# Certain Studies On The Impact Of Inspection Errors On The Performance Of Inspection Plans

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# CONTENT

ABSTRACT

This dissertation is devoted to the study of inspection errors based on performance of inspection plans follows different types of inspection errors.

The first chapter deals with basic concepts of acceptance sampling, notations and review of literature.

In the various quality control procedures the possibility of inspection errors is considered as an important issue. The presence of these errors leads to change in the operational characteristic (O.C) curve, and as a result the average outgoing quality of an industrial process. A new mathematical model that can be applied to calculate such quantities as the expected number of defective items replaced in an accepted lot.

In third chapter performance measures of inspection process are presented for single, double and repeat inspection plans are presented. The impact of inspection error on these plans are evaluated through the plan's performance measures.

In the fourth chapter the effect of Type-II inspection errors on the effectiveness of a quality inspection plan designed utilizing a risk exposure control approach is analysed. The probability of a Type-II error is integrated into the material at risk (MAR) model used to control risk exposure. A linear programming formulation, including stochastic behaviour of the model, is presented and solved. Experiments conducted to analyze the effect of inspection error on risk exposure control reveal the computational complexity of the problem.

Usually assumed that the inspector is perfect neglecting the possibility of Type –I and Type-II errors in sampling inspection.Fifth chapter deals with the impact of inspection errors on rectifying single, double and chain sampling inspection plans with average outgoing quality (AOQ) and average total inspection (ATI).

# CHAPTER - 1

## INTRODUCTION

Acceptance sampling is a statistical method which enables us to make the decision of either accepting or rejecting a shipment of items for the lot. In most situations, 100 percent inspection of all items is neither desirable nor economically feasible.

Some advantages of acceptance sampling plan are:

- It is more economical as against 100 percent inspection in terms of inspection cost.
- It is usually more accurate than 100 percent inspection, since it allows less opportunity for inspection fatigue, which can be responsible for mistakes.
- Less product damage occurs since it requires less handling of the product.
- Rejecting the entire lot on the basis of simple sample testing can motivate the suppliers of the product to improve their quality control standards and producers.
- It is the only approach in situations where quality is testing by destroying the item.

Inspections for acceptance purposes are carried out at many stages in manufacturing. There may be inspection of incoming materials and parts, process inspection at various points in the manufacturing operations, final inspection by a manufacturer of his own product, and ultimately inspection of the finished product by one or more purchasers.

Much of this acceptance inspection is necessarily on a sampling basis. All the acceptance tests that are destructive of the item tested must inevitably be done by sampling. In many other situations, sampling inspection is used because the cost of 100 percent inspection is prohibited.

The major areas of acceptance sampling according to Dodge (1969)[24] are:

- Lot –by-lot sampling by the method of attributes in which each unit in a sample is inspected on a go-not go basis for one or more charactertics.
- Lot-by lot sampling by the method of variables in which each unit in a sampling is measured for a single characteristic, such as weight or strength.
- Continuous sampling of a flow of units by the methods of attributes.
- Special purpose plans including chain sampling, skip-lot sampling, small sampling plans, etc.

**Sampling Plans, Sampling Scheme and Sampling System**

According to American National Standards Institutes /American Society for Quality Control (ANSI/ASQC). Standards $A_2$ (1987) [2] an acceptance sampling plan is a specific plan that states the sampling rule to be used, and the associated acceptance and non- acceptance criteria and an acceptance sampling scheme is a specific set of producers which usually consists of acceptance sampling plans in which lot sizes, sample sizes and acceptance criteria, or the amount of 100 percent inspection and sampling are related.

Hill (1962) [39] has also describe the difference between sampling plan and sampling scheme. According to him, a sampling scheme is whole set of sampling plans and operations included in the standard "the over-all strategy specifying the way in which sampling plans are to be used".

The MIL-STD-105D (1963) [48] is a well- known sampling scheme. Stephens and Larson(1967) [57] have described a sampling system as an assigned grouping of two or more sampling plans and the rules for using these plans for sentencing lots to achieve a blend of the advantageous features of each of the sampling plans. Tightened-Normal-Tightened sampling scheme of Calvin (1977) [15] is an example for sampling scheme.

**Operating Characteristic (OC) Curve**

Every sampling plan is associated with an operating, characteristic curve, familiarly known as OC curve of the plan. This curve when referred to two axes, the axis of p- proportion nonconforming of the material offered for inspection and the axis of $P_a(p)$-probability of acceptance of a lot or process, is the locus of (p, $P_a(p)$).The OC curve gives the practical performance of a sampling plan.

**Type A OC-Curve**

A curve showing, for a given sampling plan, the probability of accepting a lot as a function of the lot quality. This curve is for isolated or unique lots or a lot from an isolated sequence.

**Type B OC-Curve**

A curve showing, for a given sampling plan, the probability of accepting a lot as a function of the process average. This curve is for continuous stream of lots

**Binomial Model**

This model is exact for the case of nonconforming units under Type B situations. This model can also be used under Type a situation for the case of nonconforming units whenever $n / N \leq 0.10$, where n and N are the sample and lot sizes respectively.

**Poisson Model**

This model is exact for the case of nonconforming units under Type B situations. Under situation of Type A, for the case of nonconforming units, this model can be used whenever $n / N \leq 0.10$, n is large and p is small such that $np < 5$. Under situations of Type B, for the case of nonconforming units, this model can be used whenever n is large and p is small such that $np < 5$.

**Hyper –Geometric Model**

This model is exact for the case of nonconforming units under Type A situations and is useful for isolated lots.

**Acceptance Sampling Plans**

A specific plan that states the sample size or sizes to be used and the associated acceptance and non-acceptance criteria.

**Probability of Acceptance**

The probability that a lot will be accepted under a given sampling plan.

**Probability of Rejection**

The probability that a lot will not be accepted under a given sampling plan.

**Acceptance Quality Level**

The AOQL is percent defective that is the base line requirement for the quality of the producer's product. The producer would like to design a sampling plan such that there is a high probability of accepting a lot that has a defect level less than or equal to the AQL.

**Lot tolerance percent defective**

Lot tolerance percent defective is defined as a maximum percentage of defective item in a lot beyond which the lot should be rejected.

**Consumer's Risk**

For a given sampling plan, the probability of acceptance of a lot the quality of which has designated numerical vale representing a level which it is generally desired to accept.

**Producer's Risk**

For a given sampling plan, the probability of not acceptance of a lot the quality of which has designated numerical vale representing a level which it is generally desired to accept.

**Average Sample Number**

The average number of sample units per lot for making decisions (acceptance or non-acceptance).

**Average Total Inspection**

The excepted number of items inspected per lot to arrive at a decision in an acceptance-rectification sampling, inspection plan calling for 100 percent inspection of the rejection lots is called average amount of total inspection.

**Average Outgoing Quality**

The expected quality of outgoing product following the use of an acceptance sampling plan for a given value of incoming product quality.

**Average Outgoing Quality Limit**

For a given acceptance sampling plan, the maximum average outgoing quality over all possible levels of incoming quality.

**Average Total Inspection**

The average number of units inspected per lot, including all units in rejected lots.

**Fraction of items defectives**

The proportion or fraction nonconforming (defective) in a population is defined as the ratio of the number of nonconforming items in the population to the total number of items in that population. The item under consideration may have one more quality characteristics that are inspected simultaneously.

**Quality**

Quality means fitness for use and it is inversely proportioned to variability.

**Sample Size**

Sample size determination is the act of choosing the number of observations or replicates to include in a statistical sample. The sample size is an important feature of any empirical study in which the goal is to make inferences about a population from a sample.

**Lot Size**

Lot size refers to the quantity of an item ordered for delivery on a specific data or manufactured in a single production run. In other words, lot size basically refers to the total quantity of a product ordered for manufacturing.

**Single Sampling**

Sampling inspection in which the decision to accept or not to accept a lot is based on the inspection of a single of size 'n'.

**Double Sampling**

Sampling inspection in which the inspection of the first sample of size "$n_1$" leads to a decision to accept a lot, not to accept it, or to take a second sample of size "$n_2$" and the inspection of the second sample then leads to a decision to accept the lot.

**Chain Sampling**

In acceptance sampling, a plan in which the criteria for acceptance and rejection apply to the cumulative sampling results for the current lot and one or more immediately preceding lots.

**Inspection Error**

Embedded within the design of acceptance-sampling plans is an assumption that the inspection procedures are error free. However, many inspection tasks are not error free; on the contrary, they may even be error prone. Two types of errors are possible in attribute sampling. An item which is good may be classified as defective (Type I error, $e_1$), or an item that is defective may be classified as good (Type II error, $e_2$).

**Designing Sampling Plans**

In designing a sampling plan one has to accomplish a number of different purpose. According to Hamaker (1960) [37] the important ones are:

- To strike a proper balance between the consumer's requirements, the producer's capabilities inspection capacity.

- To separate bad lots from good.
- Simplicity of procedures and administration.
- Economy in number of observations.
- To reduce the risk of wrong decisions with increasing lot size.
- To use accumulated sample data as a valuable source of information.
- To exert pressure on the producer or supplier when the quality of lots received is unreliable or not up to standard and.
- To reduce sampling when the quality is reliable and satisfactory.

He further noted that these aims are partly conflicting and all of them cannot be simultaneously realized.

The design methodologies of acceptance sampling may be categorized as follows:

| | Risk Based | Economical Based |
|---|---|---|
| Non – Bayesian | 1 | 2 |
| Bayesian | 3 | 4 |

Risk based sampling plans are traditional in nature, drawing upon procedure and consumer type of risk as depicted by the OC curve. Economically based sampling plans explicitly consider such factors as costs of inspections, accepting a non conforming unit and rejection a conforming unit in an attempt to design a cost-effective plan. Bayesian plan design takes into account the past history of similar lots submitted previously for inspection purposes. Non – Bayesian plan design is not explicitly based upon the past lot history.

According to Peach (1947) [51] the following are some of the major types of designing the plans, which are classified according to types of protection

- The plan is specified by requiring the OC curve to pass through (or nearly through)two fixed points. In some cases it may be possible to impose certain additional conditions. The two points generally selected are $(p_1, 1-\alpha)$ and $(p_2, 1-\beta)$.

where

- $p_1$ Or $p_{1-\alpha}$ - the quality level that is considered to be good so that the procedure expects lots of $p_1$ quality to be accepted most of the time.
- $p_2$ Or $p_\beta$- the quality level that is considered to be poor so that the consumer expects lots of $p_2$ quality to be rejected most of the time.
- $\alpha$ −The producer's risk of rejecting $p_1$ quality.
- $\beta$ −The consumer's risk of accepting $p_2$ quality.

The tables provided by Cameron (1952), are an example for this type of designing. Schilling (1980), considered the term $p_1$ as the Producer's Quality Level (PQL) and $p_2$as the Consumers Quality Level (CQL). Earlier literature calls $p_1$ as the Acceptance Quality Level (AQL) and $p_2$ as the Limiting Quality Level (LQL) or Reject able Quality Level (RQL) or Lot Tolerance Proportion Defective (LTPD). Peach (1947) [51], have defined the ratio $p_2$ /$p_1$ associated with specified values of $\alpha$ and $\beta$ as the 'Operating Ratio'. Traditionally the values of $\alpha$ and $\beta$ are assumed to take 0.05 and 0.10 respectively.

- The plan is specified by fixing one point only, through which the OC curve is required to pass and setting up one or more conditions, not explicitly in terms of the OC curve. Dodge and Romig (1959), LTPD tables is an example for this type of design.
- The plan is specified by imposing upon the OC curve of two or more independent conditions none of which explicitly involves the OC curve. Dodge and Romig (1959), AOQL tables is an example for this type of design.

Certain additional symbols that are used in the dissertation are explained below:

**Glossary of symbols**

| | | |
|---|---|---|
| N | - | Lot size |
| n | - | Sample size |
| p | - | Lot or process quality |
| r | - | Rejection number |
| K | - | Total number of lots |
| $p_a(p)$ | - | Probability of lot acceptance |
| $p_1$ | - | Acceptable quality level |
| $p_2$ | - | Limiting quality level |
| $\alpha$ | - | Producer's risk |
| $\beta$ | - | Consumer's risk |
| c | - | Acceptable number for single sampling plan |
| d | - | Number of defective in the sample |
| AQL | - | Acceptance quality control |
| LTPD | - | Lot tolerance percent defective |
| LQL | - | Limiting quality level |
| ASN | - | Average sample number |
| AOQ | - | Average outgoing quality |
| AOQe | - | Average outgoing quality level |
| AOQL | - | Average outgoing quality limit |

ATI - Average total inspection

IQL - indifference quality level

p - Fraction defective

$E_1$ - The event that a good item is classified as a defective

$E_2$ - The event that a defective item is classified good

A - The even that an item is defective

B - The event that an item is classified as a defective

P - True fraction defective

$P_e$ - Apparent fraction defective

$e_1$ - The probability that $E_1$ occurs

$e_2$ - The probability that $E_2$ occurs

$P_{ae}$ - Probability of acceptance when inspection errors is present

Ψ - Pr [incorrectly classifying a conforming item as nonconforming]

θ - Pr [incorrectly classifying a nonconforming item as conforming]

X - Number of nonconforming items in the lot

P - X/N-proportion of nonconforming items in the lot

X - Number nonconforming items in the sample

Y - Number items in the sample classified as nonconforming

$n_i$ - Sample size at stage i

$c_i$ - Acceptance number at stage i

$X_i$ - Actual number of NCU's in sample i

$q$ - 1-p

$P_{ai}$ - Probability of accepting a lot at stage when $e_1 = e_2 = 0$

$P_e$ - Apparent fraction defective

$P_{ch}$ - Probability of acceptance (chain sampling plans)

$P_{ri}$ - Probability of rejecting a lot at stage when $e_1 = e_2 = 0$

$X'_i$ - Apparent number of NCU's in sample i ($e_1 > 0$ or $e_2 > 0$)

$p'$ - Apparent fraction nonconforming or apparent product quality

$q'$ - $1 - p'$

$P_{ai'}$ - Probability of accepting a lot at stage i when ($e_1 > 0$ or $e_2 > 0$)

$P_{a_1}$ - Probability of acceptance based on the first sample of a double sample plan

$P_{a_{1e}}$ - Probability of acceptance based on the first sample of a double sampling plan with inspection error

$P_{a_2}$ - Probability of acceptance of a lot based on the second sample of double sample plan

$P_{a_{2e}}$ - Probability of acceptance with inspection

$P_{ri'}$ - Probability rejecting a lot at stage i when $e_1>0$ or $e_2>0$

$P_{r_{1e}}$ - Probability of rejection of the lot based on the first sample in a double sample Inspection plan when inspection error is considered

$P_{d_1}$ - Probability of a decision concerning the lot first sample in a double sample Inspection plan $P_{d_1}=P_{a_1}=P_{r_1}$

$P_{d_{1e}}$ - Probability of a decision concerning the lot first sample in a double sample Inspection plan when inspector error $(P_{d_{1e}}) = P_{a_{1e}} = P_{r_{1e}}$

B(x, n P) - Binomial cumulative distribution function

NCU's - Non conforming units

$n_1$ - sample size on the first sample

$C_1$ - Acceptance number of the first sample

$n_2$ - Sample size on the second sample

$C_2$ - Acceptance number for both sample

$d_1$ - Number of defectives in the first sample

$d_2$ - Number of defective in the second sample

$P_a^I$ and $P_a^{II}$ - Probability of acceptance on the first and second sample

j - Cycle under inspection

i - Stage or characteristic under inspection

$e_i$ - Probability of misclassification an inspection an inspector can make for $i^{th}$ characteristic

PG - Probability of a component to be good while entering a new cycle

PR - Probability of a component to be rework while entering a new cycle

PS - Probability of a component to be scrap while entering a new cycle

# REVIEW OF LITERATURE

Acceptance sampling plan is an essential tool in the Statistical Quality Control and is a methodology which deals with quality contracting on product orders between the producers and the consumers and thus allows the producers to take decision to accept or reject the manufactured products based on the inspection of samples. It is the process of evaluating a portion of the product/material in a lot for the purpose of accepting or rejecting the lot as either conforming or not conforming to a quality specification.

Acceptance sampling is necessary to limit the cost of inspection and is the only available method to appraise the quality in destructive testing. Acceptance sampling itself does not improve quality, but whenever the lot is rejected it indicates the instability of the production process. Acceptance sampling is cost efficient and only admissible method of efficient tests with quick results.

