Contra ψgs - Continuous Functions in Topological Spaces

SONA J

(17PMA019)

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore - 641043

In Partial Fulfillment of the Requirements for the Degree of

Master of Science in Mathematics

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Signature of the Head

of the Department

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Signature of the Supervisor

ACKNOWLEDGEMENT

First and foremost , I am extremely thankful to the **LORD ALMIGHTY** for his graces and blessings showered on me.

I take immense pleasure in thanking **Dr.P.R.KRISHNAKUMAR,** Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for providing the conducive infrastructure for the conduct of the research study.

I would like to thank **Dr. (Tmt.) PREMAVATHY VIJAYAN,** Vice Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for providing the opportunity to develop and establish my skills.

I extend my heartfelt thanks to **Dr. (Tmt.) S.KOWSALYA,** Registrar, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for the encouragement given by her during the investigation.

My words never fail to express my deep sense of gratitude to **Hony.COL. Dr. (Tmt.) SAROJA PRABHAKARAN,** Former Vice Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and The Director, Halls of Residence, Avinashilingam Education Trust Institution Hostel for Women, Coimbatore, for all the necessary support and guidance towards the completion of the study.

I express my heartfelt thanks to **Dr. (Tmt.) K.UDAYA CHANDRIKA,** Professor of Mathematics, Dean, School of Physical Sciences and Computational Sciences, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her excellent support, unflinching encouragement and guidance during the course of the investigation.

I express my heartfelt thanks to **Dr. (Tmt.) P.JEYALAKSHMI,** Professor and Head, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her support and guidance during the course of the investigation.

I thank my thesis supervisor **Dr. (Tmt.) N.BALAMANI**, Assistant Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her inspiring guidance, innovative ideas, critical suggestions and constant encouragement throughout the completion of this work.

I would like to express my sincere thanks to all the **Staff Members of the Department of Mathematics** who were responsible for the good finish of this dissertation.

I owe my special thanks to my **Beloved Parents, Loving Sister, Friends and also the Graceful Relatives,** who helped me by providing full strength, support and encouragement to complete my project successfully.

CONTENT

INTRODUCTION

General topology is the branch of topology that deals with the basic set theoretic definitions and constructions used in topology. It is the foundation of most other branches of topology, including differential topology, geometric topology, and algebraic topology. Another name for general topology is point - set topology. In the field of mathematics topology plays a vital role.

The notion of closed sets is one of the most important concepts in topological spaces. In the study of topological spaces many concepts of topology have been generalized by introducing the regular open sets due to Stone (1937) instead of open sets. Levine (1963) introduced the semi open sets in topological spaces. Using Levine's idea many researchers have introduced various types of generalized closed sets. Gowsalya and Balamani (2016a) introduced the concept of ψgs - closed sets in topological spaces.

Continuous functions are important notions in the field of mathematics. Levine (1963) introduced semi continuous functions in topological spaces using semi open sets. Levine (1970) introduced continuous functions in topological spaces using open sets. Jain(1980) introduced totally continuous functions in topological spaces. Gowsalya and Balamani (2016b) introduced the concept of ψgs - continuous functions in topological spaces.

Dontchev (1996) introduced contra continuous functions in topological spaces. Ekici (2004) introduced the concept of almost contra continuous functions in topological spaces.

Crossley and Hildebrand (1972) introduced irresolute functions in topological spaces.

The present study focuses on the following concepts :

1. Totally ψgs - continuous functions and ψgs - totally continuous functions in topological spaces.
2. Contra ψgs - continuous functions and almost contra ψgs - continuous functions in topological spaces.
3. ψgs - irresolute functions in topological spaces.

Chapter 1 deals with the preliminary definitions and results in topological spaces that are required for the present study.

In chapter 2, two new classes of totally continuous functions called totally ψgs - continuous functions and ψgs - totally continuous functions are introduced in topological spaces.

In chapter 3, the concepts of contra ψgs - continuous functions and almost contra ψgs - continuous functions are introduced and their properties are analysed.

In chapter 4, a new form of irresolute functions called ψgs - irresolute function is defined and the association of ψgs - irresolute functions with other existing irresolute functions are obtained.

REVIEW OF LITERATURE

General topology grew out of a number of areas that includes the study of subsets of real line, manifold concept and in the area of functional analysis. General topology assumed its present from around 1940.

The topological spaces were initially characterized by open sets. Stone (1937) introduced regular openness which is stronger than openness. Levine (1963) introduced the notion of semi openness which is weaker than the notion of openness. Levine(1970) introduced the generalized closed sets in topological spaces. Bhattacharyya and Lahiri (1987) introduced semi generalized closed sets in topological spaces.

