CHAPTER – VI

CHAPTER VI

FUZZY SEMI-GENERALIZED IRRESOLUTE AND FUZZY GENERALIZED SEMI-IRRESOLUTE FUNCTIONS

Definition: 6.1

A fuzzy subset μ of a fts (X,τ) is said to be **fuzzy semi-generalized closed (Fsg-closed, for short)** if $scl(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is fuzzy semi-open set.

The complement of a Fsg-closed set is called **Fsg-open** set.

Definition: 6.2

Let (X,τ) be a fts. We define the **fuzzy semi-generalized closure (Fsgcl (μ), for short)** for any fuzzy set μ in (X,τ) as follows :

$Fsgcl(\mu) = \wedge \{ \eta : \mu \leq \eta \text{ and } \eta \text{ is Fsg-closed}\}$.

Example: 6.3

Let $X = \{x, y\}$ and $\tau = \{0_X, y_{0.7}, 1_X\}$.

If $\mu = y_{0.3}$ then μ is sg-closed set fuzzy set but not g-closed. Furthermore, if $\mu = y_{0.5}$, then μ is g-closed but not sg-closed.

Remark: 6.4

Every fuzzy semi-closed set is Fsg-closed.

Definition: 6.5

A fuzzy subset μ of a fts (X,τ) is said to be **fuzzy generalized semi-closed (Fgs-closed, for short)** if $scl(\mu) \leq \eta$ whenever $\mu \leq \eta$ and η is fuzzy open set.

The complement of a Fgs-closed set is called **Fgs-open** set.

Example: 6.6

Let $X = \{x\}$ and $\tau = \{0_X, x_{0.1}, x_{0.3}, 1_X\}$. If $\mu = x_{0.3}$, then μ is gs-closed fuzzy set.

Remark: 6.7

Every g-closed fuzzy set is Fgs-closed.

Theorem: 6.8

Every Fsg-closed set is Fgs-closed.

Proof:

Since every open fuzzy set is semi-open, then the result follows.

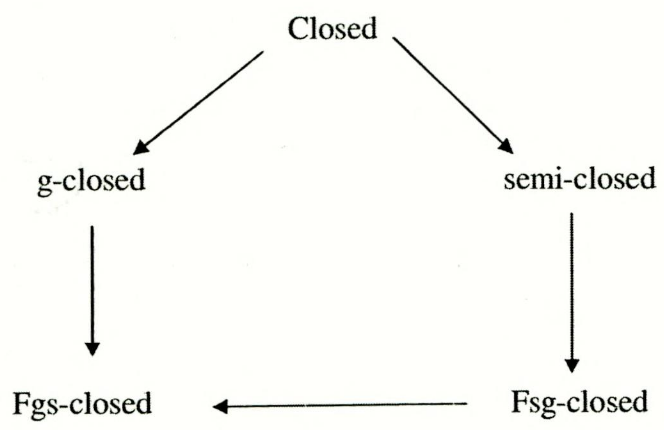
The converse of the above theorem is not true by the following counter Example .

Example: 6.9

Consider the fts (X,τ) of Example 6.3 and let $\mu = y_{0.8}$. Then μ is Fgs-closed set but not Fsg-closed.

Note: 6.10

From the above definitions the following diagram holds:



Definition: 6.11

A mapping f from a fts (X,τ) into a fts (Y,δ) is called **fuzzy semi-generalized continuous (Fsg-continuous, for short)** if the inverse image of every closed fuzzy set in Y is Fsg-closed set in X.

Remark: 6.12

Every Fs-continuous mapping is F-sg-continuous.

The converse of the above remark 6.12 is not true, in general, by the following counter example.

Example: 6.13

Let X = Y = {x, y}. Consider the fuzzy topology $\delta = \{0_Y , y_{0.5}, 1_Y \}$ and the fuzzy topology of Example 6.3 . If f : $(Y,\delta) \longrightarrow (X,\tau)$ is the identity fuzzy mapping. Then f is F-sg-continuous but not Fs-continuous and so not fuzzy continuous.

Definition: 6.14

A mapping f from a fts (X,τ) into a fts (Y,δ) is called **fuzzy generalized semi-continuous (Fgs-continuous, for short)** if the inverse image of every closed fuzzy set in Y is Fgs-closed set in X.

