outgoing quality limit [AOQL] Single Sampling Plans [SSP] by variables and specification limit. By solving the modified Kapur and Wang's model economic specification limits and the optimal inspection policy of Dodge-Romig AOQL SSP variables were obtained.

The third chapter proposes a modification of Dodge-Romig single rectifying inspection plan with average outgoing quality limit (AOQL) protection. The quality investment and inspection error are considered in the modified model. The optimal parameters of sampling inspection plan and quality investment level are simultaneously determined by minimizing the expected total cost of product under the specified AOQL value. Finally, the comparison of solution between the modified model with / without inspection error are provided for illustration.

The fourth chapter deals with the AOQL single sampling plans when the remainder of rejected lot is inspected. Two types of AOQL plans are considered – for inspection by variables and for inspection by variables and attributes. These plans compared with corresponding Dodge-Romig AOQL plans for inspection by attributes from economical point of view. From the results of numerical investigations it follows that under the same protection of consumer AOQL plans for inspection by variables are in many situations more economical than the corresponding Dodge-Romig attributes sampling plans. The analysis is made by dependence of the saving of the inspection cost on acceptance sampling characteristics.

Review of Literature

Review of Literature

Acceptance sampling is a statistical method which enables us to make the decision of either accepting or rejecting a shipment of item a shipment of item for the lot. A specific plan that states the sample size or sizes to be used and the associated acceptance and non-acceptance criteria. It is the usual practice to use Poisson distribution or hyper geometric to evaluate the OC curves.

The single-sampling plan is a decision rule to accept or reject a lot based on the results of one random sample from the lot. The procedure is to take a random sample of size(n) and each item. If the number of defects does not exceed a specified acceptance number (c), the consumer accept the entire lot. Any defects found in the sample are either repaired or returned to the producer. If the number of defects in the sample is greater than c the consumer subjects the entire lot to 100% inspection or rejects the entire lot and returns it to the producer. The single sampling plan is easy to use but usually results in a larger Average Number Inspected than the other plans

AOQL is identified with a sampling method which guarantees that the quality of the population investigated meets certain minimum quality requirements after an inspection that allows for correction of error found. Originally AOQL was designed for industrial purposes but now a days this method is also used in auditing and in control of administrative processes. The AOQL sampling system was developed by Dodge-Romig. It was originally developed for industrial purposes.

Jindrich Klufa[24] suggested the acceptance sampling plans when the remainder of rejected lots in inspected. Two plans are considered – for inspection by variables and for inspection by variables and attributes. These plans are compared by Dodge – Romig[13] with AOQL plans for inspection by attributes and also presented the algorithm allowing the calculation of these plans.Dodge H.F, Romig H.G[14] introduced the Sampling Inspection Tables and Klufa. J[25] introduced the tables of sampling inspection plans by variables. Further more Likes. J, Klufa. J, Jarosova.E [27] explained the Statistical Methods in Reliability and Quality control.

Chung – Ho Chen [10] proposed economic design of Dodge – Romig AOQL single sampling plans by variables with the quadratic loss function. Sower [33] proposed an integrated model of statistical process control (SPC) and acceptance sampling plan. The classical Dodge – Romig[13] suggested rectifying attributes sampling plans provided the lot tolerance percent defective (LTPD) on each lot or the average outgoing quality limit (AOQL) protection for the lots. Kapur and Wang's[20] model pointed out the optimal inspection policy of Dodge-Romig AOQL Single Sampling Plan by variables. Montgomery[31] studied that variables sampling plans usually involve smaller sample size than attributes sampling plans for the same levels of protection. Due to economic reasons, Klufa [25] presented the designs of Dodge – Romig LTPD and AOQL single sampling plan by variables.

Taguchi [34] refined the quality of product and presented the quadratic quality loss function to reducing total losses to the society. Tagaras [33] adopted the quadratic quality loss function for designing the parameters of acceptance sampling plan by variables. Further Kapur and Wang, Kapur[21], Kapur – cho and Chen and Chou [5] addressed the problems of quality loss function applied in the economic design of specification limits. Kapur and Wang [20] pointed out that one of the short – term approaches to reduce variance of the units shipped to the customer is to put specification limits on the process and truncate the distribution by inspection. Chen [4], Chen and Chou [6] considered the problems of the integrated designs of Dodge – Romig AOQL single sampling plan by attributes and specification limits. The design of integrating Dodge – Romig AOQL SSP by variables and specification limit is presented. By solving the modified Kapur and Wang's [20] model they have obtained the economic specification limits and also the optimal inspection policy of Dodge – Romig AOQL SSP by variables. Finally they have compared the results with those of Chen [4] and Chen and Chou [6].

Chung – Ho Chen [10] proposed Economic Selection of Dodge – Romig AOQL Sampling Plan under the quality investment and inspection error. Dodge – Romig [14] provided the rectifying SSP and double sampling plans for attributes with the protection of lot tolerance percent defective (LTPD) or average outgoing quality limit (AOQL). Klufa further presented the modified Dodge – Romig's model based on variable single sampling inspection plan. One assumption of the classical Dodge – Romig [13] AOQL SSP is perfect inspection without error. Hong et al, Ganeshan et al, and Chen and Tsou [9] have presented the exponential reduction of process mean and standard deviation as the function of quality investment.

Abdul – Kader et al. [2] further adopted Chen and Tsou quality investment function for determining the optimum quality investment and corresponding improved process mean and standard deviation. Chen [4] further extended Chen and Tsou's method for designing the sampling inspection plan under the economic selection. Chen – Ho Chen [10] proposes a modified Dodge – Romig [14] AOQL SSP under the quality investment and inspection error. It is an extension of Chen's [4] work. This chapter also presents the effect of inspection error on the modified Dodge – Romig [13] model.

Case et al. [3] demonstrated that the AOQ function with the inspection error. Nikola Kasprikova – Jindrich Klufa[26] proposed calculation of AOQL single sampling plans for inspection by variables and its software implementation. The AOQL plans for inspection by variables were introduced by Klufa [13] and further the calculation of these plans when the non – central t distribution is used for the operating characteristic is pointed out by Johnson and Welch [19] and these problems are solved and solution is givenby Klufa [14].

Chapter - I

CHAPTER I

SAMPLING INSPECTION PLANS FROM NUMERICAL POINT OF VIEW

In this chapter **"**Sampling inspection plans from numerical point of view" by JindrinchKulfa [24] have been reviwed.

The design of acceptance sampling plans when the remainder lots is inspected consists of two types of AOQL plans – for inspection by variables and for inspection by variables and attributes. These plans are compared with the corresponding Dodge – Romig AOQL plans for inspection by attributes.

The sampling plans are considered which minimize the mean number of items inspected per lot of process average quality, assuming that the remainder of rejected lots is inspected.

$$I_s = N - (N-n).L(\overline{p};n,c) \qquad (1.1)$$

Under the condition

$$\max_{0<p<1} AOQ(p) = p_L \qquad (1.2)$$

Or under the condition $L(p_t\ ;n\ ,c) = 0.10$ (LTPD single sampling plans), where N is the number of items in the lot (the given parameter), $\overline{p}$ is the process average fraction defective (the given parameter), p_L is the average outgoing quality limit (the given parameter, denoted AOQL), p_t is the lot tolerance fraction defective (the given parameter, denoted LTPD), n is the number of items in the sample(n<N), c is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than c), L(p; n, c) is the operating characteristic(the probability of accepting a submitted lot with fraction defective p), AOQ(p) is the average outgoing quality (the mean fraction defective after inspection when the fraction defective before inspection was p). The average outgoing quality (all defective items found are replaced by good ones) is approximately,

$$AOQ(p) = (1 - \frac{n}{N})pL(p; n, c). \quad (1.3)$$

Therefore condition (1.2), which protects the consumer against the acceptance of a bad lot, can be written as

$$\max_{0<p<1} (1 - \frac{n}{N}) pL(p; n, c) = p_L$$

(1.4)

The Dodge-Romig LTPD and AOQL plans can be used under the assumption that each inspected item is classified as either good or defective(acceptance sampling by attributes). The procedure of an algorithm allowing calculation of two types of AOQL plans is given below:

a) For inspection by variables – all items from the sample and all items from the remainder of rejected lot are inspected by variables.
b) For inspection by variables and attributes – all items from the sample are inspected by variables, but the remainder of rejected lots is inspected by attributes only.

1.2 AOQL plans by variables and comparison with the Dodge-Romig plans

It is assumed that measurements of a single quality characteristic X are independently identically distributed normal random variables with unknown parameters μ and σ^2. For the quality characteristic X is given either an upper specification limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L). It is further assumed that the unknown parameter σ is estimated from the sample standard deviation s (unknown standard deviation plans), no use is made of the average range as an estimator of σ. The inspection procedures is as follows,

1. Draw a random sample of n items and compute

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \ , \quad s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{1.5}$$

2. Compute $\frac{u-\bar{x}}{s}$ for an upper specification limit, or $\frac{\bar{x}-L}{s}$ for a lower specification limit.

