Chapter 7

Various Types of Somewhat Functions Using δP_S-Open Sets

7.1 Introduction

Gentry introduced and studied the concept of somewhat continuous and somewhat open functions in the year 1971. These ideas are closely related to weakly equivalent topologies which was first introduced in 2010. Benchalli and Priyanka introduced somewhat b-continuous and somewhat b-open functions in topological spaces. Inspired with these developments, in this chapter we introduced somewhat δP_S-continuous, somewhat almost δP_S-continuous, somewhat δP_S-irresolute, somewhat δP_S-open and somewhat almost δP_S-open functions. These findings result in procuring several characterizations, properties and interrelations with other types of functions.

7.2 Somewhat δP_S-Continuous Functions

Definition 7.2.1. A function f is said to be **somewhat δP_S-continuous** if for $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$, there exists a non-empty δP_S-open set V in (X, τ) such that $V \subseteq f^{-1}(U)$.

The following two examples are established to show the existence of somewhat δP_S-continuous function, and not all functions are somewhat δP_S-continuous.

Example 7.2.2. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a, b\}\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a, f(b) = a, f(c) = b$. Then f is somewhat δP_S-continuous.

Example 7.2.3. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{a, b\}\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be the identity functions. Then f is not somewhat δP_S-continuous.

Proposition 7.2.4. Every δP_S-continuous function is somewhat δP_S-continuous function.

Proof. Let $f: (X, \tau) \to (Y, \sigma)$ be δP_S-continuous. Let $x \in X$ and $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$. Therefore, there exists $x \in f^{-1}(U) = \emptyset \Rightarrow f(x) \in U$. Since f is δP_S-continuous, there exists a δP_S-open set V in (X, τ) such that $f(V) \subseteq U \Rightarrow V \subseteq f^{-1}(U)$. Hence f is somewhat δP_S-continuous.

Remark 7.2.5. The converse of above proposition is not true, in general.

The following example exhibits somewhat-δP_S-continuous function need not be δP_S-continuous.

Example 7.2.6. Let $X = \{a, b, c\}$ and $Y = \{a, b, c, d\}$ with $\tau = \{X, \emptyset, \{a\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by $f(a) =$

$d, f(b) = b, f(c) = c,\ then\ f^{-1}(a) = \emptyset, f^{-1}(c) = \{c\}, f^{-1}(a,b) = \{b\}, f^{-1}(a,c) = \{c\}, f^{-1}(a,b,c) = \{b,c\}, f^{-1}(a,c,d) = \{a,c\}$. Here, f is somewhat δP_S-continuous. Since $\delta P_S O(X) = \{X, \emptyset, \{b\}, \{c\}, \{b,c\}\}$ but not δP_S-continuous as $f^{-1}(a,c,d)$ is not δP_S-open.

Note 7.2.7. The composition of two somewhat δP_S-continuous functions need not be somewhat δP_S-continuous functions. In general, which is shown in the following example.

Example 7.2.8. Let $X = Y = Z = \{a,b,c\}$ with $\tau = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c\}\}, \sigma = \{Y, \emptyset, \{a\}, \{a,b\}\}$ and $\eta = \{Z, \emptyset, \{a\}, \{a,b\}\}$ Let $f: (X,\tau) \to (Y,\sigma)$ be defined by $f(a) = b, f(b) = a, f(c) = a$ and $g(a) = b, g(b) = a, g(c) = a$. Then the functions f and g are somewhat δP_S-continuous but their composition $g \circ f: (X,\tau) \to (Z,\eta)$ is not somewhat δP_S-continuous. Since there exists no $\delta P_S O(X,\tau)$ contained in $(g \circ f)^{-1}\{a\} = \{a\}$.

Proposition 7.2.9. If f is somewhat δP_S-continuous and g is continuous, then $g \circ f$ is somewhat δP_S-continuous.

Proof. Let $f: (X,\tau) \to (Y,\sigma)$ be a somewhat δP_S-continuous function and $g: (Y,\sigma) \to (Z,\eta)$ be a g-continuous function. Let U be open in (Z,η) then $g^{-1}(U)$ is open in (Y,σ) since g is complete continuous. Since f is somewhat δP_S-continuous, $f^{-1}(g^{-1}(U))$ will contain a non-empty δP_S-open set in (X,τ). Therefore $g \circ f$ is somewhat δP_S-continuous.

Corollary 7.2.10. If f is somewhat δP_S-continuous and g is super continuous (resp., complete continuous), then $g \circ f$ is somewhat δP_S-continuous.

Proof. Follows from the definition of super continuity (resp., complete continuity) and every δ-open set (resp., regular open) is open. The proof follows as in Proposition 7.2.9.

Proposition 7.2.11. A subset A of (X,τ) is δP_S-dense in (X,τ) if there is no proper δP_S-closed set C in (X,τ) such that $A \subseteq C \subseteq X$.

Proof. Suppose there exists C, δP_S-closed set in X such that $A \subseteq C \subseteq X$ $\longrightarrow$ (1)

Since A is δP_S-dense, $\delta P_S\, Cl(A) = X$.

$\delta P_S cl(A) = \cap \{M | A \subseteq M$ and M is δP_S-closed in $X\}$ $\longrightarrow$ (2)

(1) and (2) $\Rightarrow C$ is one set in the above intersection. Therefore $\delta P_S cl(A) \subseteq C$,

$X \subseteq C \Rightarrow C = X$.

Theorem 7.2.12. For a surjective function f, the following statements are equivalent:

a) f is somewhat δP_S-continuous.

b) If C is a closed subset of (Y,σ) such that $f^{-1}(C) \neq X$, then there is a proper δP_S-closed subset D of (X,τ) such that $f^{-1}(C) \subseteq D$.(Equivalently, if C is an open subset of (Y,σ) such that $f^{-1}(C) \neq X$, then there exists a proper δP_S-open subset D of (X,τ) such that $f^{-1}(C) \subseteq D$).

c) If A is a δP_S-dense subset of (X,τ), then $f(A)$ is a dense subset of (Y,σ).