The lot is accepted if the number of defects falls below where the acceptance number or otherwise the lot is rejected. Embedded within the design of acceptance-sampling plans is an assumption that the inspection procedures are error free. However, many inspection tasks are not error free; on the contrary, they may even be error prone. Two types of errors are possible in attribute sampling. An item which is good may be classified as defective (Type - I error, $e_1$), or an item that is defective may be classified as good (Type - II error, $e_2$).

Dodge (1955) introduced chain sampling inspection plans. Duncan A.J (1959) developed operating characteristics of fertilizer inspection plans based on rectifying two stage (Double) sampling.

Wortham and Mogg (1970) introduced nine different rectification inspection policies. Collins, Case and Bennet (1973) discussed the effects of inspection error on attribute sampling plans. Hald Case, Bennett and Schmidt (1978) developed formulae or calculating the average outgoing quality.

Collins and Case (1981) derived an expression for the probability of acceptance under inspection errors. Beainy and Case (1981) generalized the model of Wortham and Mogg(1970), and they developed nine different sample/rest of lot disposition policies for single and double sampling along explicitly developed AOQ models. Jackson (1985) developed the mathematical expression for single sampling based on hyper geometric distribution. Raz (1986) discussed a survey of models for allocating of inspection effect in multistage production systems.

Montgomery (1990) discussed that within the design of acceptance sampling plan is an assumption that the inspection procedures are error free. However, many inspection takes are not error free, on the contrary, they may even be error prone. Johnson, Kotz and Wu (1991) describes some more modern industrial approaches to inspection errors with attributes in quality control. Villalobos, Foster, Disney (1993) introduce and model the concept of flexible inspection system for serial multi-stage production systems in the field of printed circuit boards. Tang and Tang (1994) have been developed design of screening procedures. Rabinowitz and Emmons and Emmons and Rabinowitz (1997) have been presented an inspiring non linear model of the inspection allocation problem.

Verduzco, Villalobos Vega (2001) has been presented an interesting case of information based inspection allocation. Duffuaa and Khan (2002) have been discussed an optimal repeated inspection plan with several classifications. Abraham, Hadi, Fouad and Abdulla (2002) adopted to determine the proportion of detected defective and undetected defectives left in a lot after rectifying sampling inspection. Jalbout (2002) has been presented a mathematical model that can be applied to calculate such quantities as the expected number of defectives items replaced in an accepted lot and other functions. Straub and Faber (2005) have presented an inspection model based on risk modelling. Pulak and Al- Sultan's (2005) determine the optimum process mean under the rectifying inspection plan. Radhakrishnan, Sampathkumar (2006) discussed construction of mixed sampling plans indexed through MAPD and AQL with chain sampling plan as attribute plan. Chung-Ho Chen (2008) determined the economic specification limit setting for rectifying inspection plan with inspection error.

# CHAPTER-2

# THE EFFECT OF INSPECTION ERRORS ON THE AVERAGE OUTGOING QUALITY IN AN INDUSTRIAL PROCESS

In this chapter "The Effect of Inspection Errors on the Average Outgoing Quality in an Industrial Process" by Abraham Jalbout , Hadi Alkahby, Fouad Jalbout, Abdulla Darwish (2002) [1] have been reviewed.

In various quality control procedures the possibility of inspection errors is considered as an important issue. The presence of these errors leads to change in the operational characteristic (O.C) curve and as a result the average outgoing quality of an industrial process. A new mathematical model that can be applied to calculate such quantities as the expected number of defective items replaced in an accepted lot.

In attribute sampling plans the errors are generally of two kinds:

Type I error: a good item is classified as bad, with a probability $e_1$

Type II error: a bad item is classified as good, with a probability $e_2$

Collins and case (1973) [21] derive an expression for the probability of acceptance under inspection error. An expression was later derived for the marginal distribution of the observed defectives. (Hald [37] case, Bennett and Schmidt(1974) [6] developed formulas for calculating the average outgoing quality (AOQ) when attribute inspection is subject to Type-I and Type-II inspection errors. Nine different rectification inspection policies are considered. These policies were first introduced by Wortham and Mogg (1970) [61]. Beainy and case(1981) [4] later generalized these models, and they developed nine different sample/rest-of-lot disposition policies for single and double sampling alone explicitly developed AOQ models.

## 2.1 Mathematical Development

Hald(1976) [36]has derived the following form of the marginal distribution of x:

$$g_n(x) = \binom{n}{x} P^x (1-p)^{n-x}, \qquad x = 0,1,2,\ldots,n \qquad (2.1)$$

Under the assumption that the number of defectives X in a lot size N is binomially distributed with a probability density function:

$$f_N(X) = \binom{N}{X} P^x (1-p)^{N-X} \qquad X = 0,1,2,...,N \tag{2.2}$$

Where p is the process fraction defective.

The second assumption of equation (2.2) is that the number of defectives x in a sample size n given X is hyper geometric

$$f(x|X) = \frac{\binom{n}{x}\binom{N-n}{X-x}}{\binom{N}{X}} \tag{2.3}$$

which proves that the Hald's derivations of the binomial distribution is reproduced by hyper geometric sampling. Thus for the Bayesian operating characteristic (BOC) curve the probability of lot acceptance is derived from the above equations as:

$$p_a = \sum_{x=0}^{c} g_n(x) = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0,1,2,\ldots,n \quad 0 \le p \le 1 \tag{2.4}$$

where c is the acceptance number. For the inspection error analysis the observed defective from a sample is replaced by observed number of defectives $y_e$. The probability of lot acceptance is given by

$$p_{ae} = \sum_{y_e}^{c} g_n(y_e) \tag{2.5}$$

where

$$g_n(y_e) = \binom{n}{y_e} p_e^{y_e} (1-p_e)^{n-y_e} \qquad y_e = 0,1,2,\ldots,n, \tag{2.6}$$

which gives the probability of lot acceptance for perfect inspection. The probability of lot acceptance when inspection errors are present is

$$p_{ae} = \sum_{y_{e=0}}^{c} \binom{n}{y_e} p_e^{y_e} (1-p_e)^{n-y_e} \tag{2.7}$$

The AOQ can be defined as

$$AOQ = \frac{\text{expacted number of defective items remaining after inspection}}{\text{total number of items in the lot}}$$

$$= \frac{(N-n)p\, P_a}{N} \qquad (2.8)$$

An expression for AOQ can be derived by introducing the following terms:

p (N-M), the number of defectives in the uninspected portion of an accepted

p (N-M)$e_2$, the number of defective of defective items classified as being good in the screened portion of the rejected lot.$n_p e_2$ is the number of defective items classified as good in the sample, DITR is the number of defective items introduced through replacement into the lot. For an accepted lot, the expected number of defective items replaced in the lot is:

$$y = np_e \qquad (2.9)$$

The probability that an item is classified as being good is:

$$P_g = (1-p)\,(1-e_1) + pe_2 \qquad (2.10)$$

A set of $n_1$ items are selected at random, tested and classified as good or bad. A total of $np_e$ items were needed to replace the defective items in the accepted lot.

This procedure of sampling defines a negative binomial process. The expected number of items tested to obtain $np_e$ items, which are good, is $\frac{y}{P_g}$ (2.11)

The expected number of defective items replaced in an accepted lot is:

$$DITR_a = Pe_2\left(\frac{y}{P_g}\right) \qquad (2.12)$$

The expected number of defective items replaced in a rejected lot, which is screened is

$$DITR_s = pe_2\frac{(N-n)p_e}{p_g} \qquad (2.13)$$

The expected number of items to be replaced is

$$\mathrm{DITR} = pe_2\left(\frac{y}{P_g}\right) + Pe_2\frac{(N-n)p_e}{P_g}(1-P_{ae})$$

$$= \frac{pe_2}{P_g}[y + (N-n)p_e(1-P_{ae})] \quad (2.14)$$

$$\mathrm{AOQ} = \frac{p(N-n)P_{ae} + p(N-n)(1-P_{ae})e_2 + npe_2 + \mathrm{DITR}}{N} \quad (2.15)$$

$$= \frac{p(N-n)P_{ae} + p(N-n)(1-P_{ae})e_2 + npe_2 + \frac{pe_2}{P_g}[y + (N-n)p_e(1-P_{ae})]}{N}$$

$$\mathrm{AOQ} = \frac{npe_2 + p(N-n)(1-p_e)P_{ae} + p(N-n)(1-P_{ae})e_2}{N(1-p_e)} \quad (2.16)$$

AOQ for sampling with no replacement is

$$\mathrm{AOQ} = \frac{p(N-n)P_{ae} + p(N-n)(1-P_{ae})e_2 + npe_2}{N - np_e - (1-P_{ae})(N-n)p_e} \quad (2.17)$$

This is true so no defectives are introduced through the replacement process.

The expressions for the average outgoing quality were derived for both the model involving replacement of defective items in the lot and when the items are not replaced.

# CHAPTER - 3

# EFFECT OF INSPECTION ERRORS ON THE PERFORMANCE OF INSPECTION PLANS IN QUALITY CONTROL SYSTEMS

In this chapter "Effects of Inspection Errors on the Performance of Inspection Plans in Quality Control Systems" Mehmood Khan and Duffuaa (2002) [47] have been reviewed.

## 3.1 Role of inspection plans

Inspection of raw materials, semi finished products, or finished products are an important part of quality control. Inspection plans are designed for the purpose of acceptance or rejection of a product, based on adherence to specifications. There are several types of inspection plans. The widely used inspection plans are the single, double and repeat inspection plans. Tang and Tang(1994) [59] have consider several factors in designing an inspection plan. These factors include the goal to be accomplished, the nature of the performance variables, available information on the population, and economical and manufacturing environments.

Two separate objectives have been commonly used to design inspection plans. One is to optimize the expected total profit associated with an inspection procedure, and the other is to use inspection to reach certain statistical goals, such as controlling the outgoing nonconforming rate of the product. The methods using these objectives are known as economic and statistical designs of inspection plans, respectively. In an economic design three cost components are commonly considered: the cost of inspection, cost of rejection, and the cost of acceptance.

The cost of inspection include expenses of testing materials, lab or, equipment, and so forth. The cost of rejection is incurred by false rejection of good components and by corrective actions taken on these items, such as repairing, scrapping, or returning the items to the supplier. The cost of acceptance is caused by the items of imperfect quality that reach the customers and it include damage caused by product failure, warranty cost, handling cost, loss in sales, loss in goodwill, and so forth. In the statistical design of an inspection plan, the most commonly used criterion is the outgoing conforming rate.

The inspection is error-free, the outgoing conforming rate should be 100% after inspection. However, the outgoing conforming rate becomes a meaningful and important design criterion when nonconforming items may not be detected because of inspection error or for other reasons. The economic factors are usually considered implicitly in selecting statistical goals.

For example, the outgoing conforming rate should be set at a high level when the cost of accepting nonconforming items is large. In fact, it also possible to incorporate both the economical and statistical criteria in designing an inspection plan.

The performance of an inspection plan is greatly influenced by inspection errors. An inspector can commit two types of errors. Type I error is the probability of classifying a non-defective item as defective and Type II error is the probability of classifying a defective item as non-defective. These errors may have an adverse effect on the ability of an inspection plan to ensure product quality.

### 3.2 Performance Measures

The objective of the performance measures is to examine how well an inspection plan is accomplishing the required results.

### 3.3 Inspection Errors

Embedded within the design of acceptance-sampling plans is an assumption that the inspection procedures are error free. However, many inspection tasks are not error free; on the contrary, they may even be error prone. Two types of errors are possible in attribute sampling.

An item which is good may be classified as defective (Type I error, $e_1$), or an item that is defective may be classified as good (Type II error, $e_2$). So, for attribute sampling the apparent fraction of defective items in a lot is

$$\mathrm{Pe} = \mathrm{p}(1-\mathrm{e}_2)+(1-\mathrm{p})\mathrm{e}_1$$

### 3.4 Sampling Plans and the Effects of Inspection Errors

A single sampling plan for attributes is characterized by a sample size n and an acceptance number c. The procedure is: select n items at random from the lot. If there are c or fewer defectives in the sample, accept the lot, and if there are more than c defective items in the sample, reject the lot. According to Collins (1973) [21] if N and p represent the lot size and the true fraction of defective items in the lot, the average outgoing quality of the inspection with replacement is

$$AOQ = \frac{npe_2 + p(N-n)(1-p_e)P_{a_e} + p(N-n)(1-P_{a_e})e_2}{N - np_e - (1-Pa_e)(S-n)p_e}$$

where

n = Sample size

$e_1$ = Probability of Type - I error

$e_2$ = Probability of Type - II error

$p_e$ = Apparent fraction of defective items

$P_{ae}$ = Probability of acceptance with inspection error, given by

$$\sum_{x=0}^{c} \binom{n}{x} p_e^x (1-p_e)^{n-x}$$

Similarly, an expression for the average total inspection for the inspection process without replacement is

$$ATI = \frac{n + (1-Pa_e)(S-n)}{1-p_e}$$

**Example 3.1**

Let lot as size S= 4000, Sample size n=150, acceptance number c = 5. Collins (1973) [21] assumed four error-pairs as $(e_1, e_2)$ = (0,0),(0.01),(0,0.15) and (0.01.0.15) the AOQ and ATI are determined as a function of incoming fraction defective for each error-pair.

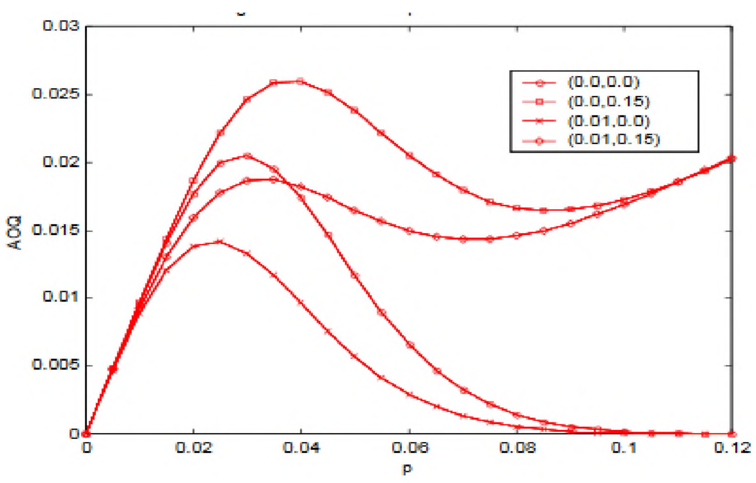



**Figure 3.1 Effects of Inspection Errors on AOQ**

Figure (3.1) examines the average outgoing quality as a function of fraction defective and errors. Type - I error reduces the AOQ due to the fact that more screening inspection takes places while Type - II error has the effect of causing higher AOQ for all values of p.

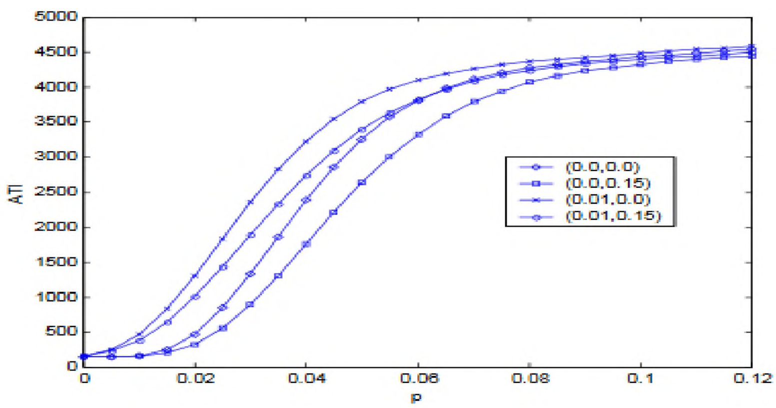



**Figure 3.2 Effect of Inspection Error on ATI**

Figure (3.2) illustrates the average total inspection as a function of fraction defective and errors. One can see that, the effect of Type - I error increases ATI and that of Type - II errors decreases it.

Procedure of double-sampling plan is in which, under certain circumstances, a second sample is required before the lot can be sentenced. A double sampling plan is defined by four parameters,

$n_1$ = Sample size on the first sample

$c_1$ = Acceptance number of the first sample

$n_2$ = Sample size on the second sample

$c_2$ = Acceptance number for both samples

1) A random sample of n1items is selected from the lot, and the number of defectives in the sample, $d_1$ observed.
2) (i) If $d_1 \leq c_1$, the lot is accepted on the first sample. If $d_1 > c_2$, the lot is rejected on the first sample.

   (ii) If $c_1 < d_1 \leq c_2$, a second random sample of size $n_2$is drawn from the lot, and the number of defectives in this second sample, $d_2$, observed.