3. Accept the lot if

$$\frac{u-\bar{x}}{s} \geq k \quad \text{or} \quad \frac{\bar{x}-L}{s} \geq k \tag{1.6}$$

The problem is to determine the sample size n and the critical value k. There are different solutions to this problem. We shall look for the acceptance plan (n,k) minimizing the mean inspection cost per lot of process average quality c_{ms}under the condition (1.4). Inspection cost per lot, assuming that the remainder of rejected lots is inspected by attributes (the inspection by variables and attributes), is nc_m^* with probability L(p; n, k) and [nc_m^* + (N – n)c_s^*] with probability [1 – L(p; n, k)], where $c_s^*$is the cost of inspection of one item by attributes, and $c_m^*$is the cost of inspection of one item by variables. The mean inspection cost per lot of process average quality is therefore

$$c_{ms} = nc_m^* + (N-n)c_s^*.[1 - L(\bar{p}; n, k)] \tag{1.7}$$

Let us denote

$$c_m = \frac{c_m^*}{c_s^*} \tag{1.8}$$

The acceptance plan for (n,k) minimizing

$$I_{ms} = nc_m + (N-n)[1 - L(\bar{p}; n, k)] \tag{1.9}$$

instead of c_{ms} (both functions c_{ms} and I_{ms} have a minimum for the same acceptance plan, $c_{ms} = I_{ms} c_s^*$) under the condition

$$\max_{0<p<1} \left(1 - \frac{n}{N}\right) p L(p; n, k) = p_L . \qquad (1.10)$$

For these AOQL plans for inspection by variables and attributes (the type (b)) the new parameter c_m was defined. This parameter must be statisticallyestimated in each real situation.

Usually, there is

$$C_m > 1 \qquad (1.11)$$

substituting formally $c_m = 1$ into (1.9) (I_{ms}in this case is denoted by I_m) we obtain

$$I_m = N - (N - n) . L(\overline{p}; n, k), \qquad (1.12)$$

that is the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of rejected lots are inspected by variables. Consequently we study the AOQL plans for inspection variables and attributes for C_m =1. From (1.12) it is evident that for the determination of AOQL plans by variables it is not necessary to estimate C_m (C_m =1 is not real value of this parameter).

For the given parameter N, $\overline{p}$, p_L and c_m , we determine the acceptance plan (n,k) for inspection by variables and attributes, minimizing I_{ms} in (1.9) under the condition (1.10).

First we shall deal with the solution of (1.10). The operating characteristic, using the normal distribution as an approximation of non-central t distribution is

$$L(p; n, k) = ¤ \left(\frac{u_{1-p} - k}{A}\right).$$

(1.13)

where

$$A = \sqrt{\frac{1}{n} + \frac{k^2}{2(n-1)}} \tag{1.14}$$

The function Φ in (2.13) is a standard normal distribution function and u_{1-p} is a quantile of order 1 – p, i.e., $\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} \exp(-x^2/2)\, dx$, $u_{1-p} = \Phi^{-1}(1-p)$ (the unique root of the equation $\Phi(u) = 1 - p$). The approximation (1.13) holds both for an upper specification limit U and for a lower specification limit L. The equation (1.10), using (1.13), has an (approximately) equivalent form .

$$\max_{0<p<1} p\,.\Phi\left(\frac{u_{1-p} - k}{A}\right) = \frac{p_L}{1 - \frac{n}{N}}. \tag{1.15}$$

Let us denote

$$G(p;n,k) = p\Phi\left(\frac{u_{1-p} - k}{A}\right),\ M(n,k) = \max_{0<p<1} G(p;n,k). \tag{1.16}$$

Let n, N, p_L be given parameters (for the given n we shall write Mn(k) instead of M(n,k)).

First look for critical value k for which (1.15) holds i.e

$$M_n(k) = p_L\left(1 - \frac{n}{N}\right) \tag{1.17}$$

Theorem 1.1

Let n, N, p_L be given parameters, $p_L < \frac{1}{4} - \frac{7}{4N}$. If

$$n \in (7, (1 - 4p_L)N), \tag{1.18}$$

then each solution k of equation (1.17) is non negative that is, $k \geq 0$.

proof

If k< 0, then $L(\frac{1}{2}) = ¤(\frac{-k}{A}) > \frac{1}{2}$ and $M_n(k) > \frac{1}{4}$, but the right hand side of (1.17) is for $n \in (7, (1-4p_L)N)$ less than or equal to $\frac{1}{4}$.

The assumption (1.18) is not limiting one from practical point of view. From numerical investigations if follows that for most of the given parameters N, $\overline{p}$, p_L , c_m . The assumption (1.18) is valid. If assumption (1.18) is not valid (very small lots), AOQL plans for inspection by variables and attributes are not considered for economical reasons.

Let us denote $k_n = \{k \geq 0; M_n(k) \geq p_L\}$ (1.19)

Theorem 1.2

Let p_L be the given parameter, $n \in (7, (1-4p_L)N)$. If for n,

$$¤(-(n-1)\sqrt{\frac{2}{n}}) \leq p_L \quad (1.20)$$

holds then the function $M_n(k)$ is decreasing in k_n.

For usually chosen p_L the assumption (1.20) hold. The left hand side of (1.20) is decreasing function of n and for n = 7 the left hand side of (1.20) is approximately 0.0007 (minimum value of AOQL is $p_L = 0.001$).

From theorem 1.2, it follows that each solution of equation (1.17) is unique. Since an explicit formula for k does not exist, we have to solve (1.17) numerically. Using Newton's third method , therefore we must determine $M_n(k)$ and derivative M_n' (k).

Using the equation (1.16) one has

$$M_n(k) = p_M\ ¤(\frac{u_{1-p_M} - k}{A}), \tag{1.21}$$

where $p_M \in$ (0.1) isthe value of p, for which the function G(p; n, k) in (1.16) has a maximum. Evidently it holds that G(0; n, k) = G(1; n, k) = 0 and G(p; n, k) > 0 for p $\in$ (0,1).

Since the function G(p; n, k) is continuous for p $\in$ (0,1), p_M exists. We determine the value p_M as a solution of the equation $G'(p) = 0$ i.e.,

$$¤(\frac{u_{1-p_M} - k}{A}) - \frac{p}{A}\exp[-\frac{1}{2A^2}[(1-A^2)u_{1-p}^2 - 2ku_{1-p} + k^2]] = 0. \tag{1.22}$$

Theorem 1.3

Let n be the given parameter, n $\in$ (7, (1- 4 p_L)N), k_r= (n-1) $\sqrt{\frac{2}{n}}$. If k = k_r, then

$p_M = ¤(-\frac{k_r}{2})$ is a solution of equation (1.22).

Proof

For k_r= (n-1) $\sqrt{\frac{2}{n}}$ one obtains A = 1. Since $u_{1-p_M} = \frac{k_r}{2}$, it is evident that

$p_M = ¤(-\frac{k_r}{2})$ satisfies the equation (1.22).

Theorem 1.4

Let n be the given parameter, n $\in$ (7, (1- 4 p_L)N), k_r= (n-1) $\sqrt{\frac{2}{n}}$. If k$\in$ $(0,\propto) - \{k_r\}$, then

solution p_M of the equation (1.22) is between p_a and p_r, where

$$p_a = ¤(\frac{-k - A\sqrt{k^2 - 2(1-A^2)\ln A}}{1-A^2}),\ p_r = ¤(\frac{-k}{1+A}). \tag{1.23}$$

Instead of p_M we shall look for $x_M = u_{1-p_M} (p_M = ¤(-x_M))$ as a solution of the equation $G'(x) = 0$, i.e.,

$$¤(\frac{x-k}{A}) - \frac{¤(-x)}{A}.\exp[-\frac{1}{2A^2}[(1-A^2)x^2 - 2kx + k^2]] = 0. \quad (1.24)$$

The equation (1.24) must be solved once more numerically. From figure 1.1, it is evident that numerical solution of the equation $G'(x) = 0$ depends on good first approximation x_0. Under the assumptions of therorem 1.4, solution x_M of equation (1.24) is between x_r and x_a,

where

$$x_r = \frac{k}{1+A}, \qquad x_a = \frac{k + A\sqrt{k^2 - 2(1-A^2)\ln A}}{1-A^2} \quad (1.25)$$

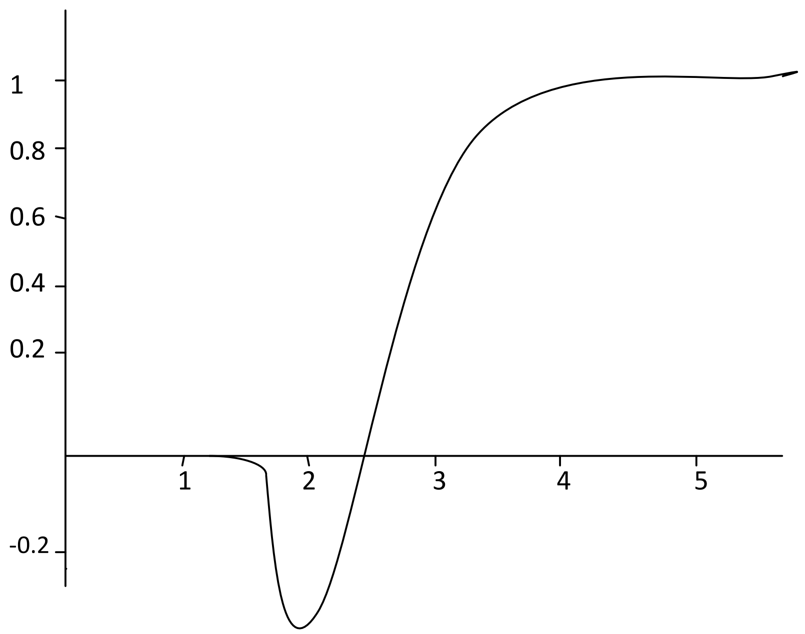

Figure 1.1 The function G'(x) for n = 60 and k = 2.2.