Remark: 6.15

Every Fsg-continuous mapping is Fgs-continuous.

The converse of the above remark 6.15 may not be true in general, by the following example.

Example: 6.16

Let X = Y = {x, y}. Consider a fuzzy topology τ of Example 6.13

Fgs-continuous mapping but not Fsg-continuous.

Definition: 6.17

A mapping f : $(X,\tau) \longrightarrow (Y,\delta)$ from a fts (X,τ) into a fts (Y,δ) is called:

(i) **fuzzy semi-generalized irresolute (Fsg-irresolute, for short)** if the inverse image of every Fsg-closed set in Y is sg-closed in X.

(ii) **fuzzy generalized semi-irresolute (Fgs-irresolute, for short)** if the inverse image of every Fgs-closed set in Y is gs-closed in X.

(iii) **fuzzy open (closed)** if $f(\alpha) \in \delta$ ($f(\alpha)^C \in \delta$), for each $\alpha \in \tau$ ($(\alpha)^C \in \tau$)

Theorem: 6.18

Let f : $(X,\tau) \longrightarrow (Y,\delta)$ be a mapping from a fts (X,τ) into a fts (Y,δ) .

Then the following statements are equivalent:

(i) f is Fsg-irresolute (resp. Fgs-irresolute).

(ii) the inverse image of each Fsg-open (resp. gs-open) set in Y is Fsg-open (resp. gs-open) set in X.

Proof:

It follows from the definitions.

Theorem: 6.19

Fsg-irresolute (resp. Fgs-irresolute) $\Longrightarrow$ Fsg-continuous(resp. Fgs-continuous).

Proof:

Since every closed fuzzy set is Fsg-closed (resp., Fgs-closed), then it is proved that f is Fsg-continuous (resp. Fgs-continuous).

The converse of the above theorem 6.19 is not true in general which is shown in the following example.

Example: 6.20

Let X=Y={x} and $\tau = \{0_X, x_{0.1}, x_{0.3}, 1_X\}$ $\delta = \{0_Y, x_{0.1}, 1_Y\}$. If f : (X,$\tau$) ⟶ (Y,$\delta$) is the identity fuzzy mapping, then f is Fsg-continuous mapping but not Fsg-irresolute. Also, f is Fgs-continuous mapping but not Fgs-irresolute.

Theorem: 6.21 (Composition of two Fsg-irresolute maps is Fsg-irresolute)

If f : (X,τ) ⟶ (Y,δ) and g : (Y,δ) ⟶(Z,σ) are both Fsg-irresolute mappings, then g ∘ f : (X,τ) ⟶ (Z,σ) is F-sg-irresolute.

Proof:

Suppose that η is sg-closed fuzzy set in Z, then $g^{-1}(\eta)$ is Fsg-closed in Y and $f^{-1}(g^{-1}(\eta))$ is Fsg-closed in X, since g and f are Fsg-irresolute. Thus

$(g \circ f)^{-1}(\eta) = f^{-1}(g^{-1}(\eta))$ is sg-closed and therefore g ∘ f is Fsg-irresolute.

Theorem: 6.22 (Composition of two Fgs-irresolute maps is Fgs-irresolute)

If f : (X,τ) ⟶ (Y,δ) and g : (Y,δ) ⟶(Z,σ) are both F-gs-irresolute mappings, then g ∘ f : (X,τ) ⟶ (Z,σ) is Fgs-irresolute.

Proof:

Similar to the proof of Theorem 6.21 .

Theorem: 6.23

Let (X,τ), (Y,δ) and (Z,σ) be any fuzzy topological spaces such that f : $(X,\tau) \longrightarrow (Y,\delta)$ is Fsg-irresolute (resp. Fgs-irresolute) and g : $(Y,\delta) \longrightarrow (Z,\sigma)$ is Fsg-continuous (resp. Fgs-continuous). Then g ∘ f is Fsg-continuous (resp. Fgs-continuous).

Proof:

Similar to the proof of Theorem 6.21.

Theorem: 6.24

Let (Y,δ) be a fuzzy topological space such that every semi-closed fuzzy subset of Y is closed. If f : $(X,\tau) \longrightarrow (Y,\delta)$ is bijective, pre-semi-open and Fsg-continuous, then f is Fsg-irresolute.