Proof. $(a) \Rightarrow (b)$ Let C be a closed subset of (Y,σ) such that $f^{-1}(C) \neq X$. Then $Y \backslash C$ is an open set Y such that $f^{-1}(Y \backslash C) = X \backslash f^{-1}(C) \neq \emptyset$. By (a), there exists a non-empty subset $U \in \delta P_S O(X,\tau)$ such that $U \neq \emptyset$ and $U \subseteq f^{-1}(Y \backslash C) = X \backslash f^{-1}(C)$. This means that $f^{-1}(C) \subseteq X \backslash U$ and $X \backslash U = D$ is a proper δP_S-closed set in (X,τ).

$(b) \Rightarrow (a)$ Let $U \in \sigma$ and $f^{-1}(U) \neq \emptyset$ then $Y \backslash U$ is closed and $f^{-1}(Y \backslash U) = X \backslash f^{-1}(U) \neq X$. By (b), there exists a proper δP_S-closed set D such that $f^{-1}(Y \backslash U) \subseteq D,\ X \backslash f^{-1}(U) \subseteq D$. This implies that $X \backslash D \subseteq f^{-1}(U)$ and $X \backslash D$ is δP_S-open and $X \backslash D \neq \emptyset$.

$(b) \Rightarrow (c)$ Let A be a δP_S-dense set in (X,τ). Suppose that $f(A)$ is not dense in Y. Then there exists a proper closed set C in Y such that $f(A) \subseteq C \subseteq Y$. Clearly $f^{-1}(C) \neq X$. By (b), there exists a proper δP_S-closed set D such that $A \subseteq f^{-1}(C) \subseteq D \subseteq X$. This is a contradiction to the fact that A is δP_S-dense in (X,τ).

$(c) \Rightarrow (b)$ Suppose (b) is not true there exists a closed set C in Y such that $f^{-1}(C) \neq X$ but there is no proper δP_S-closed set D in (X,τ) such that $f^{-1}(C) \nsubseteq D$. This means that $f^{-1}(C)$ is δP_S-dense in (X,τ). But by (c), $f(f^{-1}(C)) = C$ must be dense in Y, which is a contradiction to the choice of C.

Definition 7.2.13. If X is a set and τ and σ are topologies on X, then τ is said to be **δP_S-equivalent** to σ provided if $U \in \sigma$ and $U \neq \emptyset$, then there is a non-empty δP_S-open set V in (X,τ) and $V \subseteq U$.

Proposition 7.2.14. Let X be a set, τ and σ are δP_S -equivalent topologies on X. When f is identity then $f\colon (X,\tau) \to (Y,\sigma)$ and $f^{-1}\colon (Y,\sigma) \to (X,\tau)$ are somewhat δP_S-continuous. Conversely, if the identity function f is somewhat δP_S-continuous in both directions, then τ and σ are δP_S-equivalent.

Proof. Let τ and σ be two δP_S -equivalent topologies on X. Let $f\colon (X,\tau) \to (Y,\sigma)$ be an identity function. Then for an open set C in $Y, f^{-1}(C) = C$ in (X,τ) such that $D \subseteq C = f^{-1}(C)$. By Theorem 7.2.12, f is somewhat δP_S continuous function.

Conversely, Retracing the steps the converse can be proved.

Proposition 7.2.15. Let $f\colon (X,\tau) \to (Y,\sigma)$ be a surjective somewhat δP_S -continuous function and τ^* be a topology for X, which is δP_S-equivalent to τ. Then $f\colon (X,\tau^*) \to (Y,\sigma)$ is somewhat δP_S-continuous.

Proof. Let $V \in \sigma$ such that $f^{-1}(V) \neq \emptyset$. Since $f: (X, \tau) \to (Y, \sigma)$ is somewhat δP_S-continuous, there exists a non-empty δP_S-open set U in (X, τ) such that $U \subseteq f^{-1}(V)$. But by hypothesis, τ^* is δP_S-equivalent to τ. Therefore, there exists a δP_S-open set $U^* \in (X, \tau^*)$ such that $U^* \subseteq U$. But $U \subseteq f^{-1}(V)$ then $U^* \subseteq f^{-1}(V)$, hence $f: (X, \tau^*) \to (Y, \sigma)$ is somewhat δP_S -continuous.

Proposition 7.2.16. Let $f: (X, \tau) \to (Y, \sigma)$ be a surjective somewhat δP_S-continuous function and σ^* be a topology for Y, which is equivalent to σ. Then $f: (X, \tau) \to (Y, \sigma^*)$ is somewhat δP_S -continuous.

Proof. Let $V^* \in \sigma^*$ such that $f^{-1}(V^*) \neq \emptyset$. Since σ^* is equivalent to σ, there exists a non-empty open set V in (Y, σ) such that $V \subseteq V^*$. Now $\emptyset \neq f^{-1}(V) \subseteq f^{-1}(V^*)$. Since $f: (X, \tau) \to (Y, \sigma)$ is somewhat δP_S -continuous, there exists a non-empty δP_S -open set U in (X, τ) such that $U \subseteq f^{-1}(V)$. Then $U \subseteq f^{-1}(V^*)$, hence $f: (X, \tau) \to (Y, \sigma^*)$ is somewhat δP_S-continuous.

7.3 Somewhat Almost δP_S- Continuous Functions

Definition 7.3.1. A function f is said to be **somewhat almost δP_S-continuous** if for every $U \in \delta PO(Y, \sigma)$ and $f^{-1}(U) \neq \emptyset$ there exists a non-empty $V \in \delta P_S O(X, \tau)$ such that $V \neq \emptyset$ and $V \subseteq f^{-1}(U)$.

Proposition 7.3.2. Every somewhat almost δP_S-continuous function is a somewhat δP_S-continuous.

Proof. Let $f: (X, \tau) \to (Y, \sigma)$ is a somewhat almost δP_S-continuous. Consider $U \in \sigma$ such that $f^{-1}(U) \neq \emptyset$. Then $U \in \delta PO(\sigma)$. Since f is somewhat almost δP_S-continuous there exists a δP_S-open set V in (X, τ) such that $V \subseteq f^{-1}(U)$.

$\Rightarrow f$ is somewhat δP_S-continuous.

Example 7.3.3. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \emptyset, \{a, b\}\}, \sigma = \{Y, \emptyset, \{a\}\}$ and $f: (X, \tau) \to (Y, \sigma)$ such that $f(a) = b, f(b) = c$ and $f(c) = a$. Then $\delta P_S O(\tau) = \{X, \emptyset, \{c\}\}$ and $\delta PO(X, \tau) = \mathcal{P}(X)$. Hence f is somewhat δP_S but not somewhat almost δP_S-continuous.

