6

Designing Group Acceptance Sampling Plan for Truncated Life Tests Using Minimum Angle Method

6.1 Introduction

Acceptance sampling plan has been one of the most practical tools in quality control. In a time truncated acceptance sampling plan, a random sample is chosen at random from a submitted lot of items and put on the test where the number of failures is observed until the pre-specified time. If the number of failures is higher than the specified acceptance number, then the submitted lot will be rejected. Two risks are associated to a time-truncated acceptance sampling plan. The probability of accepting a bad lot is known as the producer's risk and the probability of rejecting a good lot is called the consumer's risk. Usually in many life tests, consumer's risk is only considered but not producer's risk. An acceptance sampling plan should be schemed so that both the risks are minimum. An acceptance sampling procedure which is used to protect the producer's and consumer's is called a well designed acceptance sampling procedure.

An ordinary acceptance sampling plan in the decorum is used to examine a single product in a tester. In practice, more than one tester can be used to test the multiple products simultaneously. When multiple items are inspected at a time by availability of testers, it is called Group acceptance sampling plan. In Group acceptance sampling plan cost and time can be saved than the ordinary experiment, since we are testing a multiple number of items simultaneously.

Aslam and Jun (2009 a), have designed a Group acceptance sampling plan for a truncated life test when the life time of an item follows either Inverse Rayleigh or a log-logistic distribution. Further many researcher like Srinivasa Rao (2009 a) for Generalized Exponential distribution, (2011 a), for Marshall Olkin extended Lomax distribution, Aslam, et.al. (2009 b), for Gamma distribution and. Aslam, et.al.

(2011 a), for the Generalized Rayleigh distribution presented Group acceptance sampling plans based on truncated life tests.

In this chapter author proposed a Group acceptance sampling plan for a truncated life test when the life time of the test items follows different distributions. The distributions considered in this chapter are Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution. Minimum angle method is applied to determine the design parameter group size g by satisfying both the risks at the specified quality levels simultaneously and at the same time minimizing the sum of risks. Tables of design parameters are provided, and the results are explained with some examples. The comparisions are made among the distributions considered.

6.2 Operating procedure for a Group sampling plan for life test

The following is the operating procedure of the Group acceptance sampling plan based on the truncated life test given by Aslam and Jun (2009 a),

1) Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be $n = gr$.
2) Select the acceptance number c for a group and specify the experiment time t_0.
3) Perform the experiment for g groups simultaneously and record the number of failures for each group.
4) Accept the lot if at most c failures occurs in each of all groups by the experiment time.
5) Terminate the experiment as soon as more than *c* failures occur in any group and reject the lot.

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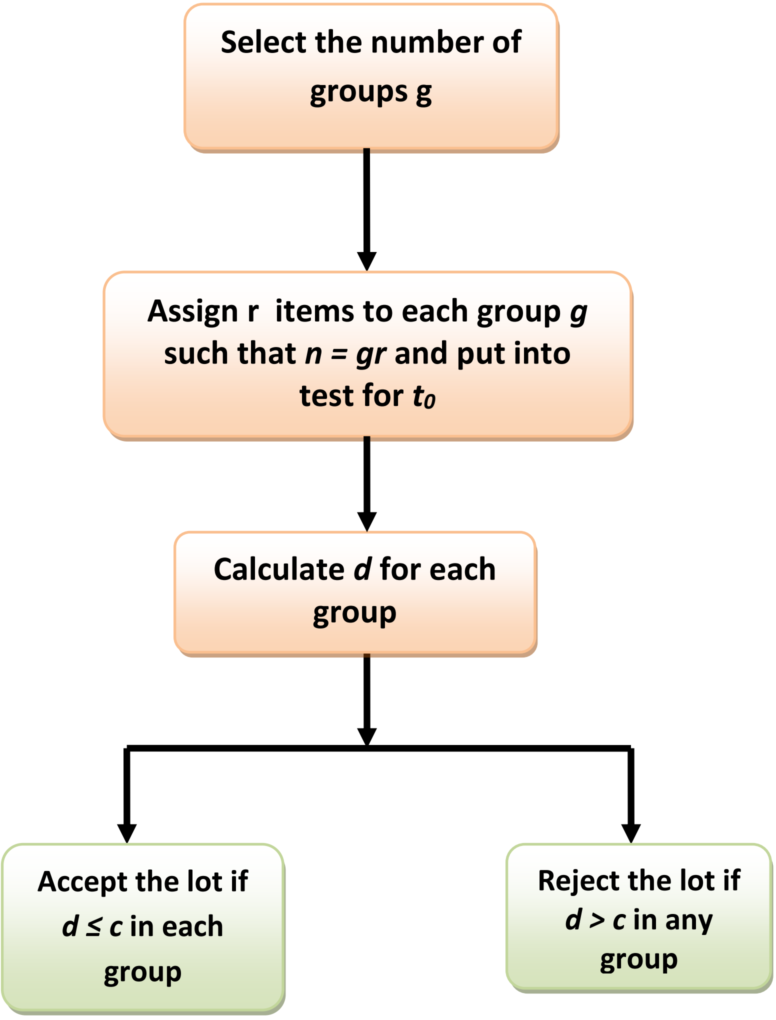
The following is the operating procedure for Group acceptance sampling plan for life test in the form of a flow chart

Flow -Chart



Select the number of groups g
Assign r items to each group *g* such that *n = gr* and put into test for t_0
Calculate *d* for each group
Accept the lot if $d \leq c$ in each group
Reject the lot if $d > c$ in any group

The Group acceptance sampling plan is used to determine the number of groups g, when the number of items, acceptance number *c,* mean ratio and the termination time t_0 are assumed to be specified.

