Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test I – August 2025

Semester III

Class : II UG **Time : 2 Hours**

Branch : Mathematics **Max.Marks : 60**

23BMAC06 Group Theory

Course Outcomes:

CO1: Recognize the mathematical objects called groups.

CO2: Link the fundamental concepts of groups and symmetries of geometrical objects.

CO3:Explain the significance of the notions of cosets, normal subgroups, and factor groups.

CO4: Analyze consequences of Lagrange's theorem.

CO5: Learn about structure preserving maps between groups and their consequences.

Part A **6 x 1 = 6**

Choose the Correct Answer

1. The order of the dihedral group D_5 is CO1K1

a.10 b. 5 c.8 d.6

2. U(12) = ------ CO1K2

a.{1,5,7,11} b.{2, 4,6,8,10} c.{1,2,3,6,8,11} d. 4

3.The set {1, 2, . . . , n -1} is a group under multiplication modulo n if and only if n is CO1K2

a.prime b.composite c.odd d. even

4.If G ={0,1,2,3,4} is a group under addition modulo 5,then |3| is CO2 K2

a.4 b.5 c.3 d.2

5. If G is a group of order 8 and H is a subgroup of order 2, then index of H in G is CO2 K2

a.1 b.4 c.2 d.8

6. Let G be a group of integers under addition and H be a subset consisting of multiples of 3. Then the distinct right cosets of H in G are CO3 K1

a. H, H+1 b.H+1, H+2 c.H, H+1, H+2 d. H, H+2

Part B **3 x 6 = 18**

Answer ALL questions

7. a. Define U(n) and construct a Cayley table for U(10) CO1K3

(or)

7. b. Show that {1, 2, 3, 4, 5, 6} under multiplication modulo 7 is a group. CO1K3

8. a. Prove that if G is an Abelian group and H and K are subgroups of G. Then

HK = {hk | h ∈ H, k∈ K}is a subgroup of G. CO1K3

(or)

8. b. State and prove Fermat's Little Theorem CO2K4

(or)

9. a. Define Euler phi function, ϕ(n) and find first 10 values of ϕ(n) CO2K3

(or)

9. b. Prove that a subgroup H of G is normal in G if and only if $xHx^{-1} \subseteq H$ for all x in G.

CO3K4

Part C **3 x 12 = 36**

Answer ALL questions

10. a. (i) Prove that in a group G, there is only one identity element CO1K4

(ii) State and prove Socks-Shoes Property

(iii) Find the determinant of $\begin{pmatrix} 2 & 5 \\ 6 & 3 \end{pmatrix}$ in GL(2, Z_7)

(or)

10. b. (i) Prove that the set of all 2 x 2 matrices with real entries is a group under matrix addition.

(ii)Find the inverse of $\begin{pmatrix} 4 & 5 \\ 6 & 3 \end{pmatrix}$ in GL(2, Z_7) CO1K4

11. a. (i) Prove that if H is a nonempty finite subset of a group G and if H is closed under the operation of G, then H is a subgroup of G.

(ii) Find all generators of Z_8 CO2K4

(or)

11. b. (i)State and proveLagrange's Theorem CO2K5

(ii)Let G be a finite group, and let a∈ G. Then prove that $a^{|G|} = e$.

12. a. State and prove any five properties of cosets CO2K5

(or)

12. b. (i) Prove that if G is a group and H is a normal subgroup of G, then G/H = {aH | a∈ G} is a group under the operation (aH)(bH) =abH.

(ii) Find G/H. where G= Z_{18} and H ={0, 6, 12}

CO3K5

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