Remark 7.3.4. δP_S continuous and somewhat almost δP_S-continuous are independent.

The following examples show that both are independent.

Example 7.3.5. Let X, Y, τ, σ and f be as in Example 7.3.3. It is δP_S-continuous but not somewhat almost δP_S-continuous.

Example 7.3.6. Let $X = \{a, b, c\}, Y = \{a, b, c, d\}$ with $\tau = \{X, \emptyset, \{b\}\}$ and $\sigma = Y, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\} \}$ and $\delta PO(Y) = \sigma$, and $f: (X, \tau) \to$

$(Y,\sigma), f(a) = a, f(b) = b$ and $f(c) =$ c Then f is somewhat almost δP_S-continuous but not δP_S-continuous.

Proposition 7.3.7. The composition of two somewhat almost δP_S-continuous functions is somewhat almost δP_S-continuous.

Proof. Let $f: (X,\tau) \to (Y,\sigma)$ and $g: (Y,\sigma) \to (Z,\eta)$ be somewhat almost δP_S-continuous. Let $U \in \delta PO(Z,\eta)$ such that $(g \circ f)^{-1}U \neq \emptyset \Rightarrow f^{-1}(g^{-1}(U)) \neq \emptyset \Rightarrow g^{-1}(U) \neq \emptyset$. Since g is somewhat almost δP_S-continuous, there exists a non-empty $V \in \delta P_S O(Y,\sigma)$ such that $V \neq \emptyset$ and $V \subseteq g^{-1}(U)$. Since every δP_S-open set is δP-open set, we get $V \in \delta PO(Y,\sigma)$ and since f is somewhat almost δP_S-continuous, there exists $W \in \delta P_S O(X,\tau)$ and $W \neq \emptyset$ and $W \subseteq f^{-1}(V) = f^{-1}\big(g^{-1}(U)\big) = (g \circ f)^{-1}(U)$. Hence $g \circ f$ is somewhat almost δP_S-continuous.

Remark 7.3.8. If f is somewhat almost δP_S-continuous and g is δ^*-continuous, then $g \circ f$ is somewhat almost δP_S-continuous.

Proof. Since g is an δ^* almost continuous functions, by Theorem 5.2.7(b) for every δP_S-open set of $V, g^{-1}(V)$ is δP_S-open in (Y,σ). Now f is somewhat almost δP_S-continuous implies there exists a δP_S-open set U in (X,τ) such that $U \subseteq f^{-1}\big(g^{-1}(V)\big) = (g \circ f)^{-1}(V)$. Hence $g \circ f$ is somewhat almost δP_Scontinuous function.

Proposition 7.3.9. If $f: (X,\tau) \to (Y,\sigma)$ is somewhat almost δP_S-continuous and $g: (Y,\sigma) \to (Z,\eta)$ is pre-irresolute, then $g \circ f: (X,\tau) \to (Y,\sigma)$ is somewhat almost δP_S-continuous function.

Proof. Since $\delta PO(Y,\sigma) = PO(Y,\sigma_S)$ and $\delta PO(Z,\eta) = PO(Z,\eta_S)$ by Lemma 1.1.21(b) and g is pre-irresolute, we get for every $V \in \delta PO(\mathrm{Z},\eta), g^{-1}(V) \in \delta PO(Y,\sigma)$**.** Now f is somewhat almost δP_S-continuous function and hence $f^{-1}(g^{-1}(V)$ contains a δP_S-open set W in (X,τ) (i.e.,) $W \subseteq (g \circ f)^{-1}(V) \Longrightarrow g \circ f$ is somewhat almost δP_S- continuous function.

Theorem 7.3.10. For a surjective function f, the following statements are equivalent:

(a) f is somewhat almost δP_Scontinuous.

(b) If C is a δ-semi closed subset in (Y,σ) such that $f^{-1}(C) \neq X$, then there is a non-empty $D \in \delta P_S \mathcal{C}(X,\tau)$ and $f^{-1}(C) \subseteq D$.

(c) If A is a δP_S-dense subset of (X,τ), then f(A) is a δP-dense subset of (Y,σ).

Proof. (a) $\Rightarrow$ (b) Let C be a δ-preclosed subset of (Y,σ) such that $f^{-1}(C) \neq X$. Then Y \ C is a δ-preopen set in (Y,σ) such that $f^{-1}(Y\backslash C) = X\backslash f^{-1}(C) \neq \emptyset$. By (a), there exists a non-empty $V \in \delta P_S O(X,\tau)$ and $V \subseteq f^{-1}(Y\backslash C) = X\backslash f^{-1}(C)$. Thus, $f^{-1}(C) \subseteq X\backslash V$ and $X\backslash V = D$ is a proper δP_S-closed set in (X,τ).

(b) $\Rightarrow$ (a) Let $U \in \delta PO(Y,\sigma)$ and $f^{-1}(U) \neq \emptyset$ then $Y\backslash U \in \delta PC(Y,\sigma)$ and $f^{-1}(Y\backslash U) = X\backslash f^{-1}(U) \neq X$. By (b), there exists a proper $D \in \delta P_S C(X,\tau)$ such that $f^{-1}(Y\backslash U) D$. This implies that $X\backslash D \subseteq f^{-1}(U)$ and $X\backslash D$ is δP_S-open and $X\backslash D \neq \emptyset$.

(b) $\Rightarrow$ (c) Let A be a δP_S-dense set in (X, τ). Suppose that f(A) is not δP_S-dense in (Y, σ). Then there exists a proper subset C ∊ δPC(Y, σ) such that f(A) ⊆ C ⊆ Y. Clearly $f^{-1}(C) \neq X$. By (b), there exists a proper subset $D \in \delta P_S C(X,\tau)$ such that $A \subseteq f^{-1}(C) \subseteq D \subseteq X$. This is a contradiction to the fact that A is δP_S-dense in (X,τ).

