INTRODUCTION

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Topology is the mathematical study of shapes and spaces. It is an area in mathematics concerned with the properties of spaces that are preserved under continuous deformations. General topology establishes the foundational aspects of topology and investigates the properties and the concepts inherent to topological spaces. Later the generalization of closed sets was introduced by Levine[33]. Since then they are the research topics of several researchers worldwide. As a generalization of open sets, the notion of b-open sets was introduced by Andrijervic [10]. This type of sets was discussed by Ekici and Caldas[26] under the name of γ-open sets. The class of b-open sets is contained in the class of semipre-open sets and contains the class of semi-open sets and the class of pre-open sets. Moreover, it generates the same topology as the class of pre-open sets. Since the advent of these notions, several research papers with interesting results in different aspects came into existence.

The present study focusses on the following concepts:

1) On generalized b-closed sets in topological spaces [3]
2) On generalized b-closed sets and their relationships [2]
3) On πgb closed-sets in Topological spaces [49]
4) On πgb-closed sets and related topics [48]
5) On πgb*-closed sets in topological spaces [18]
6) On πgb*-continuous functions in topological spaces [19]
7) πgb*-compact spaces and πgb*-neighborhoods.

In chapter I the idea cherished by Al-Omari and Noorani on generalized b-closed sets (briefly, gb-closed sets) were discussed. The class of gb-closed sets is finer than that of closed, α-closed, pre-closed, b-closed, g-closed, αg-closed, gp-closed, b-closed sets and coarser than that of gsp-closed sets. Some of the interesting characterizations of these sets were discussed. A necessary and sufficient condition for a gb-closed set to be a b-closed set was studied. Since every open set is b-open, it is always true that b-d(A) $\subset$ d(A) for any subset A of a topological space X (where d(A) denotes the derived set of A and b-d(A) denotes the b-derived set of A). Moreover, if d(A) = b-d(A), then cl(A) = bcl(A). Also the union of gb-closed sets A and B is gb-closed provided d(A) $\subset$ b-d(A) and d(B) $\subset$ b-d(B) and intersection of an open gb-closed set and a b-closed set is gb-closed. The b-closure of a gb-closed set is a gb-closed set. A characterization of extremely disconnected spaces in terms of gb-closed sets was added. The relationship between approximately b-closed maps and contra b-closed maps were studied. The notion of pre-closed maps were discussed. Some weaker and stronger forms of continuity, namely, ap-b-continuity, contra-b-continuity were discussed and their relationship with other forms of continuity were analyzed. The concepts of irresolute map, b-irresolute map and contra b-irresolute map were studied and the behaviors of gb-closed sets under these mappings were analyzed. The chapter was concluded with the characterizations of T_{gs}-spaces using the concepts of ap-b-continuous maps and ap-b-closed maps. Finally the characterizations of $T_{1/2}$-space were discussed.

In chapter II the relationship between gb-closed sets with the other forms of closed sets furnished by Adea Khaliefa Hussien were discussed. Several equivalence relationships between pre-closed, semi-closed, b-closed, semipre-closed, gs-closed, gp-closed, sg-closed and gb-closed sets were studied. The equivalency between extremely disconnected spaces and the above given closed sets were analyzed. The chapter was concluded with the equivalence relationships between the already existing closed sets, the T_{gs}-spaces and sg-submaximal spaces.

