Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination –August 2020
VI Semester

Class : III UG **Time : 2 Hours**
Major : Special Education and Mathematics **Max. Marks : 50**

15BSMC15 Complex Analysis II

Part A **10 x 1 = 10**
Choose the Correct Answer

1. $1 + z + z^2 + z^3 + \ldots =$
   a. sin z b. $(1-z)^{-1}$ c. Cos z d. Tan z

2. If $C: |z| = 1$ then $\int_C \frac{z}{z-2} dz$
   a. $2\pi i$ b . πi c. $-\pi i$ d. 0

3. The converse of Cauchy's fundamental theorem is
   a. Lioville's theorem b. Morera's theorem
   c. Goursat's theorem d. Taylor's theorem

4. What is the singularity of $f(z) = \frac{1}{sin\ z}$?
   a. 0 b. 1 c. 2 d. 3

5. A pole of order two is called
   a. Simple pole b. Double pole c. Zero d. Multipole

6. The zero(s) of sin Z
   a. π b. ∞ c. 0 d. (2n + 1)$\frac{\pi}{2}$

7. The Taylor series expansion of f(z) about the point zero is called
   a. Euler series b. Laurent series
   c. Maclaurin's series d. Rouches series

8. I$f(z)$ is analytic in a region D and is not identically zero in Dthen the set of all zeros of $f(z)$ is
   a. Isolated b. convergence c. mobius d. Limit

9. Find the residue of $\frac{1}{z^{2+}a^2}$ at $z = ai$
   a. $i/4a3$ b. $-i/4a^3$ c. $1/4a3$ d. $-1/4a3$

10. If a is a pole of order m>1 for f(z) and $f(z) = \frac{g(z)}{(z-a)^m}$ where g(z) is analytic at a and g(a)≠ 0 then res {f(z):a}=
    a. $\frac{g^m(a)}{m!}$ b. $\frac{g^{m-1}(a)}{(m-1)!}$
    c. $gm-1(a)/m!$ d. $gm-1(a)/m-1$

Part B **3 x 6 = 18**

Answer any **Three** questions

Each answer should not exceed 400 words or two pages

11.State and prove Liouville's Theorm.

12. Prove that zeros of an analytic function are isolated.

13. Construct i. cos z and ii. sin z into a Taylor's series expansion about the point $z = \pi/2$ and $z = {}^{\pi}/_{4}$ respectively. Also determine their region of convergence.

14. Find the Laurent's series for z/(z-1)(z-2) in the region i. |z-1|>1 ii. |z|<1.

15. Define singularity and its types with suitable examples.

16. Obtain the singularities and classify the singular points of f(z) i. $\frac{z}{e^z-1}$ ii. $\frac{\sin z}{z}$.

17.. Find the poles and determine residue at poles for the following:

i. $(z+1)/(z^2-2z)$ ii. $\frac{z^2+4}{z^3+2z^2+2z}$

18. Evaluate using residue theorem $\int_C tanz\, dz$ where C is the circle $|z| = 2$.

19. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\,sin\theta}$.

20. State and prove Cauchy's Residue Theorem.

Part C **2 x 11 = 22**

Answer any **Two** questions

Each answer should not exceed 800 words or four pages

21. State and prove Maximum Modules Theorem.

22. State and Prove Laurent's series.

23. State and prove Taylor's theorem.

24. Find the Laurent's series expansion of the function $\frac{z^2-1}{(z+2)(z+3)}$ valid in the annular region 2 < |z| < 3.

25. State and prove Weierstrass Theorem.

26. i. If $z = a$ is apole of a function $f(z)$, then prove that $\lim_{Z\to a} f(z) = \propto$.
ii. If a function $f(z)$ is analytic in a deleted neighbourhood of $z = a$ and if $\lim_{z\to a} f(z) = \propto$ then prove that $z = a$ is a pole of $f(z)$.

27. Find the residue of the function $\frac{Z^4}{(Z-1)^4\,(Z-2)(Z-3)}$ at its singularties.

28. Evaluate using (i) Cauchy's integral formula (ii) Residue theorem $\int_C \frac{z+1}{z^2+2z+4} dz$ where C is the circle |z+(1+i) | = 2.

29. Using Contour Integration Prove that $\int_0^{\infty} \frac{dx}{x^6+1} = \frac{\pi}{3}$.

30. Prove that $\int_{-\infty}^{\infty} \frac{(x^2-x+2)dx}{x^4+10x^2+9} = \frac{5\pi}{12}$ using contour integration.

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