(c) $\Rightarrow$ (b) Suppose (b) is not true there exists C ∊ δPC(Y, σ) such that f^{-1} (C) ≠ X but there is no proper subset $D \in \delta P_S C(X,\tau)$ such that $f^{-1}(C) \subseteq D$. Thus $f^{-1}(C)$ is δP_S-dense in (X,τ). But by (c), $f\left(f^{-1}(C)\right) = C$ is δP-dense in (Y, σ), which contradicts the choice of C.

Proposition 7.3.11. Let f be a function and X = A∪B, where $A, B \in RO(X,\tau)$. Then, if the restriction functions $f|_A$ and $f|_B$ are somewhat almost δP_S-continuous, then f is somewhat almost δP_S-continuous.

Proof. Let U ∊ δPO(Y, σ) such that $f^{-1}(U) \neq \emptyset$. Then $(f|_A)^{-1}(U) \neq \emptyset$ or $(f|_B)^{-1}(U) \neq \emptyset$ or both $(f|_A)^{-1}(U) \neq \emptyset$ and $(f|_B)^{-1}(U) \neq \emptyset$. Suppose $(f|_A)^{-1}(U) \neq \emptyset$, since $f|_A$ is somewhat almost δP_S-continuous, there exists a δP_S-open set V in A such that V ≠ ∅ and $V \subseteq (f|_A)^{-1}(U) \subseteq f^{-1}(U)$. Since V is δP_S-open in A and A is Regular open in (X, τ), V is δP_S-open in (X, τ), by Proposition 2.2.15. Thus, f is somewhat almost δP_S-continuous.

The proof of other cases is similar.

Proposition 7.3.12. Let $f: (X,\tau) \to (Y,\sigma)$ be a somewhat almost δP_S-continuous surjective and τ^* be a topology for X, which is δP_S-equivalent to τ. Then f: $(X, \tau^*) \to (Y, \sigma)$ is somewhat almost δP_S-continuous.

Proof. Let $V \in \delta PO(\sigma)$ such that $f^{-1}(V) \neq \emptyset$. Since f is somewhat almost δP_S-continuous, there exists a non-empty δP_S-open set U in (X, τ) such that $U \subseteq f^{-1}(V)$. But by hypothesis τ^* is δP_S-equivalent to τ. Therefore, there exists $U^* \in \delta P_S O(X, \tau^*)$ such that $U^* \subseteq U$. But $U \subseteq f^{-1}(V)$ then $U^* \subseteq f^{-1}(V)$, hence $f: (X,\tau^*) \to (Y,\sigma)$ is somewhat almost δP_S-continuous.

Definition 7.3.13. If X is a set and τ and τ^* are topologies on X, then τ is said to be **δ-pre-equivalent** to τ^* provided if U ∈ δPO(X, τ) and U ≠ ∅ then there exists $U^* \in \delta PO(X, \tau^*)$ such that $U^* \neq \emptyset$ and $U^* \subseteq U$ and if $U \in \delta PO(X, \tau^*)$ and U ≠ ∅ then there exists V ∈ δPO(X, τ) such that V ≠ ∅ and U ⊇ V.

Proposition 7.3.14. Let $f: (X, \tau) \to (Y, \sigma)$ be a somewhat almost δP_S-continuous surjective and σ^* be a topology for Y, which is δ-pre-equivalent to σ. Then f: (X, τ) → (Y, σ^*) is somewhat almost δP_S -continuous.

Proof. Let $V^* \in \delta PO(Y, \sigma^*)$ such that $f^{-1}(V^*) \neq \emptyset$. Since σ^* is δ-pre-equivalent to σ, there exists a non-empty δ-preopen set V in (Y, σ) such that $V \subseteq V^*$. Now $\emptyset \neq f^{-1}(V) \subseteq f^{-1}(V^*)$. Since f is somewhat almost δP_S-continuous, there exists a non-empty δP_S-open set U in (X, τ) such that $U \subseteq f^{-1}(V)$. Then $U \subseteq f^{-1}(V^*)$, hence $f: (X, \tau) \to (Y, \sigma^*)$ is somewhat almost δP_S-continuous.

7.4 Somewhat δP_S - Irresolute Functions

Definition 7.4.1. A function $f: (X, \tau) \to (Y, \sigma)$ is said to be **somewhat δP_S-irresolute** if for $U \in \delta P_S O$ (Y, σ) and $f^{-1}(U) \neq \emptyset$, there exists a non-empty δP_S-open set V in (X, τ) such that $V \subseteq f^{-1}(U)$.

Example 7.4.2. Let X = Y = {a,b,c} with τ = {X, ∅, {a,b}} and σ = {Y, ∅, {a}, {a,b}, {a,c}}. Let $f: (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is somewhat δP_S-irresolute but not somewhat irresolute as for the semi open set {a} in (Y, σ), $f^{-1}\{a\} = \{a\}$ doesn't contain any non-empty semiopen set in (X, τ).

Example 7.4.3. Let X = Y = {a,b,c} with τ = {X, ∅,{a},{b},{a,b}} and σ = {Y, ∅, {a}, {a,b}, {a,c}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) = b, f(b) = c, f(c) = a. Then f is not somewhat δP_S-irresolute as $f^{-1}\{a\} = \{c\}$ contains no non-empty δP_S-open set in (X, τ), but f is somewhat irresolute.

Proposition 7.4.4. If f and g are somewhat δP_S-irresolute functions from $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ then $g \circ f$ is δP_S-irresolute.

Proof. Let $W \in (Z, \eta)$ be in $\delta P_S O(\eta)$. Then, since g is somewhat δP_S-irresolute then there exists $V \in \delta P_S O(\sigma)$ such that $V \subseteq g^{-1}(W)$. Since f is somewhat δP_S-irresolute there exists $U \in \delta P_S O(\tau)$ such that $U \subseteq f^{-1}(V)$.

$\Rightarrow U \subseteq f^{-1}(V) \subseteq f^{-1}\big(g^{-1}(W)\big) = (g \circ f)^{-1}(W)$

$\therefore g \circ f$ is somewhat δP_S-irresolute.

Theorem 7.4.5. For a surjective function f, the following statements are equivalent:

(a) f is a somewhat δP_S-irresolute function.

(b) If C is a δP_S-closed subset of (Y, σ) such that $f^{-1}(C) \neq X$, then there is a proper δP_S-closed subset D of (X, τ) such that $f^{-1}(C) \subseteq D$.