   (i) If $d_1+d_2 \leq c_2$, the lot is accepted.

   (ii) If $d_1+d_2 > c_2$, the lot is rejected.

When rectifying inspection is performed with double sampling, according to Montgomery(1990)[51] the average outgoing quality, is given by

$$AOQ = \frac{[P_a^I(N - n_1) + P_a^{II}(N - n_1 - n_2)]p_e}{N}$$

where $P_a^I$ and $P_a^{II}$ denote the probability of acceptance on the first and second samples, respectively. The probability of acceptance of the lot, $P_a$would therefore be the sum of the above two probabilities. Assuming that all the defectives are replaced with good ones, the average total inspection is given by

$$ATI = n_1 P_a^I + (n_1 + n_2)P_a^{II} + N(1 - P_a)$$

**Example 3.2**

Let $n_1$= 50, $c_1$= 1, $n_2$= 100, $c_2$= 3. For the same set of four error-pairs as in the single sampling plan, one can determine the AOQ and ATI as a function of incoming fraction defective.

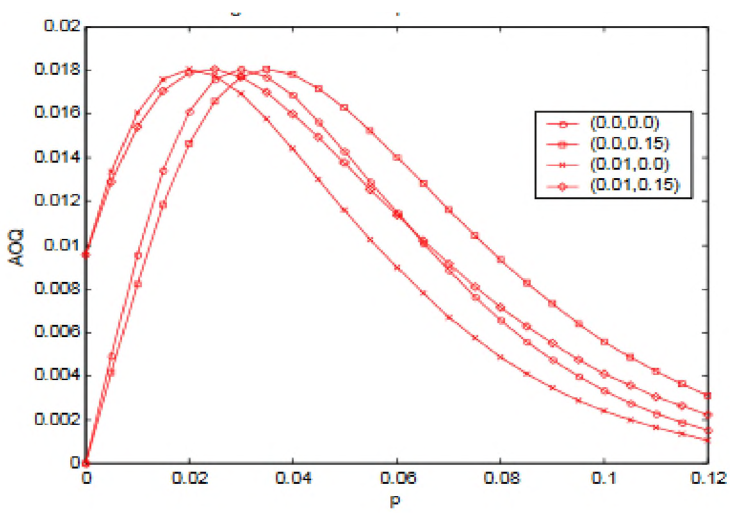



**Figure 3.3 Effect of inspection Error on AOQ**

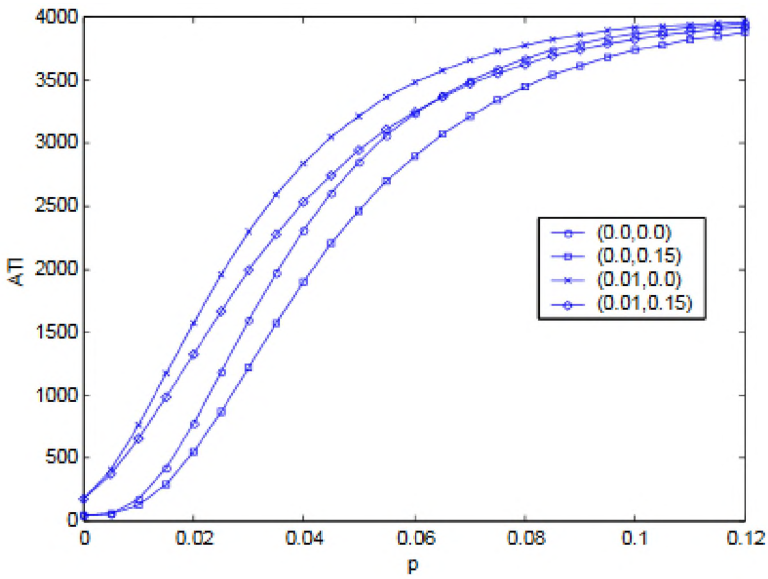



**Figure 3.4 Effect of inspection Error on ATI**

From figure (3.3) one can conclude that the effect of Type - I error increases average total inspection, while the Type - II errors decreases average total inspection.

While inspection components with several characteristics that can cause high cost repeatedly doing inspection is likely to reduce the expected total cost of inspection. The repeat inspection plan for such components, where an inspector has to make a classification of good, rework and scrap components is applied as follows: an inspector inspects one particular characteristic for each component entering the inspection process and classifies it as meeting specifications scrap or rework. All the accepted components and the ones that are found to be meeting specifications at rework station, go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected once.

Thus one cycle of inspection is completed. All accepted components, if necessary go to the next cycle of inspection, and this process is repeated a total of n times before the components are finally accepted. Here n is the optimal number of inspections necessary to minimize the total cost per accepted component.

The average outgoing quality for this plan is given by the ratio of the number of defective components after inspection and the total number of accepted components.

The average total inspection here is defined as the total number of inspections conducted in the optimal inspection plan. For a batch of M components ATI is computed as:

$$ATI = \sum_{j=1}^{n}(\sum_{i=1}^{N} M_{i,j} + M_j^j PG[\prod_{k=0}^{i-1}(1 - e_{kgs})]e_{igr} + M_{i,j}PS_{i,j}e_{isr} + M_{i,j}PR_{i,j}(1 - e_{irg} - e_{irs}))$$

where

J = cycle under inspection

I = stage or characteristic under inspection

$e_i$ = probability of misclassification an inspection can make for $i^{th}$ characteristic

N = number of characteristic

PG = probability of a component to be good while entering a new cycle

PR = probability of a component to be rework while entering a new cycle

PS = probability of a component to be scrap while entering a new cycle

**Example: 3.3**

A assume an inspection of a lot of 100 components with 3 characteristic. The probabilities of misclassification are taken to be 0.01,0.10 and 0.15. The AOQ and ATI are determined as a function of type II error at a fixed value of type I error. The other misclassification errors are fixed at 0.01 or 0.15.Figure 2.3 shows that AOQ increases as egs increases. For egs level of 0.01, 0.10 and 0.15 the increase in AOQ is similar where egs varies from 0.01 to 0.15. The other errors of misclassification are taken to be at 0.01

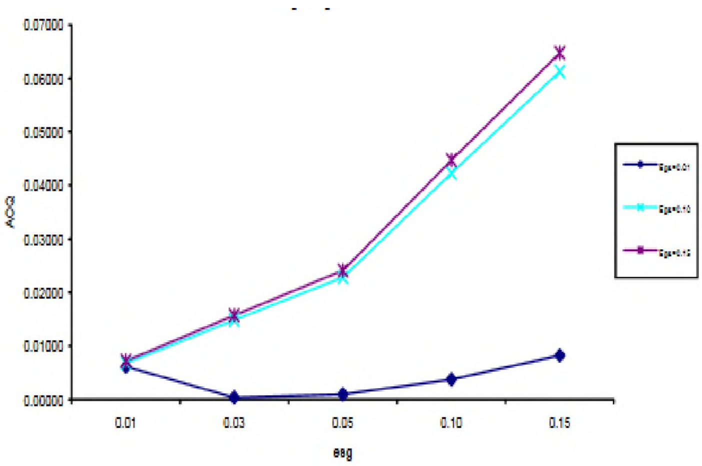



**Figure 3.5 Effect of the Error on the Average Outgoing Quality(AOQ) Egr=Erg=Ers=Esr=0.01**

Figure (3.5) shows that AOQ increases as Egs increases. However, at Egs level of 0.01 or 0.03, AOQ decreases by 82 per cent when Esg goes from 0.05 to 0.10. The other errors of misclassification are taken to be at 0.15.

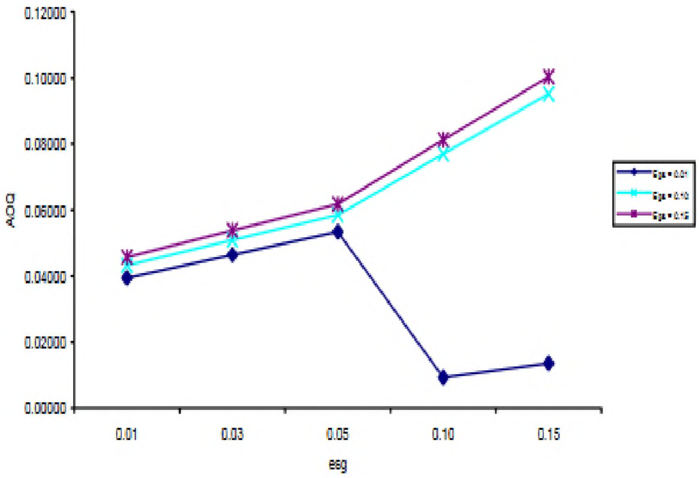



**Figure 3.6 Effect of the Error Egs on the Average Outgoing Quality (AOQ)Egr=Erg=Ers=Ers=0.15**

Figure (3.6) shows that the inspection load decreases with the increase in Egs. On the other hand, inspection load increases in a piece wise linear fashion as Esg increases. The other errors of misclassification are taken to be at 0.01.

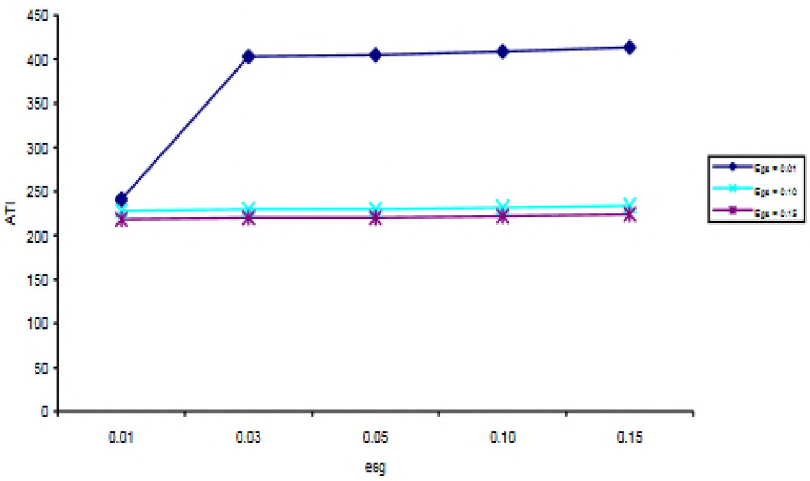



**Figure 3.7 Effect of the Error on the Average Total Inspection (ATI) Egr=Egr=Ers=Esr=0.01**

Figure (3.7) shows that the inspection load decrease with the inspection load decrease with the increase in Egs. On the other hand, inspection load increase as Esg increases. At Egs level of 0.01.

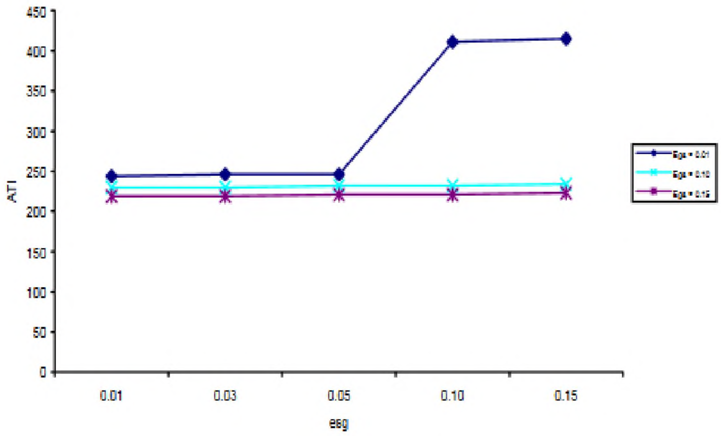



**Figure 3.8 Effect of the Error on the Average Total Inspection (ATI) Egr =Erg=Ers=Esr=0.15**

Similarly, Figure(3.8) shows that the inspection load decreases with the increase in Egs while it increases with the variation in Esg from 0.01 to 0.15.

One can conclude that the inspection error has a drastic impact on the performance of inspection and could result to misleading conclusions about product quality.

# CHAPTER - 4

# IMPACT OF TYPE-II INSPECTION ERRORS ON A RISK EXPOSURE CONTROL APPROACH BASED QUALITY INSPECTION PLAN

In this chapter " Impact of type-II inspection errors on a risk exposure control approach based quality inspection plan " by Bettayeb, Basseto (2016) [8] have been reviewed.

The model presented is an extension of the model of quality inspection planning based on risk exposure control approach, proposed in Bettayeb (2014) [7]. The model add an insurance perspective to quality control planning, by embedding a simplified model of the MAR concept used to control the amount of quality uncertain products, while judiciously allocating the available inspection capacity over a finite planning horizon. Refines the model of MAR by taking into account Type-II inspection error and analyses its impact on the risk exposure control approach based quality inspection plan is presented. The MAR concept, which was first introduced by Bean (1997) [5] for the semiconductor industry, seems very appropriate in this context. It refers to the production of potentially faulty products, which is directly influenced by production and quality control plans. It can also be strongly influenced by operations management, and particularly by the quality produced between two controls. The search for a way to reduce the amount of MAR in manufacturing can be seen as an opportunity to incorporate a mean to monitor, control, and manage massive scraps into the traditional approaches to production line assessment. A generic model of MAR and its usefulness in a risk exposure control approach based quality inspection plan is give in Bettayeb and Bassetto(2016) [8]. Their objective is to judiciously allocate inspection capacity, in order to maximize the effectiveness of quality control activities while guaranteeing a minimum level of risk exposure. If one can suppose that corrective actions are always beneficial for the system, even if they are initiated following a false alarm, Type-II errors would then be more harmful in terms of actual losses.

This is because the actual losses will continue to increase if failure has occurred since the last inspection and it is not detected by the current inspection, while the MAR will have been incorrectly reset to zero. In fact, the probability of Type-II error make the MAR stochastic as in Fig (4.1). After each inspection, the MAR continues to increase with a probability β and falls to zero with a probability 1-β.

The consequences of Type-II error on the risk exposure control approach are illustrated in example in Fig. (4.2). The MAR proposed model issues the inspection plan, as in Fig. (4.2(a)). In this model, there is no possibility of exceeding the $R_L$ threshold. The constraint generated by the prohibited zone is always considered. By considering β≠0, as in Fig (4.2(b)), there is a non null prohibited zone. For instance. if the second control (at $t_2$) fails, the MAR exceeds $R_L$ and reaches level C. Then, if the third control (at $t_3$ ) also fails, the MAR reaches level B, far exceeding $R_L$. When β≠0, the bold dashed path has a non-null probability of being created.

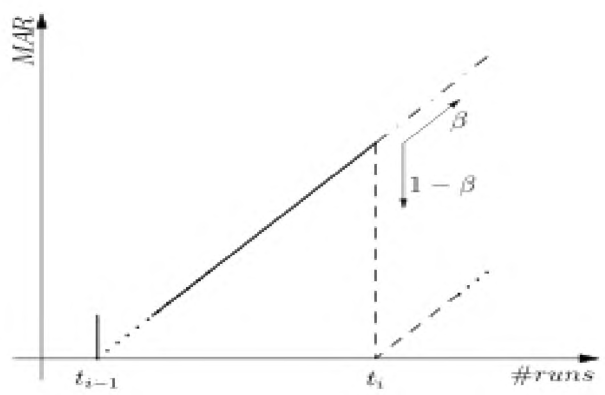
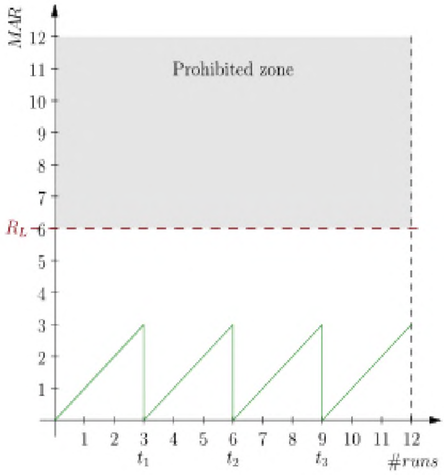
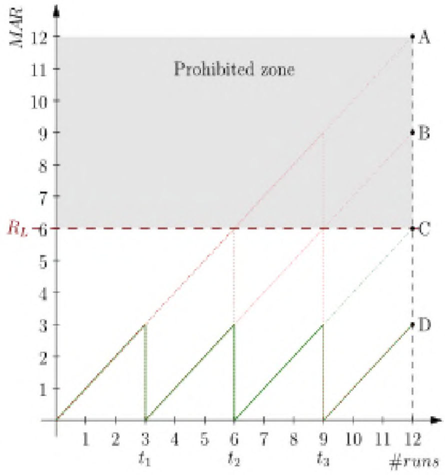



**Figure 4.1 Stochastic variation of the MAR under the β- risk effect.**




(a) Inspection plan with $\beta = 0$




(b) Inspection plan with $\beta \neq 0$

**Figure 4.2 Effect of β-risk on MAR control.**

### 4.1 The model

To take Type-II errors into account in quality control planning from an insurance point of view, one have to examine a finite set of MAR paths, each of which correspond to a combination of successful and failed planned inspection occurring during the planning horizon. The MAR paths can be enumerated using a decision tree, as in Fig.(4. 3). The increase in the number of paths (denoted by p) is exponential with the number of planned inspection denoted by n;p $\in \{1, \dots, 2^n\}$.