Arya and Nour (1990) introduced the notion of generalized semi closed sets in topological spaces using semi - open sets. Veera Kumar (2000) introduced ψ - closed sets in topological spaces. Ramya and Parvathi (2013) introduced ψg - closed sets in topological spaces. Gowsalya and Balamani (2016) introduced ψgs - closed sets as a stronger form of ψg - closed sets in topological spaces.

Continuous functions are an important notion in the study of mathematical sciences. Many generalizations of continuous functions have been introduced over years and many interesting results are obtained. Levine (1963) introduced semi continuous functions in topological spaces. Levine (1970) introduced continuous functions in topological spaces.

Arya and Gupta (1974) introduced completely continuous functions in topological spaces. Balachandran et al.(1991) introduced generalized continuous functions in topological spaces. Sundaram et al.(1991) introduced semi generalized continuous functions in topological spaces. Veera Kumar (2000) introduced ψ - continuous functions in topological spaces. Gowsalya and Balamani (2016b) introduced ψgs - continuous functions in topological spaces.

Jain (1980) introduced the notion of totally continuous functions using the closed sets in topological spaces. The study of contra continuous functions in topological spaces was introduced by Dontchev (1996).Dontchev and Noiri (1999) introduced contra semi continuous functions in topological spaces.

Singal and Singal (1968) introduced almost continuous functions in topological spaces. Ekici (2004) introduced almost contra continuous functions in topological spaces. Iyappan and Nagaveni (2010) introduced almost contra semi continuous functions in topological spaces.

Crossley and Hildebrand (1972) introduced irresolute functions in topological spaces. Bhattacharyya and Lahiri (1987) introduced semi generalized irresolute functions in topological spaces. Devi et al.(1995) introduced generalized semi irresolute functions in topological spaces. Ramya and Parvathi (2013) introduced ψg - irresolute functions in topological spaces.

CHAPTER 1

PRELIMINARIES

Definition 1.1 [18]

A subset A of a topological space (X, τ) is called ***regular open*** if A = int(cl(A)) and ***regular closed*** if A = cl(int(A)).

Definition 1.2 [14]

A subset A of a topological space (X, τ) is called ***semi open*** if A ⊆ cl(int(A)) and ***semi - closed*** if int(cl(A)) ⊆ A .

Definition 1.3 [15]

A subset A of a topological space (X, τ) is called ***generalized closed*** if cl(A)⊆U whenever A⊆ U and U is open in (X,τ).

Definition 1.4 [4]

A subset A of a topological space (X,τ) is called ***semi generalized closed*** if scl(A) ⊆ U whenever A ⊆ U and U is semi - open in (X,τ).

Definition 1.5 [2]

A subset A of a topological space (X,τ) is called ***generalized semi closed*** if scl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ).

Definition 1.6 [20]

A subset A of a topological space (X,τ) is called **ψ - *closed*** if scl(A)⊆U whenever A⊆ U and U is sg - open in (X,τ).

Definition 1.7 [16]

A subset A of a topological space (X,τ) is called **ψg - *closed*** if ψcl(A)⊆U whenever A⊆ U and U is open in (X,τ).

Definition 1.8 [10]

A subset A of a topological space (X,τ) is called ***ψgs* - *closed*** if ψcl(A)⊆U whenever A⊆ U and U is semi - open in (X,τ).

Definition 1.9 [10]

A subset A of a topological space (X,τ) is called ***ψgs* - *clopen*** if it is both ψgs-closed and ψgs - open in (X,τ).

Result 1.10

1. Every closed (open) subset in (X,τ) is ψgs - closed (ψgs - open).

2. Every clopen subset in (X,τ) is ψgs - clopen .

3. Every semi open (semi closed) subset in (X,τ) is ψgs - open (ψgs - closed).

4. Every regular open (regular closed) subset in (X,τ) is open (closed).

Definition 1.11[17]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***almost continuous*** if $f^{-1}(V)$ is closed in (X,τ) for every regular closed set V of (Y,σ) .

Definition 1.12 [15]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***continuous*** if $f^{-1}(V)$ is closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.13 [1]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***completely continuous*** if $f^{-1}(V)$ is regular open in (X,τ) for every open set V of (Y,σ).

Definition 1.14 [13]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***totally continuous*** if $f^{-1}(V)$ is clopen in (X,τ) for every open set V of (Y,σ) .

Definition 1.15 [3]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***g - continuous*** if $f^{-1}(V)$ is g - closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.16 [7]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***contra continuous*** if $f^{-1}(V)$ is closed in (X,τ) for every open set V of (Y,σ).