6.3 Distributions

The following are the life time distributions used in this chapter :

6.3.1 Rayleigh distribution

The cumulative distribution function (cdf) of the Rayleigh distribution is given by

$$F(t,\lambda) = 1 - e^{-\frac{1}{2}\left(\frac{t}{\lambda}\right)^2}, \quad t>0, \lambda>0 \qquad (6.1)$$

where λ is the scale parameter

6.3.2 Generalized Exponential distribution

The cumulative distribution function (cdf) of the Generalized Exponential distribution is given by

$$F(t,\lambda) = \left(1 - e^{-\frac{t}{\lambda}}\right)^{\alpha}, \quad t>0, \lambda>0 \qquad (6.2)$$

where λ is the scale parameter and α is the shape parameter and it is fixed as 2

6.3.3 Weibull distribution

The cumulative distribution function (cdf) of the Weibull distribution is given by

$$F(t,\lambda) = 1 -- e^{-\left(\frac{t}{\lambda}\right)^m}, \quad t>0, \lambda>0 \qquad (6.3)$$

where λ is the scale parameter and m is the shape parameter and it is fixed as 2

6.3.4.Gamma distribution

The cumulative distribution function (cdf) of the Gamma distribution is given by

$$F(t,\lambda)=1-e^{-\frac{t}{\lambda}}\sum_{j=0}^{\gamma-1}\left(\frac{t}{\lambda}\right)^{j}/\,j!\quad,\quad t>0,\lambda>0 \qquad (6.4)$$

where λ is the scale parameter and $\gamma > 0$ is the shape parameter and it is fixed as 2

If some other parameters are involved, then they are assumed to be known. The failure probability of an item by time t_0 is given by

$$p = F(t_0, \lambda), \qquad t>0, \lambda>0 \qquad (6.5)$$

The quality of an item is usually represented by its true mean lifetime. Let us assume that the true mean λ can be represented by the scale parameter. In addition, it is convenient to specify the test time as a multiple of the specified life so that $a\lambda_0$ and the quality of an item as a ratio of the true mean to the specified life **(λ/λ_0)**.

Then we can rewrite (6.5) as a function of '*a*' (termination time) and the ratio **(λ/λ_0)**.

$$p = F(a\lambda_0; \lambda/\lambda_0), \qquad t>0, \lambda>0 \qquad (6.6)$$

when the underlying distribution is the Rayleigh distribution, the failure probability is

$$p = 1 - e^{-\frac{1}{2}\left(\frac{a}{\lambda/\lambda_0}\right)^2}, \qquad t>0, \lambda>0 \qquad (6.7)$$

when the underlying distribution is the Generalized Exponential distribution, the failure probability is

$$p = \left(1 - e^{-\frac{a}{\lambda/\lambda_0}}\right)^{\alpha}, \quad t>0, \lambda>0 \qquad (6.8)$$

when the underlying distribution is the Weibull distribution, the failure probability is

$$p = 1 - e^{-\left(\frac{a}{\lambda/\lambda_0}\right)^m}, \quad t>0, \lambda>0 \qquad (6.9)$$

when the underlying distribution is the Gamma distribution, the failure probability is

$$P = 1 - e^{-a/(\lambda/\lambda o)} \sum_{j=0}^{\gamma-1} (a/(\lambda/\lambda o))^{j} \Big/ j! \quad , \quad t > 0, \lambda > 0 \qquad (6.10)$$

where λ is the scale parameter and $\gamma > 0$ is the shape parameter and it is fixed as 2.

6.4 Construction of Tables

The probability of acceptance of the Group acceptance sampling plan is given by

$$L(p) = \left(\sum_{i=0}^{c} \binom{r}{i} p^{i} (1-p)^{r-1} \right)^{g} \qquad (6.11)$$

where *p* is the probability that an item in a group fails before the termination time $t_0 = a\lambda_0$.

The time termination ratio $\mathbf{t}/\lambda_0$ values are fixed as 0.7, 0.8, 1.0, 1.2, 1.5 and 1.8, and the mean ratio λ/λ_0 values are fixed as 4,6,8,10 and 12. The failure probability p is obtained such that it satisfies the following inequality at worst case $\lambda = \lambda_0$, $L(p) \leq \beta$ where β is less than 0.10. The number of groups is determined using minimum angle method for predetermined values of acceptance numbers, number of testers, time mutlipilier and the mean ratios and also satisfying the conditions $L(p_1) \geq 0.95$ and $L(p_2) \leq 0.10$. The minimum number of groups is determined for Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution, the results are presented in Table 6.1, Table 6.2 Table 6.3 and Table 6.4 respectively. The value of θ and $\tan\theta$ are also provided in each table. The number of groups is selected corresponding to the minimum value of θ.

Table – 6.1 The number of groups and probability of acceptance for Minimum angle method Group sampling plan when the life time of the items follows Rayleigh distribution (r=6, c=2) Section 6.1.1

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.7	4	7	0.984427	0.083618	0.224354	12.64513
	4	8	0.982222	0.05866	0.218826	12.34326
	6	7	0.996828	0.083618	0.23052	12.98106
	6	8	0.996375	0.05866	0.224496	12.65288
	8	7	0.998986	0.083618	0.233212	13.12742
	8	8	0.998842	0.05866	0.227057	12.79251
	10	7	0.999583	0.083618	0.23456	13.20064
	10	8	0.999523	0.05866	0.228352	12.86308
	12	7	0.999798	0.083618	0.23532	13.24192
	12	8	0.99977	0.05866	0.229087	12.90305
0.9	4	3	0.982283	0.098303	0.348456	19.21121
	4	4	0.976447	0.045369	0.33083	18.30574
	6	3	0.996333	0.098303	0.35838	19.71667
	6	4	0.995114	0.045369	0.338866	18.71976
	8	3	0.998822	0.098303	0.362808	19.94116
	8	4	0.998429	0.045369	0.342806	18.92203
	10	3	0.999514	0.098303	0.365044	20.05429
	10	4	0.999352	0.045369	0.34485	19.02676
	12	3	0.999765	0.098303	0.36631	20.1183
	12	4	0.999686	0.045369	0.346022	19.08673
1.2	4	2	0.964886	0.029371	0.50159	26.63789
	4	3	0.957794	0.005034	0.497735	26.46115
	6	2	0.992478	0.029371	0.512348	27.12825
	6	3	0.988739	0.005034	0.50162	26.63927
	8	2	0.997554	0.029371	0.51856	27.40943
	8	3	0.996333	0.005034	0.506467	26.86072
	10	2	0.998986	0.029371	0.521933	27.56152
	10	3	0.998479	0.005034	0.509413	26.99487
	12	2	0.999507	0.029371	0.523906	27.6503
	12	3	0.999261	0.005034	0.511211	27.07662
1.5	4	1	0.959847	0.041119	0.661186	33.47212
	6	1	0.991103	0.041119	0.678517	34.15757
	6	2	0.982286	0.001691	0.657337	33.31838