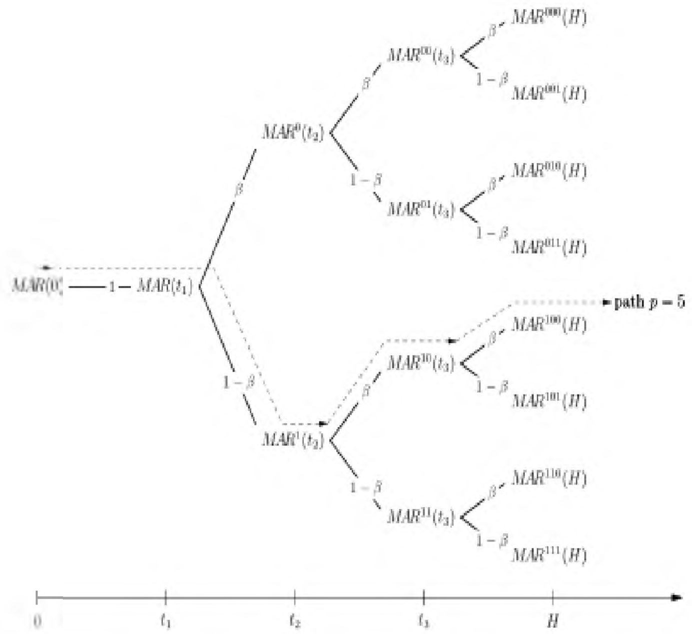


**Figure 4.3 Tree of MAR variation according to β-risk.**

Process and quality controls are based either on information obtained by inspecting the process parameters during the various process steps, or from the quality characteristics of semi finished or finished products. The given model is designed to take into account the β-risk of non detecting process drifts or failures that may impact all subsequent products until they are detected and the processing tool is fixed.

This development is valid for all failure modes, the occurrence of any of which will, bring the process to an out-of-control state.

Let $R_L$ be the threshold of risk exposure that should never be exceed if a major disturbance of the production organization is to be avoided. This means that the production organization could manage an actual loss below $R_L$, as revealed the measurement result. If the actual loss exceeds $R_L$, the organization will need significant resources to compensate for the delays caused, and perhaps to recall products already delivered and compensate affected customers. Control plans are usually designed to mitigate risks related to process or tool failures, which are commonly the causes of major product losses.

The control plan for a subsystem specifies the way in which each risk is monitored and controlled, depending on its ranking and a predefining threshold of risk acceptance. A risk based control plan design is proposed that will be suitable for either an entire production system or one of its subsystems, or a single production entity like a tool an assembly device, for instance. The planning horizon, which refers to the number of products to be processed, will be denoted by H.

Let X be a quality and (or process inspection plan characterized by the triptych

$(n^x, T^x, y^x)$.

**4.2 Mathematical formulization**

Knowing the probability of making a Type-II error at each inspection, the objective is to minimize the overall probability of the quality risk exposure (MAR) over a predefined threshold ($R_L$),i.e the probability that the MAR overlaps threshold $R_L$

Minimize

$$\sum_{p=1}^{P^x} \Pr(\mathrm{MAR}^{x,p} \text{ overlaps } R_L) \qquad (4.1)$$

Using this objective function makes it possible to determine, for a fixed number of inspections, their optimal positions in the planning horizon such that the probability of being exposed to a quality risk exceeding $R_L$is minimized. The probability of occurrence of the pth MAR path in inspection plan x is:

$$\Pr(Y^x = y_p^x) = \Pr(Y_1^x = y_{p,1}^x, \ldots, Y_k^x = y_{p,k}^x, \ldots, = Y_{n^x}^x = y_{p,n^x}^x)$$

$$= \prod_{k=1}^{n^x} \beta \,.\, (1 - y_{p,k}^x) + (1 - \beta).\, y_{p,k}^x \qquad (4.2)$$

In order to check whether or not the MAR path p exceeds $R_L$,it is sufficient merely to check the value of the MAR at the times of inspection (MAR peaks) and the MAR at the end of the planning horizon $MAR^{x,p}(H)$. The MAR peaks can be expressed recursively for all $k\in\{1,\dots,n^{x+1}\}$, as presented in eq.(4.3). At each inspection time $t_k$, the vale of the MAR peak depends on the distance $t_k$-$t_{k-1}$and the success or failure of the previous inspection at $t_{k-1}$. If the (k-1)th inspection is Type-II error-free(successful, or$y^x_{p,k}$= 1), then the k[th] MAR peak ($MAR(t_k)$) will be equal to the distance ($t_k$-$t_{k-1}$). ?If the (k-1)th inspection fails ($y^x_{p,k}$) = 0), the k[th] MAR peak will be equal to the distance ($t_k$-$t_{k-1}$)plus the value of the MAR peak at $t_{k-1}$:

$$MAR^{x,p}(t_k) \ = \ (1-y^x_{p,k-1}).MAR^{x,p}(t_{k-1}) + (t_k - t_{k-1})$$

$$= \ (t_k - t_{k-1}) + \sum_{j=1}^{k-1}(t_j - t_{j-1}).\prod_{m=j}^{k-1}(1-y^n_{p,m})$$

where $t_0 = 0$ and $t_{n^x+1}$= H The problem is mathematically formalized by an integer linear program as follows:

- **Decision variables**

  Decision variables are designed to define the position of each inspection

  $k\in\{1,\dots,n\}$ in H:

$$u_{i,k} = \begin{cases} 1, & \text{if } t_k = i, \\ 0, & \text{otherwise.} \end{cases} \qquad (4.4)$$

- **Intermediary variables**

  In order to characterize and evaluate the stochastic behaviour of the MAR when taking into account Type-II error made during inspection, some intermediary variables are needed. These variables depend on the of the inspections, which are represented by the various MAR paths.

  $t_k$: the kth inspection time, which corresponds to the index of the item to be inspected and is expressed as follows:

$$t_k = \sum_{i=1}^{H-1} i, u_{i,k} \quad \forall \quad k$$

$$= 1, \ldots . n \qquad (4.5)$$

P: the number of MAR paths, which corresponds to the number of all possible combinations regarding the state of success or failure of each inspection; $P=2^n$.

$y_{p,k}$=: the state of success or failure of the k th inspection of path $p \in \{1, \ldots, P\}$

$$y_{p,k} = \begin{cases} 1, & \text{if the kth inspection in path p is type} - \text{II error} - \text{free;} \\ 0, & \text{otherwise.} \end{cases} \qquad (4.6)$$

$v_{p,k}$ = the effect of success or failure of the kth inspection of path p on the subsequent MAR peak of that path $\forall \quad p = 1, \ldots P \qquad \forall \quad k = 1, \ldots, n$:

$$v_{p,k} = \begin{cases} 1 & \text{if } MAR^p(t_{k+1}) > R_{L;} \\ 0 & \text{otherwise.} \end{cases} \qquad (4.7)$$

where $MAR^p(t_{k+1})$ is expressed, by using Eqs (4.3) and (4.5), as follows:

$$MAR^p(t_{k+1}) = \sum_{i=1}^{H} i.(\, u_{i,k+1} - u_{i,k})$$
$$+ \sum_{j=1}^{k-1} \sum_{i=1}^{H} u_{i,j+1} - u_{i,j}) \,. \prod_{m=j}^{k-1} (1 - y_{p,m} \qquad (4.8)$$

$w_p$ = the state of path p with represent to the constrain related to the threshold $R_{L;}$ $\forall \quad p = 1, \ldots P$

$$w_p = \begin{cases} 1, & \text{if MAR path p } (MAR^p)\text{has at least one peck above } R_L \text{;} \\ 0, & \text{otherwise.} \end{cases} \qquad (4.9)$$

$$\text{i.e } w_{p,k} = \max_{1,\dots,n} v_{p,k}$$

- **Objective function**

  The aim is to minimize the overall probability of violating the constraint of the threshold $R_L$, i.e. the sum of probabilities of the paths p of the MAR for which $w_p$=1.

Minimize

$$\sum_{p=1}^{P} w_p \prod_{k=1}^{n} \beta(1-y_{p,k}) + (1-\beta)y_{p,k} \qquad (4.10)$$

The quantity corresponding to the probability that

$$\prod_{k=1}^{n} \beta(1-y_{p,k}) + (1-\beta)\, y_{p,k}$$

the path p occurs. Each MAR path is there realization of a Bernoulli

process composed of a sequence of successes and failures of the n-inspections.

- **Constraints**

  Constraints, are used to warrant the integrity of the decision variables and to express, algebraically, that certain 'logical' statements are true Eq.(4.15) implies that, at a given time, it is not possible to inspect more products that have been produced up to that time. Eq.(4.16) warrants that each product can be inspected once at most .Eq.(4.17) signifies that an inspection is used to inspect only one product. Eqs. (4.18) and (4.19) are used to determine $v_{p,k}$, the value of which depends on the validity of the logical statement: $MAR^p(t_{k+1})$ exceeds $R_L$. In fact, if the last statement is true, then m equals a very small positive real ($m=\epsilon^+$) and then $v_{p,k}$ =1, because Eq. (4.18)$\Rightarrow$ $v_{p,k} \le 1+\epsilon^+$.In the other case ($MAR^p(t_{k+1}) \le R_L$),m equals a very small negative real ($m=\epsilon^-$) and then $v_{p,k}$ =0, because Eq.(3.18)$\Rightarrow$ $v_{p,k} > \epsilon^-$ and Eq. (4.19) $\Rightarrow$ $v_{p,k} \le 1+\epsilon^-$. Finally, Eq.(4.20) expresses the statement that verifies whether or not MAR path p has at least one peak that exceeds $R_L$.

$$u_{i,k} \in \{0,1\} \qquad \forall\ i = 1, \dots, H-1 \qquad \forall\, k = 1, \dots, n \qquad (4.11)$$

$$v_{p,k} \in \{0,1\} \qquad \forall\ p = 1, \dots., P \qquad \forall\, k = 1, \dots, n \qquad (4.12)$$

$$w_p \in \{0,1\} \qquad \forall\ p = 1, \dots., P \qquad (4.13)$$

$$u_{H,n+1}=1 \qquad (4.14)$$

$$u_{i,k}=0 \qquad \forall\ i < k \qquad \forall\, k = 1, \dots, n \qquad (4.15)$$

$$\sum_{k=1}^{n} u_{i,k} \leq 1 \qquad \forall\ i = 1, \dots, H-1 \qquad (4.16)$$

$$\sum_{i=1}^{H} u_{i,k} = 1 \qquad \forall\, k = 1, \dots, n \qquad (4.17)$$

$$v_{p,k}>m \qquad \forall\, k = 1, \dots, n \qquad (4.18)$$

$$v_{p,k}>m+1 \qquad \forall\ p = 1, \dots., P \qquad \forall\, k = 1, \dots, n \qquad (4.19)$$

$$w_{p,k} = \max_{1, \dots, n} v_{p,k} \qquad \forall\ p = 1, \dots., P \qquad (4.20)$$

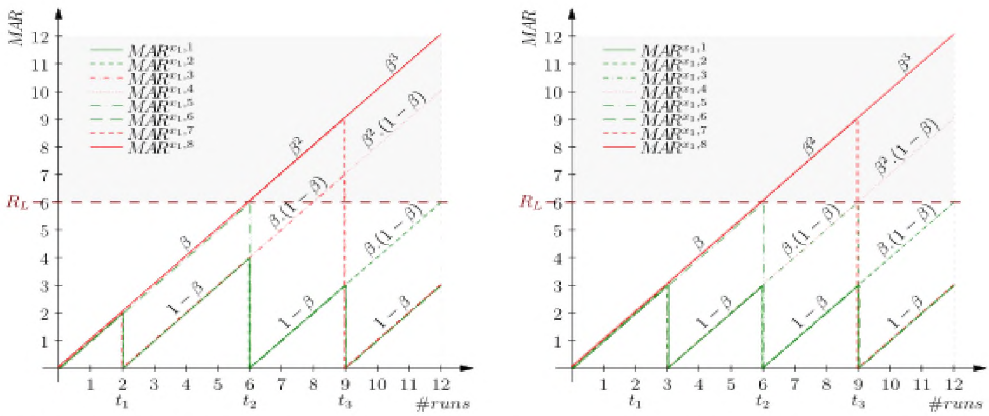
Where $m = \frac{MAR^{p}(t_{k+1}) - R_L}{M}$, with: M a large positive integers.

### 4.3 Complexity

The problem studied here has a complexity of $O(c_n^{H-1}$ $O(c_n^{H-1}2^n)$. where $c_n^{H-1} = \binom{H-1}{n} = \frac{(H-1)!}{n!(H-n-1)!}$ For a given number of inspections n<H, there are $c_n^{H-1}$ possibilities of choosing the inspection dates (k ∈) {1,...,n}, and there are $2^n$ MAR paths corresponding to each of them. Each MAR path $MAr^p$ has a probability $\Pr(Y = y_p)$ of containing a MAR peak that exceeds $R_L$.

### 4.4 Illustration with 3 inspections

To illustrate the purpose of the method , an example with 3 inspections to be executed during a planning horizon of 12 runs, applying two different inspection plans $x_1$ and $x_2$ is taken as illustrated in Fig.(4.4).The threshold $R_L$ is set to 6.




(a) First inspection plan: $t_1 = 2$; $t_2 = 6$ and $t_3 = 9$

(b) Second inspection plan: $t_1 = 3$; $t_2 = 6$ and $t_3 = 9$

**Figure 4.4 MAR variation according to β-risk for two different inspection plans.**

The corresponding decision variable matrices ($U^{x1}$and $U^{x2}$) and intermediary variables (y,$V^{x1}$,$V^{x1}$,$W^{x1}$and $W^{x1}$) as given below:

$$U^{x1}=\begin{pmatrix} u_{1,1}^{x1} & u_{1,2}^{x1} & u_{1,3}^{x1} \\ u_{2,1}^{x1} & u_{2,2}^{x1} & u_{2,3}^{x1} \\ \vdots & \vdots & \vdots \\ u_{i,1}^{x1} & u_{i,2}^{x1} & u_{i,3}^{x1} \\ \vdots & \vdots & \vdots \\ u_{12,1}^{x1} & u_{12,2}^{x1} & u_{12,3}^{x1} \end{pmatrix}=\begin{pmatrix} 0\,0\,0 \\ 1\,0\,0 \\ 0\,0\,0 \\ 0\,0\,0 \\ 0\,0\,0 \\ 0\,1\,0 \\ 0\,0\,0 \\ 0\,0\,0 \\ 0\,0\,1 \\ 0\,0\,0 \\ 0\,0\,0 \end{pmatrix}; \quad u^{x2}=\begin{pmatrix} 0\,0\,0 \\ 0\,0\,0 \\ 1\,0\,0 \\ 0\,0\,0 \\ 0\,0\,0 \\ 0\,1\,0 \\ 0\,0\,0 \\ 0\,0\,0 \\ 0\,0\,1 \\ 0\,0\,0 \\ 0\,0\,0 \end{pmatrix};$$

$$y^{x1}=y^{x2}=y=\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \\ \vdots \\ y_P \end{pmatrix}=\begin{pmatrix} y_{1,1} & y_{1,2} & y_{1,3} \\ y_{2,1} & y_{2,2} & y_{2,3} \\ \vdots & \vdots & \vdots \\ y_{p,1} & y_{p,2} & y_{p,3} \\ \vdots & \vdots & \vdots \\ y_{P,1} & y_{P,2} & y_{P,3} \end{pmatrix}=\begin{pmatrix} 1\,1\,1 \\ 1\,1\,0 \\ 1\,0\,1 \\ 1\,0\,0 \\ 0\,1\,1 \\ 0\,1\,0 \\ 0\,0\,1 \\ 0\,0\,0 \end{pmatrix};$$