Proof:

Let μ be an Fsg-closed set in Y and let $f^{-1}(\mu) \leq \eta$ where η is fuzzy semi-open set in X. Then $\mu \leq f(\eta)$. Since f (η) is fuzzy semi-open in Y and μ is Fsg-closed in Y , then scl $(\mu) \leq f(\eta)$and hence f^{-1}(scl $(\mu)) \leq f^{-1}(f(\eta)) = \eta$. Since f is Fsg-continuous, and scl (μ) is closed in Y , then $f^{-1}($ (scl$(\mu))$ is Fsg-closed. Therefore scl($f^{-1}($ scl$(\mu))) \leq \eta$ and so scl $(f^{-1}(\mu)) \leq \eta$. Hence $f^{-1}(\mu)$ is Fsg-closed in X. Then f is Fsg-irresolute.

Example: 6.25

Let X = Y = {x}, τ be the fuzzy topology of Example 6.6 and $\delta = \{0_Y, x_{0.5}, 1_Y\}$. Let f : $(X,\tau) \longrightarrow (Y,\delta)$ be the identity fuzzy mapping. We have that in (Y,δ) every fuzzy semi-closed subset is closed. Thus f is Fsg-continuous bijective but it is not pre-semi-open and so f is not Fsg-irresolute.

Theorem: 6.26

Let (Y,δ) be a fuzzy topological space such that every fuzzy semi-closed subset of Y is closed. If f : $(X,\tau) \longrightarrow (Y,\delta)$ is bijective , fuzzy open and Fgs-continuous, then f is Fgs-irresolute.

Proof:

Similar to the proof of Theorem 6.24.

Example: 6.27

In Example 6.25 we have f is Fgs-continuous bijective but it is not fuzzy open and so f is not Fgs-irresolute.

Theorem: 6.28

If a mapping f : $(X,\tau) \longrightarrow (Y,\delta)$ is Fsg-irresolute, then f (sgcl(μ)) $\leq$ scl(f(μ)) for every fuzzy subset μ of X.

Proof:

Let $\mu \in I^X$. Since scl (f(μ)) is Fsg-closed in Y , then f^{-1}(scl(f(μ))) is Fsg-closed in X. Furthermore, $\mu \leq f^{-1}$ (f (μ)) $\leq f^{-1}$(scl (f(μ))) and hence sgcl(μ) $\leq f^{-1}$(scl (f(μ))), and consequently, f (sgcl(μ)) $\leq$ f(f^{-1}(scl (f(μ)))) $\leq$ scl (f(μ)).

The following two examples 6.29 and 6.30 show that the concepts of fuzzy irresolute mapping and Fsg-irresolute mappings are independent of each other.

Example: 6.29

Let X = Y = {x, y}, $\tau = \{ 0_X, x_{1/3}, 1_X\}$, and $\delta = \{0_Y , y_{0.3}, 1_Y\}$.If f : $(X,\tau) \longrightarrow (Y,\delta)$ is the identity fuzzy mapping, then f is Fsg-irresolute. However $\mu = y_{0.7}$ is fuzzy semi-open set in Y but f^{-1} ($y_{0.7}$) = $y_{0.7}$ is not fuzzy semi-open in X.

Therefore f is not fuzzy irresolute.

Example: 6.30

Let X, Y and f be as in Example 6.25. Then f is fuzzy irresolute but not Fsg-irresolute.

Definition: 6.31

A fts $(X,\tau) \longrightarrow (Y,\delta)$ be a mapping from a fts (X,τ) into a fts (Y,δ). If (X,τ) is **fuzzy semi-$T_{1/2}$** , then f is Fsg-continuous if and only if it is Fs-continuous.

Theorem: 6.32

Let f : $(X,\tau) \longrightarrow (Y,\delta)$ be Fsg-irresolute. Then f is fuzzy irresolute if (X,τ) is fuzzy semi-$T_{1/2}$.

Proof :

Let $\nu \in I^Y$ be semi-closed. Since ν is Fsg-closed in Y and f is Fsg-irresolute, then $f^{-1}(\nu)$ is Fsg-closed in X. But (X,τ) is fuzzy semi-$T_{1/2}$ and so $f^{-1}(\nu)$ is semi-closed. Hence f is fuzzy irresolute.