Definition 1.17 [8]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***contra semi - continuous*** if $f^{-1}(V)$ is semi closed in (X,τ) for every open set V of (Y,σ).

Definition 1.18 [20]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***ψ - continuous*** if $f^{-1}(V)$ is ψ - closed in (X,τ) for every closed set V of (Y,σ).

Definition 1.19 [9]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***almost contra continuous*** if $f^{-1}(V)$ is closed in (X,τ) for each regular open set V of (Y,σ).

Definition 1.20 [12]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***almost contra semi - continuous*** if $f^{-1}(V)$ is semi - closed in (X,τ) for every regular open set V of (Y, σ).

Definition 1.21 [11]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***ψgs - continuous*** if $f^{-1}(V)$ is ψgs - closed in (X,τ) for every closed set V of (Y,σ) .

Definition 1.22 [5]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***irresolute*** if $f^{-1}(V)$ is semi closed in (X,τ) for every semi closed set V of (Y,σ).

Definition 1.23 [4]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***sg - irresolute*** if $f^{-1}(V)$ is sg - closed in (X,τ) for every sg - closed set V of (Y,σ) .

Definition 1.24 [6]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***gs - irresolute*** if $f^{-1}(V)$ is gs - closed in (X,τ) for every gs - closed set V of (Y,σ) .

Definition 1.25 [16]

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***ψg - irresolute*** if $f^{-1}(V)$ is ψg - closed in (X,τ) for every ψg - closed set V of (Y,σ).

CHAPTER 2

Totally ψgs - Continuous Functions in Topological Spaces

2.1 Introduction

Levine (1970) introduced the idea of continuous functions in topological spaces. Jain (1980) introduced the concept of totally continuous functions in topological spaces. Balachandran et al.(1991) introduced generalized continuous functions in topological spaces. Sundaram et al.(1991) introduced semi generalized continuous functions in topological spaces. Devi et al.(1995) introduced generalized semi continuous functions in topological spaces. Veera Kumar (2000) introduced and studied ψ - continuous functions in topological spaces. Gowsalya and Balamani (2016b) introduced ψgs - continuous functions in topological spaces.

In this chapter we introduce totally ψgs - continuous functions and ψgs - totally continuous functions in topological spaces.

2.2 Totally ψgs - continuous functions

In this section a new type of totally continuous functions called totally ψgs - continuous functions in topological spaces is introduced and studied some of their properties.

Definition 2.2.1

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called ***totally ψgs - continuous*** if $f^{-1}(V)$ is ψgs-clopen in (X,τ) for every open set V of (Y,σ).

Example 2.2.2

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b,c\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$.Let $f:(X,\tau) \to (Y,\sigma)$ be a function defined by $f(a) = b$, $f(b) = c$,$f(c) = a$. Then f is totally ψgs - continuous.

Theorem 2.2.3

A function $f: (X,\tau) \to (Y,\sigma)$ is totally ψgs - continuous if and only if the inverse image of every closed subset of (Y,σ) is a ψgs - clopen subset of (X,τ).

Proof :

Let $f: (X,\tau) \to (Y,\sigma)$ be a totally ψgs - continuous function. Let V be any closed set in (Y,σ). Then Y-V is open in (Y, σ). Since f is totally ψgs - continuous, $f^{-1}(Y- V) = X - f^{-1}(V)$ is ψgs - clopen in (X,τ) which implies that $f^{-1}(V)$ is ψgs - clopen in (X,τ).

Conversely, assume that U is any open set in (Y, σ). Then $Y – U$ is closed in (Y, σ). By assumption, $f^{-1}(Y – U) = X – f^{-1}(U)$ is ψgs - clopen in (X,τ) which implies that $f^{-1}(U)$ is ψgs - clopen in (X,τ). Hence f is totally ψgs - continuous.

Proposition 2.2.4

Every totally continuous function is a totally ψgs - continuous function but not conversely.

Proof :

Let V be any open set in (Y,σ). Since f is totally continuous ,$f^{-1}(V)$ is clopen in (X,τ). By result 1.10, $f^{-1}(V)$ is ψgs - clopen in (X,τ). Hence f is totally ψgs - continuous.

Example 2.2.5

Let X = Y = {a,b,c}, τ = {ϕ, {a},{b},{a,b},X} and σ = {ϕ, {a,b},Y}.Let f:(X,τ)→(Y,σ) be a function defined by f(a) = a, f(b) = c , f(c) = b .Then f is totally ψgs- continuous but not totally continuous, since for the open set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,c} is ψgs - clopen in (X,τ) but not clopen subset in (X,τ) .