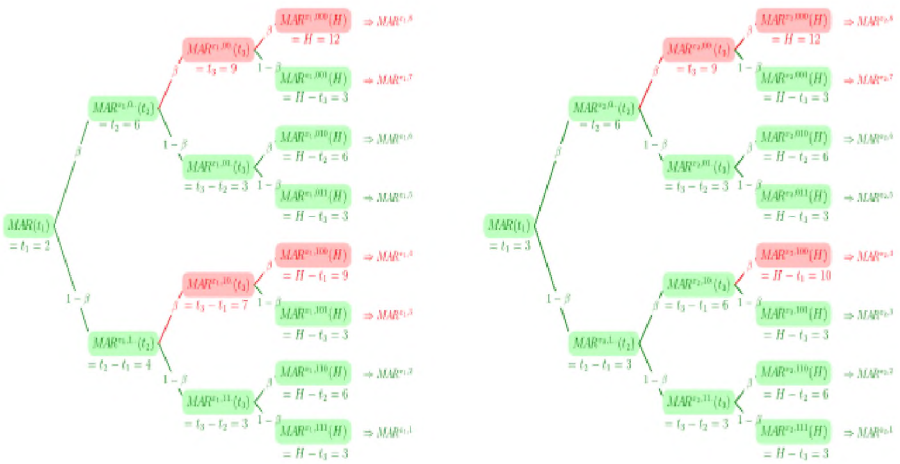
$$V^{x1}=\begin{pmatrix} v_{1,1}^{x1} & v & v_{1,3}^{x1} \\ v_{2,1}^{x1} & v_{2,2}^{x1} & v_{2,3}^{x1} \\ \vdots & \vdots & \vdots \\ v_{p,1}^{x1} & v_{p,2}^{x1} & v_{p,3}^{x1} \\ \vdots & \vdots & \vdots \\ v_{P,1}^{x1} & v_{P,2}^{x1} & v_{P,3}^{x1} \end{pmatrix}=\begin{pmatrix} 0\,0\,0 \\ 0\,0\,0 \\ 0\,1\,0 \\ 0\,1\,1 \\ 0\,0\,0 \\ 0\,0\,0 \\ 0\,1\,0 \\ 0\,1\,1 \end{pmatrix}; \quad V^{x2}=\begin{pmatrix} 0\,0\,0 \\ 0\,0\,0 \\ 0\,0\,0 \\ 0\,1\,1 \\ 0\,0\,0 \\ 0\,0\,0 \\ 0\,1\,0 \\ 0\,1\,1 \end{pmatrix};$$

$W^{x1} = (0\;0\;1\;1\;0\;0\;1\;1)$ and $W^{x2} = (0\;0\;0\;1\;0\;0\;1\;1)$

In matrices $U^{x1}$ and $U^{x2}$, corresponding to inspection plans $x_1$ and $x_2$ respectively, the number of columns equals the number of inspections and the number of lines corresponds to all their possible positions during the planning horizon (H-1).

Within inspection plan $x_1$ (respectively, $x_2$), the first inspection is planned at the second run (respectively, the third run), i.e. $u_{2,1}^{x1} = 1$ (respectively, $u_{3,1}^{x2} = 1$). The matrix y represents the sample space of the both random binary vectors $y^{x1}$ and $y^{x2}$ which depends only on the number of inspection planned. This matrix has $2^{n^{x_1}}$ rows, which correspond to all the possible combinations (paths) of success/failure of the planned inspections. The number of columns equals the number of inspections.

For example, the second row of matrix $y(y_2=(110))$ corresponds to the second MAR path, where inspections 1 and 2 are successful (error-free) and inspection 3 is $R_L(v_{p,k} = 1)$ or not ($v_{p,k} = 0$) for each path (line) at each inspection time (column). The dimensions of vectors $W^{x1}$ and $W^{x2}$ are the total number of paths of their respective inspection plans. An element of these vectors equal to 1 implies that the corresponding MAR path over laps the prohibited zone.



(a) First inspection plan: $t_1 = 2$; $t_2 = 6$ and $t_3 = 9$

(b) Second inspection plan: $t_1 = 3$; $t_2 = 6$ and $t_3 = 9$

**Figure 4.5 Tree of MAR according to β- risk of inspection plans $x_1$ and $x_2$**

For any inspection plan x with $n^x = 3$, the probability of each path of the tree (regardless of whether or not it contains a MAR peak that exceeds $R_L$ ) is expressed as follows:

$$\Pr(Yx = y_1^x) = \Pr(Y^x = (111)) = R_L;$$

$$\Pr(Yx = y_2^x) = \Pr(Y^x = (110)) = \beta(1-\beta)^2;$$

$$\Pr(Yx = y_3^x) = \Pr(Y^x = (101)) = \beta(1-\beta)^2;$$

$$\Pr(Yx = y_4^x) = \Pr(Y^x = (100)) = \beta^2(1-\beta);$$

$\Pr(Yx = y_5^x) = \Pr(Y^x = (011)) = \beta(1-\beta)^2;$

$\Pr(Yx = y_6^x) = \Pr(Y^x = (010)) = \beta^2(1-\beta);$

$\Pr(Yx = y_7^x) = \Pr\big(Y^x = (001)\big) = \beta^2(1-\beta);$

$\Pr(Yx = y_8^x) = \Pr(Y^x = (000)) = \beta^2;$

For each inspection plan ($X_1$ and $X_2$), the probability of exceeding the threshold $R_L$ can be computed by summing the probabilities of the paths that contain at least one MAR peak that exceeds $R_L$:

$\Pr(MAR^{x_1} \text{ overlaps } R_L) = \sum_{p=1}^{8} \Pr(MAR^{x_1-p} \text{ overlaps } R_L)$

$= \Pr(Yx = y_3^x) + \Pr(Yx = y_4^x) + \Pr(Yx = y_7^x) + \Pr(Yx = y_8^x)$

$= \beta(1-\beta)^2 + \beta^2(1-\beta) + \beta^2(1-\beta) + \beta^2$

$= \beta$

In this example, Pr ($MAR^{x_1}$ overlaps $R_L$) > Pr($MAR^{x_2}$ overlaps $R_L$) $\beta \in [0,1]$, which means that inspection plan $x_2$ dominates inspection plan $x_1$. In fact, $x_2$ is one of the optimal solutions for this example, since it minimizes the probability of exposure to an MAR exceeding $R_L$. However, no conclusion can be drawn about the optimality of $x_2$ with the respect to other values of $R_L$. Experiments are needed to characterize and analyze the behaviour of optimal solutions with the various parameters of the model (H, $R_L$, β and n) which is our aim in the following section.

### 4.5 Experiments

The experiments were carried out using small instances generated in order to analyze the effect of Type –II errors on mastery of the stochastic behaviour of MAR. The experiments were conducted by solving the ILP resulting from each combination of the following parameter ranges: H=10; $n \in \{1,2,\ldots,9\}$; $R_L \in \{1,2,\ldots,9\}$ and

$\beta \in$ { 0.01,0.03,0.05,0.07,0.09,0.1,0.3,0.5,0.7,0.9}.

**4.6 Computation time**

The evolution of the computation time of the optimal inspection plan is summarized in Fig.(4.6).It shows the variation of the average CPU time with the various values of β for each value of the ratio N/H. For instance, when N/H = 0.3, the average value of CPU time, among the various values of β varies from 0.85 s to 2.53 s, with an overall average of 1.47 s. The average CPU time increases slowly until it reaches its maximum at N/H = 0.7.where $Av_{R_L}/H$ $Avg_\beta$ CPU($R_L$/H, β) = 2.59 s, and then drops rapidly to 1.47 s. This is because the solution procedure explores the entire solution space, the cardinality of which grows with n as the multiplication of a concave function by an exponential function: $C_n^{H-1}.2^n$.

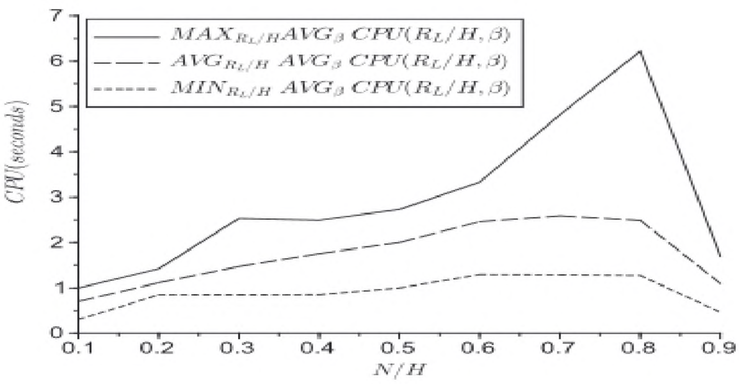



**Fig.4.6 Variation of CPU time.**

## 4.7 Behaviour of the objective function

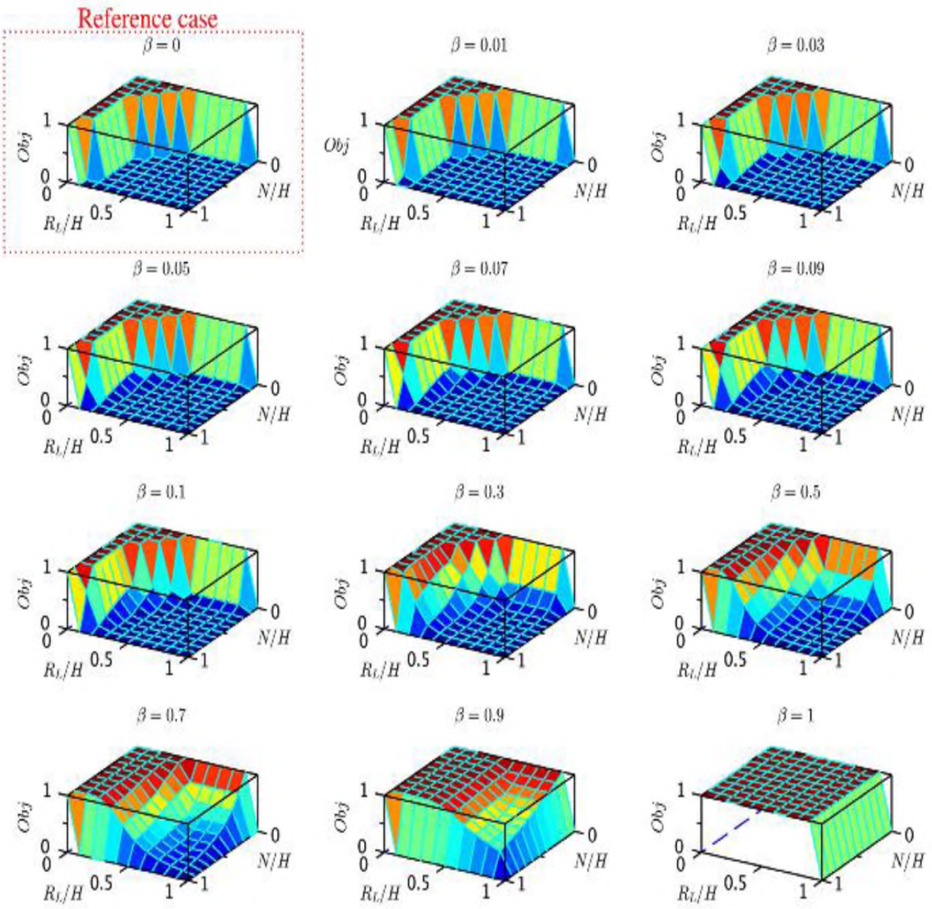



**Figure 4.7 Optimal solution surfaces for different values of β.**

Figure (4.7) shows the evolution of the objective function (a minimum probability of MAR exceeding $R_L$) with N/H and $R_L$ / H for various values of β. With this set of data, N/ H and $R_L$ / H vary within the range {0,0.1,0.2,...,0.9,1}. Note that the values of the objective function for the extreme values of these parameters (N/ H =0,N / H =1, $R_L$ / H=0 and $R_L$ / H=1) are immediate. For instance, when N / H=0 (no quality control planned), the probability of exceeding $R_L$ is always equal to 1 until $R_L$ becomes $\geq$ H.

In the latter case ($R_L$ / H$\geq$1 ), the probability of exceeding $R_L$ is always equal to 0, whatever the parameter N/H is. Each surface portion is obtained by four adjacent points from the experiments, and is characterized by the triptych (N / H, $R_L$ / H Obj (N / H,$R_L$ / H)).

The colour of the surface portion is the codification of the mean value of the objective function among its four constituent points. Note that these surfaces represent the optimum values ,i.e. they constitute a lower bound to the objective function, and the fact that no solution could be found with an objective function below the lower bound. For a given combination of N / H and $R_L$ / H, the quality of the solution will depend only on the inspections' locations in the planning horizon.

When there are no Type-II errors (reference case with β=0: perfect control), the objective function is either zero or one , and there is a minimum number of inspections

$n_{min} =[\frac{H}{R_L} -1]$ from which the objective function is minimized. When the probability of Type –II errors increases (β increases), the lower bound surface is increased globally but not homogeneously. The gradient is steeper when $R_L$ / H is low and N / H is high. However, when $R_L$ / H is high the gradient is steeper when N / H is low. Minimizing the overall probability of exposure to a loss exceeding $R_L$ has a lower bound surface that rises with β, whatever the rations $R_L$ / H and N / H are. This surface tends to a plan with a value of 1.

If there is a probability threshold to be respected, the intersection of the plan representing the value of this threshold and the lower bound surfaces will constitute the contour that defines the zone where the values of $R_L$ / H and N / H should be. Fig (4.8) shows the contours of optimal objective function surfaces of Fig (4.7) for different probability thresholds, from 0 (in dark blue) to 1 (in dark brown) with a 0.1 pace.

Each contour represents the frontiers of the N / H and $R_L$ / H values that warrant a predefined minimum value of the objective function. For each value of β, the pole of each contour results in two separate zones, as presented in the reference case.

The first zone, which is above the curve of the contour corresponding to the probability threshold $Pr_L$= 0,...,1, defines the combinations of $R_L$ / H and N / H values with which the probability of exposure to a risk exceeding RL is greater than or equal to $Pr_L$. The second zone corresponds to the combinations of $R_L$ / H and N / H values with which there exists at least one inspection plan that warrants a probability of exposure to a risk exceeding RL less than or equal to $Pr_L$. The first zone expands when β increases, at which point the second zone shrinks. This leads to fewer possibilities in the choice of $R_L$ and/ or N to guarantee a given threshold of the overall probability of risk exposure.

For instance, if β = 0.05 and $R_L$ / H=0.2,N / H should be greater than or equal to 0.4, in order to be able to warrant that an optimized inspection plan with an objective function less than or equal to $Pr_L$=0.2. If N/H is less than 0.4 there is a probability of more than 0.2 of exposure to a risk of loss of more than $R_L$ products.

**4.8 Exploitation of the results**

These experiments show that Type-II inspection error has an impact on the optimal inspection plan that minimizes the probability of risk exposure from an insurance perspective. Although the tested instance is relatively short, the experiments provide some insights into how to design risk-based inspection plans when inspection resources are not error free. To illustrate, let us take an example showing how these results can be exploited. The following steps are required:

1. Specify the framework and the input parameters: subsystem and failures to be monitored, quantities of products to be manufactured during the planning horizon, and inspection resource effectiveness.
2. Knowing the value of β, draw a corresponding sub-plot similar to those in Fig.3.8. β should reflect the ability of the inspection resource to detect the occurrence of a failure event. In fact, Type-II errors can be caused by any or all of the following: measuring tool ineffectiveness ,failure diagnosability, sampling size.
3. Knowing the threshold of risk exposure $R_L$.draw a vertical line corresponding to $(\mathrm{R_L/H})_{\mathrm{target}}$.As shown in the example in Fig.3.9 for β=0.07, the intersection points of this line with the various contours of objective function levels make it possible to determine the minimum values of N/H to warrant each of them. For instance, with N/H ≥ $(\mathrm{N/H}_{\mathrm{min}}^{0.3})$,there is an inspection plan that has a probability that is less than or equal to 0.3 of leading to an MAR that exceeds the threshold $R_L$that corresponds to $(\mathrm{R_L/H})_{\mathrm{target}}$. Similarly, this probability is greater than or equal to 0.9 when N/H≤ $(\mathrm{N/H})_{\mathrm{min}}^{0.9}$.