Using (1.25) we choose for x_0 the following point (numerical investigation show that this point is a good start value for solution of equation (1.24))

$$x_0 = \frac{(100+n)x_r + nx_a}{2n+100} \tag{1.26}$$

If we find x_M for which (1.24) holds, then we determine $M_n(k)$ from the formula.

$$M_n(k) = ¤(-x_M)¤(\frac{x_M - k}{A}) \tag{1.27}$$

and the derivative $M_n^{'}(k)$ from the formula[4]

$$M_n^{'}(k) = \frac{¤(-x_M)}{A^3\sqrt{2®}}.[\frac{1}{n} + \frac{kx_M}{2(n-1)}]\exp[-\frac{1}{2A^2}(x_M - k)^2] \tag{1.28}$$

Determination of the acceptance plans (n,k) for which (1.17) holds is in comparison with the solution of the equation $L(p_t; n, c) = 0.10$. From these plans we must choose the acceptance plan (n,k) minimizing $I_{ms} = nc_m + (N-n)\sim$, where

$$\sim = 1 - L(\overline{p}; n, k) = ¤(\frac{k - u_{1-\overline{p}}}{A})$$

(1.29)

is producer's risk (the probability of rejecting a lot of process average quality).

1.2 Numerical solution

Now solving the problem once more numerically.

For calculation of the AOQL plans by variables and attributes we shall use software mathematica.

Example. Let N=1000, $p_L = 0.0025$, $\overline{p} = 0.001$, $c_m = 1.8$ (the cost of inspection of one item by variables is higher by 80% than the cost of inspection of one item by attributes).

We shall look for the AOQL plan for inspection by variables and attributes. Furthermore we shall compare this plan and the corresponding Dodge-Romig AOQL plan for inspection by attributes.

According to (1.14), (1.24), (1.25) and (1.26) we have

```
In [1] :=  << statistics normal distribution

In [2] :=  ndist = Normal Distribution [0,1]

In [3] :=  cm = 1.8

In [4] :=  pL  = 0.0025

In [5] :=  pbar = 0.001

In [6] :=  nbig = 1000

In [7] :=  A[n_, k_ ] := sqrt[1/n + k^2 / (2n – 2)]

In [8] := G′ [ x_, n_, k_ ]:= CDF [ndist, (x – k) /A[n,k]] – CDF [ndist, - x] *

              Exp[- ((1 –A[n,k]^2)  x^2 – 2k x + k^2) / (2A [ n, k] ^ 2)] /A[n, k]

In [9]  := xr [n_, k_ ]  :=  k / (1 + A[n, k])

In [10]  := xa [n_, k_ ] := (k + A[n, k] * sqrt[k^2 – 2(1 – A[n,k] ^ 2) *

                    Log [A[n, k] ] ] ) / (1 – A[n,k] ^ 2)

In [11] := x0[n_, k] := ( (100 + n ) * xr [n, k] + n*xa [n,k] ) / (2n + 100)


In [12]  := FR[n_, k_ ] :=  FindRoot [G′ [x, n, k] = =  0,  {x, x0 [n, k] } ]

In  [13]  := xM[n_, k_] :=  x / FR[n, k]
```

Using Newton’s method with start point 0 = 1.6 and (2.29) we have

```
In  [14]  := c[n_, k] := -(CDF (ndist, -xM[n, k] – k) /A[n, k] ] – pL/(1 – n /nbig) ) /
```

```
                    (- CDF [ndist, -xM[n, k] ] * (1 / n + k xM [n, k] /(2n – 2) ) *
                     Exp[ -xM[n, k] – k) ^ 2 / (2A [n, k] ^ 2 / (2A[n, k] ^ 2) ] /
                     (A[n, k] ^ 3 * sqrt[2Pi] ) )

In  [15] := o = 1.6

In  [16]  := fRecAux [n_, i_ ] := fRecAux [n, i] = fRecAux [n, i – 1] +
                      c [n, fRecAux [n, i-1 ] ];  fRecAux [n_, 0] = o

In  [17]  := k [n_ ] := fRecAux [n, 7]

In  [18]  := a[n_ ] := CDF[ndist, (k[n] – Quantile[ndist, 1 – pbar] )
               /sqrt [1 / n + k[n] ^2 / (2n – 2) ] ]

In  [19]  := Ims [n_ ]  := n cm + (nbig – n) * a[n]

In  [20]  := plot [Ims [n], {n, 10, 100} ]
```

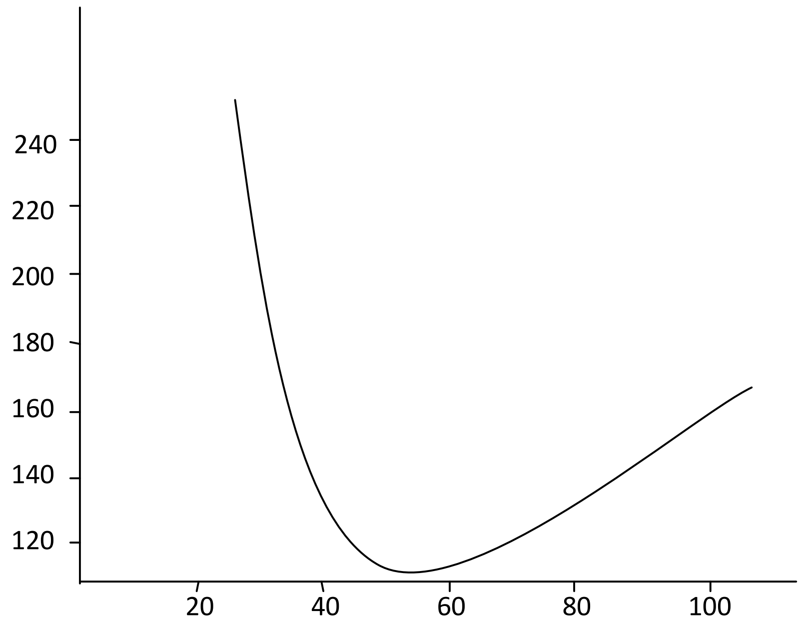

Figure 1.2 Sampling inspection plans from numerical point of view.

In [21] := Table [{n, k[n], Ims [n] }, {n, 40, 50, 1}]

In [22] := TableForm[%]

Out [22] / / TableForm = 40 2.56734 126.755

41	2.56613	125.879
42	2.56501	125.157
43	2.56397	124.579
44	2.56302	124.135
45	2.56214	123.815
46	2.56133	123.61
47	2.56058	123.515
48	2.55988	123.52
49	2.55923	123.621

50 2.55863 123.81

The AOQL plans for inspection by variables and attributes is (minimum of the function I_{ms} is I_{ms}= 123.515)

$$n_1 = 47, \qquad k = 2.56058.$$

The corresponding AOQL plan for inspection by attributes can be found in (2.2).

For given parameters N, p_L and $\overline{p}$ we have

$$n_2 = 130, \qquad c = 0.$$

For the comparison of these two plans from an economical point of view we use parameter e (1.30). The mathematica gives

```
In[23] := n1 = 47

In[24] := k = 2.56058

In[25] := Ims = 123.515

In[26] := n2 = 130

In[27] := c = 0

In[28] := e = 100*Ims / (nbig – n2) Sum[Binomial [nbig*pbar, i]*Binomial[nbig –
          nbig*pbar, n2 – i] / Binomial[nbig, n2], {i, 0, c} ] )

Out[28] := 50.8083
```

Since e = 50.8083%, using the AOQL plan for inspection by variables and attributes(47, 2.56058) there can be expected approximately 49% saving of the inspection cost in comparison with the corresponding Dodge-Romig plan (130,0).