Proposition 2.2.6

Every totally ψgs - continuous function is a ψgs - continuous function but not conversely.

Proof :

Let f: (X,τ) →(Y,σ) be a totally ψgs - continuous function**.** Let V be any open set in (Y,σ). Since f is totally ψgs - continuous, f^{-1}(V) is ψgs - clopen in (X,τ) which implies that f^{-1}(V) is ψgs - open in (X,τ) **.** Hence f is ψgs - continuous.

Example 2.2.7

Let X = Y = {a,b,c},τ = {ϕ, {a,b},X} and σ = {ϕ, {a},Y}. Let f:(X,τ) →(Y,σ) be the identity function. Then f is ψgs - continuous but not totally ψgs - continuous, since for the open set {a} in (Y,σ), f^{-1}({a})= {a} is ψgs - open but not ψgs - closed in (X,τ).

Remark 2.2.8

Continuous function is independent from totally ψgs - continuous function as seen from the following examples.

Example 2.2.9

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$.Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be the identity function.Then f is continuous but not totally ψgs - continuous, since for the open set {a,b} in (Y,σ), $f^{-1}(\{a,b\}) = \{a,b\}$ is open but not ψgs - clopen in (X,τ).

Example 2.2.10

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b,c\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$.Let $f: (X,\tau)\rightarrow(Y,\sigma)$ be a function defined by f(a) = c, f(b) = b, f(c) = a. Then f is totally ψgs - continuous but not continuous, since for the open set {a} in (Y,σ), $f^{-1}(\{a\}) = \{c\}$ is ψgs - clopen but not open in (X,τ).

Proposition 2.2.11

If $f:(X,\tau) \rightarrow (Y,\sigma)$ is totally ψgs - continuous and X is ψgs - connected, then Y is an indiscrete space.

Proof :

Suppose Y is not indiscrete space. Let V be a non - empty open subset of Y. Since f is totally ψgs - continuous, $f^{-1}(V)$ is non - empty ψgs - clopen subset of X. Then $X = f^{-1}(V) \cup (f^{-1}(V))^C$. Thus X is union of two non - empty disjoint ψgs - open sets which is a contradiction to the fact that X is ψgs - connected. Therefore Y must be indiscrete space.

Theorem 2.2.12

Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be any function from discrete space (X, τ) into a topological space (Y,σ).Then f is totally ψgs - continuous function if and only if

(i) f is a continuous function.

(ii) f is a ψgs - continuous function.

Proof:

(i) Assume that f is a totally ψgs - continuous function. Let V be any open set in (Y,σ). Then $f^{-1}(V)$ is ψgs - clopen in (X,τ). Since (X,τ) is discrete space, every subset of (X,τ) is open and closed in (X,τ) which implies that $f^{-1}(V)$ is open in (X,τ). Therefore f is continuous.

Conversely, assume that f is continuous. Let V be any open set in (Y,σ). Then $f^{-1}(V)$ is open in (X,τ). Since (X,τ) is discrete space, every subset of (X,τ) is clopen in (X,τ) which implies that $f^{-1}(V)$ is ψgs - clopen in (X,τ). Therefore f is totally ψgs-continuous.

(ii) Assume that f is a totally ψgs - continuous function. Let V be any open set in (Y,σ). Then $f^{-1}(V)$ is ψgs - clopen in (X,τ). Since (X,τ) is discrete space, $f^{-1}(V)$ is open in (X,τ).By result 1.10, $f^{-1}(V)$ is ψgs - open in (X,τ).Therefore f is ψgs - continuous.

Conversely, assume that f is ψgs - continuous. Let V be any open set in (Y,σ). Then $f^{-1}(V)$ is ψgs - open in (X,τ). Since (X,τ) is discrete space, $f^{-1}(V)$ is clopen in (X,τ).By result 1.10, $f^{-1}(V)$ is ψgs - clopen in (X,τ). Therefore f is totally ψgs - continuous.

Remark 2.2.13

The composition of two totally ψgs - continuous functions need not be a totally ψgs - continuous function as seen from the following example.

Example 2.2.14

Let X = Y = Z = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X} , σ = {ϕ,{a},{b,c},Y} and η = {ϕ, {a,b},Z}. Let f:(X,τ)→(Y,σ) be the identity function and g:(Y,σ) →(Z, η) be the identity function. Then f and g are totally ψgs - continuous but their composition g o f :(X,τ)→(Z,η) is not totally ψgs - continuous , since {a,b} is open in (Z,η), whereas (g o f)$^{-1}$({a,b})={a,b} is not ψgs - clopen in (X,τ).