(c) If A is a δP_S-dense subset of (X, τ), then f(A) is a δP_S-dense subset of (Y, σ).

Proof. (a) ⇒ (b) Let C be a δP_S-closed subset of (Y, σ) such that $f^{-1}(C) \neq X$. Then Y \ C is a δP_S-open set in (Y, σ) such that $f^{-1}(Y \setminus C) = X \setminus f^{-1}(C) \neq \emptyset$. By (a), there exists a non-empty δP_S-open set V in (X, τ) and $V \subseteq f^{-1}(Y \setminus C) = X \setminus f^{-1}(C)$. This means that $f^{-1}(C) \subseteq X \setminus V$ and X \ V = D is a proper δP_S-closed set in (X, τ).

(b) ⇒ (a) Let $U \in \delta P_S O(Y, \sigma)$ and $f^{-1}(U) \neq \emptyset$ then Y \ U is δP_S -closed and $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U) \neq X$. By (b), there exists a proper δP_S-closed set D such that $f^{-1}(Y \setminus U) \subseteq D$. This implies that $X \setminus D \subseteq f^{-1}(U)$ and X \ D is δP_S-open in (X, τ) and $X \setminus D \neq \emptyset$.

(b) ⇒ (c) Let A be a δP_S-dense set in (X, τ). Suppose that f(A) is not δP_S-dense in (Y, σ). Then there exists a proper δP_S-closed set C in (Y, σ) such that $f(A) \subseteq C \subseteq Y$. Clearly $f^{-1}(C) \neq X$. By (b), there exists a proper δP_S-closed set D such that $A \subseteq f^{-1}(C) \subseteq D \subseteq X$. This is a contradiction to the fact that A is δP_S-dense in (X, τ).

(c) ⇒ (b) Suppose (b) is not true, there exists a δP_S-closed set C in (Y, σ) such that $f^{-1}(C) \neq X$ but there is no proper δP_S-closed set D in (X, τ) such that $f^{-1}(C) \subseteq D$. This means that $f^{-1}(C)$ is δP_S-dense in (X, τ). But by (c), $f(f^{-1}(C)) = C$ must be δP_S-dense in (Y, σ), which is a contradiction to the choice of C.

Proposition 7.4.6. Let f be a function and $X = A \cup B$, where $A, B \in RO(X, \tau)$. Then, if the restriction functions $f|_A: (A; \tau|_A) \to (Y, \sigma)$ and $f|_B: (B; \tau|_B) \to (Y, \sigma)$ are somewhat δP_S-irresolute, then f is a somewhat δP_S-irresolute function.

Proof. Let $U \in \delta P_S O(Y, \sigma)$ such that $f^{-1}(U) \neq \emptyset$. Then $(f|_A)^{-1}(U) \neq \emptyset$ or $(f|_B)^{-1}(U) \neq \emptyset$ or both $(f|_A)^{-1}(U) \neq \emptyset$ and $(f|_B)^{-1}(U) \neq \emptyset$. Suppose $(f|_A)^{-1}(U) \neq \emptyset$, since $f|_A$ is somewhat δP_S-irresolute, there exists a δP_S-open set V in A such that $V \neq \emptyset$ and $V \subseteq (f|_A)^{-1}(U) \subseteq f^{-1}(U)$. Since A is regular open in (X, τ) and V is δP_S-open in (X, τ), by Proposition 2.2.15. Thus, f is somewhat δP_S-irresolute.

The proof of other cases is similar.

Proposition 7.4.7. If f is the identity function on a set X. The two topologies on X, τ and σ are δP_S-equivalent. Then f and f^{-1} are somewhat δP_S-irresolute functions. Conversely, if the identity function is somewhat δP_S irresolute in both directions, then τ and σ are δP_S-equivalent.

Proof. Let τ and σ are δP_S-equivalent.

For every $U \in \delta P_S O(X, \tau)$ there exists $V \in \delta P_S O(X, \sigma)$ such that $V \subseteq U$ → (1)

and for every $U \in \delta P_S O(X, \sigma)$, there exists $V \in \delta P_S O(X, \tau)$ such that $U \supseteq V$ → (2)

If $f: (X, \tau) \to (X, \sigma)$, Let $U \in \delta P_S O(X, \sigma)$ then $f^{-1}(U) = U$ since f is identity. By (2), there exists $V \in \delta P_S O(X, \tau)$ such that $V \subseteq U = f^{-1}(U)$. Therefore, f is somewhat δP_S-irresolute.

If $f^{-1}: (X, \tau) \to (X, \sigma)$, Let $U \in \delta P_S O(X, \tau)$ then $(f^{-1})^{-1}(U) = U$ since f is identity. By (1), there exists $V \in \delta P_S O(X, \sigma)$ such that $V \subseteq U = f(U)$. Therefore, f is somewhat δP_S-irresolute.

Proposition 7.4.8. Let $f: (X, \tau) \to (Y, \sigma)$ be a somewhat δP_S-irresolute surjective and τ^* be a topology for (X, τ), which is δP_S-equivalent to τ. Then f: $(X, \tau^*) \to (Y, \sigma)$ is somewhat δP_S-irresolute.

Proof. Let $V \in \delta P_S O(Y, \sigma)$ such that $f^{-1}(V) \neq \emptyset$. Since $f: (X, \tau) \to (Y, \sigma)$is somewhat δP_S-irresolute, there exists a non-empty δP_S-open set U in (X, τ) such that $U \subseteq f^{-1}(V)$. Since τ^* is δP_S-equivalent to τ. By (2) in the above Proposition, there exists $U^* \in \delta P_S O(X, \tau^*)$ such that $U^* \subseteq U$. But $U \subseteq f^{-1}(V)$ then $U^* \subseteq f^{-1}(V)$, hence f : $(X, \tau^*) \to (Y, \sigma)$ is a somewhat δP_S-irresolute function.

Proposition 7.4.9. Let $f: (X, \tau) \to (Y, \sigma)$ be a somewhat δP_S-irresolute surjective function and σ^* be a topology for Y, which is equivalent to σ. Then f : $(X, \tau) \to (Y, \sigma^*)$ is somewhat δP_S-irresolute.