Further we compare the operating characteristics of these plans(1.13).

```
In[29] := L1[p_ ] := CDF[ndist,  (N[Quantile[ndist,  1 – p],  16] – k) /
                    Sqrt [1/n1 + k ^ 2 / (2*n1 – 2) ] ]

In[30] := L2[p_ ] := Sum[Binomial [nbig*p, i]* Binomial[nbig – nbig*p, n2 – i] /
                    Binomial[nbig, n2], {I, 0, c} ]

In[31] := Table [ {p,  N[L1[p],   5],  N[L2[p],  5] },  {p,  0.001,  0.031,  0.002} ]

In[32] := TableForm[%]

Out[32] / / TableForm =
```

0.001	0.959165	0.87
0.003	0.730845	0.658207
0.005	0.51999	0.497674
0.007	0.36707	0.376067
0.009	0.260801	0.284003
0.011	0.187205	0.214346
0.013	0.135854	0.161675
0.015	0.0996376	0.121872
0.017	0.0738028	0.0918112
0.019	0.0551687	0.0691225
0.021	0.0415875	0.0520083
0.023	0.0315927	0.039107
0.025	0.0241711	0.0293876
0.027	0.0186145	0.0220699

0.029	0.0144223	0.0165638
0.031	0.0112372	0.0124235

For example, we get $L_1(\bar{p}) = L_1(0.001) = 0.959165,$ i.e., the producer's risk for the AOQL plan for inspection by variables and attributes is therefore approximately

$$\sim \; = 1 - L_1(\bar{p}) = 0.04.$$

The producer's risk for the corresponding Dodge-Romig plan is

$$\sim \; = 1 - L_2(\bar{p}) = 1 - 0.87 = 0.13.$$

Finally, the graphic comparison of the operating characteristics of these plans:

```
In[33] := oc1 = plot[L1[p], {p,  0,  0.03},  AspectRatio - > 1.3,
                    AxesLable - >  {“p”,  “L(p)”} ]

In[34] := oc2 = plot[L2[p],  {p,  0,  0.03},  AspectRatio - > 1.3,
                    AxesLabel - > {“p”, “L(p)” } ]

In[35] := show[oc1, oc2]
```

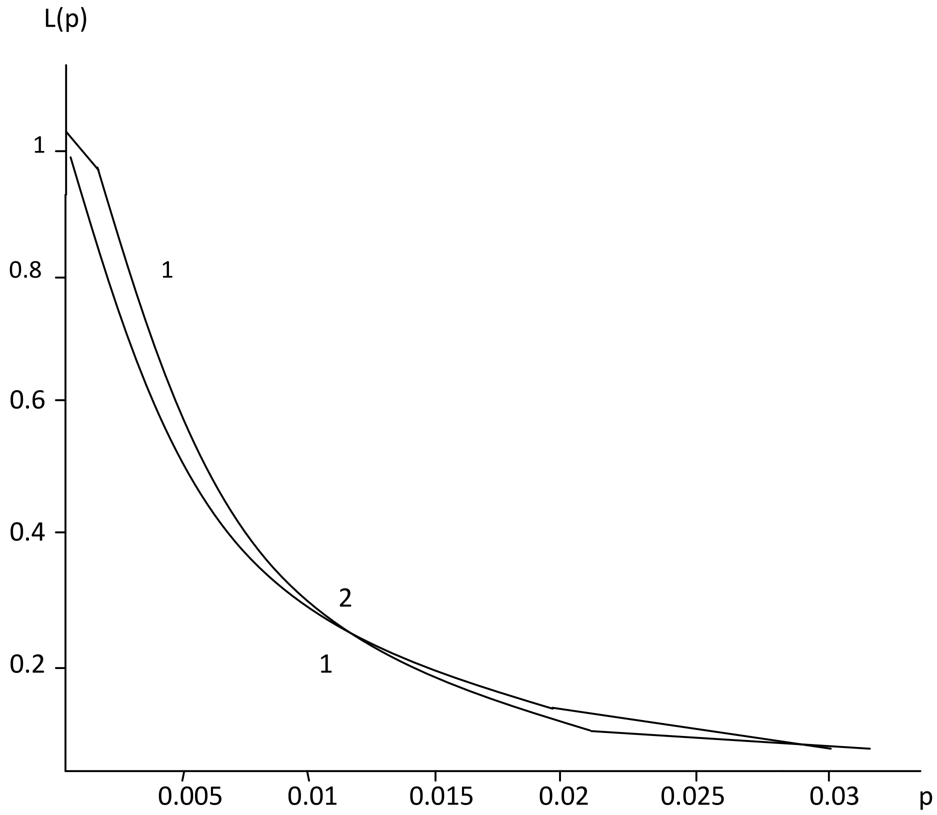

Figure 1.3 OC curves for the AOQL sampling plans

1 - for inspection by variables and attributes (47, 2.560)

2 – for inspection by attributes (130, 0)

From these results it follows that the AOQL plan for inspection by variables and attributes is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan (49% saving of the inspection cost). Furthermore, the OC curve for the AOQL plan by variables and attributes is better than corresponding OC curve for the AOQL plan by attributes(figure 1.2) (for example, the producer risk for the AOQL plan by variables and attributes α = 0.04 is less than that for the corresponding Dodge-Romig plan α = 0.13).

Chapter - II

CHAPTER II

ECONOMIC DESIGN OF DODGE-ROMIG AOQL SINGLE SAMPLING PLANS BY VARIABLES WITH THE QUADRATIC LOSS FUNCTION

In this chapter "Economic Design of Dodge-Romig AOQL Single sampling plans by variables with the quadratic loss function" by Chung-Ho-Chen[10] have been reviewed.

Acceptance sampling plan is field of statistical quality control. Tagaras[34] pointed out that in recent years more emphasis was placed on process control and off-line quality control methods, but acceptance sampling plan still remained important functions in many practical quality control systems. Sower [33] proposed an integrated model of statistical process control (SPC) and acceptance sampling plan. Acceptance sampling plan might be useful in an SPC environment under the following three situations:

1. When a supplier has not yet achieved statistical control of the process.
2. When a process may shift from an in-control state to an out-of-control state.
3. When a customer specifies that a standard acceptance sampling plan be used.

Although there exist situations in which screening inspection (i.e 100% inspection) are feasible, there are also many cases where sampling is inevitable, either because inspection is destructive, or because lot sizes are large and inspection is expensive, time-consuming and high error rate.

The classical Dodge-Romig [13] rectifying attributes sampling plans provide the lot tolerance percent defective (LTPD) on each lot or the average outgoing quality limit(AOQL) protection for the lots. Montgomery [31] pointed out that variables sampling plans usually involve smaller sample size than attributes sampling plans for the same levels of protection. Because of economic reasons, Klufa [25] presented the designs of Dodge-Romig LTPD and AOQL SSP by variables. The traditional concept of conformance of specification is that items meet the specification limits. Taguchi [35] refined the quality of product and presented the quadratic quality loss function to reducing total losses to the society.

In general, there is an optimal target value for every measurable quality characteristic. Any deviation from this target value incurs an economic loss, even if the value of the quality characteristic lies within the specification limits. The losses of quadratic function are expressed in monetary terms and are easy to understand and to apply in the evaluation of product or process improvement. The quadratic quality loss function has been succeeded in the application of SPC, sampling plans and specification limits design.

Tagaras [34] adopted the quadratic quality loss function for designing the parameters of acceptance sampling plan by variables. Kapur and Wang [20], Kapur [21] Kapur-cho [22] and Chen and Chou[6] addressed the problems of quality loss function applied in the economic design of specification limits. Kapur and Wang [20] pointed out that one of the short-term approaches to reduce variance of the units shipped to the customer is to put specification limits on the process and truncate the distribution by inspection. Chen [4], Chen and Chou[6] considered the problems of the integrated designs of Dodge-Romig AOQL SSP by attributes and specification limits and Dodge-Romig LTPD SSP by variables and specification limits, respectively. If turns out that not only the sampling plan parameters, but also the specification limits have to be looked upon as design parameters in order to minimize the expected total cost. The design of integrating Dodge-Romig AOQL SSP by variables and specification limit is presented. By solving the modified Kapur and Wang's model they have obtained the economic specification limits and also the optimal inspection policy of Dodge-Romig AOQL SSP by variables. Finally, they have compared the result with those of Chen[4] and Chen and Chou [5].

2.1 Mathematical model

The following are the assumption for formulating a mathematical programming model.

1. Quality characteristic y, follows a normal distribution with parameters mean « and variance ±^2.
2. A loss incurred when y deviates from the target value and this loss is governed by the quadratic loss function.
3. Process mean is centered at the target value m.

The expected quality loss per unit with quadratic loss function for 100% inspection of Kapur and Wang model is,

$$E[L(y)] = E[(y-m)^2]$$

$$= \int_{\text{«}-Z\text{±}}^{\text{«}+Z\text{±}} K(y-m)^2 \, f(y)\,dy$$

$$= \text{K} \int_{\text{«}-Z\text{±}}^{\text{«}+Z\text{±}} (y-m)^2 \, f(y)\,dy$$

$$= \text{K} \int_{\text{«}-Z\text{±}}^{\text{«}+Z\text{±}} \left(y^2 + m^2 - 2my\right) f(y)\,dy$$

$$= K\text{±}^2 \left[1 - \frac{2\text{¤}(Z)Z}{2\text{¤}(Z)-1}\right] \tag{2.1}$$

where

y is the quality characteristic;

Z is the Co-efficient of specification limit;

k is the Co-efficient of quality loss;

m is the target values;

f(y) is the probability density function of the truncated normal random variable.

$$\frac{1}{\left[2¤(Z)-1\right]± \sqrt{2®}}\, e^{-\frac{1}{2}\left[\frac{y-m}{±}\right]^2} \; ; « - Z± \le y \le « + Z±$$

$¤(Z)$ is the cumulative distribution function for the standard normal random variable with density function $¤(Z)$.