Proposition 2.2.15

If f: (X,τ) →(Y,σ) is a totally ψgs - continuous function and g:(Y,σ) →(Z, η) is a continuous function, then g o f : (X,τ) →(Z,η) is a totally ψgs - continuous function .

Proof :

Let V be any open set in (Z,η). Since g is continuous, g $^{-1}$(V) is open in (Y,σ). Since f is totally ψgs - continuous, (g o f)$^{-1}$(V) = f $^{-1}$(g $^{-1}$(V)) is ψgs - clopen in (X,τ).Hence g o f is a totally ψgs - continuous function .

Proposition 2.2.16

Let f:(X,τ) →(Y,σ) and g:(Y,σ)→(Z,η) be two functions, then g o f : (X,τ) →(Z,η) is a totally ψgs - continuous function if

(i) f is a totally ψgs - continuous function and g is a totally continuous function.

(ii) f and g are totally continuous functions.

Proof:

(i) Let V be any open set in (Z,η).Since g is totally continuous, g^{-1}(V) is clopen in (Y,σ) which implies g^{-1}(V) is open in (Y,σ).Since f is totally ψgs - continuous,

$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - clopen in (X,τ) . Hence g o f is a totally ψgs - continuous function .

(ii) Let V be any open set in (Z,η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies $g^{-1}(V)$ is open in (Y,σ) . Since f is totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.10, $(g \circ f)^{-1}(V)$ is ψgs - clopen in (X,τ). Hence g o f is a totally ψgs - continuous function.

2.3 ψgs - Totally continuous functions

In this section we introduce ψgs - totally continuous functions in topological spaces and study some of their properties.

Definition 2.3.1

A function f: $(X,\tau) \to (Y,\sigma)$ is called ***ψgs - totally continuous*** if $f^{-1}(V)$ is clopen in (X,τ) for every ψgs - open set V of (Y,σ).

Example 2.3.2

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b,c},X} and σ = {ϕ,{a,b},Y}.Let f:(X,τ) →(Y,σ) be a function defined by f(a) = f(b) = f(c) = a. Then f is ψgs - totally continuous.

Theorem 2.3.3

A function f: $(X,\tau) \to (Y,\sigma)$ is ψgs - totally continuous if and only if the inverse image of every ψgs - closed subset of (Y,σ) is a clopen subset of (X,τ).

Proof :

Let f: $(X,\tau) \to (Y,\sigma)$ be a ψgs - totally continuous function **.** Let V be any ψgs-closed set in (Y,σ).Then Y-V is ψgs - open in (Y,σ). Since f is ψgs - totally continuous**,** $f^{-1}(Y- V) = X - f^{-1}(V)$ is clopen in (X,τ) which implies that $f^{-1}(V)$ is clopen in (X,τ).

Conversely, assume that U is any ψgs - open set in (Y, σ). Then Y – U is ψgs - closed in (Y, σ). By assumption, $f^{-1}(Y – U) = X – f^{-1}(U)$ is clopen in (X,τ) which implies that $f^{-1}(U)$ is clopen in (X,τ). Hence f is ψgs - totally continuous.

Proposition 2.3.4

If f:(X,τ) →(Y,σ) is a ψgs - totally continuous function, then

(i) f is a totally continuous function.

(ii) f is a totally ψgs - continuous function.

Proof:

(i) Let V be any open set in (Y,σ). By result 1.10, V is ψgs - open in (Y, σ). Since f is ψgs - totally continuous, $f^{-1}(V)$ is clopen in (X,τ). Therefore f is totally continuous.

(ii) Let V be any open set in (Y,σ). By result 1.10, V is ψgs - open in (Y, σ).Since f is ψgs - totally continuous, $f^{-1}(V)$ is clopen in (X,τ).By result 1.10, $f^{-1}(V)$ is ψgs - clopen in (X,τ). Therefore f is totally ψgs - continuous.

The converse of proposition 2.3.4 (i) need not be true in general as seen from the following example.

Example 2.3.5

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},{a,c},X} and σ = {ϕ,{a},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = c, f(b) = a, f(c) = b. Then f is totally continuous but not ψgs - totally continuous, since for the ψgs - open set {a,b} in (Y,σ), $f^{-1}(\{a,b\})$ ={ b,c} is not clopen in (X,τ).

The converse of proposition 2.3.4 (ii) need not be true in general as seen from the following example.

Example 2.3.6

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},{a,c},X} and σ = {ϕ,{a,b}, Y}. Let f: (X,τ)→(Y,σ) be a function defined by f(a) = a, f(b) = c, f(c) = b. Then f is totally ψgs - continuous but not ψgs - totally continuous, since for the ψgs - open set {a} in (Y,σ), f^{-1}({a}) ={a} is not clopen in (X,τ).