4. Knowing the maximum inspection capacity, draw a horizontal line corresponding to $(N/H)_{target}$ . As shown in Fig. 9, the intersection points of this line with the various contours of the objective function levels make it possible do determine the minimum values of ($R_L$/H) that could be achieved, with a given probability, by an optimized inspection plan. For instance, with $(N/H)_{target}$ =0.2, the optimized inspection plan has a probability that is less than or equal to 0.9 of leading to an MAR that exceeds 0.31. H, a probability that is less than or equal to 0.2 of leading to an MAR that exceeds 0.39.H and a probability that is less than or equal to 0.1 to leading to an MAR that exceeds 0.46.H.

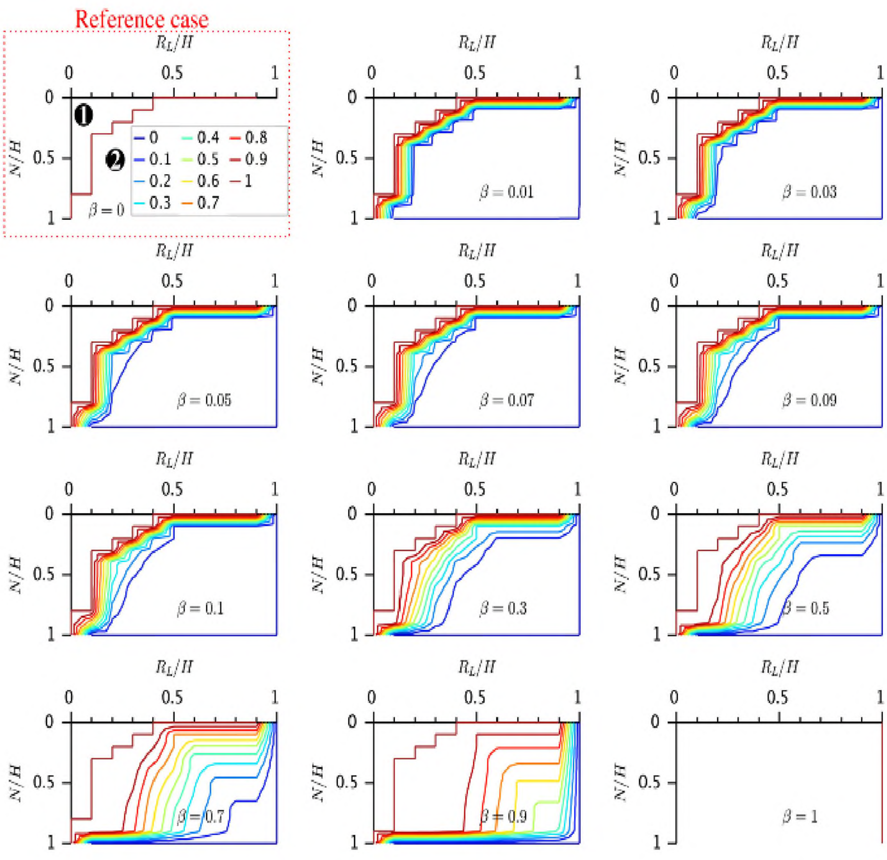



**Figure 4.8 Contours of the optimal objective function: $R_L$/H vs N/H**

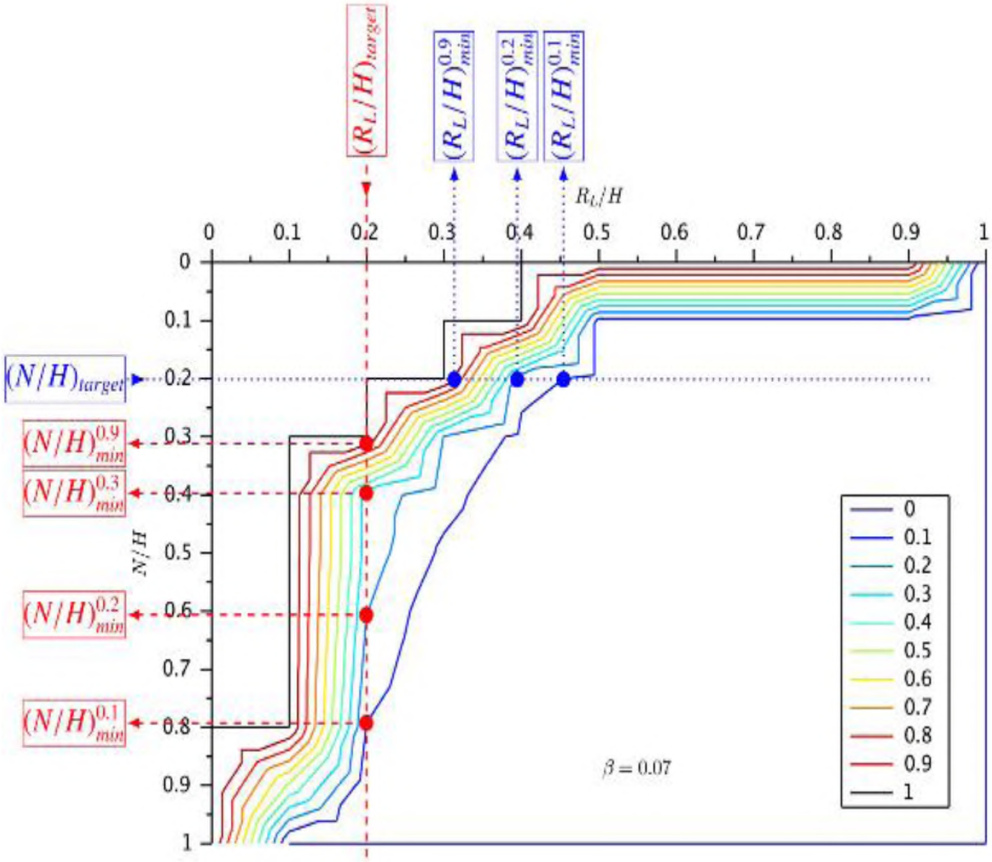



**Figure 4.9 Contours of optimal objective function $R_L$/H vs N/H for β = 0.07.**

Using these experiments one can analyze the effect of Type-II error on the optimized inspection plan form insurance perspective. The impact of Type-II inspection error on the material-at-risk control is found to be significant. It was expected that the greater the risk of non-detection is, the greater the number of inspections must be increased to ensure a given probability of falling in the prohibited zone of over-exposure. However ,the merit of this document is to provide an advance in Risk-Exposure control approach based Quality Inspection Plan which is adjusted according to the degree of confidence required when considering Type-II inspection error.

# CHAPTER - 5

# IMPACT OF INSPECTION ERRORS ON SINGLE, DOUBLE AND CHAIN INSPECTION SAMPLING PLANS

In this chapter "Impact of Inspection Errors on Single, Double and Chain Inspection Sampling Plans" by Edokpa Idemudia Waziri and Odunayo Joseph Braimah(2016) [32] have been reviewed.

It is usually assumed that the inspector is perfect neglecting the possibility of

Type –II errors in sampling inspection. In this study, the authors the impact of inspection errors on rectifying single, double and chain sampling inspection plans with average outgoing quality (AOQ) and average total inspection (ATI) used to assess their performances is studied.

Sampling Inspection is an important aspect of quality control. However, for practical and economic considerations it is not feasible to inspect each item in every lot completely. The usual practice is to draw a random sample from the lot and take a decision on lot disposition based on information obtained from the sample. When a sample is inspected for the purpose of making a decision to either accept or reject the lot, the procedure is called Acceptance Sampling. A typical application of Acceptance Sampling is as follows: Lots are inspected immediately after production or before the product is shipped to the customer. Also lots are inspected as they are received from the supplier. While the first case is referred to as outgoing inspection, the second case is called incoming inspection.

Some sampling inspection programs may require corrective action when lots are rejected. This generally takes the form of 100% inspection or screening of rejected lots, with all discovered defective items either removed for subsequent rework or return to the supplier or replaced by good items. Such sampling programs are called rectifying inspection because the inspection activity affects the final quality of the outgoing product.

## 5.1 Rectifying inspection

Suppose that incoming lots to the inspection activity have fraction defective $p_0$. Some of these lots will be accepted, and others will be rejected. The rejected lots will be screened, and their final fraction defective will be zero. However, accepted lots have fraction defective $p_0$. Consequently, the outgoing lots from the inspection activity are a mixture of lots with the fraction defective $p_0$ and fraction defective zero, so the average fraction defective in the stream of outgoing lots in $p_1$, which is less than $p_0$. Thus, a rectifying inspection program serves to "correct" lot quality. For the purpose of this work, single, double and chain rectifying inspection plans are studied.

In rectifying single sampling inspection plan, a decision to accept or reject a lot is based on the result of one random sample from the lot. The procedure is to take a random sample of size n and inspect each item. If the number of defectives in the sample do not exceed a specified acceptance number c, the lot is accepted. If the number of defectives in the sample is more than the acceptance number c, the lot is rejected and the rejected lot is subjected to 100% inspection and any defectives found is replaced by non-defective items.

For rectifying double sampling plan, a first sample of size $n_1$ is taken from a lot for inspection and the number of defectives$d_1$ is compared with acceptance number $c_1$ and $c_2$, if the number of defectives in the first sample is less than or equal to the acceptance number in the first sample $c_1$,the lot is accepted. If the number of defectives in the first sample is more than the acceptance number $c_2$, the lot is rejected and screened where all the defectives are replaced by non-defectives. Also if the number of defectives in the first sample lies between $c_1$ and $c_2$ , then a second sample of size $n_1$ is drawn. If the combined number of defectives in both samples is more than $c_2$ the lot is accepted. However, if the combined number of defectives in both samples is more than $c_2$ the lot is rejected and inspected 100% to replace all defectives by non-defective items.

Chain sampling inspection plan makes use of previous lots results and the current lot information to accept or reject a lot. This plan makes use of two acceptance numbers $c_1$ and $c_2$ instead of 0 and 1 of ChSP-1. Under this plan two stages are involved, in the first stage, a sample size n is selected from each lot and tested for conformance to the specified requirements.

If the observed number of defectives $d_0 \leq c_1$ the lot is accepted but if $d_0 \geq r$ the lot is rejected. Therefore, all the rejected lots are subjected to 100% screening. The unique feature here is that a lot can also be accepted when more than ($c_1$and up to $c_2$) defective units are observed in the sample, provided that the total number of defectives, including those in the current sample plus those from the samples of specified number of successive nearby lots is less than or equal to $c_2$.

The performance of any inspection plan is greatly influenced by inspection errors. An inspector can commit two types of errors. This can be true in the sample as well as in the rectified units. The authors consider rectifying inspection for lots vulnerable to inspection error. The impact of inspection errors on the performance measure of some rectifying sampling inspection plans will be assessed. The performance measures used are: Average outgoing Quality(AOQ) and Average total Inspection (ATI). The authors aimed at studying the impact of inspection errors on some Rectifying Inspection Plans.

Jalbout (2002) [41] discussed a mathematical model that can be applied to calculate such quantities as the expected number of defectives items replaced in an accepted lot and other functions of this process. A modified model for determining the optimum process mean under the rectifying inspection plan was presented by Pulak and Al-sultan's(2005) [17].

The modified model for determining the economic specification limit under the single sampling inspection plan with inspection error was given by pulak and Al-Sultan's (2008) [18]. However, these authors do not consider the effects of inspection errors on the performance measures of rectifying sampling inspection plans. In this chapter the impact of inspection errors on some rectifying inspection plans using performance assessments are analyzed, i.e Average outgoing Quality (AOQ) and Average total Inspection (ATI).

## 5.2 Errors in Inspection Process

To ease the derivation of Type - I & Type - II, Let T (true) and A (apparent) represent the true and the observed items respectively Define:

T=0 when the inspected item is truly non-defective, T=1when the inspected item is truly defective, and A=0when the inspected item is observed (or classified) as non-defective, A=1when the inspected item is observed (or classified) as defective.

The following table gives all possible combinations of the realization of the two random variables and the two types of errors:

**Table 5.1 Types of Inspection Errors**

True State

| **Inspection** | **T=0** | **T=1** |
|---|---|---|
| **A=0** | No inspection error | Type II error ($e_2$) |
| **A=0** | Type I error ($e_1$) | No inspection Error |

As long as the lot size is sufficiently large the probability

$P_r$(T=1) that a particular unit is actually 'defective' as defined by the process, is approximately the true fraction defective (p)in the lot. Similarly, the empirical fraction defective ($p_e$)is defined as the probability $P_r$ (A=1)that a randomly selected unit is classified defective.

**Table 5.2 Probability of the realisation of two state variables**

| State Variable | Realisation | Probability of occurrence | Inspection of the probabilities |
|---|---|---|---|
| T | 1 | P | True fraction defective |
| | 0 | 1-p | True proportion of 'good' units |
| A | 1 | $P_e$ | Observed fraction 'defective' |
| | 0 | $1-P_e$ | Observed proportion of 'good'units |

The probabilities of the inspection errors in Table (5.1) are conditional probabilities. The probability of Type I error is given by:

$$\Pr(e_1) = \Pr\left(A = \frac{1}{T} = 0\right) = \frac{\Pr(A = 1, T = 0)}{\Pr(T = 0)} \tag{5.1}$$

, (5.1)

and that of Type - II inspection error by

$$\Pr(e_2) = \Pr\left(A = \frac{0}{T} = 1\right) = \frac{\Pr(A = 0, T = 1)}{\Pr(T = 1)} \tag{5.2}$$

Inspection errors occur with probability $e_1$for the type I error and with probability $e_2$for the type II error. This determines the probability that a defective unit is actually classified defective to be Pr (A=1/T=1) = $1-e_2$. Type II inspection error has the effect that instead of observing the actual fraction defective p, we observe a lower fraction defective, namely $p(1-e_2)$.Type I inspection error has the opposite effect, since amongst the proportion 1-p of good units each is classified defective with probability $e_1$. Combining these two types of errors results in an observed fraction defective which is the fraction of incoming items which will be judged defective by the inspector and it's written as:

$$P_e = p(1 - e_2) + (1 - p)e_1 \tag{5.3}$$

## 5.3 Rectifying Single sapling plan

In a single sampling plan, the probability of acceptance is synonymous with the probability that the number defective in the sample is less than or equal to acceptance number c. Under perfect inspection, the probability of acceptance ($P_a$) is calculated using the binomial model:

$$Pa = p(x \leq c) = \sum_{x=0}^{c} \binom{n}{x} P^x (1-P)^{n-x} \qquad (5.4)$$

where c= the acceptance number, n = sample size, p= the true fraction defective. The consideration of inspector error in the formula for the probability of acceptance causes the value of the true fraction defective p to be replaced by the value of the apparent fraction defective ($p_e$) in equation (5.3) Thus equation (5.4) becomes

$$P_{a_e} = \sum_{x=0}^{c} \binom{n}{x} [e_1(1-p) + p(1-e_2)]^x . [1 - e_1(1-p) + p(1-e_2)]^{n-x}$$

$$P_{a_e} = \sum_{x=0}^{c} \binom{n}{x} p_e^x (1-p_e)^{n-x} \qquad (5.5)$$

## 5.4 Average Outgoing Quality

This is widely used for the evaluation of a rectifying sampling plan. The average outgoing quality in the lot that results form the application of rectifying inspection. It is the average value of lot quality that would be obtained over a long sequence of lots from a process with fraction defective p.

It is assumed that the lot size is N and that all discovered defectives are replaced with good units. The number of defective items in the unscreened portion of the accepted lot is:

$P(N-n)p_a$ Which may be expressed as an average fraction defective, called the average outgoing quality or

$$AOQ = \frac{p(N-n)p_a}{N}$$

When considering inspector error, the actual average outgoing quality with replacement of all the items classified as defective is written as:

$$AOQ_e = \frac{npe_2 + p(N-n)(1-p_e)P_{ae} + p(N-n)(1-P_{ae})e_2}{N - np_e - (1-P_{ae})(N-n)p_e} \quad (5.6)$$

where $P_{a_e}$ the probability of acceptance, considering inspector error, $p_e$ is the apparent fraction defective and $e_2$ is Type - II error.

### 5.5 Average Total Inspection

If the lot quality is $0 < p < 1$, the average amount of inspection per lot will vary between the sample size n and the lot size N. If the lot is of quality p and the probability of lot acceptance is $p_a$, then the average total inspection per lot will be

$$ATI = n + (1-p_a)(N-n) \quad (5.7)$$

When inspection error are considered, the value of the probability of acceptance changes from $P_a$ to $P_{a_e}$. Than the average amount of inspection may be calculated as:

$$ATI_e = n + (1-p_a)(N-n) \quad (5.8)$$

### 5.6 Rectifying Double Sampling Plan

When calculating the probability of acceptance for a double sample inspection plan, the possibility of using either one samples must be taken into account. The probability of acceptance for the perfect inspection plan is equivalent to probability of finding $c_1$ or less defective in the first sample, plus the probability of finding $c_2$ or less in both samples, provided there was no decision made on the first sample.