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
	8	1	0.997068	0.041119	0.68824	34.53732
	8	2	0.994145	0.001691	0.662925	33.5414
	10	1	0.998776	0.041119	0.693526	34.74232
	10	2	0.997554	0.001691	0.666919	33.70009
	12	1	0.999404	0.041119	0.696625	34.86204
	12	2	0.998808	0.001691	0.669496	33.80213

Table - 6.1 The number of groups and probability of acceptance for Minimum angle method Group sampling plan when the life time of the items follows Rayleigh distribution (r=9, c=2) Section 6.1.2

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.7	4	3	0.98186	0.093392	0.22747	12.815
	4	4	0.975887	0.042372	0.216493	12.21563
	6	3	0.996244	0.093392	0.233164	13.12484
	6	4	0.994995	0.042372	0.220982	12.4611
	8	3	0.998793	0.093392	0.235779	13.26685
	8	4	0.998391	0.042372	0.223295	12.58739
	10	3	0.999502	0.093392	0.237111	13.3391
	10	4	0.999336	0.042372	0.22451	12.65368
	12	3	0.999759	0.093392	0.237868	13.38016
	12	4	0.999679	0.042372	0.225211	12.69186
0.9	4	2	0.968603	0.03826	0.331091	18.31925
	4	3	0.953276	0.007484	0.325683	18.03954
	6	2	0.99331	0.03826	0.336984	18.62297
	6	3	0.989982	0.007484	0.327569	18.13721
	8	2	0.997828	0.03826	0.340481	18.80274
	8	3	0.996744	0.007484	0.330262	18.27642
	10	2	0.9991	0.03826	0.342389	18.90065
	10	3	0.99865	0.007484	0.331913	18.36169
	12	2	0.999563	0.03826	0.343507	18.95796
	12	3	0.999345	0.007484	0.332924	18.41385
1.2	4	1	0.954577	0.029732	0.507377	26.90221
	6	1	0.989859	0.029732	0.513939	27.20038
	6	2	0.979822	0.000884	0.504063	26.75099
	8	1	0.996649	0.029732	0.519239	27.44008
	8	2	0.99331	0.000884	0.505892	26.8345
	10	1	0.9986	0.029732	0.522335	27.57965
	10	2	0.997201	0.000884	0.507944	26.92803
	12	1	0.999317	0.029732	0.524204	27.6637
	12	2	0.998635	0.000884	0.509406	26.99456
1.5	6	1	0.976575	0.002177	0.661517	33.48531
	6	2	0.953699	0.000474	0.675878	34.05389
	8	1	0.992072	0.002177	0.664639	33.60957
	8	2	0.984207	0.000474	0.668483	33.76206

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
	10	1	0.996649	0.002177	0.667852	33.73707
	10	2	0.99331	0.000474	0.668637	33.76815
	12	1	0.998356	0.002177	0.670126	33.82706
	12	2	0.996715	0.000474	0.669769	33.81293
1.8	6	1	0.954577	0.00078	0.794238	38.45794
	8	1	0.984176	0.000786	0.789664	38.29688
	8	2	0.968603	0.000618	0.802296	38.73994
	10	1	0.993221	0.000786	0.79146	38.36019
	10	2	0.986487	0.000618	0.796799	38.5478
	12	1	0.996649	0.000486	0.793636	38.43678
	12	2	0.99331	0.000618	0.796241	38.52825

Table – 6.2 The number of groups size and probability of acceptance for Minimum angle method Group sampling plan when the life time of the items follows Generalized Exponential distribution (r=6, c=2) Section 6.2.1

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.7	4	3	0.961612	0.098949	0.342107	18.88619
	6	3	0.990843	0.098949	0.352844	19.43507
	6	4	0.98781	0.045767	0.33406	18.47243
	6	5	0.984785	0.021168	0.326581	18.08606
	8	3	0.996839	0.098949	0.358769	19.73641
	8	4	0.995788	0.045767	0.339082	18.73087
	8	5	0.994737	0.021168	0.33088	18.30837
	10	3	0.998639	0.098949	0.362046	19.90259
	10	4	0.998185	0.045767	0.342002	18.88078
	10	5	0.997732	0.021168	0.333546	18.44591
	12	4	0.999096	0.045767	0.343779	18.9719
	12	3	0.999322	0.098949	0.363999	20.00145
0.9	6	3	0.97748	0.017624	0.436555	23.58393
	6	4	0.970087	0.004586	0.434003	23.46101
	6	2	0.98493	0.067723	0.456855	24.55352
	6	3	0.97748	0.017624	0.436555	23.58393
	6	4	0.970087	0.004586	0.434003	23.46101
	8	3	0.991981	0.017624	0.442015	23.84616
	8	4	0.989323	0.004586	0.437356	23.62247
	8	5	0.986672	0.001194	0.437027	23.60665
	10	2	0.997657	0.067723	0.469261	25.13882
	10	3	0.996488	0.017624	0.445804	24.02751
	10	4	0.99532	0.004586	0.440463	23.77173
	12	2	0.998821	0.067723	0.472117	25.27281
	12	3	0.998231	0.017624	0.448281	24.14577
	12	4	0.997642	0.004586	0.442661	23.87711
1.2	8	2	0.98493	0.008378	0.579538	30.09391
	8	1	0.992436	0.091534	0.628202	32.1371
	10	1	0.996607	0.091534	0.635832	32.44952
	10	2	0.993226	0.008378	0.584329	30.29899
	12	1	0.998264	0.091534	0.640646	32.64551
	12	2	0.996532	0.008378	0.587858	30.44947

1.5	6	1	0.957652	0.02783	0.68638	34.46491
	8	1	0.983579	0.02783	0.696615	34.86167
	8	2	0.967428	0.000774	0.688758	34.55742
	10	1	0.992436	0.02783	0.704727	35.1734
	10	2	0.98493	0.000774	0.690729	34.63397
	12	1	0.996064	0.02783	0.710402	35.39006
	12	2	0.992144	0.000774	0.693824	34.75383