Proposition 2.3.7

If f:(X,τ) →(Y,σ) is a ψgs - totally continuous function, then

(i) f is a continuous function.

(ii) f is a ψgs - continuous function.

Proof:

(i) Let V be any open set in (Y,σ). By result 1.10,V is ψgs - open in (Y, σ).Since f is ψgs - totally continuous, f^{-1}(V) is clopen in (X,τ) which implies that f^{-1}(V) is open in (X,τ). Therefore f is continuous.

(ii) Let V be any open set in (Y,σ). By result 1.10, V is ψgs - open in (Y, σ). Since f is ψgs - totally continuous, f^{-1}(V) is clopen in (X,τ) which implies that f^{-1}(V) is open in (X,τ). By result 1.10, f^{-1}(V) is ψgs - open in (X,τ). Therefore f is ψgs - continuous.

The converse of proposition 2.3.7 (i) need not be true in general as seen from the following example.

Example 2.3.8

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b\},\{a,b\},\{a,c\},X\}$ and $\sigma = \{\phi,\{a\},\{a,b\},Y\}$.Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by f(a) = a, f(b) = c, f(c) = b. Then f is continuous but not ψgs - totally continuous, since for the ψgs - open set {a,c} in (Y,σ), $f^{-1}(\{a,c\}) = \{a,b\}$ is not clopen in (X,τ).

The converse of proposition 2.3.7 (ii) need not be true in general as seen from the following example.

Example 2.3.9

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b\},\{a,b\},\{a,c\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by f(a) = a, f(b) = c, f(c) = b. Then f is ψgs - continuous but not ψgs - totally continuous, since for the ψgs - open set {b} in (Y,σ), $f^{-1}(\{b\}) = \{c\}$ is not clopen in (X,τ).

Theorem 2.3.10

Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be any function from discrete space (X,τ) into a topological space (Y,σ). If f is a ψgs - totally continuous function, then

(i) f is a continuous function.

(ii) f is a ψgs - continuous function.

Proof:

(i) Let V be any open set in (Y,σ).By result 1.10,V is ψgs - open in (Y, σ). Since f is ψgs - totally continuous, $f^{-1}(V)$ is clopen in (X,τ) which implies that $f^{-1}(V)$ is open in (X,τ). Therefore f is continuous.

(ii) Let V be any open set in (Y,σ).By result 1.10,V is ψgs - open in (Y, σ). Since f is ψgs - totally continuous, $f^{-1}(V)$ is clopen in (X,τ) which implies that $f^{-1}(V)$ is open in (X,τ). By result 1.10, $f^{-1}(V)$ is ψgs - open in (X,τ). Therefore f is ψgs - continuous.

Proposition 2.3.11

Let f:(X,τ) →(Y,σ) be a ψgs - totally continuous function and g: (Y,σ)→(Z, η) be any function. Then g o f :(X,τ) →(Z, η) is a totally ψgs - continuous function if

(i) g is a totally ψgs - continuous function.

(ii) g is a totally continuous function.

Proof:

(i) Let V be any open set in (Z, η).Since g is totally ψgs - continuous, $g^{-1}(V)$ is ψgs- clopen in (Y,σ) which implies that $g^{-1}(V)$ is ψgs - open in (Y, σ). Since f is ψgs- totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.10, $(g \circ f)^{-1}(V)$ is ψgs - clopen in (X,τ). Therefore g o f is totally ψgs - continuous.

(ii) Let V be any open set in (Z, η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies that $g^{-1}(V)$ is open in (Y, σ). By result 1.10, $g^{-1}(V)$ is ψgs - open in (Y, σ). Since f is ψgs - totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ).By result 1.10, $(g \circ f)^{-1}(V)$ is ψgs - clopen in (X,τ). Therefore g o f is totally ψgs - continuous.

Proposition 2.3.12

Let f:(X,τ) →(Y,σ) be a ψgs - totally continuous function and g: (Y,σ)→(Z, η) be any function. Then g o f :(X,τ) →(Z, η) is a totally ψgs - continuous function if

(i) g is a continuous function.

(ii) g is a ψgs - continuous function.

Proof:

(i) Let V be any open set in (Z, η). Since g is continuous, g^{-1} (V) is open in (Y,σ). By result 1.10,g^{-1} (V) is ψgs – open in (Y, σ).Since f is ψgs – totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.10, $(g \circ f)^{-1}(V)$ is ψgs - clopen in (X,τ).Therefore g o f is totally ψgs – continuous.