$« - Z±$ is the lower specification limit for screening product, $« + Z±$ is the upper specification limit for screening product.

$$¤(Z) = \frac{1}{\sqrt{2®}}\, e^{-\frac{Z^2}{2}} \quad ; \quad -\infty < Z < \infty \tag{2.2}$$

Assume that the process mean («) is equal to the target value m. From Kapur and Wang, we have the following two equations,

$$TC_o = k±^2 \tag{2.3}$$

$$TC_1 = k±^2 \left[1 - \left(\frac{2Z}{2¤(Z)-1}\right)¤(Z)\right] + 2\left(1 - ¤(Z)\right)C_r + C_i \tag{2.4}$$

where

TC_0 is the expected total cost per unit for no inspection;

TC_1 is the expected total cost per unit for 100% inspection;

C_r is the scrap per unit;

C_i is the inspection cost per unit;

Studying the behavior of TC_1 on varying Z shows that TC_1 has a unique minimum for $Z \in [0, \infty]$. Using direct search method one can obtain the optimum Z value which minimizes the expected total cost.

In the Dodge-Romig rectifying inspection plan, the rejection lots need for to the 100% inspection and the accepted lot only takes sampling inspection. The ATI denotes the average amount of inspection per lot. Hence, the ratio of the ATI and lot size (N) becomes the average fraction inspected per lot.

The following designations are used:

N – Lot size

P_L – The specified AOQL value

n– Sample size

t – Critical value.

The expected total cost for Dodge-Romig AOQL SSP by variables include the expected cost for 100% inspection $\left(\frac{ATI}{N} * TC_1\right)$ and the expected cost for no inspection $\left(1 - \frac{ATI}{N}\right) * TC_0$

Now the problem is to design the optimal Dodge-Romig AOQL SSP by variables with the minimum expected total cost per unit.

The non- linear mathematical programming model of this problem is as follows,

Minimize,

$$TC_S = \frac{ATI}{N} * TC_1 + \left[1 - \frac{ATI}{N}\right] * TC_0 \tag{2.5}$$

subject to,

$$\max_{0<p<1} \left(1 - \frac{n}{N}\right) .\ p\ .\ L(p; n, t) = p_L \tag{2.6}$$

where

TC_S is the expected total cost per unit for Dodge-Romig AOQL SSP by variables.

N is the lot size;

p_L is the specified AOQL value;

n is the sample size;

t is the critical value;

ATI is the average total inspection $0 < \frac{ATI}{N} < 1$

P is the fraction defective of the process

L(p; n, t) is the operating characteristic [probability of accepting a submitted lot with fraction defective p].

According to Klufa (1994),

$$ATI = N - (N - n)\, L(\bar{p}, n, t)$$

$$= N - (N - n) \left[1 - ¤ \left(\frac{t - u_{1-\bar{p}}}{A} \right) \right] \qquad (2.7)$$

where $\bar{p}$ is the process average fraction defective; $U_{1-\bar{p}}$ is a quintile of order (1 - $\bar{p}$) for a standard normal distribution function,

$$A = \sqrt{\frac{1}{n} + \frac{t^2}{2(n-1)}} \qquad (2.8)$$

For n[≥ 6] and $0 < \bar{p} < 0.5$ equation [2.7] is convex.

Substituting equation [2.7] into equation[2.5] and [2.6] can be written as, Minimize

$$TC_S = \frac{\left[N-(N-n)\left(1-\text{¤}\left(\frac{t-U_{1-\bar{p}}}{A}\right)\right)\right]}{N}TC_1 + \left[1-\frac{\left[N-(N-n)\left(1-\text{¤}\left(\frac{t-U_{1-\bar{p}}}{A}\right)\right)\right]}{N}\right]TC_0 \qquad (2.9)$$

$$= \frac{\left[N-(N-n)\left(1-\text{¤}\left(\frac{t-U_{1-2(1-\text{¤}(Z)}}{A}\right)\right)\right]}{N}TC_1 + \left[1-\frac{\left[N-(N-n)\left(1-\text{¤}\left(\frac{t-U_{1-2((1-\text{¤}(Z))}}{A}\right)\right)\right]}{N}\right]TC_0$$

subject to,

$$\max_{0<p<1}\left(1-\frac{n}{N}\right).\,p\,.L(p;n,t) = p_L \qquad (2.10)$$

where $\bar{p} = 2[1-\text{¤}(Z)]$ is the area under the standard normal distribution beyond LSL or USL. For given n, N, p_L and p equation (3.10) has the only one solution t, for any z ≥ 0, ∀ n either

$TC_1 \geq TC_0$ or $TC_1 < TC_0$ .now suppose that it is possible to find the absolute minimum of $\frac{ATI}{N}$ and that it corresponds to a pair of values $0 < z^* < \infty$, $0 < n^* \leq N$. Then there arise two situations,

1. Suppose that $TC_1 \geq TC_0$, then one can further reduce the expected total cost per unit by choosing $n^* = 0$ (0% inspection) and correspond to acceptance without control.

2. Suppose that $TC_1 < TC_0$, then one can further reduce the expected total cost per unit by choosing $n^* = N$ (100% inspection) and define the economic specification limits throughout the value z^* that minimizes TC_1 .

Hence, the optimum inspection policy is either acceptance without control or 100% inspection. The 100% inspection arises when the minimum of TC_1 is less than TC_0 . The type of the chosen acceptance sampling procedure becomes irrelevant.

Example 2.1

Assume that k = 5, « = m = 10, $\pm^2 = 0.25$, $C_r = 2$ and $C_i = 0.1$

Let N = 2000, $p_L = 0.01$

One can find Dodge-Romig [9] AOQL SSP by variables which minimize the expected cost per unit.

By solving (2.4), one can obtain the $z^* = 1.3269$ which minimizes TC_1.

The optimal inspection policy is to do 100% inspection, because the minimum of TC_1.

$$TC_1 = k\pm^2 \left[1 - \left(\frac{2z}{2¤(z) - 1} \right) ¤(z) \right] + 2\left(1 - ¤(z)\right) C_r + C_i$$

$$= 1.0501$$

$$TC_0 = k\pm^2$$

$$= 1.25$$

$\therefore TC_1 < TC_0$

Set LSL = 9.335 and USL = 10.665 to screen the product and skip the product between the specification limits to the customers.

From Chen and Chou's [5] model one obtain the 100% inspection policy for the above numerical example. Solution is the same as that of Chen and Chou's [5] Dodge-Romig LTPD Single Sampling Plan by variables. Both of them adopt the 100% inspection for screening the product. From Chen's [2] model, the optimum solution of Dodge-Romig [9] AOQL SSP by attributes is the acceptance number $c^* = 0$, the sample size $n^* = 36$ and the co-efficient of specification limits $z^* = 1.3269$. Its inspection policy is different from ours for the above numerical example.

Example : 2.2

Let k= 5

$« = m = 10$,

$\pm^2 = 0.25$,

$C_r = 2$ and $C_i = 0.35$

By solving equation (3.4),

Minimum

$$TC_1 = k\pm^2 \left[1 - \left(\frac{2z}{2¤(z) - 1}\right)¤(z)\right] + 2\left(1 - ¤(z)\right)C_r + C_i$$

= 1.30 and

$TC_0 = k\pm^2$

= 1.25

Because the minimum of TC_1 is less than TC_0, the optimum is to adopt 0% inspection [acceptance without control]. According to Kapur and Wang[16], if we have specification limits and a process is under control but not capable of meeting specifications then inspection in an on-line quality control system may be a short term approach to reduce the variance of the units shipped to the customer. Kapur and Wang[16] have addressed the cost model which minimizes the loss of customer.

This study is an extension of Kapur and Wang's[16] work and the proposed model is a generalization of Kapur and Wang's[16] one. The optimum inspection policy is either acceptance without control or 100% inspection. The 100% inspection arises when the minimum of the expected total cost per unit 0% inspection. The type of the chosen acceptance sampling procedure becomes irrelevant. Further study will extend to an integrated model of Bayesian SSP for variables and section limits.

Chapter - III

CHAPTER III

ECONOMIC SELECTION OF THE DODGE-ROMIG AOQL SAMPLING PLAN UNDER THE QUALITY INVESTMENT AND INSPECTION ERROR.

In this chapter " Economic selection of the Dodge-Romig AOQL sampling plan under the quality investment and inspection error" by Chung-Ho Chen[10] have been reviewed.

Sampling inspection plan is one of the fields for statistical quality control. It can be applied in procurement, production and shipment of product when the product quality is not stable. The lot by lot attribute single sampling plans (SSP) are easy to adopt for evaluating the quality of lot. In 1959, Dodge-Romig[13] provided the rectifying SSP and double sampling plans for attributes with the protection of lot tolerance percent defective (LTPD) or average outgoing quality limit(AOQL). For attribute rectifying sampling plans with AOQL protection, they are derived to provide assurance that the long-run average lots will be more worse than the indexed AOQL value. Klufa[24] further presented the modified Dodge-Romig's model based on variable single sampling inspection plan. The classical Dodge-Romig[13] AOQL SSP are based on the following assumptions:

1. The manufacturing process is normally in binomial control with a process average fraction defective equal to $\bar{p}$.
2. Inspection is rectifying and rejected lots are totally inspected.
3. To make sure that the average quality of his product is satisfactory, the producer chooses an AOQL value and considers only sampling plans satisfying this specification.
4. Among plans having the specified AOQL , the producer chooses the one minimizing average total inspection(ATI) for product of process average fraction defective.
5. Inspection is perfect without error.