Table - 6.2 The number of groups and probability of acceptance for Minimum angle method Group sampling plan when the life time of the items follows Generalized Exponential distribution (r=9, c=2) Section 6.2.2

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.7	6	2	0.983523	0.038589	0.333038	18.41972
	6	3	0.975387	0.00758	0.325167	18.01283
	8	2	0.994223	0.038589	0.33709	18.62846
	8	3	0.991347	0.00758	0.327451	18.13106
	10	2	0.997494	0.038589	0.339689	18.76203
	10	3	0.996243	0.00758	0.329464	18.2352
	12	2	0.998746	0.038589	0.341334	18.84653
	12	3	0.99812	0.00758	0.330865	18.30756
0.9	6	1	0.979986	0.065067	0.457997	24.60763
	6	2	0.960373	0.004234	0.438252	23.66554
	8	1	0.992742	0.065067	0.464259	24.90349
	8	2	0.985536	0.004234	0.438887	23.69605
	10	1	0.996789	0.065067	0.46836	25.09652
	10	2	0.993588	0.004234	0.441077	23.80117
	12	1	0.998374	0.065067	0.471	25.22043
	12	2	0.99675	0.004234	0.442902	23.88864
1.2	8	1	0.979986	0.009343	0.583065	30.24496
	8	2	0.960373	0.000873	0.589354	30.51313
	10	1	0.99085	0.009343	0.586318	30.38385
	10	2	0.981783	0.000873	0.586205	30.37904
	12	1	0.995268	0.009343	0.589186	30.506
	12	2	0.990559	0.000873	0.586482	30.39085
1.5	8	1	0.957749	0.001076	0.695943	34.83573
	10	1	0.979986	0.001076	0.69443	34.77727
	12	1	0.989418	0.001076	0.69595	34.83597
1.8	10	1	0.963032	0.000108	0.782359	38.03818
	12	1	0.979986	0.000108	0.780075	37.95689

Table – 6.3 The number of groups and probability of acceptance for Minimum angle method Group sampling plan when the life time of the items follows Weibull distribution (r=6, c=2) Section 6.3.1

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.7	4	7	0.996419	0.019666	0.365715	20.08821
		8	0.995908	0.011219	0.362767	19.93911
		10	0.994888	0.003651	0.360371	19.81769
		11	0.994378	0.002083	0.359986	19.79819
	6	8	0.999617	0.011219	0.378243	20.71879
		7	0.999665	0.019666	0.381485	20.88111
		9	0.999569	0.006401	0.376426	20.62766
		10	0.999521	0.003651	0.375405	20.5764
	8	8	0.99993	0.011219	0.384083	21.01091
		6	0.999948	0.034473	0.393326	21.47102
		7	0.999939	0.019666	0.387389	21.17581
		9	0.999922	0.006401	0.382223	20.91801
	10	7	0.999984	0.019666	0.390165	21.31399
		10	0.999977	0.003651	0.383896	21.00161
		8	0.999982	0.011219	0.386833	21.1481
		9	0.999979	0.006401	0.384958	21.05459
	12	10	0.999992	0.003651	0.385387	21.07601
		11	0.999991	0.002083	0.384781	21.0458
		12	0.999991	0.001188	0.384437	21.02861
		9	0.999993	0.006401	0.386453	21.12916
0.8	4	8	0.991213	0.000607	0.437608	23.63458
		6	0.993402	0.003869	0.438082	23.65737
		5	0.994499	0.009765	0.440217	23.75992
		9	0.99012	0.000241	0.437929	23.65001
	6	4	0.999579	0.024645	0.466787	25.02255
		5	0.999474	0.009765	0.459818	24.69385
		7	0.999264	0.001533	0.456122	24.51876
		9	0.999054	0.000241	0.455628	24.49531
	8	8	0.999846	0.000607	0.46311	24.84933
		6	0.999884	0.003869	0.464609	24.91999
		5	0.999904	0.009765	0.467366	25.04979
		4	0.999923	0.024645	0.474487	25.38375
	10	6	0.999969	0.003869	0.468154	25.08681
		7	0.999964	0.001533	0.467061	25.03543
		5	0.999974	0.009765	0.470939	25.21755
		8	0.999959	0.000607	0.466631	25.01519

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
	12	5	0.999991	0.009765	0.472895	25.30922
		9	0.999984	0.000241	0.468393	25.09806
		8	0.999986	0.000607	0.468564	25.1061
		6	0.99999	0.003869	0.470097	25.17806
0.9	4	8	0.982936	0.000138	0.514565	27.22875
		6	0.987174	0.000226	0.512466	27.13358
		5	0.9893	0.000916	0.511721	27.09978
		9	0.980823	0.000134	0.515668	27.27869
	6	4	0.999163	0.003712	0.535328	28.16141
		5	0.998953	0.000916	0.533941	28.0996
		7	0.998535	0.000128	0.533705	28.08906
		9	0.998117	0.000134	0.5339	28.09778
	8	8	0.999691	0.000124	0.542741	28.49049
		6	0.999768	0.000226	0.542814	28.49373
		5	0.999807	0.000916	0.543168	28.5094
		4	0.999845	0.003712	0.544671	28.57586
	10	6	0.999938	0.000226	0.547232	28.6889
		7	0.999928	0.000125	0.547145	28.68504
		5	0.999948	0.000916	0.547605	28.70531
		8	0.999918	0.000138	0.547127	28.68427
	12	5	0.999983	0.000916	0.550046	28.81283
		9	0.999969	0.000134	0.549552	28.79108
		8	0.999972	0.000138	0.549556	28.79124
		6	0.999979	0.000226	0.549669	28.79621
1.2	4	3	0.968982	0.000130	0.698697	34.94188
		2	0.979213	0.000968	0.69206	34.68554
		5	0.948839	0.000129	0.713507	35.50813
		4	0.958858	0.000138	0.706053	35.22411
	6	4	0.995597	0.000137	0.727064	36.01954
		5	0.994499	0.000192	0.727866	36.04959
		2	0.997796	0.000968	0.726165	35.98585
		3	0.996696	0.000134	0.726283	35.99028
	8	2	0.999581	0.000968	0.741853	36.56996
		2	0.999581	0.000968	0.741853	36.56996
		5	0.998953	0.000192	0.7416	36.56062
		4	0.999163	0.000137	0.741445	36.55491
	10	3	0.99983	0.000102	0.748925	36.83046
		2	0.999887	0.000968	0.749586	36.85472
		5	0.999717	0.000137	0.748987	36.83275
		2	0.999887	0.000968	0.749586	36.85472