Proof. Let $V^* \in \delta P_S O(Y, \sigma^*)$ such that $f^{-1}(V^*) \neq \emptyset$. Since σ^* is equivalent to σ, by (1) in Proposition 7.4.7, there exists a non-empty $V \in \delta P_S O(Y, \sigma)$ such that $V \subseteq V^*$. Now $\emptyset \neq f^{-1}(V) \subseteq f^{-1}(V^*)$. Since f is somewhat δP_S-irresolute, there exists a non-empty δP_S-open set U in (X, τ) such that $U \subseteq f^{-1}(V)$. Then $U \subseteq f^{-1}(V^*)$, hence f : $(X, \tau) \to (Y, \sigma^*)$ is a somewhat δP_S-irresolute function.

7.5 Somewhat δP_S-Open Functions

Definition 7.5.1 A function f is said to be **somewhat δP_S-open** provided that if $U \in \tau$ and $U \neq \emptyset$, then there exists a non-empty δP_S-open set V in (Y, σ) such that $V \subseteq f(U)$.

The following function f is both somewhat open and somewhat δP_S-open.

Example 7.5.2 Let X = Y = {a,b,c} with τ = {X, ∅, {a}, {a,b}} and σ = {Y, ∅,{a},{b},{a,b}}. Let $f: (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is somewhat δP_S-open and somewhat open.

Remark 7.5.3. The following examples exhibits somewhat openness and somewhat δP_S-openness are independent.

Example 7.5.4. Let X = Y = {a, b, c} with τ = {X, ∅,{a},{b},{a,b},{a,c}} and σ = {Y, ∅, {a}}. Then $f: (X, \tau) \to (Y, \sigma)$ is neither somewhat open and nor somewhat δP_S- open.

Example 7.5.5. Let X = Y = {a, b, c} with τ = {X, ∅,{a},{b},{a,b}} and σ = {Y, ∅, {a},{b},{a,b},{a,c}}. Then the identity function $f: (X, \tau) \to (Y, \sigma)$ is somewhat open but not somewhat δP_S- open.

Example 7.5.6. Let X = Y = {a, b, c} with τ = {X, ∅, b},{c},{b,c}} and σ = {Y, ∅, {a}}. Then the identity function $f: (X, \tau) \rightarrow (Y, \sigma)$ is not somewhat open but it is somewhat δP_S-open.

Remark 7.5.7. The composition of two somewhat δP_S-open functions need not be somewhat δP_S-open.

Example 7.5.8. Let X = Y = Z = {a,b,c} with τ = {X, ∅, {a}, {a,b}}, σ = {Y, ∅, {a,b}} and η = {Z, ∅,{a},{b},{a,b}}. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = c, f(b) = a, f(c) = b$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ such that $g(a) = g(b) = a$ and $g(c) = c$. Then the functions f and g are somewhat δP_S-open but their composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is not somewhat δP_S-open function.

Proposition 7.5.9. Let f be an open function and g be somewhat δP_S-open. Then $g \circ f$ is somewhat δP_S-open.

Proof. Let U be open in (X, τ). Since f is an open function, f(U) is open in (Y, σ) and since g is somewhat δP_S-open, g(f(U)) contains non-empty somewhat δP_S-open set in (Z, η). Hence $g \circ f$ is somewhat δP_S-open.

Theorem 7.5.10. For a bijective function f, the following are equivalent:

(a) f is somewhat δP_S-open

(b) If C is a closed subset of (X, τ), such that f(C) ≠ Y, then there is a δP_S-closed subset D of (Y, σ) such that D ≠ Y and f(C) ⊆ D.

Proof. (a) ⇒ (b) Let C be any closed subset of (X, τ) such that f(C) ≠ Y. Then X \ C is open in (X, τ) and X \ C ≠ ∅ as f is bijective. Since f is somewhat δP_S-open, there exists a δP_S-open set V ≠ ∅ in (Y, σ) such that V ⊆ f(X \ C). Put D = Y \ V. Clearly, D is δP_Sclosed in (Y, σ) and we claim D ≠ Y. If D = Y, then V = ∅, which is a contradiction. Since V ⊆ f(X \ C), D = Y \ V ⊇ (Y \ f(X \ C)) = f(C).

(b) ⇒ (a) Let U be any non-empty open subset of (X, τ). Then C = X \ U is a closed set in (X, τ) and since f is bijective, f(X \ U) = f(C) = Y \ f(U) implies f(C) ≠ Y. Therefore by (b), there is a δP_S-closed set D of (Y, σ) such that D ≠ Y and f(C) ⊆ D. Clearly V = Y \ D is a δP_S-open set and V ≠ ∅. Also, V = Y \ D ⊆Y \ f(C) = Y \ f(X \ U) = f(U).

Proposition 7.5.11. f is somewhat δP_S-open if and only if for a non-empty open subset A⊆ (X, τ), δP_SInt(f(A)) ≠ ∅.

Proof. Let A ≠∅ in (X, τ) and A is open. Since f is somewhat δP_S-open, there exists a non-empty δP_S-open set V of (Y, σ) such that V ⊆ f(A).

⇔ That is δP_SInt(V) = V ≠ ∅

$\Leftrightarrow$ Now δP_SInt(V) ⊆ δP_SInt(f(A))

$\Leftrightarrow$ δP_SInt(f(A)) ≠ ∅.

Theorem 7.5.12. The following statements are equivalent:

(a) f is somewhat δP_S-open

(b) If A is a δP_S-dense subset of (Y, σ), then f^{-1}(A) is a dense subset of (X, τ).

Proof. (a) ⇒ (b) Suppose A is a δP_S-dense set in (Y, σ). If f^{-1}(A) is not dense in (X, τ), then there exists a closed set B in (X, τ) such that f^{-1}(A) ⊆ B ⊆ X. Since f is somewhat δP_S-open and X \ B is open, there exists a non-empty δP_S-open set C in Y such that C⊆ f (X \ B). Therefore C ⊆f(X \ B) ⊆ f(f^{-1}(Y \ A)) ⊆ Y \ A. That is A ⊆Y \ C ⊆ Y. Now, Y \ C is a δP_S-closed set and A ⊆ Y \ C ⊆ Y. This implies that A is not a δP_S-dense set in (Y, σ), which is a contradiction. Therefore f^{-1}(A) is a dense set in (X, τ).