In chapter III the concepts of πgb-closed sets in topological spaces were discussed. The class of πgb- closed sets is finer than that of closed, α-closed, pre-closed, gb-closed, g-closed, πg-closed, πgα-closed πgs-closed and πgp-closed sets but coarser than that of πgsp-closed sets. Some of the interesting properties of these sets were discussed. Finite union (intersection) of πgb-closed sets need not be πgb-closed. Since every closed set is b-closed, b-d(A) $\subset$ d(A) for any subset A of a topological space X. Also, the union of πgb-closed sets A and B is πgb-closed provided d(A) ⊂ b-d(A) and d(B) ⊂ b-d(B) and intersection of πgb-closed set and a b-closed set is πgb-closed. A π-open and πgb-closed is b-closed. The idea of Q-set introduced by Bhattacharyya and Lahiri[16] was discussed. The concept of extremely disconnected spaces and hyper connected spaces were studied. The idea of πgb-$T_{1/2}$ spaces, πgb-spaces, πgb-open sets and their characterizations were discussed. The characterization and the relationship among πgb-continuous functions, πgb-irresolute functions and almost πgb continuous functions were analyzed. The implication relationship of πgb- continuous and other continuous functions were analyzed. The composition of two πgb-continuous functions need not be πgb-continuous was substantiated by an example. The notion of almost πgb-continuity is a weaker form of almost b-continuity. The concept of pre b-closed function was introduced and its properties were discussed.

Finally the concepts of πgb-compact spaces and their behaviour under πgb-continuous functions and almost πgb-continuous functions were studied. Every compact set is πgb-compact. The relations between πgb-compact, b-compact, gb-compact spaces were discussed. The chapter was concluded with the definition of quasi b-normal spaces and its characterizations.

In chapter IV the author had defined a new class of generalized b-closed sets called π generalized b*-closed sets (briefly, πgb*-closed) in topological spaces. By the definition, a subset S of a topological space is said to be πgb*-closed if int(bcl(S)) ⊂ U, whenever S ⊂ U and U is π-open. Every closed, α-closed, pre-closed, semi-closed, b-closed, g-closed, gp-closed, gs-closed, gb-closed, gα-closed, b*-closed, π-closed, πg-closed, πgα-closed, πgp-closed, πgs-closed and πgb-closed set is πgb*-closed. Counter examples were given to show that the reverse implication need not be true. The characterizations of πgb*-closed sets were discussed. Finite union and finite intersection of πgb*-closed sets need not be πgb*-closed. The concept of πgb*-open sets, πgb*-$T_{1/2}$ spaces and its characterizations were added.

In chapter V the author had cherished the idea of πgb*-continuity in topological spaces. Every α -continuous, pre-continuous, semi-continuous, b-continuous, g-continuous, gα-continuous, gp-continuous, gs-continuous, gb-continuous, πα-continuous, πg-continuous, πgp-continuous, πgs-continuous, πgb-continuous map is πgb*-continuous. Counter examples were given to show that the reverse implication need not be true. Also the concept of πgb*-irresolute and πgb*-closed maps and its characterizations were studied. The concept of πgb*-space was discussed and proved that every πgb*-space is a πgb*-$T_{1/2}$ space. The chapter was concluded with the definition and the characterization of almost πgb*-continuous functions.

Chapter VI dealt with the concepts of πgb*-compact spaces and πgb*-neighborhoods. The idea of πgb*-compact spaces and its properties were studied. Every πgb*-closed subset of a πgb*-compact space is πgb*-compact. Some of the interesting characterizations and properties were discussed. Further πgb*-Hausdorff, πgb*-regular and πgb*-normal space were defined and their properties were analyzed. Finally the idea of πgb*-neighborhood at a point was studied.

It is worth mentioning that the author of this thesis had defined a new class of closed sets called "π- generalized b*-closed sets (briefly πgb*-closed sets) in topological spaces" and published an article entitled ***"π- GENERALIZED b*-CLOSED SETS IN TOPOLOGICAL SPACES"*** in the ***INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN SCIENCE ENGINEERING AND TECHNOLOGY*** with the ISSN number 2319-8753 in Volume 3 under the Issue 5 on May 2014.

The author had extended the study to πgb*-continuous functions in topological spaces and defined a new class of continuous functions called "π- generalized b*-continuous functions (briefly πgb*-continuous function) in topological spaces" and published an article entitled ***"π- GENERALIZED b*- CONTINUOUS FUNCTIONS IN***

TOPOLOGICAL SPACES" in the ***INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH IN SCIENCE ENGINEERING AND TECHNOLOGY*** with the ISSN number 2319-8753 in Volume 3 under the Issue 6 on June 2014.