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
	12	5	0.999904	0.000129	0.753195	36.98686
		2	0.999961	0.000968	0.753881	37.01195
		3	0.999942	0.000134	0.753188	36.98663
		4	0.999923	0.000138	0.753181	36.98636
1.5	4	1	0.966827	0.001553	0.790879	38.33975
	6	1	0.996129	0.001553	0.838562	39.9819
		3	0.988433	0.000174	0.843774	40.15679
		2	0.992274	0.000142	0.84051	40.04738
		3	0.988433	0.000137	0.843774	40.15679
	8	2	0.998476	0.000241	0.86137	40.74062
		1	0.999238	0.001553	0.862051	40.76301
		1	0.999238	0.001553	0.862051	40.76301
		2	0.998476	0.000241	0.86137	40.74062
	10	3	0.999372	0.000137	0.8729	41.11772
		2	0.999581	0.000124	0.87272	41.11184
		1	0.999791	0.001553	0.873892	41.14995
		2	0.999581	0.000127	0.87272	41.11184
	12	3	0.999784	0.000174	0.879287	41.32475
		3	0.999784	0.000137	0.879287	41.32475
		2	0.999856	0.000124	0.879226	41.32277
		2	0.999856	0.000141	0.879226	41.32277

Table 6.3 The number of groups and probability of acceptance for Minimum angle method Group sampling plan when the life time of the items follows Weibull distribution (r=9, c=2) Section 6.3.2

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.7	4	7	0.986014	0.000126	0.362306	19.91578
		8	0.984032	0.000122	0.363016	19.95172
		4	0.991984	0.004314	0.361672	19.88366
		3	0.993982	0.016833	0.365567	20.08073
	6	8	0.998439	0.000182	0.374446	20.52822
		7	0.998634	0.000126	0.374393	20.52556
		9	0.998244	0.000124	0.374514	20.53163
		5	0.999024	0.001106	0.374635	20.53768
	8	8	0.999712	0.000182	0.379863	20.79994
		6	0.999784	0.000283	0.379936	20.8036
		7	0.999748	0.000124	0.37987	20.80028
		9	0.999676	0.000174	0.379871	20.80036
	10	7	0.999933	0.000126	0.382539	20.93381
		4	0.999962	0.004314	0.384158	21.01466
		8	0.999923	0.000186	0.382522	20.93296
		9	0.999914	0.000147	0.38252	20.93287
	12	3	0.99999	0.016833	0.390555	21.33337
		4	0.999987	0.004314	0.385645	21.0889
		5	0.999984	0.001106	0.384408	21.02715
		9	0.999971	0.000177	0.38399	21.00627
0.8	4	3	0.987341	0.001717	0.43982	23.74084
		6	0.974843	0.000195	0.444685	23.97402
		5	0.978991	0.000152	0.442811	23.88428
		4	0.983157	0.000206	0.441016	23.79822
	6	4	0.998304	0.000206	0.455954	24.51081
		5	0.99788	0.000126	0.456065	24.51607
		7	0.997033	0.000001	0.456442	24.53393
		6	0.997456	0.000195	0.456249	24.52478
	8	4	0.999684	0.000206	0.462999	24.84409
		6	0.999525	0.000124	0.462979	24.84312
		5	0.999604	0.000146	0.462952	24.84187
		3	0.999763	0.001717	0.463664	24.87544
	10	6	0.999873	0.000195	0.466389	25.00381

a	λ/λ_0	g	L (p1)	L(p2)	tanθ	θ
		3	0.999936	0.001717	0.46716	25.04009
		5	0.999894	0.000146	0.466389	25.00382
		4	0.999915	0.000206	0.466463	25.00733
	12	5	0.999964	0.000126	0.468301	25.09376
		3	0.999978	0.001717	0.469089	25.13073
		2	0.999986	0.014339	0.475092	25.41202
		6	0.999957	0.000195	0.468295	25.09344
0.9	4	4	0.968108	0.000171	0.522441	27.58441
		2	0.983925	0.002171	0.515177	27.25649
		3	0.975985	0.000101	0.518276	27.39663
	6	4	0.996658	0.000171	0.534683	28.13264
		3	0.997493	0.000101	0.534287	28.11501
		5	0.995824	0.000001	0.535128	28.15249
		2	0.998328	0.002171	0.534949	28.14453
	8	3	0.999526	0.000101	0.542877	28.49654
		2	0.999684	0.002171	0.543918	28.54258
		5	0.999211	0.00001	0.542999	28.50193
		4	0.999369	0.000171	0.542911	28.49801
	10	4	0.99983	0.000176	0.54717	28.68617
		3	0.999872	0.000101	0.5472	28.68747
		2	0.999915	0.002171	0.548312	28.73648
		5	0.999787	0.00001	0.547196	28.68732
	12	4	0.999942	0.000171	0.549567	28.79175
		3	0.999957	0.000101	0.549612	28.79373
		2	0.999971	0.002171	0.550745	28.84353
		5	0.999928	0.00001	0.549578	28.79222
1.2	4	1	0.963939	0.000001	0.702331	35.08156
	6	1	0.995762	0.000949	0.727636	36.04097
		3	0.987341	0.0001	0.733217	36.24949
		2	0.991543	0.000101	0.730036	36.1308
	8	2	0.998328	0.000101	0.742065	36.57782
		1	0.999163	0.000949	0.742149	36.58091
		3	0.997493	0.0001	0.74276	36.60349
	10	1	0.99977	0.000949	0.749659	36.85741
		2	0.99954	0.000124	0.749121	36.83765
		3	0.99931	0.00001	0.7493	36.84422
	12	1	0.999921	0.000949	0.753897	37.01254
		2	0.999842	0.000127	0.753242	36.98859
		3	0.999763	0.0001	0.753376	36.9935