One assumption of the classical Dodge-Romig [13] AOQL single sampling plan is perfect inspection without error. However, the inspection error usually occurs in industrial or medical application.

If we just use the product inspection for providing the quality assurance, then it is a short term method. For modern industrial statistics, we usually address the preventive method for quality improvement and adopt the process control and quality design for improving product quality and satisfying the need of the customer. The on-line 100% rectifying inspection can be used as a short term method for controlling the product quality shipped to the customer. However, quality investment is an available method for improving the process parameters in the long-term. For example, one can buy a new machine for manufacturing the product and address the continuous education training for personnel. Hong et al., Ganeshan et al., and Chen and Tsou [9] have presented the exponential reduction of process mean and standard deviation as the function of quality investment. Abdul-kader et al [2] further adopted Chen and Tsou's [9] quality investment function for determining the optimum quality investment and corresponding improved process mean and standard deviation. Chen[4] further extended Chen and Tsou's [9] method for designing the sampling inspection plan under the economic selection.

In this chapter, the modified Dodge-Romig [13] AOQL single sampling plan under the quality investment and inspection error is proposed by the author. It is an extension of Chen's work. The objective of this research is to integrate the process improvement and sampling inspection for obtaining the minimum total cost of product when the inspection error occurs. The motivation behind this work stems from the fact that neglecting the effect of inspection error should under estimate the expected total cost of product. The effect of inspection error on the modified Dodge-Romig [13] model is discussed in this chapter. The solution procedure for obtaining the optimal parameters of Dodge-Romig AOQL single sampling plan and quality investment level are presented. Finally the comparison of solution between the modified model with/without inspection error is given for illustration.

3.1 Dodge-Romig AOQL SSP with inspection error

Consider the inspection error exists for Dodge-Romig[13] AOQL single sampling plan.

The determination of the parameters (c,n) with maximum AOQ_e function.

From Beaing and Case[3], we have

$$AOQ_e = \frac{npe_2 + p(N-n)(1-p_e)p_{ae}}{N(1-p_e)} + \frac{p(N-n)(1-p_{ae})e_2}{N(1-p_e)}) \tag{3.1}$$

$$P_{ae} = \sum_{x=0}^{c} \frac{e^{-np_e}(np_e)^x}{x!} \tag{3.2}$$

Let A = $npe_2 + p(N-n)(1-p_e)p_{ae}$ + $p(N-n)(1-p_{ae})e_2$. Differentiating Equation (3.1) with respect to p and equating the result to zero, we obtain

$$\{N(1-p_e)[ne_2 + (N-n)(1-p_e)p_{ae} - p(N-n)p_{ae}(1-e_1-e_2) + p(N-n)(1-p_e)\frac{dp_{ae}}{dp} + (N-n)(1-p_{ae})e_2 - p(N-n)e_2\frac{dp_{ae}}{dp}] + AN\frac{dp_e}{dp}\}/[N^2(1-p_e)^2] = 0 \tag{3.3}$$

where $\frac{dp_e}{dp} = 1 - e_1 - e_2$ and $\frac{dp_{ae}}{dp} = -\frac{e^{-np_e}(np_e)^c}{c!}n(1-e_1-e_2)$.

equation(3.3) can be rewritten as,

$$N(1-p_e)[ne_2 + (N-n)(1-p_e)p_{ae} - p(N-n)p_{ae}(1-e_1-e_2) - p(N-n)(1-p_e)\frac{e^{-np_e}(np_e)^c}{c!}. n(1-e_1-e_2) + (N-n)(1-p_{ae})e_2 + p(N-n)e_2\frac{e^{-np_e}(np_e)^c}{c!}n(1-e_1-e_2)] + AN(1-e_1-e_2) = 0 \tag{3.4}$$

Let p = p_1 is the incoming fraction defective when AOQ_e reaches a maximum value p_L . That is $\max_{0 \le p \le 1} AOQ_e = \frac{A}{N(1-p_e)} = p_L$. Hence we have

$$A = p_L N (1 - p_e) \tag{3.5}$$

Substituting equation (3.5) into equation (3.4), we obtain

$$\begin{aligned} &ne_2 + (N-n)(1-p_e)p_{ae} - p(N-n).\, p_{ae}(1-e_1-e_2) - p(N-n)(1-p_e). \\ &c\frac{e^{-np_e}(np_e)^c}{c!} n(1-e_1-e_2) + (N-n)(1-p_{ae})e_2 + p(N-n)e_2 \frac{e^{-np_e}(np_e)^c}{c!} n(1-e_1-e_2)] + \\ &Np_L(1-e_1-e_2) = 0 \end{aligned} \tag{3.6}$$

Equation (3.6) can be rewritten as

$$\begin{aligned} AOQ_e &= ne_2 + (N-n)(1-p_e)p_{ae} - (N-n)p_{ae}(p_e-e_1) + (N-n)(1-e_1-e_2). \\ &(e_2 - 1 + p_e)\frac{e^{-np_e}(np_e)^{c+1}}{c!} + (N-n)(1-p_{ae})e_2 + Np_L(1-e_1-e_2) = 0 \end{aligned} \tag{3.7}$$

Assume that the maximum AOQ_e function occurs when $p = p_1 = \frac{x}{n}$, where $n = \frac{yN}{p_L N + y}$.

Hence, we have $p_1 = \frac{x(p_L N + y)}{yN}$.

According to Beaing and Case[3], we have the modified Dodge-Romig[13] AOQL single sampling plan as follows:

Minimize

$$ATI_e = \frac{n + (N-n)(1-P_{ae})}{1-p_e} \tag{3.8}$$

Subject to

$$\max_{0 \le p \le 1} AOQ_e = p_L \tag{3.9}$$

where

$$P'_{ae} = \sum_{x=0}^{c} \frac{e^{-n\overline{p}_e} (n\overline{p_e})^x}{x!} \tag{3.10}$$

$$P_{ae} = \sum_{x=0}^{c} \frac{e^{-np_e} (np_e)^x}{x!} \tag{3.11}$$

$$\overline{p_e} = \overline{p}\,(1-e_2) + (1-\overline{p})e_1 \tag{3.12}$$

P_{ae} is the acceptance probability of inspection lot with average fraction defective $\overline{p_e}$;

$$AOQ_e = \frac{npe_2 + p(N-n)(1-p_e)P_{ae} + p(N-n)(1-P_{ae})e_2}{N(1-p_e)} ;$$

P is the true fraction defective; p_e is the apparent fraction defective, $p_e = p(1-e_2) + (1-p)e_1$; e_1 is the probability that a good item classified as a defective; e_2 is the probability that a defective item classified as good; $\overline{p}$ is the average fraction defective; n is the sample size; X is the non- conforming items found in the sample size n; c is the acceptance number; p_L is the specified AOQL value.

According to Case et al.[3], in general , the AOQ function with the inspection error, AOQ $_{e,}$ will not be unimodal. The accepted definition of the AOQL $_e$ is used as the first mode of the AOQ $_e$ function, even though higher AOQ $_e$ values may be realized as the process fraction defective increases. Some combinations of parameters (c, n) that satisfies equation (3.9) are given. The unique combination of parameters (c^*, n^*) can minimize the objective function ATI $_e$ is the optimal solution.

3.2 Modified Dodge-Romig AOQL SSP with quality investment and inspection error

Similarly to Chen[4], we have the following modified Dodge-Romig (1959) AOQL SSP with quality investment and inspection error as follows:

Minimize

$$TC_f = ATI_e . TC_1 + I \tag{3.13}$$

Subject to

$$\max_{0 \le p \le 1} AOQ_e = p_L \tag{3.14}$$

where

$$ATI_e = \frac{\left[n + (N-n)(1-P_{ae}) \right]}{(1-p_e)} \tag{3.15}$$

$$P_{ae} = \sum_{x=0}^{c} \frac{e^{-n p_e} (n p_e)^x}{x!} \tag{3.16}$$

$$p_e = (1-p)e_1 + p(1-e_2) \tag{3.17}$$

$$p = 1 - \int_{LSL}^{USL} f(y,I)\,dy = 1 - \left[\text{¤}\left(\frac{USL - \text{«}_1}{\pm_1} \right) - \text{¤}\left(\frac{LSL - \text{«}_1}{\pm_1} \right) \right] \tag{3.18}$$

$$TC_1 = \int_{LSL}^{USL} k(y-y_0)^2 f(y,I)dy + (1-p).C_r + C_i$$

(3.19)

$$\int_{LSL}^{USL} k(y-y_0)^2 f(y,I)dy = k\left\{ \left[\left(\text{«}_1 - y_0 \right)^2 + \pm_1^2 \right] . \left[\text{¤}\left(\frac{USL - \text{«}_1}{\pm_1} \right) - \text{¤}\left(\frac{LSL - \text{«}_1}{\pm_1} \right) \right] + \right.$$

$$\left. \pm_1 \left[\left(\text{«}_1 - 2y_0 + LSL \right) \text{¤}\left(\frac{LSL - \text{«}_1}{\pm_1} \right) - \left(\text{«}_1 - 2y_o + USL \right) \text{¤}\left(\frac{LSL - \text{«}_1}{\pm_1} \right) \right] \right\} \tag{3.20}$$

$$f(y,I)=\frac{1}{\sqrt{2®}\,±_1}\exp\left[-\frac{1}{2}\left(\frac{y-«_1}{±_1}\right)^2\right] \tag{3.21}$$

$$«_1^2=«_T^2+(«_0^2-«_T^2)\exp(-\ \ I) \tag{3.22}$$

$$±_1^2=±_T^2+(±_0^2-±_T^2)\exp(-\sim I) \tag{3.23}$$

$\sim$ is the exponential reduction coefficient of process standard deviation for the function of quality investment, $\sim > 0$; is the exponential reduction coefficient of process mean for the function of quality investment, > 0; ¤ (.) is the cumulative distribution function of standard normal random variable; ¤ (.) is the probability density function of standard normal random variable; k is the quality loss coefficient; Y, is normally distributed with known process mean $«_o$ and process standard deviation $±_0$; y_0 is the target value of product; $«_T$ is the target value of process mean; $±_T$ is the target value of process standard deviation; $«_1$ is the improved process mean; $±_1$ is the improved process standard deviation; LSL is the lower specification limit of product; USL is the upper specification limit of product; C_r is the replacement cost per unit for an non-conformance product; C_i is the inspection cost per unit; I is the quality improvement.