(ii) Let V be any open set in (Z, η). Since g is ψgs – continuous, g^{-1} (V) is ψgs-open in (Y,σ). Since f is ψgs – totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ).By result 1.10, $(g \circ f)^{-1}(V)$ is ψgs – clopen in (X,τ). Therefore g o f is totally ψgs – continuous.

Proposition 2.3.13

Let f:(X,τ) →(Y,σ) and g: (Y,σ)→(Z, η) be any two functions. If g is a ψgs - totally continuous function and if

(i) f is a totally continuous function, then g o f :(X,τ) →(Z, η) is a ψgs - totally continuous function.

(ii) f is a continuous function , then g o f :(X,τ) →(Z, η) is a continuous function.

(iii) f is a ψgs - continuous function, then g o f :(X,τ) →(Z, η) is a ψgs - continuous function.

Proof:

(i) Let V be any ψgs - open set in (Z, η).Since g is ψgs - totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies that g^{-1} (V) is open in (Y,σ). Since f is totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). Therefore g o f is ψgs - totally continuous.

(ii) Let V be any open set in (Z, η). By result 1.10, V is ψgs - open in (Z, η). Since g is ψgs- totally continuous, g^{-1} (V) is clopen in (Y,σ) which implies that g^{-1} (V) is open in (Y,σ).Since f is continuous,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in (X,τ). Therefore g o f is continuous.

(iii) Let V be any open set in (Z, η).By result 1.10, V is ψgs - open in (Z, η). Since g is ψgs - totally continuous, g^{-1}(V) is clopen in (Y,σ) which implies that g^{-1} (V) is open in (Y,σ).Since f is ψgs - continuous,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - open in (X,τ). Therefore g o f is ψgs - continuous.

Proposition 2.3.14

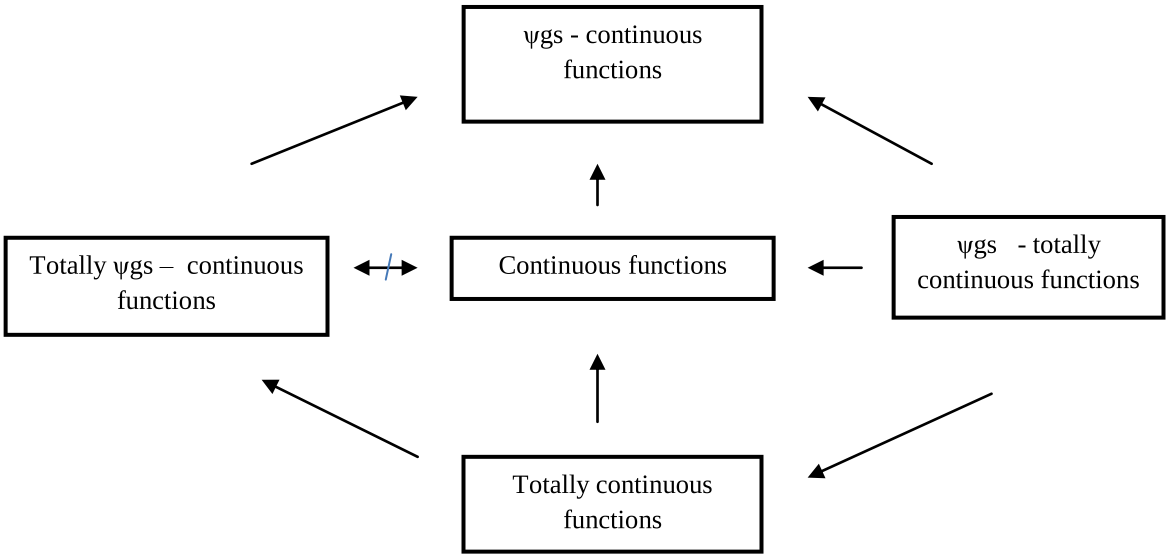
Let f:(X,τ) →(Y,σ) and g: (Y,σ)→(Z,η) be two ψgs - totally continuous function, then g o f :(X,τ) →(Z,η) is also a ψgs - totally continuous function.

Proof :

Let V be any ψgs - open set in (Z,η).Since g is ψgs - totally continuous, g^{-1} (V) is clopen in (Y,σ). By result 1.10,g^{-1} (V) is ψgs - clopen in (Y,σ) which implies that g^{-1} (V) is ψgs - open in (Y,σ). Since f is ψgs - totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). Therefore g o f is ψgs - totally continuous.