The probability of acceptance on the first sample ($P_{a_1}$) is calculated as:

$$P_{a_1} = p\{x_1 \le c_1\} = \sum_{x_1=0}^{c_1} \binom{n_1}{n_1} p^{x_1}(1-p)^{n_1-x_1}$$

The probability of acceptance on the second sample ($P_{a_2}$) may be calculated by combining the probabilities of the following mutually exclusive conditions:

$P_{a_1} = \Pr(x_1 + x_2 \leq c_2, \text{given } c_1 < x_1 \leq c_2)$, at any given p where $n_1$ and $n_2$ represented the number of defectives observed in the combined samples of $n_1$ and $n_2$ respectively. Therefore under perfect inspection, the probability of acceptance on the basis of the second sample is written as:

$$P_{a_2} = \sum_{x_1=c_1+1}^{c_2} \left\{ \left[ \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} \right] \times \left[ \sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} p^{x_2} (1-p)^{n_2-x_2} \right] \right\}$$

The probability of acceptance for the combined samples ($P_a$) is the sum of $P_{a_1}$ and $P_{a_2}$ i.e

$$P_a = P_{a_1} + P_{a_2} = \sum_{x_1=0}^{c_1} \binom{n_1}{n_2} p^{x_1} (1-p)^{n_1-x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[ \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} \right] \times \left[ \sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} p^{x_2} (1-p)^{n_2-x_2} \right] \right\}$$

The probability of acceptance of the lot when inspector error is considered ($P_{a_e}$) is calculated as:

$$P_{a_e} = \sum_{x_1=0}^{c_1} \binom{n_1}{n_2} p^{x_1} (1-p)^{n_1-x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[ \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1} \right] \times \left[ \sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} p^{x_2} (1-p)^{n_2-x_2} \right] \right\}$$

The probability of rejection on the first sample ($P_{r_1}$) which is the same as the probability of finding more than $c_2$ defective items in the first sample ($n_1$) is:

$$P_{r_1} = 1 - \sum_{x_1=c_1+1}^{c_2} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1-x_1}$$

The actual probability of rejection on the first sample considering inspector error is:

$$P_{r_{1e}} = 1 - \sum_{x_1=c_1+1}^{c_2} \binom{n_1}{x_1} p_e^{x_1} (1 - p_e)^{n_1 - x_1} \qquad (5.9)$$

When rectifying inspection is carried out with double sampling, all the defective items encountered in the rejected lot are replaced by good ones.

### 5.7 Average Outgoing Quality (AOQ)

The number of defectives in the lot when it is accepted on the basis of the first sample is $pP_{a_1}(N - n_1)$ and that remaining in the lot when accepted on the basis of the second sample, $pP_{a_1}(N - n_1 - n_2)$. Therefore the average outgoing quality (AOQ) under perfect inspection is $\frac{p[P_{a_1}(N-n_1)+P_{a_2}(N-n_1-n_2)]}{N}$

On considering the inspection error, the expected number of defectives remaining is obtaining as follows:

i. The number of defective items classified as non-defective in the first sample $n_1 e_2 p$

ii. The number of defective items classified as non-defective in the second sample if no decision is made on the first sample; $n_2(1-Pd_{1_e})e_2\, p$

iii. The number of defective items in unscreened portion of an acceptance lot on the basis of first sample under inspection error:$(N-n_1)Pa_{1_e}p$

iv. The number of defective items in the unscreened portion of the lot accepted on the second sample: :$(N-n_1-n_2)Pa_{2_e}p$

v. The number of items classified as non-defective items in the screened portion of the rejection lot on the first sample: :$(N-n_1-n_2)Pr_{1_e}e_2p$

vi. The number of items classified as non-defective in screened portion of the lot not rejected on the first sample but is rejected on the second sample is $(N-n_1 - n_2)((1-p_{a_e} - pr_{1_e})e_2p$

Therefore the Average outgoing Quality when inspector error is considered is

$$AOQ_e = \frac{[n_{1e_2}p+n_2(1-pd_{1e})e_2p+(N-n_1)p_{a_{1e}}p+(N-n_1-n_2)p_{a_{2e}}p+(N-n_1)p_{r_{1e}}e_2p+(N-n_1-n_2)(1-p_{ae}-p_{r_{1e}})e_2p]}{N}$$

$$(5.10)$$

## 5.8 Average Total Inspection (ATI)

The average total inspection under perfect inspection is calculated as:

$$ATI = n_1 + n_2(1 - P_{d_1}) + (N - n_1)P_{r_1} + (N - n_1 - n_2)(1 - P_a - P_{r_1})$$

This is reduced as :

$$ATI = n_1 + n_2(1 - P_{a_1}) + (N - n_1 + n_2)P_{r_1}(1 - P_a)$$

The average total inspection for double sample inspection when inspector error is considered is :

$$ATI_e = n_1 + n_2\left(1 - P_{a_{1e}}\right) + (N - n_1 + n_2)P_{r_1}(1 - P_{a_e}) \quad (5.11)$$

## 5.9 Chain Sampling Plan

In order to develop a mathematical expression of chain sampling plans, it is essential to first develop a mathematical model for single stage sampling plans in the presence of inspection errors. The mathematical expression for single sampling based on hyper geometric distribution

$$\Pr[Z = d|n, D, N;\ \rho\ \rho'] = \sum_{y} \frac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}} \sum\nolimits_{y} \binom{y}{w}\binom{n-y}{d-w} \rho^{w}\,(1-\rho)^{y-w}\rho'^{d-w}$$
$$(1-\rho')^{n-y-d+w} \quad (5.12)$$

where $\rho'$ and $\rho$ to stand for the type I and type II inspection errors respectively.

Therefore the probability of acceptance for a single stage acceptance sampling is given by:

$$P_s = \sum_{d=0}^{c} (\Pr(Z = d|n, D, e_1, e_2)),$$

where $P_{s(c,n,D,e_1,e_2)} =$

$$\sum_{d=0}^{c}\left[\sum_{y}\frac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}}\left(\sum_{w}\left(\binom{Y}{W}\binom{n-Y}{d-W} e_1^{d-w} e_2^{y-w(1-e_1)^{n-y-d+w}}(-e_2)^{w}\right)\right)\right] \qquad (5.13)$$

Biegel (1974) [15] found out that error rate is related to the process quality or process fraction of defectives p. He proposed linear model given below:

$$e_1(P) = \left.\begin{matrix} a_1 + b_1 P \\ a_2 + b_2 P \end{matrix}\right\} \qquad (5.14)$$

where p is the process fraction of defectives ranging from zero to one.

Mathematical expression of the probability of acceptance under this linear model can be obtained for the single stage sampling plan as:

$$P_s = \sum_{d=0}^{c} (\Pr(Z = d|n, D, e_1(p), e_2(p)))$$

$$= \sum\nolimits_{d=0}^{c}\left[\sum_{y}\frac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}}\left(\sum_{w}\left(\binom{Y}{w}(1-a_2-b_2p)^{w}a_2\right.\right.\right.$$

$$\left.\left.\left. + b_2 p^{y-w}\binom{n-Y}{d-w}(a_1+b_1p)^{d-w}(1-a_1-b_1p)^{n-y-d+w}\right)\right)\right] \qquad (5.15)$$

and probability of acceptance for chain sampling plan

$$(P_{ch}) = \sum_{d_0=c_1+1}^{r+1}\left[\sum_{y}\frac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}}\left(\sum_{w}\left(\begin{array}{c} *\binom{Y}{w}\left(\frac{N-Na_2-Db_2}{N}\right)^{w}\left(\frac{Na_2+Db_2}{N}\right)^{Y-w}\binom{n-Y}{d-w}\left(\frac{Na_1+Db_1}{N}\right)^{d-w}\left(\frac{N-Na_1-Db}{N}\right)*^{n-Y-d+w} \\ \sum_{d_{pre}=c}^{c_2-d_0}\left(\sum_{i}\frac{\binom{(k-1)D}{i}\binom{(k-1)(N-D)}{(k-1)n-i}}{\binom{(k-1)N}{(k-1)n}}\left(\sum_{w_2}\binom{i}{w_2}\binom{(k-1)n-i}{z_{pre}-w_2}\left(\frac{N-Na_2-Db_2}{N}\right)^{w_2}\left(\frac{Na_2-Db_2}{N}\right)^{i-w_2}\right.\right. \\ \left.\left. *\left(\frac{Na_1+Db_1}{N}\right)^{d_{pre}-w_2}\left(\frac{N-Na_1-Db}{N}\right)^{(k-1)n-i-d_{pre}+w_2}\right)\right) \end{array}\right)\right)\right]$$

(5.16)

Under verifying inspection error AOQ is given as

$$AOQ = \frac{(N-n)pP_{ch} + (N-n)pe_2(p)(1-P_{ch}) + npe_2(p)}{N}$$

$$\frac{\left((N-n)(p-p(e_1(p)+e_2(p))+e_1(p))\frac{pe_2(p)}{1-p+p(e_1(p)+e_2(p))-e_1(p)}\right)(1-P_{ch})}{N}$$

$$+\frac{n(N-n)(p-p(e_1(p)+e_2(p))+e_1(p))\frac{pe_2(p)}{1-p+p(e_1(p)+e_2(p))-e_1(p)}}{N}$$

$$= \frac{De_2(p)}{(1-e_1(p))(N-D)+De_2(p)} + \frac{(N-n)}{N}\frac{D}{N}\left(\frac{(N-D)(1-e_1(p)-e_2(p))}{(1-e_1(p))(N-D)+De_2(p)}\right)P_{ch}$$

$$= \frac{D(a_2+b_2p)}{(1-a_1-b_1p)(N-D)+D(a_2+b_2p)} + \frac{(N-n)}{N}\frac{D}{N}\left(\frac{(N-D)(1-a_1-b_1p-a_2-b_2p)}{(1-a_1-b_1p)(N-D)+D(a_2+b_2p)}\right)P_{ch}$$

$$AOQ = \frac{D(Na_1+Db_2)}{(N-Na_1-Db_1)(N-D)+D(Na_1+Db_2)} + \left(\frac{N-n}{N}\right)\left(\frac{D}{N}\right)\left(\frac{N-D((1-a_1-a_2)N-(b_1-b_2)D)}{N-Na_1-Db_1(N-D)+D(Na_1+Db_2)}\right)P_{ch}$$

(5.17)

The value for $P_{ch}$ in (5.16) can be substituted in (5.17) to get the average outgoing quality of chains sampling plans.

$$ATI = P_{ch}\left(n+\frac{NP_e}{1-NP_e}\right)+(1-P_{ch})\left(N+\frac{NP_e}{1-NP_e}\right)$$
$$= \frac{N^3-N^2(N-n)P_{ch}}{(N-Na_1-nb_{2p})(N-D)D(Na_1+Db_2)} \qquad (5.18)$$

Similarly $P_{ch}$ in (4.16) can be substituted in (4.18) to given the formula for ATI.

## 5.10 Rectifying Chain Sampling Inspection

$$\text{PDR}=(\text{N-n})\text{pe}_2(\text{p})(1-\text{P}_{\text{ch}})+\text{npe}_2(\text{p})+\left((\text{N}-\text{n})\left(\text{p}-\text{p}(\text{e}_1(\text{p})+\text{e}_2(\text{p}))+\text{e}_1(\text{p})\right)\frac{\text{pe}_2(\text{p})}{1-\text{p}+\text{p}(\text{e}_1(\text{p})+\text{e}_2(\text{p}))-\text{e}_1(\text{p})}\right)(1-\text{P}_{\text{ch}})+\text{n}(\text{p}-\text{p}(\text{e}_1(\text{p})+\text{e}_2(\text{p}))+(\text{e}_1(\text{p}))\frac{\text{pe}_2(\text{p})}{1-\text{p}+\text{p}(\text{e}_1(\text{p})+\text{e}_2(\text{p}))-\text{e}_1(\text{p})} \quad (5.19)$$

$$\text{PDA}=\text{P}_{\text{ch}}(\text{N}-\text{n})\text{p} \quad (5.20)$$

The Tables and Figures below show the results of the effects of inspection errors on the AOQ and ATI of rectifying single sampling inspection, rectifying double sampling inspection plans and rectifying chin sampling inspection plan. The proportions of undetected and detected defectives in the plans are also obtained.

**Table 5.3: Selected values of parameters from the original Rectifying Single Sampling Inspection plan with inspection error.**

| Type I ($e_1$) | Type II ($e_2$) | AOQL | ATI |
|---|---|---|---|
| **0.00** | **0.00** | 0.0199 | 167.8922 |
| **0.01** | **0.00** | 0.0134 | 468.8227 |
| **0.00** | **0.15** | 0.0252 | 158.6816 |
| **0.01** | **0.15** | 0.0181 | 387.8272 |

**Effects of inspection errors on AOQ and ATI in a rectifying single sampling inspection plan.**

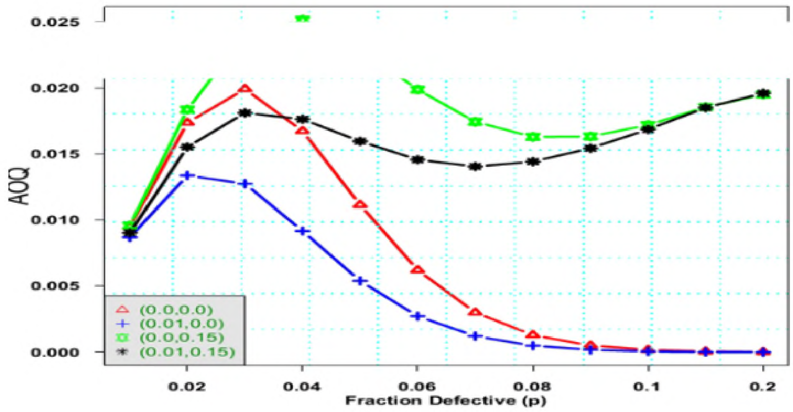



**Figure 5.1:Effects of Inspection Errors on ATI in RSSP**

From Table (5.3) and Figure (5.1) above, it can be seen that type I inspection error results in increase in average outgoing quality limit (AOQL) at a lower level of true fraction defective (p)than under perfect inspection. This is because the quality of the items is judged to be worse than it actually is giving rise to high probability of lot rejection. This situation resulted to one hundred percent inspection of the lot which in the long run improves the outgoing lot quality. Type II inspection error($e_1 = 0.00$ , $e_2 = 0.15$) on the other hand causes the outgoing quality to be worse than under perfect inspection ($e_1 = 0.00$ , $e_2 = 0.00$). This can be seen in the highest value of 0.0252 for average outgoing quality limit (AOQL) of the lots which represents the worst possible average outgoing quality that results from rectifying single sampling inspection plan. This bad situation is due to the fact that more defective items in the lot are erroneously observed as non-defective as a result of Type II inspection error even though the actual number of non-defective items is fewer. The resultant economic effect of this situation is increase in quality failure cost such as rework cost and scrap cost which is the cost of correcting the defective item and the cost of labour spent in producing defective product.

**Average Total Inspection (ATI)**

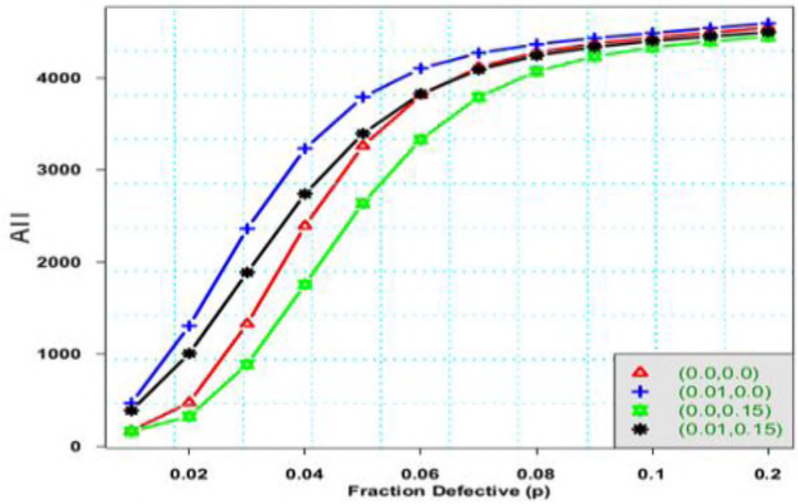



**Figure 5.2: Effects of Inspection Errors on ATI in RSSP**

From Table (5.3), it can be seen that the average total amount of items inspected when rejected lots are screened 100% stands at 468.8 when Type - I inspection error is committed. This is also seen in Figure (5.2) where the average total inspections (ATI) curve for $e_1 = 0.01, e_2 = 0.00$ is above the other inspection error pairs. This is because more non-defective items are wrongly classified as defective due to Type - I inspection error. It therefore means that the probability of accepting the lot by the consumer would decrease as such more lots would be rejected on account of being defective. This development would mean carrying out 100% inspection of all the rejected lots with a view to replacing defective items with non-defective ones. It should also be noted that producer's risk (Type- I error) is high in this case because of the high probability of the consumers erroneously rejecting lots containing acceptable quality level (AQL) from the producer.

