Avinashilingam Institute for Home Science and Higher Education for Women (Deemed to University) Coimbatore- 641043.

Master's Degree Examination – November 2018
Semester - III

Class : II PG **Time: 3 hours**
Major : Mathematics **Max. Marks: 60**

17MMAC13 – TOPOLOGY I

Part A

Choose the correct answer

10 x 1/2 = 5

1. The Closed interval [a, b] = -------

   a) $\{ x: a<x < b\}$ b) $\{ x: a>x > b\}$ c) $\{ x: a \le x \le b\}$ d) $\{ x: a<x > b\}$

2. Let A be subset of X ,the -------------- is the set of all limit points of A

   a) dense b) derived set c) nowhere dense d) none

3. A subset A of a topological space is said to be a----------- if A=D(A).

   a) perfect set b) dense set c) derived set d) none

4. An open sub base is a class of open subsets of X whose ----------------- form an open base.

   a) finite intersections b) intersections c)union d) infinite intersections

5. Neighborhood of X is -----------

   a) an open set U containing X b) a null set
   c) an closed set U containing X d) an open interval

6. Any continuous mapping of a compact metric space into a metric space is ---------

   a) uniformly continuous b) bounded c) continuous d) discontinuous

7. Let X be a metric space with metric d,the open sphere in A subset of X is defined by---

   a) $\{x:d(x,x_0) < r \}$ b) $\{x:d(x,x_0) > r \}$ c) $\{x:d(x,x_0) = r \}$ d) $\{x:d(x,x_0) < = r \}$

8. A -------------- is a topological space X it cannot be as the union of two disjoint non- empty open sets.

   a) Connected space b) T_1 space c) completely regular d) Hausdorff space

9. A ---------of t he real line R is connected if and only f it is an interval. In particular R is Connected

   a) dense set b) derived set c) subspace d) closed subset

10. Any ------------- image of a connected space is connected

    a) continuous b) equicontinuous c) completely regular d) discontinuous

Part B **5 x 4 = 20**

Answer ALL questions

Answer should not exceed 200 words or one page

11 .a. Compare Isolated Points and Limit Points with Examples

(Or)

b. List all possible topologies for the set X ={a,b,c}.

12.a. Let X be a non empty set and there be a given closure operation which assign to each subset A of X . Then check out the following

(i) $\overline{\varphi} = \varphi$ (ii) $A \subseteq \overline{A}$ (iii) $A = \overline{A}$ (iv) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

(Or)

b. Let f : X → Y be a mapping of one topological space into another and let there be given an open subspace with its generated open base in Y .

Then, prove that f is continuous if and only if the inverse of each basic open set is open.

13. a . Demonstrate the statement : Let X and Y be Metric spaces and f is a mapping of X into Y. Then f is continuous if and only if f^{-1} (G) is open in X whenever G is open in Y.

(Or)

b. Illustrate : In any metric space X, prove that each closed sphere is a closed set.

14.a .(i) Define Continuous Mapping with suitable example .

(ii) Relate Cauchy Sequence and Convergent Sequence with suitable examples .

(Or)

b. Explain Uniform limit Theorem with few applications .

15. a. Let X be a topological Space and A is a connected subspace of X.

If B is a subspace of X such that $A \subseteq B \subseteq \overline{A}$ then, prove that B is connected .

(Or)

b. Prove that a topological Space X is disconnected if there exist a continuous mapping of X onto the discrete two point space { 0,1}.

Part C **5 x 7 = 35**

Answer ALL questions

Answer should not exceed 600 words or three pages

16.a. (i) Express the product topology X x Y in terms of a subbasis.

(ii) Write the definition of Product topology. Give an example for Product topology.

(Or)

b.(i) Define open base and open Subspace with proper examples.

(ii) Compare order topology and product topology with proper examples.

17. a. Check out : If Y is a subspace of X, and A is a subset of Y, then the topology A inherits as a Subspace of Y is the same as the topology it inherits as a subspace of Y .

(Or)

b. Examine the following statement : Let Y be a subspace of X ; Let A be a subset of Y ; Let $\bar{A}$ denotes the closure of A in X. Then the closure of A in Y equals $\bar{A} \cap Y$.

18 a. Show that ($X_1 \times X_2 \times X_3 \times \ldots\ldots\ldots \times X_{n-1}$) x X_n is homeomorphic with $X_1 \times X_2 \ldots\ldots\ldots \times X_n$

(Or)

b. Construct the following :

Let $\bar{d} = \min \{ |a - b| ,1\}$ be the standard bounded metric on R . If x and y are two points of R^{ω} ,define $D(x,y) = \sup \{ \frac{\bar{d}(x_i, y_i)}{i} \}$ .then prove that D is a metric that induces the product topology on R^{ω}.

19.a. Prove that the composite function of two continuous functions is continuous.

(Or)

b. Let f : X-→Y be continuous. Then prove that for every convergent sequence $x_n \to x$ in X , the sequence $f(x_n) \to f(x)$. The converse holds if X is metrizable .

20.a. Prove that a finite Cartesian product of connected space is connected.

(Or)

b. (i) Check whether every locally connectedness is connectedness

(ii) Prove that a space X is locally connected iff for every open set U of X , each component of U is open in X .