For equation (3.13), TC_1 denotes the expected product cost per unit which includes the expected quality loss within specification limits, the expected replacement cost per unit out of specification limits, and the unit inspection cost. TC_f denotes the expected total cost of product which includes the quality investment cost and the expected product cost per inspection lot.

The solution procedure of equations (3.13) to (3.14) is as follows:

Step 1. From Appendix, find the possible combination of parameters (c, n) satisfying equation (3.14).

Step 2. For a given c and its corresponding n from Step1, one can adopt the direct search method for obtaining the optimal quality investment level satisfying equation (3.13).

Step 3. Let c = c + 1. Repeat step 2 until obtaining the optimal parameters (c^*, n^*, «$_1^*$, ±$_1^*$, I^*) with minimum TC_f .

Example 3.1

Assume that the declining exponential function of the quality investment can be used for describing the improved process mean and standard deviation. By regression analysis of the historical production data, the exponential reduction coefficients are ~ = 0.01 and = 0.05. consider the normal quality characteristic with known process mean «$_0$ = 10 and standard deviation ±$_0$ = 0.5. The lower specification limit of

product is LSL = 9.24, the upper specification limit of product is USL = 10.56, the target value of product y_0 = 9.9, and the quality loss coefficient k = 5. The replacement cost for non-conformance product is C_r = 2 and the inspection cost per unit is C_i = 0.1. The product lot size N = 2000. The average outgoing quality limit AOQL = 1%. The probability for classifying a good unit as a defective is e_1 = 0.01 and the probability for classifying a defective unit as a good one is e_2 = 0.02. The long term improved process mean is «$_T$ = 9.9 and improved process standard deviation is ±$_T$ = 0. We would like to adopt Dodge-Romig [13] AOQL SSP for controlling the output quality of product by integrating the concept of quality investment.

By the aforementioned solution procedure, the solution for modified Dodge-Romig[13] AOQL SSP with quality investment and inspection error is (c^*, n^*, «$_1^*$, ±$_1^*$, I^*) = (4.146, 9.9, 0.212, 171.73) with TC_f = 238.31. the solution for Chen(2011) is (c^*, n^*, «$_1^*$, ±$_1^*$, I^*) = (1, 81, 9.9, 0.214, 169.60) with TC_f = 204.04.

The modified Dodge- Romig [13] AOQL SSP under the quality investment and inspection error considers an integrated optimum problem for the process improvement and inspection plan. From the aforementioned numerical results, one has the conclusions that

(1) The exponential reduction coefficient of process standard deviation for quality improvement has a major effect on the quality investment level and the expected total cost of product;

(2) The modified model with quality investment and inspection error needs more acceptance number, sample size, quality investment level, and expected total cost of product than those of one without inspection error.

Chapter - IV

CHAPTER IV

ECONOMIC EFFICIENCY OF THE AOQL PLANS BY VARIABLES WHEN THE REMAINDER OF REJECTED LOTS IS INSPECTED.

In this chapter "Economic efficiency of the AOQL plans by variables when the remainder of rejected lot is inspected" by JindrichKlufa[24] have been reviewed .

AOQL single sampling plans when the remainder of rejected lot is inspected consists of two types of AOQL plans are considered – for inspected by variables and for inspection by variables and attributes. These plans are compared with corresponding Dodge-Romig AOQL plans for inspection by attributes from economical point of view.

Under the assumption that each inspected item is classified as either good or defective (inspection by attributes (Hald [18]), Dodge and Romig[13] consider sampling plans which minimize the mean number of items inspected per lot of process average quality.

$$I_s = N - (N-n).L(\overline{p};n,c) \tag{4.1}$$

Under the condition

$$\max_{0<p<1} AOQ(p) = p_L \tag{4.2}$$

(AOQL single sampling plans), where N is the number of items in the lot (the given parameter), $\overline{p}$ is the process average fraction defective (the given parameter), p_L is the average outgoing qualiy limit (the given parameter denoted by AOQL), n is the number of items in the sample (n<N), c is the acceptance number (the lot is rejected when the number of defective items in the sample is greater than c), L(p) is operating characteristic (the probability of accepting a submitted lot with fraction defective p), AOQ (p) is average outgoing quality (the mean fraction defective after inspection when the fraction defective before inspection was p). condition (4.2) protects the consumer against the acceptance of

bad lot, average outgoing quality is less or equal to p_L (the chosen value) for each fraction defective p before inspection. The AOQL plans for inspection by attributes are extensively tabulated (Dodge and Romig [13]).

4.1 AOQL plans by variables and attributes

The problem to find AOQL plans for inspection by variables has been solved under following assumptions:

Measurements of a single quality characteristic X are independent, identically distributed normal random variables with unknown parameters « and $\pm^{2}$. For the quality characteristic X is given either an upper specification limit U (the item is defective if its measurement exceeds U), or a lower specification limit L (the item is defective if its measurement is smaller than L). It is further assumed that the unknown parameter ± is estimated from the sample standard deviation s.

The inspection procedure is as follows: Draw a random sample of n items and compute $\bar{x}$ and s. Accept the lot if

$$\frac{U-\bar{x}}{s} \geq k \text{, or} \frac{\bar{x}-L}{s} \geq k \qquad (4.3)$$

Now to determine the sample size n and critical value K.

The problem is formulated as follows. Let us consider AOQL plans inspection by variables and attributes – all items from the sample are inspected by variables, but the remainder of rejected lots is inspected by attributes. Let us denote,

c_s^* - the cost of inspection of one item by attribute.

c_m^* - the cost of inspection of one item by variables.

Inspection cost per lot, assuming that the remainder of rejected lots is inspected by attributes (the inspection by variables and attributes), is $n.c_m^*$ with the probability $L(p;n,k),$ and $n.c_m^* + (N-n)\,.\,c_s^*$ with probability $1-L(p;n,k)$. The mean inspection cost per lot of process average quality is therefore

$$c_{ms} = n \cdot c_m^* + (N - n) \cdot c_s^* [1 - L(\overline{p}; n, k)] \tag{4.4}$$

The acceptance plan (n,k) minimizing the mean inspection cost per lot of process average quality c_{ms} under the condition (4.2). The condition (4.2) is same one as used for protection the consumer Dodge and Romig .

Let us introduce the function

$$I_{ms} = n \cdot c_m + (N - n)[1 - L(\overline{p}; n, k)] \tag{4.5}$$

where

$$c_m = \frac{c_m^*}{c_s^*} \tag{4.6}$$

since

$$c_{ms} = I_{ms} \cdot c_s^* \tag{4.7}$$

Both functions C_{ms} and I_{ms} have a minimum for the same acceptance plan (n,k). Therefore , the acceptance plan (n,k) minimizing (4.5) instead of (4.4) under the condition (4.2). For these AOQL plans for inspection by variables and attributes the new parameter c_m. The parameter must be estimated in each real situation. Usually it is

$$c_m > 1 \tag{4.8}$$

substituting formally $c_m = 1$ into (4.5) (I_{ms} in this case is denoted by I_m) we obtain

$$I_m = N - (N - n) \cdot L(\overline{p}; n, k), \tag{4.9}$$

i.e. the mean number of items inspected per lot of process average quality, assuming that both the sample and the remainder of rejected lots is inspected by variables. Consequently the AOQL plans for inspection by variables are a special case of the AOQL plans by variables and attributes for $c_m = 1$. From (4.9) is evident that for the determination AOQL plans variables it is not necessary to estimate c_m ($c_m = 1$ is not real value of this parameter).