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
1.5	6	1	0.985834	0.000129	0.846002	40.23127
		2	0.97187	0.000001	0.858155	40.63471
	8	1	0.99704	0.000129	0.862613	40.78147
		2	0.994088	0.000001	0.865171	40.86542
	10	1	0.999163	0.000145	0.873086	41.12376
		2	0.998328	0.00001	0.873822	41.14768
	12	1	0.999708	0.000126	0.879358	41.32703
		2	0.999416	0.000001	0.879611	41.33523

Table – 6.4 The number of groups and probability of acceptance for Minimum angle method Group sampling plan when the life time of the items follows Gamma distribution (r=6, c=2) Section 6.4.1

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.7	4	43	0.997887	0.099329	0.158217	8.990635
	4	44	0.997838	0.094136	0.157316	8.940281
	6	43	0.999788	0.099329	0.166033	9.42702
	6	44	0.999783	0.094136	0.165082	9.37397
	8	43	0.99996	0.099329	0.168985	9.591513
	8	44	0.999959	0.094136	0.168016	9.53754
	10	43	0.999989	0.099329	0.170393	9.669945
	10	44	0.999989	0.094136	0.169416	9.615543
	12	43	0.999996	0.099329	0.171171	9.713259
	12	44	0.999996	0.094136	0.17019	9.658621
0.8	4	25	0.997417	0.098947	0.193312	10.94099
	4	26	0.997314	0.090203	0.19147	10.83924
	6	25	0.999736	0.098947	0.203234	11.488
	6	26	0.999725	0.090203	0.201283	11.38057
	8	25	0.999949	0.098947	0.207024	11.69637
	8	26	0.999947	0.090203	0.205035	11.58702
	10	25	0.999986	0.098947	0.208841	11.79614
	10	25	0.999986	0.098947	0.208841	11.79614
	12	25	0.999995	0.098947	0.209847	11.85137
	12	26	0.999995	0.090203	0.20783	11.74064
0.9	4	16	0.996842	0.097281	0.228667	12.88021
	4	17	0.996645	0.084097	0.225413	12.70289
	6	16	0.99967	0.097281	0.240841	13.54126
	6	17	0.999649	0.084097	0.237378	13.35359
	8	16	0.999936	0.097281	0.245547	13.79589
	8	17	0.999932	0.084097	0.242014	13.60477
	10	16	0.999982	0.097281	0.247815	13.91833
	10	17	0.999981	0.084097	0.244248	13.72563
	12	16	0.999994	0.097281	0.249073	13.98627
	12	17	0.999994	0.084097	0.245488	13.79269
1.2	4	6	0.994454	0.092405	0.33306	18.42085
	4	7	0.993532	0.062131	0.322564	17.87783
	6	6	0.99938	0.092405	0.352655	19.42548
	6	7	0.999276	0.062131	0.341302	18.84488

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
	8	6	0.999876	0.092405	0.360548	19.82666
	8	7	0.999855	0.062131	0.348916	19.23469
	10	6	0.999965	0.092405	0.364409	20.02222
	10	7	0.999959	0.062131	0.352648	19.42512
	12	6	0.999988	0.092405	0.366571	20.13148
	12	7	0.999986	0.062131	0.354739	19.5316
1.5	4	3	0.991235	0.095692	0.43236	23.38174
	4	4	0.988331	0.043769	0.409923	22.28984
	6	3	0.998949	0.095692	0.460196	24.71172
	6	4	0.998599	0.043769	0.43534	23.52542
	8	3	0.999783	0.095692	0.471907	25.26294
	8	4	0.999711	0.043769	0.44631	24.0517
	10	3	0.999938	0.095692	0.477734	25.53538
	10	4	0.999917	0.043769	0.451801	24.3135
	12	3	0.999978	0.095692	0.481024	25.68869
	12	4	0.999971	0.043769	0.454908	24.46116
1.8	4	2	0.985622	0.076674	0.507976	26.92949
	4	3	0.97851	0.021231	0.48233	25.74939
	6	2	0.998148	0.076674	0.542855	28.49555
	6	3	0.997223	0.021231	0.512532	27.13658
	8	2	0.999605	0.076674	0.558379	29.17809
	8	3	0.999407	0.021231	0.526843	27.7822
	10	2	0.999885	0.076674	0.566265	29.52136
	10	3	0.999827	0.021231	0.534216	28.11186
	12	2	0.999959	0.076674	0.570764	29.71615
	12	3	0.999938	0.021231	0.538442	28.2999

Table 6.4 The number of groups and probability of acceptance for Minimum angle method Group sampling plan when the life time of the items follows Gamma distribution (r=9, c=2) Section 6.4.2

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
0.7	4	14	0.997199	0.097019	0.157932	8.974696
	4	15	0.996999	0.082128	0.155396	8.832866
	6	14	0.999714	0.097019	0.165622	9.404079
	6	15	0.999694	0.082128	0.162938	9.254326
	8	14	0.999945	0.097019	0.168555	9.567573
	8	15	0.999942	0.082128	0.165821	9.415183
	10	14	0.999985	0.097019	0.169958	9.645709
	10	15	0.999984	0.082128	0.167201	9.492098
	12	14	0.999995	0.097019	0.170734	9.688892
	12	15	0.999995	0.082128	0.167964	9.534614
0.8	4	9	0.996248	0.085758	0.19076	10.79997
	4	10	0.995832	0.065276	0.186646	10.57239
	6	9	0.999607	0.085758	0.20033	11.32809
	6	10	0.999564	0.065276	0.195947	11.08647
	8	9	0.999924	0.085758	0.204043	11.53249
	8	10	0.999916	0.065276	0.199573	11.28641
	10	9	0.999979	0.000001	0.188178	10.65716
	10	10	0.999977	0.065276	0.201319	11.38261
	12	9	0.999993	0.085758	0.20682	11.68516
	12	10	0.999992	0.065276	0.202288	11.43593
0.9	4	6	0.995267	0.085293	0.22605	12.73765
	4	7	0.99448	0.056589	0.219322	12.37035
	6	6	0.999491	0.085293	0.237729	13.37266
	6	7	0.999407	0.056589	0.230513	12.98068
	8	6	0.999901	0.085293	0.242339	13.62236
	8	7	0.999884	0.000001	0.221671	12.49869
	10	6	0.999972	0.085293	0.244569	13.74302
	10	7	0.999968	0.056589	0.237129	13.3401
	12	6	0.99999	0.085293	0.24581	13.81008
	12	7	0.999989	0.056589	0.238331	13.40531
1.2	4	4	0.98576	0.018202	0.31051	17.2501
	4	5	0.982231	0.006686	0.307968	17.11714
	6	4	0.998331	0.018202	0.326334	18.07329
	6	5	0.997914	0.006686	0.32268	17.88386

