INTRODUCTION

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Soft set theory is one of the recent topics gaining significance in finding rational and logical solutions to various real life problems which involve uncertainty, impreciseness and vagueness.

Most of the real life problems have various uncertainties. A number of theories have been proposed for dealing with uncertainties in an efficient way.

In 1965, Zadeh introduced the concept of Fuzzy set theory which provides us with an intuitively pleasing method of representing one form of uncertainty. In1999, Molodtsov initiated a novel concept of Soft set theory, which is completely a new approach for modeling vagueness and uncertainty.

"Given an initial universe set U and the set of parameters E, a Soft set over U is a pair (F, A) where $A \subseteq E$ and F is a mapping given by $F: A \rightarrow P(U)$. (Here $P(U)$ is the power set of U)"

Maji et al (2001) initiated the concept of Fuzzy Soft sets with some properties regarding union, intersection and complement of a Fuzzy Soft set, De Margan's Law etc.

"Given an initial universe set U and the set of parameters E, a Fuzzy Soft set over U is a pair (F, A) where $A \subseteq E$ and F is a mapping given by $F: A \rightarrow \widetilde{P}(U)$. (Here $\widetilde{P}(U)$ is the set of all fuzzy subsets of U)".

The topological structures of set theories dealing with uncertainties were first studied by Chang. Chang (1968) introduced the notion of Fuzzy Topology and also studied some of its basic properties.

Shabir and Naz (2011) introduced the notion of Soft Topology (Definition 1.2.1) which is defined over an initial universe with a fixed set of parameters. They studied some basic concepts of Soft Topological spaces and also some related concepts such as soft interior, soft closure, soft subspace and soft separation axioms.

Aygunoglu – Aygun (2011) introduced Soft Product Topology and defined the concept of compactness in Soft Topological Spaces named as soft compactness.

Peyghan et al. (2012) introduced the concept of soft connectedness and studied some properties related to these spaces.

Topological studies of Fuzzy Soft sets were started by Tanay and Kandemir (2011). They gave the notion of Fuzzy Soft Topology for the first time by defining it as a collection of Fuzzy Soft subsets of an arbitrary Fuzzy soft set. They also introduced some basic definitions in this new space. Mahanta and Das (2012) continued working on Fuzzy Soft Topology and studied separation axioms and connectedness in Fuzzy Soft Topological spaces.

Roy and Samanta (2013) initiated the concept of Fuzzy Soft Topology in a different manner. They gave the notion of fuzzy soft topology as a collection of fuzzy soft subsets of the initial universal set and then proposed the concept of base and subbase for this space.

The main aim of this thesis is to study Soft sets, Soft Topological spaces, Fuzzy soft sets and Fuzzy soft Topological spaces.

The plan of study is as follows:

1) Soft Sets and Soft Topological Spaces
2) Fuzzy Soft Sets and Fuzzy Soft Topological Spaces
3) Fuzzy Soft Mappings
4) Fuzzy Soft Separation Axioms and Fuzzy Soft Compactness
5) Semiopen and Semiclosed Fuzzy Soft Sets in Fuzzy Soft Topological Spaces
6) Soft Regular Weakly Closed Sets in Soft Topological Spaces
7) Soft $\hat{g}$-Closed Sets in Soft Topological Spaces

The first chapter is devoted to the study of Soft sets in Soft Topological spaces. In this chapter preliminary definitions regarding Soft sets, Soft open sets, Soft closed sets, Soft interior, Soft closure, Soft neighborhood, Soft subspace, Soft mappings, Soft compactness and Soft connectedness are studied with some interesting properties. The important properties are given in theorems 1.3.3, 1.3.5, 1.4.4, 1.4.6, 1.4.7, 1.5.3, 1.5.5, 1.5.6 and 1.5.7.

The second chapter deals with preliminary definitions and notations regarding Fuzzy Soft sets and Fuzzy Soft Topological spaces. In this chapter the concepts of Fuzzy Soft sets, Fuzzy Soft open sets, Fuzzy Soft closed sets, Fuzzy Soft closure, Fuzzy Soft interior, Fuzzy Soft subspace and Fuzzy Soft neighborhood are studied with some interesting properties. The important results are given in theorems 2.1.10, 2.2.19 and 2.2.24.

The third chapter is devoted to the study of fuzzy soft mappings. In this chapter Fuzzy Soft mapping, Fuzzy Soft bijective mapping, fuzzy soft identity mapping are studied with examples. Fuzzy Soft continuous mapping on Fuzzy Soft Topological spaces are studied with interesting properties. An interesting characterization proved regarding Fuzzy Soft continuous mapping is given in theorem 3.13.