4.1 Economic efficiency of the AOQL plans by variables and attributes

For the comparison of the AOQL single sampling plans for inspection by variables and attributes with the corresponding Dodge – Romig AOQL plans for inspection by attributes from economical point of view we use parameter e defined by relation

$$e = \frac{I_{ms}}{I_s} \cdot 100 \tag{4.10}$$

Using the equation (4.7) is

$$e = \frac{I_{ms}}{I_s} \cdot 100 = \frac{I_{ms} \cdot c_s^*}{I_s \cdot c_s^*} \cdot 100 = \frac{c_{ms}}{c_s} \cdot 100 ,$$

where $c_s = I_s c_s^*$ is the mean cost of inspection by attributes (c_s^* is the cost of the inspection of one item by attributes). Therefore the AOQL plan for inspection by variables and attributes is more economical than the corresponding Dodge – Romig[13] plan when

$$e < 100$$

The expression (100 – e) represents the percentage of savings in inspection cost when sampling plan for inspection by variables and attributes is used instead of the corresponding plan for inspection by attributes.

Economic efficiency measured by parameter e is a function of four variables $p_L, N, \overline{p}, c_m$,

$$e = e(p_L , N , \overline{p} , c_m). \tag{4.11}$$

Some values of this function are presented in Table 4.1 and Table 4.2.

Table 4.1 Values of the parameter e for p_L= 0,001

P_L = 0001	c_m = 2			c_m = 3			c_m = 4			c_m = 5		
$\frac{\bar{p}}{N}$	500	4000	50000	500	4000	50000	500	4000	50000	500	4000	50000
0.000100	34	26	19	46	36	26	57	45	33	67	54	40
0.000200	42	29	19	56	39	27	68	49	34	79	58	41
0.000300	48	31	22	63	42	31	76	51	39	87	60	47
0.000400	53	33	24	69	44	33	82	54	41	94	63	49
0.000500	58	38	27	74	50	36	88	60	45	99	70	53
0.000600	62	43	29	79	55	39	92	66	48	104	76	56
0.000700	66	48	33	83	61	43	96	72	53	108	81	61
0.000800	70	53	38	87	66	49	100	77	58	111	87	66
0.000900	74	58	45	90	72	57	103	82	66	114	91	74
0.001000	77	64	56	93	77	67	106	87	76	117	96	83

Table 4.2: Values of the parameter e for p_L= 0,0025

p_L= 0,0025	c_m= 2			c_m= 3			c_m= 4			c_m= 5	
$\frac{\bar{p}}{N}$	500	4000	50000	500	4000	50000	500	4000	50000	4000	50000
0,000250	46	34	28	63	47	40	78	59	51	71	61
0,000500	54	32	22	73	44	31	89	55	39	66	47
0,000750	60	43	31	79	58	43	96	71	54	84	65
0,001000	65	47	28	84	62	39	101	76	48	89	57
0,001250	69	50	38	88	66	51	105	80	64	92	75
0,001500	72	53	39	92	69	52	108	82	64	94	75
0,001750	76	56	42	95	72	56	111	85	68	96	79
0,002000	79	60	45	98	75	58	114	88	69	99	79
0,002250	82	68	56	101	84	70	116	97	82	108	93
0,0002500	85	74	67	103	89	80	117	101	90	111	99

From the results of numerical investigations it follows that under the same protection of consumer the AOQL plans for inspection by variables are in many situations **more economical (** saving of the inspection cost is 70% in any cases) than the corresponding Dodge –Romig[13] attribute sampling plans.

Example: 4.1 when p_L= 0,001, N = 4000, $\overline{p}$ = 0,0004 and c_m= 3 is parameter e = 44 (in table 4.1), using the AOQL plan for inspection by variables and attributes it can be expected approximately 56% saving of the inspection cost in comparison with the corresponding Dodge – Romig[13] plan.

The dependence of economic efficiency measured by parameter e on the lot size N can be studied. Let p_L, $\overline{p}, c_m$ be given parameters. Function (4.11) for given p_L, $\overline{p}, c_m$ is a function of one variable N, that is

$$e = e_{pL,\overline{p},c_m}(N) \qquad (4.12)$$

From the results of the numerical investigation it follows (table 4.1 and table 4.2) that function (4.12) has decreasing trend in N, which means that when lot size N increases, then saving of the inspection cost (100 – e) increases (using the AOQL plan for inspection by variables and attributes instead of the corresponding plan for inspection by attributes).

In the second step we shall study dependence of the economic efficiency measured by parameter e on the process average fraction defective $\overline{p}$. Let p_L, N, c_m be given parameters. Function (4.11) for given p_L, N, c_m is a function of one variable $\overline{p}$, that is

$$e = e_{pL,N,c_m}(\overline{p}) \qquad (4.13)$$

From the results of numerical investigations it follows that (see also table 4.1 and table 4.2) that function (4.13) has increasing trend in $\overline{p}$,which means that the process average fraction defective $\overline{p}$ increases, then saving of the inspection cost

(100-e) decreases(using the AOQL plan for inspection by variables and attributes instead of the corresponding plan for inspection by attributes).

Finally, dependence of economic efficiency measured by parameter e on fraction of the cost of inspection of one item by variables to cost of inspection of one item by attributes c_m. Let $p_L , N, \overline{p}$ be given parameter. Function (4.11) for given $p_L , N , \overline{p}$ is a function of one variables c_m. that is

$$e = e_{pL,N,\overline{p}}(c_m) \tag{4.14}$$

From the result of numerical investigation it follows that function (4.14) has increasing trend in c_m, which means that when the fraction of the cost of inspection of one item by variables to the cost of inspection of one item by attributes c_m increases, then saving of the inspection cost (100 – e) decreases (using the AOQL plan for inspection by variables and attributes instead of the corresponding plan for inspection by attributes).

From the results of numerical investigation it follows that under the same protection of consumer the AOQL plans for inspection by variables and attributes are in many situations more economical than the corresponding Dodge-Romig AOQL attribute sampling plans. For the chosen value of average outgoing quality limit p_L this conclusion is valid especially when

1. the number of items in the lot N is large,
2. the process average fraction defective $\overline{p}$
3. the cost of inspection one item by variables is not much greater than the cost of inspection one item by attributes, i.e. c_m is not large.
4.

Similar conclusion were obtained also for the comparison of the AOQL plans for inspection by variables (special case of the AOQL plans for inspection by variables and attributes) with the Dodge-Romig AOQL plans, but saving of the inspection cost is here less than for the AOQL plans for inspection by variables and attributes. It can be proved that under assumption $c_m > 1$ the AOQL plans for inspection by variables and attributes are always more economical than the corresponding AOQL plans for inspection by variables.

Summary and Conclusion

SUMMARY AND CONCLUSION

This thesis is devoted to the study of designing AOQL single sampling plans.

First chapter deals with the design of acceptance sampling plans when the remainder of rejected lots is inspected. Two types of AOQL plans are considered- for inspection by variables and for inspection by variables and attributes. These plans are compared with corresponding Dodge-Romig AOQL plans for inspection by attributes. From these results it follows that the AOQL plan for inspection by variables and attributes is more economical than the corresponding Dodge-Romig AOQL attribute sampling plan. Furthermore the OC curve for the AOQL plan by variables and attributes is better than corresponding OC curve for the AOQL plan by attributes.

Second chapter deals with the design of integrating Dodge-Romig average outgoing quality limit [AOQL] single sampling plans [SSP] by variables and specification limits. By solving the modified Kapur and Wang's model economic specification limits and the optimal inspection policy of Dodge-Romig AOQL Single Sampling Plan by variables were obtained. This study is an extension of Kapur and Wang's work and the proposed model is a generalization of Kapur and Wang's one. The optimum inspection policy is either acceptance without control 100 percent inspection. The 100 percent inspection arises when the minimum of the expected total cost per unit for 100% inspection is less than the expected total cost per unit for 0% inspection. The type of chosen acceptance sampling procedure becomes irrelevant.

Third chapter deals with Dodge-Romig AOQL sampling plan under the quality investment and inspection error. In this chapter, the modified Dodge-Romig AOQL SSP under the quality investment and inspection error has been proposed. It considers an integrated optimum problem for the process improvement and inspection plan. From the aforementioned numerical results, one can see that (1) the exponential reduction coefficient of process standard deviation for quality improvement has a major effect on the quality investment level and the expected total cost of product; (2) the modified model with quality investment and inspection error needs more acceptance number, sample size, quality investment level, and expected total cost of product than those of one without inspection error. The managerial implications of this work are that the joint optimization of product inspection and quality investment can improve the product quality/service quality shipped to the customer and decrease the expected total cost of product for the manufacturer.

Fourth chapter deals with the AOQL plans by variables when the remainder of rejected lots is inspected. From the results of numerical investigation it follows that under the same protection of consumer the AOQL plans for inspection by variables and attributes are in many situations more economical than the corresponding Dodge-Romig AOQL attribute sampling plans. For the chosen value of average outgoing quality limit p_L this conclusion is valid especially when (1) the number of items in the lot N is large, (2) the process average fraction defective $\bar{p}$ is small, (3) the cost of inspection one item by variables is not much greater than the cost of inspection one item by attributes, i.e. c_m is not large. Similar conclusions were obtained also for the comparison of the AOQL plans for inspection by variables with Dodge-Romig AOQL plans, but saving of the inspection cost is here less than for the AOQL plans for inspection by variables and attributes. It can be proved that under assumption $c_m > 1$ the AOQL plans for inspection by variables and attributes are always more economical than the corresponding AOQL plans for inspection by variables.

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