a	λ/λ_0	g	L (p_1)	L(p_2)	tanθ	θ
	8	4	0.999661	0.018202	0.333368	18.43674
	8	5	0.999576	0.006686	0.32953	18.23859
	8	4	0.999661	0.018202	0.333368	18.43674
	10	5	0.99988	0.006686	0.33299	18.41724
	10	4	0.999904	0.018202	0.336888	18.61806
	10	5	0.99988	0.006686	0.33299	18.41724
	12	4	0.999966	0.018202	0.338874	18.72016
	12	5	0.999958	0.006686	0.334947	18.51814
1.5	4	2	0.978393	0.025869	0.406496	22.12154
	4	3	0.967766	0.004161	0.401822	21.89132
	6	2	0.997229	0.025869	0.427931	23.1676
	6	3	0.995847	0.004161	0.41916	22.7415
	8	2	0.999414	0.025869	0.43824	23.66497
	8	3	0.999121	0.00001	0.427026	23.12376
	10	2	0.99983	0.025869	0.443538	23.91909
	10	3	0.999746	0.004161	0.433904	23.45624
	12	2	0.99994	0.025869	0.446563	24.06375
	12	3	0.999909	0.004161	0.436841	23.59767
1.8	6	1	0.996421	0.058417	0.533289	28.0705
	6	2	0.992854	0.003413	0.505565	26.81955
	8	1	0.99921	0.058417	0.547778	28.71296
	8	2	0.99842	0.003413	0.517931	27.38106
	10	1	0.999766	0.058417	0.555354	29.04579
	10	2	0.999532	0.003413	0.524819	27.69133
	12	1	0.999915	0.058417	0.559722	29.2367
	12	2	0.99983	0.003413	0.528872	27.8731

6.5 Example

Assume that an experimenter wants to establish that the lifetime of the electrical devices produced in the factory ensures the true unknown mean life as at least 1000 hours when the ratio of the time mutlipilier is a = 0.7 and specified life is λ/λ_0 = 6 and acceptance numbers c = 2. Following are the results obtained when the lifetime of the test items follows the Rayleigh distribution, Generalized Exponential distribution, Weibull distribution and Gamma distribution respectively. A Group acceptance sampling plan is developed to test if the true unknown mean is greater than 1,000 hours based on a testing time of 700 hours and using testers equipped with 6 items each. Based on producer's risk, consumer's risk values and the test termination time multiplier, the number of groups of Group acceptance sampling plan can be found using equations (6.1) to (6.4).

6.5.1 Rayleigh distribution

Suppose that the lifetime of a product follows the Rayleigh distribution. From Table 6.1, the minimum number of groups required for r = 6 Items are g = 8 and one can observe that the minimum angle is $\theta = 12.65288^0$ and also $\alpha = 0.0036$, $\beta = 0.0586$ which is very much less than the specified risk. Thus, we will draw a random sample of size n = 48 ($n = gr$) items and allocate 6 items to each of the 8 groups to put on test for 700 hrs. The lot will be accepted if not more than 2 failures occur before 700 hrs in each of 8 groups. The experiment is truncated as soon as the 3rd failure occurs before the 700th hr also it satisfies the condition of producer's risk and consumer's risk $\alpha \leq 0.05$, $\beta \leq 0.10$. Thus, the required Group sampling plan has parameters as (6,8). For the same conditions when the time of experiment is 1500 hours, the probability of acceptance is 0.98228, the producer's risk is 0.0171 and consumer's risk 0.0016. The sample size is 6 and number of groups is 2 which is very much less. Thus it is clear that as the time of experiment increases, the sample size decreases. When the ratio of unknown average life to specified life as 12, there is no change in the sample size but there is increase in the probability of acceptance. The probability of acceptance is 0.9997 which is almost equal to 1 and the consumer's risk is 0.0586 which shows that there is a reduction in consumer's risk.

6.5.2 Generalized Exponential distribution

Suppose that the lifetime of a product follows the Generalized Exponential distribution. From Table 6.2, the minimum number of groups required for r = 6 items are g = 5 and one can observe that the minimum angle is θ = 18.086^0 and α = 0.0152, β = 0.0211 which is very much less than the specified risk. Thus, we will draw a random sample of size n = 30 ($n = gr$) items and allocate 6 items to each of the 5 groups to put on test for 700 hrs. The lot will be accepted if not more than 2 failures occur before 700 hrs in each of the 5 groups. The experiment is truncated as soon as the 3rd failure occurs before the 700th hr, also it satisfies the condition of producer's risk and consumer's risk α ≤ 0.05 β ≤ 0.10. Thus, the required Group sampling plan has parameters as (6,5) For the same conditions when the time of experiment is 1500 hours, the probability of acceptance is 0.9576, the producer's risk is 0.0423 and consumer's risk 0.0278. The sample size is 6 and number of groups is 1 which is very much less. Thus it is clear that as the time of experiment increases, the sample size decreases. When the ratio of unknown average life to specified life is 12, there is slight change in the sample size but there is increase in the probability of acceptance. The probability of acceptance is 0.9990 which is almost equal to 1 and the consumer's risk is 0.0457 which shows that there is a reduction in consumer's risk.