(b) ⇒ (a) Suppose A is a non-empty open subset of (X, τ). To show f is somewhat δP_S-open, using the above lemma, we want to show that δP_SInt(f(A)) ≠ ∅. Suppose δP_Sint(f(A)) = ∅. Then, $\delta P_S Cl(f(A)) = Y$. Therefore by (b), f^{-1}(Y \ f(A)) is dense in (X, τ). But f^{-1}(Y \ f(A)) $\subseteq X\backslash A$. Now, X \ A is closed. Therefore f^{-1}(Y \ f(A)) ⊆ X \ A gives X = Cl[f^{-1}(Y \ f(A))] $\subseteq X\backslash A$. This implies that A = ∅, which is a contradiction to A ≠ ∅. Therefore δP_SInt(f(A)) ≠ ∅. Hence f is somewhat δP_S-open.

Proposition 7.5.13. Let f be somewhat δP_S-open and A be any open subset of (X, τ). Then $f|_A$: (A, $\tau|_A$) → (Y, σ) is somewhat δP_S-open.

Proof. Let U ∈ $\tau|_A$ such that U ≠ ∅. Since U is open in A and A is open in (X, τ), U is open in (X, τ) and since by hypothesis f is somewhat δP_S-open function, there exists a δP_S-open set V in (Y, σ), such that V⊆ f(U). Thus, for any open set U of A with U ≠ ∅, there exists a δP_S-open set V in (Y, σ) such that V ⊆ f(U). Hence $f|_A$ is a somewhat δP_S-open function.

Proposition 7.5.14. Let f be a function and X = A∪B, where $A, B \in \tau$. Then, if the restriction functions $f|_A$ and $f|_B$ are somewhat δP_S-open, then f is somewhat δP_S-open.

Proof. Let U be any open subset of (X, τ) such that U ≠ ∅. Since X = A∪B, either A∩U ≠ ∅ or B∩U ≠ ∅. Since U is open in (X, τ), $A \cap U$ is open in A and $B \cap U$ is open in B.

Case (i) If A∩U ≠ ∅. Since $f|_A$ is somewhat δP_S-open, there exists $V \in \delta P_S O(Y, \sigma)$ such that V ⊆ f(U∩A) ⊆ f(U), which implies that f is somewhat δP_S-open.

Case (ii) If B∩U ≠ ∅. Since $f|_B$ is somewhat δP_S-open, there exists W∈ δP_SO(Y, σ) such that W ⊆ f(U∩B) ⊆ f(U), which implies that f is somewhat δP_S-open.

Case (iii) If both A∩U ≠ ∅ and B∩U ≠ ∅. Then, by case (i) and case (ii), f is somewhat δP_S-open.

Proposition 7.5.15. Two topologies τ and τ^* for X are δP_S-equivalent if and only if the identity functions $i_\tau : (X, \tau) \to (X, \tau^*)$ and $i_{\tau^*} : (X, \tau^*) \to (X, \tau)$ are both somewhat δP_S-open.

Proof. Let i_τ and i_{τ^*} be both somewhat δP_S-open

To Prove: τ is δP_S-equivalent to τ^*.

Consider $U \in \tau^*$ such that $U \neq \emptyset$, then since i_{τ^*} is somewhat δP_S-open. Then there exists $V \in \tau$ such that $V \neq \emptyset$ such that $V \subseteq i_{\tau^*}(U) = U$.

∴ τ is δP_S-equivalent to τ^*.

Similarly, for all $A \in \tau$ such that $A \neq \emptyset$, since i_τ is somewhat δP_S-open, there exists $B \in \tau^*$ such that $B \neq \emptyset$ and $B \subseteq i_\tau(A) = A$

∴ τ^* is δP_S-equivalent to τ. Hence τ and τ^* are δP_S-equivalent.

7.6 Somewhat Almost δP_S - Open Functions

Definition 7.6.1 A function $f: (X, \tau) \to (Y, \sigma)$ is said to be **somewhat almost δP_S-open** provided that if U ∈ δPO(X, τ) and U ≠ ∅, then there exists a non-empty δP_S-open set V in (Y, σ) such that V ⊆ f(U).

Example 7.6.2 Let X = {a,b,c}, τ = {X, ∅, {a}, {b}, {a,b}} and σ = {Y, ∅, {c}}. The identity function $f: (X, \tau) \to (Y, \sigma)$ is somewhat almost δP_S-open and somewhat δP_S-open.

Proposition 7.6.3 Every somewhat almost δP_S-open function is a somewhat δP_S-open function.

Proof. Let f be a somewhat almost δP_S-open function. Consider $U \in \tau$ such that $U \neq \emptyset$. Then U is δPO in τ. Since f is somewhat almost δP_S-open, there exists a non-empty δP_S-open set V in σ such that $V \subseteq f(U)$,

$\Longrightarrow f$ is a somewhat δP_S-open function.

Example 7.6.4. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a, b\}\}$ and $\sigma = \{X, \emptyset, \{a\}\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be an identity function. Then f is somewhat δP_S-open but not somewhat almost δP_S-open function.

Proposition 7.6.5. The composition of two somewhat almost δP_S-open functions is a somewhat almost δP_S-open function.

Proof. Let f and g be somewhat almost δP_S-open function.

To Prove: $g \circ f$ is somewhat almost δP_S-open function

Let $U \in \delta PO(\tau)$. Since f is somewhat almost δP_S-open such that $V \in \delta P_S O(\sigma)$ such that

$V \subseteq f(U) \longrightarrow$ (1)

Since, $\delta P_S O(\sigma) \subseteq \delta PO(\sigma)$, here V is $\delta PO(Y, \sigma)$ and moreover g is somewhat almost δP_S-open, then there exists $W \in \delta P_S(\eta)$ such that $W \subseteq g(V)$

(i.e.,) $W \subseteq g(V) \subseteq g(f(U)) = (g \circ f)(U)$

$\Rightarrow g \circ f$ is somewhat almost δP_S-open functions.

Theorem 7.6.6. For a bijective function $f: (X, \tau) \to (Y, \sigma)$, the following are equivalent:

(a) f is a somewhat δP_S-open function

(b) If C is a δ-preclosed set in (X, τ), such that f(C) ≠ Y, then there is a δP_S-closed subset D of (Y, σ) such that D ≠ ∅ and f(C) ⊆ D.

