**Avinashilingam Institute for Home Science and Higher Education for Women**
**Coimbatore – 641 043**
**Bachelor's Degree Examination – November 2017**

**V Semester**

**Class : III UG** **Time : 3 Hours**
**Major : Special Education and Mathematics** **Max. Marks : 100**

**15BSMC13 Complex Analysis - I**
**Part – A** **10 x 1 = 10**
**Choose the Correct Answer**

1. A polynomial in the finite plane is
   a. an entire function b. a limit function
   c. an inverse function d. exponential function

2. The function f(z) = $\begin{cases} \frac{Re z}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$
   a. continuous at z = 0 b. analytic at z = 0
   c. not continuous at z = 0 d. limit exist at z = 0

3. Which one of the following is not an entire function?
   a. (1+z) sin z b. $\frac{z^5}{|z|^4}$
   c. $ze^z$ d. sin z

4. The function f(z) = xy + iy is
   a. not continuous b. constant
   c. not analytic any where d. analytic any where

5. If u and v are conjugate harmonic functions then v and u are
   a. conjugate harmonic b. not conjugate harmonic
   c. entire function d. differentiable

6. The value of m such that $2x - x^2 - my^2$ may be harmonic is
   a. 3 b. 1
   c. -1 d. 0

7. The transformation w = $\frac{az+b}{Cz+d}$ is called bilinear transformation if
   a. ad – bc = 0 b. ad – bc ≠ 0
   c. ad – bc = w d. ad – bc ≠ w

8. The fixed points of the transformation w = z + b are
   a. z = ∞ b. z = 0, 1
   c. z = 1, ∞ d. z = -1

9. Integrals along simple closed rectifiable curves are called
   a. curve b. contour integral
   c. path integral d. closed

10. If c is positively oriented circle | z – 2 | = 3 then $\int_c dz$ is
    a. $2\pi$ b. $2\pi i$
    c. $\pi$ d. $\pi i$

## Part – B

**5 x 6 = 30**

**Answer the following**

**Answer should not exceed 400 words or two pages**

11.a. Show that the limit function is unique.

(Or)

11.b. Show that the function Re z is no where differentiable.

12.a. Derive the condensed from of the C – R equations of an analytic function f(z).

(Or)

12.b. Show that $f(z) = z^2$ is analytic

13.a. If $e^z = u + iv$, then show that u and v are harmonic function.

(Or)

13.b. If u(x,y) = 2x(1-y) then find f(z).

14.a. Prove that under a bilinear transformation no two points in z plane go to the same point in w – plane.

(Or)

14.b. Find the bilinear transformation that maps the points 2,1, 0 onto 1,0,i respectively.

15.a. State and prove Cauchy's theorem

(Or)

15.b. Evaluate $\int_C \frac{2Z-1}{z^3-z}\,dz$, where C is |z| = 2

## Part – C

**5 x 12 = 60**

**Answer the following**

**Answer should not exceed 800 words or four pages**

16.a. Show that the function f(z) = z is not differentiable at zero.

(Or)

16.b. Derive the Cauchy – Riemann equations in Cartesian coordinates.

17.a. If f(z) is a function of x, y and x, y are functions of z, $\bar{z}$ then show that

$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^z}{\partial y^2}\right) | f(z)|^2 = 4\, |f'(z)|^2$

(Or)

17.b. If f(z) is a function of x, y and x, y are functions of z, $\bar{z}$ then show that

$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) \log | f'(z) | = 0$

18.a. Verify whether $v = e^x$ ( x cos y – y sin y) as a harmonic function and construct the analytic function f(z).

(Or)

18.b. If u and v are harmonic functions then prove that $(u_y - v_x) + i(u_x - v_y)$ is an analytic function.

19.a. Show that $w = \frac{z-i}{z+i}$ maps the real axis in the z plane onto |w| = 1 in the w – plane. Show also that the upper half of the z – plane, Imz ≥ 0 goes onto the circular disc |w|≤1

(Or)

19.b. Find the fixed points under the following transformations:

i. $w = \frac{2z-5}{z+4}$ ii. $w = \frac{4z-4}{z}$ iii. $w = \frac{z-2}{z+3}$ iv. $w = \frac{6z-9}{z}$ v. $w = \frac{z-1}{z+1}$

20.a. If c is the positively oriented circle | z – i | = 2, then show that $\int_C \frac{e^z}{z^2+4}\,dz = \frac{\pi}{2}\, e^{2i}$

(Or)

20.b. State and prove Cauchy's integral formula for first derivative.

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