6.5.3 Weibull distribution

Suppose that the lifetime of a product follows the Weibull distribution. From Table 6.3, the minimum number of groups required for r = 6 items are g = 10 and one can observe that the minimum angle is θ = 20.5764^0 and also α = 0.0004, β = 0.0036 which is very much less than the specified risk. Thus, we will draw a random sample of size n = 60 ($n = gr$) items and allocate 6 items to each of 10 groups to put on test for 700 hrs. The lot will be accepted if not more than 2 failures occur before 700 hrs in each of 10 groups. The experiment is truncated as soon as the 3rd failure occurs before the 700th hr, also it satisfies the condition of producer's risk and consumer's risk α ≤ 0.05, β ≤ 0.10. Thus, the required Group sampling plan has parameters as (6,10). For the same conditions when the time of experiment is 1500 hours, the probability of acceptance is 0.9961, the producer's risk is 0.0038 and consumer's risk 0.0015. The sample size is 6 and number of groups is 1 which is very much less. Thus it is clear that as the time of experiment increases, the sample size decreases. When the ratio of unknown average life to specified life as 12, there is slight change in the sample size but there is increase in the probability of acceptance. The probability of acceptance is 0.9999 which is almost equal to 1 and the consumer's risk is 0.0011 which shows that there is a reduction in consumer's risk.

6.5.4 Gamma distribution

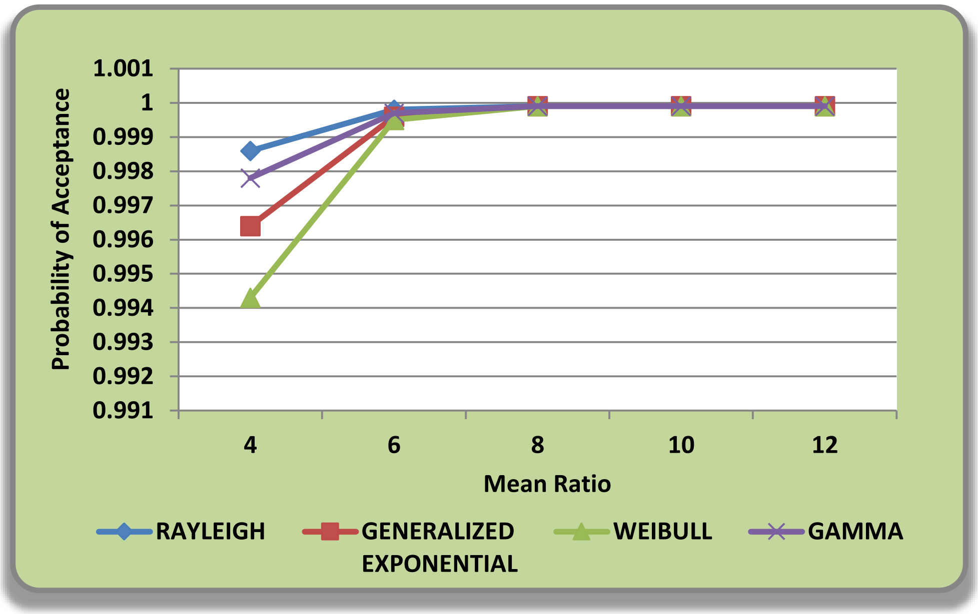
Suppose that the lifetime of a product follows the Gamma distribution. From Table 6.4, the minimum number of groups required for r = 6 items are g = 44 and one can observe that the minimum angle is θ = 9.37397^0 and also α = 0.0002, β = 0.0941 which is very much less than the specified risk. Thus, we will draw a random sample of size n = 264 where ($n = gr$) items and allocate 6 items to each of the 44 groups to put on test for 700 hrs. The lot will be accepted if not more than 2 failures occur before 700 hrs in each of the 44 groups. The experiment is truncated as soon as the 3rd failure occurs before the 700th hr also it satisfies the condition of producer's risk and consumer's risk α ≤ 0.05, β ≤ 0.10. Thus, the required Group sampling plan has parameters as (6, 44) For the same conditions when the time of experiment is 1500 hours, the probability of acceptance is 0.9985, the producer's risk is 0.0014 and consumer's risk 0.0437. The sample size is 24 and number of groups is 4 which is very much less. Thus it is clear that as the time of experiment increases the sample size decreases. When the ratio of unknown average life to specified life is 12, there is no change in the sample size but there is increase in the probability of acceptance. The probability of acceptance is 0.9999 which is almost equal to 1 and consumer's risk is 0.0941 which shows that there is a reduction in consumer's risk.

Comparison of the results of Producer's risk, Consumer's risk and sample size for Group sampling plan when the life time of the items follows different distributions is provided in Table 6.5.

Table – 6.5 Comparison of results of Producer's risk, Consumer's risk and sample size for Minimum angle method Group sampling plan when the life time of the items follows different distributions (a=0.7, r=6, c=2)

S.No	λ/λ_0	Distribution	Producer's risk	Consumer's risk	g
1	6	Rayleigh	0.0036	0.0586	8
		Generalized Exponential	0.0152	0.0211	5
		Weibull	0.0004	0.0036	10
		Gamma	0.0002	0.0941	44
2	8	Rayleigh	0.0011	0.0586	8
		Generalized Exponential	0.0052	0.0211	5
		Weibull	0.0001	0.0064	9
		Gamma	0.0001	0.0941	44
3	10	Rayleigh	0.0005	0.0586	8
		Generalized Exponential	0.0022	0.0211	5
		Weibull	0.0001	0.0036	10
		Gamma	0.0001	0.0941	44
4	12	Rayleigh	0.0002	0.0586	8
		Generalized Exponential	0.0009	0.0457	4
		Weibull	0.0001	0.0011	12
		Gamma	0.0001	0.0941	44

Figure-6.1 OC curve of Minimum angle method Group sampling Plan when the life time of the items follows different distributions with (a=0.7, r=6, c=2)



From Table 6.5 one conclude that when the Generalized Exponential distribution is followed, the sample size is very much less than the sample size of all other distribution. And at the same time producer's risk and consumer's risk are also less and the sum of the risks is also very much less for Generalized Exponential distribution. Figure 6.1 shows the OC curves of all four distributions. From the figure, one can observe that Probability of acceptance of Rayleigh distribution is more than any other distributions. It can be seen that the application of the minimum angle method minimizes simultaneously the consumer's risk and producer's risk. This minimum angle method plan provides better discrimination of accepting good lots.