The resultant effect of this inspection error (Type- I error) on the average total inspection (ATI) is that more inspectors are required to identify and remove defective items from the lots during inspection.

**Table 5.4: Selected Values of parameters from the original Rectifying Double Sampling Inspection Plan with Inspection Errors.**

| Type I ($e_1$) | Type II ($e_2$) | AOQL | ATI |
|---|---|---|---|
| | | | |
| **0.00** | **0.00** | 0.0186 | 154.4490 |
| **0.01** | **0.00** | 0.0132 | 703.5459 |
| **0.00** | **0.15** | 0.0244 | 113.9292 |
| **0.01** | **0.15** | 0.0190 | 594.5410 |

**Effects of inspection errors on AOQ and ATI in a rectifying double sampling inspection plan**

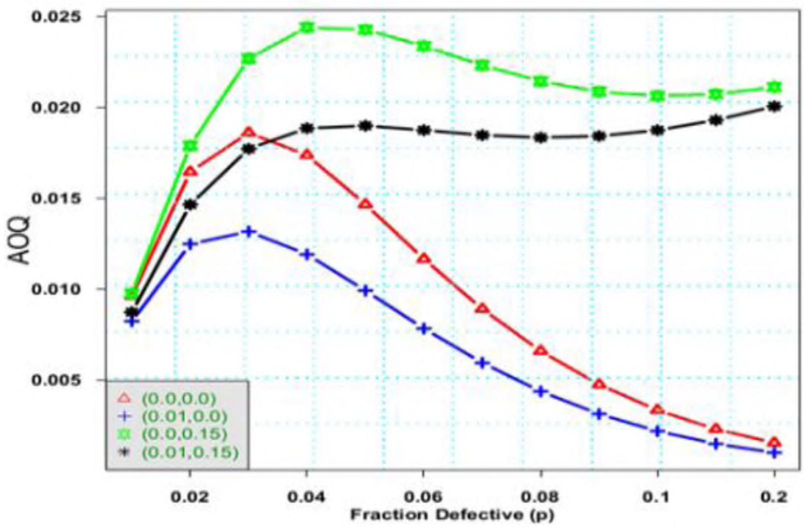



**Figure 5.3: Effect of Inspection Errors on AOQ in RDSP**

The trend is the same as explained above. From Table( 5.4)above, it is seen that Type I inspection error causes the outgoing quality to be better than that under perfect inspection ($e_1 = 0.00$, $e_2 = 0.00$).

This is because non-defective items are observed as defective which results in more lots rejection (one hundred percent inspection), than under perfect inspection. Type II inspection error ($e_1 = 0.00$, $e_2 = 0.15$) gives the highest value of 0.0244 for average outgoing quality limit (AOQL) followed by the combined Type- I and Type- II inspection error ($e_1$=0.00,$e_2$=0.15) with 0.0190.

This is depicted in average outgoing quality (AOQ) curve shown in fig.(5.3).Due to Type- II inspection error, many defective items have escaped from being inspected resulting in the worst average outgoing quality (AOQ) condition of the lot than under perfect inspection. The effect of Type- II inspection error here is that there is higher probability of consumers accepting lots with more than the tolerable defective percentage level (Consumer's risk) due to error. This type of error will bring about high quality failure cost.

**Average Total Inspection(ATI)**

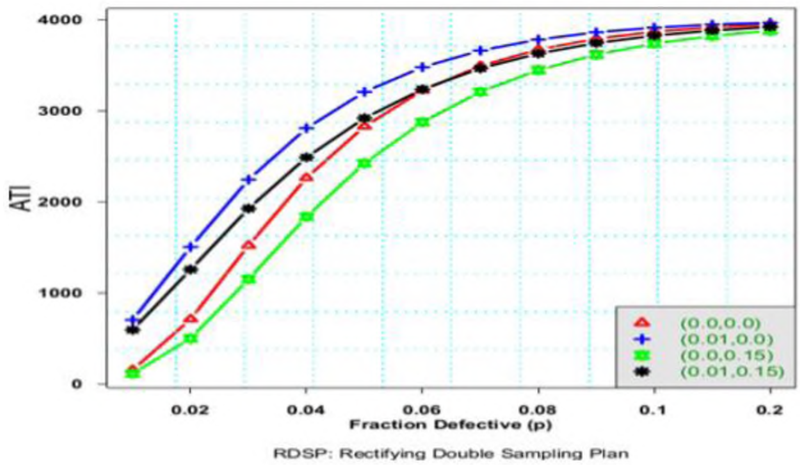



**Figure5. 4: Effect of Inspection Errors on ATI in RDSP**

From Table(5.4) and Figure (5.4), it can be seen that Type - I inspection error reduces the probability of acceptance from the perfect inspection value and results in an increase in ATI value of 703.54 because of the larger amount of lots that are rejected on account of misclassifying non-defective items as defective. Type - II inspection error increases the probability of lot acceptance and results in a lower ATI of 113.92 than under perfect inspection. This is because few lots are inspected because few are rejected.

**Table 5.5: Selected Values of parameters from the original Rectifying Double Sampling Inspection Errors.**

| Type I ($e_1$) | Type II ($e_2$) | AOQL | ATI |
|---|---|---|---|
| | | | |
| 0.00 | 0.00 | 0.009662 | 35.604 |
| 0.01 | 0.00 | 0.009663 | 35.963 |
| 0.00 | 0.15 | 0.009664 | 35.530 |
| 0.01 | 0.15 | 0.009665 | 35.908 |

**Effects of inspection errors on AOQ and ATI in a rectifying double sampling inspection plan**

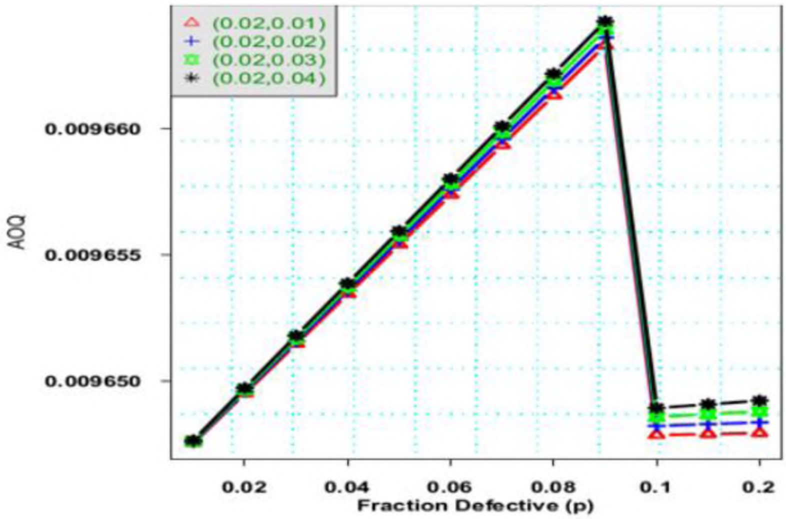



**Figure5. 5: Effects of Inspection Errors on AOQ in Rectifying Chain Sampling Plan**

From Table (5. 5) and Figure (5.5) above, it can be seen that Type - I inspection error causes average outgoing quality limit (AOQL) to occur at a lower level of true fraction

Defective (p) than under perfect inspection. This is because the quality of the items is judged to be worse than it actually is giving rise to high probability of lot rejection. This situation resulted to one hundred percent inspection of the lot which in the long run improves the outgoing lot quality.

Type - II inspection error ($e_1$ = 0.00, $e_2$ = 0.15)on the other hand caused the outgoing quality to be worse than under perfect inspection ($e_1$ = 0.00, $e_2$ = 0.00). This can be seen in the highest value of 0.09664 for average outgoing quality limit (AOQL) of the lots which represents the worst possible average outgoing quality that results from rectifying chain sampling inspection plan.

It can also be noted that the misclassification of defective items as non-defective would lead to high consumer's risk (Type - II error) this is because lots with more than acceptable defective percentage level stand a chance of been accepted by the consumer with a wrong impression that they contain the acceptable quality level (AQL).The resultant economic effect of this situation is, increase in quality failure cost such as rework cost and scrap cost which is the cost of correcting the defective item and the cost of labour spent in producing defective product.

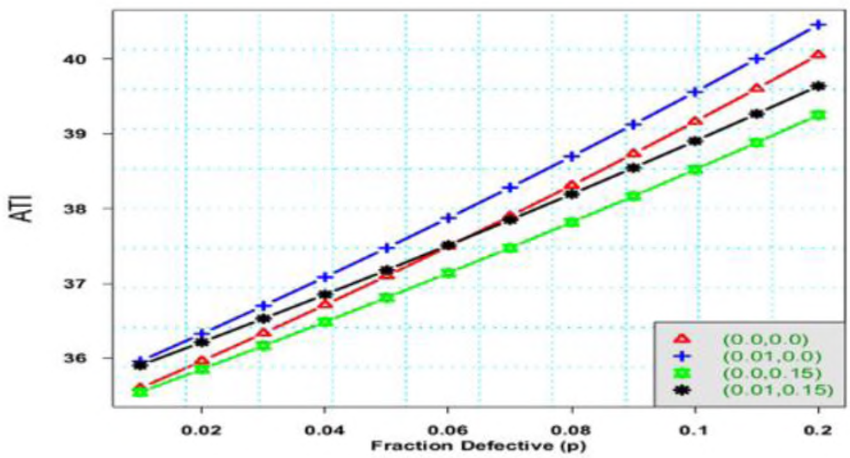



**Figure 5.6 :Effect of Inspection on ATI in RCSP**

From Table(5.6) and Figure (5.6), it can be seen that type I inspection error reduces the probability of acceptance from the perfect inspection value and results in an increase in ATI value of 35.963 because of the larger amount of lots that are rejected on account of misclassifying non-defective items as defective. This in turn increased the level of inspection of the lot. It can also be seen from the table and figure above that type II inspection error has increased the probability of lot acceptance this is because defective lots have been misclassified as non-defective. This has therefore reduced the Average.

**Table 5.6: Table of proportion of undetected defectives and detected defectives for RSSP,RDSP and RCSP ($e_1 = 0.00$, $e_2 = 0.15$)**

| P | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.0 | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| PDA | 9.6475 | 19.2950 | 28.9426 | 38.5901 | 48.2376 | 57.8851 | 67.5326 | 77.1802 | 86.8277 | 96.4752 | 106.1227 | 115.7702 |
| PDR | 0.0027 | 0.0108 | 0.0245 | 0.0440 | 0.0694 | 0.1010 | 0.1390 | 0.1834 | 0.2347 | 0.2929 | 0.3583 | 0.4311 |
| PDA | 38.8607 | 38.8607 | 38.8607 | 38.8607 | 38.8607 | 38.8607 | 38.8607 | 38.8607 | 38.8607 | 38.8607 | 38.8607 | 38.8607 |
| PDR | 0.1709 | 0.3418 | 0.5127 | 0.6836 | 0.8545 | 1.0254 | 1.1963 | 1.3672 | 1.5380 | 1.7089 | 1.8798 | 2.0507 |
| PDA | 9.6475 | 19.2950 | 28.9426 | 38.5901 | 48.2376 | 57.8851 | 67.5326 | 77.1802 | 86.8277 | 96.4752 | 106.1227 | 115.7702 |
| PDR | 0.0027 | 0.0108 | 0.0245 | 0.0440 | 0.0694 | 0.1010 | 0.1390 | 0.1834 | 0.2347 | 0.2929 | 0.3583 | 0.4311 |

Total inspection (ATI) of the lot to 35.550 lower than the ATI value of 35.604 under perfect inspection. This is because few lots are inspected because few are rejected. From Table (5.6) above, rectifying single sampling inspection Plan(RSSP) and rectifying chain Sampling Plan(RCSP) have the same proportion of undetected defectives that is accepted and detected defectives that is rejected under inspection error while rectifying double sampling plan has different proportion of undetected defectives that is accepted and proportion defectives that is rejected. The results revealed that Type-I inspection error showed better average outgoing quality than under perfect inspection because it caused non-defective items to be misclassified as defective. Type - II inspection error however, indicated the worst average outgoing quality than perfect inspection because it caused many defective items to escaped being screened. On the other hand Type - I inspection error caused an increase in the average total inspection (ATI) of the lots more than when perfect inspection is assumed while Type - II inspection error caused a reduction in the average total inspection (ATI) than perfect inspection. Conclusively, rectifying single sampling and rectifying chain sampling plans have the same proportion of undetected defectives and proportion of detected defectives when inspection.

# SUMMARY AND CONCLUSION

Acceptance sampling plan is an essential tool in the Statistical Quality Control and is a methodology which deals with quality contracting on product orders between the producers and the consumers and thus allows the producers to take decision to accept or reject the manufactured products based on the inspection of samples. It is the process of evaluating a portion of the product/material in a lot for the purpose of accepting or rejecting the lot as either conforming or not conforming to a quality specification.

Acceptance sampling is necessary to limit the cost of inspection and is the only available method to appraise the quality in destructive testing. Acceptance sampling itself does not improve quality, but whenever the lot is rejected it indicates the instability of the production process. Acceptance sampling is cost efficient and only admissible method of efficient tests with quick results.

The lot is accepted if the number of defects falls below where the acceptance number or otherwise the lot is rejected. Embedded within the design of acceptance-sampling plans is an assumption that the inspection procedures are error free. However, many inspection tasks are not error free; on the contrary, they may even be error prone. Two types of errors are possible in attribute sampling. An item which is good may be classified as defective (Type - I error,$e_1$), or an item that is defective may be classified as good (Type - II error, $e_2$).

The first chapter deals with basic concepts of acceptance sampling, notations and symbols and Review of literature.

In various quality control procedures  the possibility of inspection errors is considered as an important issue. The presence of these errors leads to change in the operational characteristic (O.C) curve, and as a result the average outgoing quality of an industrial process. A new mathematical model that can be applied to calculate such quantities as the expected number of defective items replaced in an accepted lot, and other functions of this process is presented in chapter two. The expressions for the average outgoing quality were derived for both the model involving replacement of defective items in the lot and when the items are not replaced. The potential application of this work lies in the ability of industrial researchers to calculate both of these quantities and decide the loss of such events, which are so very common in real life inspections. This simple model will be fruitful for any industrial process involving a constant inspection process.

In third chapter performance measures of the inspection process are presented for single, double and repeat inspection plans are studied. The impact of inspection error on these plans are evaluated through the plans, performance measures. Performance measures of inspection are presented for single, double and repeat inspection plans. The inspection error has a drastic impact on the performance of inspection and could result to misleading conclusions about product quality.

Chapter four deals with, studies the effect of Type - II inspection errors on the effectiveness of a quality inspection plan designed utilization risk exposure control approach. The probability of Type - II errors is integrated into the Material At Risk (MAR) model used to control risk exposure. A linear programming formulation, including the stochastic behaviour of the model, is discussed and solved. Experiments are conducted to analyze the effect of inspection error on risk exposure control reveals the computational complexity of the problem. The impact of Type - II inspection error on the material-at-risk control is significant. When the risk of non-detection is greater the number of inspections must be increased to ensure a given probability of falling in the prohibited zone of over-exposure.

Fifth chapter deals with the impact of inspection errors on rectifying single, double and chain sampling inspection plans with average outgoing quality (AOQ) and average total inspection (ATI) . The results revealed that Type - I inspection error showed better average outgoing quality than under perfect inspection because it caused non-defective items to be misclassified as defective. Type - II inspection error however, indicated the worst average outgoing quality than perfect inspection because it caused many defective items to escaped being screened. On the other hand Type - I inspection error caused an increase in the average total inspection (ATI) of the lots more than when perfect inspection is assumed while Type - II inspection error caused a reduction in the average total inspection (ATI) than perfect inspection. Conclusively, rectifying single sampling and rectifying chain sampling plans have the same proportion of undetected defectives and proportion of detected defectives when inspection.

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