Proof. (a) ⇒ (b) Let C be any δ-preclosed subset of (X, τ) such that f(C) ≠ Y. Then X \ C is δ-preopen in (X, τ) and X \ C ≠ ∅. Since f is a somewhat almost δP_S-open function, there exists a δP_S-open set V ≠ ∅ in (Y, σ) such that V ⊆ f(X \ C). Put D = Y \ V. Clearly, D is δP_S-closed in (Y, σ) and D ≠ ∅. If D = Y, then V ≠ ∅, which is a contradiction. Since V ⊆ f(X \ C), D = Y \ V ⊇ (Y \ f(X \ C)) = f(C).

(b) ⇒ (a) Let U be any non-empty δ-preopen subset of (X, τ). Then C = X \ U is a δ-preclosed set in (X, τ) and f(X \ U) = f(C) = Y \ f(U) implies f(C) ≠ Y. Then by (b), there is a δP_S-closed set D of (Y, σ) such that D ≠ ∅ and f(C) ⊆ D. Clearly V = Y \ D is a δP_S-open set and V ≠ ∅. Also, V = Y \ D ⊆ Y \ f(C) = Y \ f(X \ U) = f(U).

Proposition 7.6.7. If $f: (X, \tau) \to (Y, \sigma)$ is somewhat almost δP_S-open then for a subset A ∈ $\delta PO(X, \tau)$, $\delta P_S I$nt(f(A)) ≠ ∅.

Proof. The proof is same as Proposition 7.5.11.

Theorem 7.6.8. The following statements are equivalent:

(a) $f: (X, \tau) \to (Y, \sigma)$ is a somewhat almost δP_S-open function

(b) If A is a δP_S-dense subset of (Y, σ), then f^{-1}(A) is a δ-pre-dense subset of (X, τ).

Proof. (a) ⇒ (b) Let A is a δP_S-dense set in (Y, σ). If f^{-1}(A) is not δ-pre-dense in (X, τ), then there exists a δ-preclosed set B in (X, τ) such that f^{-1}(A) ⊆ B ⊆ X. Since f is somewhat almost δP_S-open and X \ B is δ-preopen in (X, τ), there exists a non-empty δP_S-open set C in Y such that C ⊆ f(X \ B). Now X \ B ⊆ X \ f^{-1}(A). Therefore C⊆ f(X \ B) ⊆ f(X \ f^{-1}(A)) ⊆Y \ A. That is A ⊆ Y \ C ⊆ Y. Now, Y \ C is a δP_S-closed set and A ⊆ Y \ C ⊆ Y. This implies that A is not a δP_S-dense set in (Y, σ), which is a contradiction. Therefore f^{-1}(A) is a δ-pre-dense set in (X, τ).

(b) ⇒ (a) If A is a non-empty δ-pre-open subset of (X, τ). To show f is somewhat almost δP_S-open function using Proposition 7.6.7, Also we want to show that δP_SInt(f(A)) ≠ ∅. Suppose $\delta P_S Int(f(A)) = \emptyset$. Then, $\delta P_S Cl\{(Y \backslash f(A))\} = Y$. Then by (b), $f^{-1}(Y \backslash f(A))$ is δ-pre-dense in (X, τ). But $f^{-1}(Y \backslash f(A)) \subseteq X \backslash A$. Now, $X \backslash A$ is δ-pre-closed in (X, τ). Therefore

$f^{-1}(Y\backslash f(A)) \subseteq X\backslash A$ gives $X = \delta\, pCl\{(f^{-1}(Y\backslash f(A))\} \subseteq X\backslash A$. Thus A = ∅, which contradicts $A \neq \emptyset$. Therefore $\delta P_S Int(f(A)) \neq \emptyset$. Hence f is a somewhat almost δP_S-open function.

Proposition 7.6.9. Let f be a function and X = A∪B, where $A, B \in \delta PO(X,\tau)$. Then if $f|_A$ and $f|_B$ are somewhat almost δP_S-open, then f is somewhat almost δP_S-open.

Proof. Let U be any δ-pre-open subset of (X, τ) such that U ≠ ∅. Since X = A∪B, either $A \cap U \neq \emptyset$ or $B \cap U \neq \emptyset$ or both $A \cap U \neq \emptyset$ and $B \cap U \neq \emptyset$. Since U is δ-pre-open in (X, τ), $A \cap U$ is δ-pre-open in A and $B \cap U$ is δ-preopen in B.

Case (i) Now $A \cap U \in \delta PO(A)$ such that $A \cap U \neq \emptyset$. Since $f|_A$ is somewhat almost δP_S-open, there exists $V \in \delta P_S O(Y)$ such that $V \subseteq f(U \cap A) \subseteq f(U)$, which implies that f is somewhat almost δP_S-open.

Case (ii) Now $B \cap U \in \delta PO(B)$ such that $B \cap U \neq \emptyset$. Since $f|_B$ is somewhat almost δP_S-open, there exists $V \in \delta P_S O(Y)$ such that V ⊆ f(U∩B) ⊆ f(U), which implies that f is somewhat almost δP_S-open.

Case (iii) If both $A \cap B \neq \emptyset$ and $B \cap U \neq \emptyset$.Then, by case (i) and (ii), f is somewhat almost δP_S-open.

Proposition 7.6.10. Two topologies τ and σ for X are said to be δP_S-equivalent if and only if the identity function $f: (X,\tau) \to (X,\sigma)$ is somewhat almost δP_S-open in both directions.

Proof. The proof is same as Proposition 7.5.15.

Conclusion:

In general, the composition of continuity holds good. But for weaker and stronger forms of continuous functions it may not hold. For δP_S-continuous functions, the composition fails by Vidhyapriya et al [2021]. In this paper it proved that somewhat δP_S-continuous functions and somewhat δP_S-openness functions fails to hold composition whereas somewhat almost δP_S-continuous and somewhat almost δP_S-opennes satisfy composition property which is given below. For all irresoluteness the composition holds good which is true for somewhat δP_S-irresolute.

Functions	Composition
Somewhat δP_S-continuous	✗
Somewhat almost δP_S-continuous	✓
Somewhat δP_S-irresolute	✓
Somewhat δP_S-open	✗
Somewhat almost δP_S-open	✓