The fourth chapter deals with Fuzzy Soft Separation axioms and Fuzzy Soft Compactness. In this chapter the definitions of Fuzzy Soft T_o-Space, T_1-Space, T_2-Space, T_3-Space and T_4-Space are introduced and studied. The important results proved in this chapter are given in theorems 4.1.3, 4.1.5, 4.1.6, 4.1.8, 4.1.9, 4.1.10, 4.1.14 and 4.1.17.

In this chapter, Fuzzy Soft Compact spaces are also defined with examples. Interesting theorems proved regarding Fuzzy Soft Compactness are

1) Continuous image of a Fuzzy Soft Compact space is Fuzzy Soft Compact space.
2) Let (U, τ_1, E) be a Fuzzy Soft Topological spaces and (U, τ_2, E) be a fuzzy soft Hausdroff space. Fuzzy soft mapping (φ, ψ) is closed if fuzzy soft mapping $(\varphi, \psi): (U, \tau_{1,}E) \rightarrow (U, \tau_2, E)$ is continuous.

Chapter V is devoted to the study of generalization of Fuzzy Soft open and Fuzzy Soft Closed sets. In this chapter Semiopen and Semiclosed Fuzzy Soft sets in Fuzzy Soft Topological spaces are introduced. Various properties of these sets are studied along with some characterizations. The structures like interior and closure via Semiopen and

Semiclosed Fuzzy soft sets are generalized and some of their properties are studied. Interesting results are given in theorems 7.7, 7.9 and 7.10.

Any Research work should result in addition to the existing knowledge of a particular concept. Such an effort not only widens the scope of the concept but also encourages others to explore new and newer ideas.

Here the author of this thesis has succeeded in her knowledge building effort by introducing two new classes of soft sets called Soft Regular Weakly closed sets and Soft $\hat{g}$-closed sets in Soft Topological spaces.

In 2007, Benchalli and Wali introduced the concept of Regular Weakly closed sets in Topological spaces and studied some of their properties. In 2003, Veerakumar introduced $\hat{g}$-closed sets in Topological spaces and studied some of their properties and applications.

Using these two concepts of closed sets in Topological Spaces, the author of this thesis introduced soft Regular weakly closed sets (briefly SRW-Closed sets) and Soft $\hat{g}$-Closed sets in Soft Topological spaces.

Chapter VI deals with Soft Regular Weakly Closed sets in Soft Topological spaces.

"Let (X, τ, E) be a Soft Topological Space. A soft set (F, E) is called Soft Regular Weakly Closed set (briefly SRW-Closed set) if $Cl(F, E) \subseteq (U, E)$ whenever $(F, E) \subseteq (U, E)$ and (U, E) is soft regular semi open set in (X, τ, E)"

Interesting results proved here are

1) Every soft closed set is a SRW-Closed set but not conversely
2) Union of to SRW-Closed sets is SRW-Closed set but intersection of two SRW-Closed sets is not SRW-Closed set.
3) If a soft set (F, E) is SRW-Closed set in (X, τ, E) then the difference $Cl(F, E) \backslash (F, E)$ does not contain any non-empty soft regular semi open set in (X, τ, E).
4) If (F, E) is a SRW-Closed set in (X, τ, E) such that $(F, E) \subseteq (G, E) \subseteq Cl(F, E)$ then (G, E) is SRW-Closed set in (X, τ, E).

Chapter VII deals with Soft $\hat{g}$-Closed sets in Soft Topological spaces.

"Let (X,τ,E) be a soft topological space. A soft set (F,E) is called a Soft $\hat{g}$-Closed set if $Cl(F,E) \subseteq (G,E)$ whenever $(F,E) \subseteq (G,E)$ and (G,E) is soft semi open set in (X,τ,E)"

Interesting results proved here are given in theorems 7.3, 7.4, 7.5, 7.6, 7,8, 7,9, 7.10 and 7.11.

This new class of sets widens the scope to do further research in the areas like Bitopological Spaces, Smooth topological Spaces and Fuzzy Soft Topological Spaces.

It is worth mentioning that the author of the thesis published two articles related to Soft Regular Weakly Closed sets and Soft $\hat{g}$-Closed sets in Soft Topological spaces as detail below:

1) "SRW-Closed Sets in Soft Topological spaces", *International Journal of Innovative Research in Science, Engineering and Technology,* Vol. 3, No. 6, 2014, pp. 13343-13347 [43].

2) "Soft $\hat{g}$-Closed Sets in Soft Topological spaces", *International Journal of Innovative Research in Science, Engineering and Technology,* Vol. 3, No. 7, 2014, pp. 14595-14600 [44].