Remark 2.2.15

The relations between various types of continuous functions are given in the following diagram


ψgs - continuous functions
Totally ψgs – continuous functions
Continuous functions
ψgs - totally continuous functions
Totally continuous functions

CHAPTER 3

Contra ψgs - Continuous Functions and Almost Contra ψgs - Continuous Functions in Topological Spaces

3.1 Introduction

Singal.M.K and Singal.A.R (1968) introduced almost continuous functions in topological spaces. Levine (1970) introduced the idea of continuous functions in topological spaces. Dontchev (1996) introduced the notion of contra continuity. Dontchev and Noiri (1999) introduced contra semi continuous functions in topological spaces. Ekici (2004) introduced almost contra continuous functions in topological spaces. Iyappan and Nagaveni (2010) introduced almost contra semi continuous functions in topological spaces.

In this chapter we introduce contra ψgs - continuous functions and almost contra ψgs - continuous functions in topological spaces.

3.2 Contra ψgs - continuous functions

In this section we introduce a new type of totally continuous functions called contra ψgs - continuous functions in topological spaces and derived their properties.

Definition 3.2.1

A function $f:(X,\tau) \rightarrow (Y,\sigma)$ is called **contra ψgs - continuous** if $f^{-1}(V)$ is ψgs - open in (X,τ) for every closed set V of (Y,σ).

Example 3.2.2

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is contra ψgs- continuous.

Theorem 3.2.3

A function $f:(X,\tau) \rightarrow(Y,\sigma)$ is contra ψgs - continuous if and only if $f^{-1}(V)$ is ψgs - closed in (X,τ) for every open set V of (Y,σ).

Proof:

Let $f:(X,\tau) \rightarrow(Y,\sigma)$ be a contra ψgs - continuous function. Let V be any open set in (Y,σ). Then $Y - V$ is closed in (Y,σ).Since f is contra ψgs - continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is ψgs- open in (X,τ) which implies that $f^{-1}(V)$ is ψgs - closed in (X,τ).

Conversely, assume that U is any closed set in (Y,σ). Then $Y-U$ is open in (Y,σ). By assumption, $f^{-1}(Y - U) = X - f^{-1}(U)$ is ψgs - closed in (X,τ) which implies that $f^{-1}(U)$ is ψgs - open in (X,τ). Hence f is contra ψgs - continuous.

Proposition 3.2.4

Every totally continuous function is a contra ψgs - continuous function but not conversely.

Proof:

Let V be any open set in (Y,σ). Since f is totally continuous , $f^{-1}(V)$ is clopen in (X,τ). By result 1.10 ,$f^{-1}(V)$ is ψgs - clopen in (X,τ) which implies that $f^{-1}(V)$ is ψgs - closed in (X,τ). Hence f is contra ψgs - continuous.

Example 3.2.5

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b,c\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a) = b, f(b) = a, f(c) = c$. Then f is contra ψgs - continuous but not totally continuous ,since for the open set $\{a\}$ in (Y,σ), $f^{-1}(\{a\}) = \{b\}$ is ψgs - closed but not clopen in (X,τ).

Proposition 3.2.6

Every contra continuous function is a contra ψgs - continuous function but not conversely.

Proof:

Let V be any open set in (Y,σ). Since f is contra continuous, $f^{-1}(V)$ is closed in (X,τ). By result 1.10 ,$f^{-1}(V)$ is ψgs - closed in (X,τ). Hence f is contra ψgs - continuous.

Example 3.2.7

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b,c\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by $f(a) = c, f(b) = b, f(c) = a$. Then f is contra ψgs- continuous but not contra continuous ,since for the open set $\{a\}$ in (Y,σ), $f^{-1}(\{a\}) = \{c\}$ is ψgs - closed but not closed in (X,τ).

Proposition 3.2.8

Every contra semi continuous function is a contra ψgs - continuous function but not conversely.

Proof:

Let V be any closed set in (Y,σ). Since f is contra semi continuous, $f^{-1}(V)$ is semi- open in (X,τ) . By result 1.10 , $f^{-1}(V)$ is ψgs - open in (X,τ). Hence f is contra ψgs - continuous.

Example 3.2.9

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b,c},X} and σ = {ϕ,{a},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = b, f(b) = a ,f(c) = c. Then f is contra ψgs- continuous but not contra semi continuous , since for the closed set {b,c} in (Y,σ), f^{-1}({b,c}) = {a,c} is ψgs - open but not semi - open in (X,τ).

Proposition 3.2.10

Every totally ψgs - continuous function is a contra ψgs - continuous function but not conversely.

Proof:

Let f:(X,τ) →(Y,σ) be a totally ψgs - continuous function. Let V be any closed set in (Y,σ). Since f is totally ψgs - continuous, $f^{-1}(V)$ is ψgs - clopen in (X,τ) which implies that $f^{-1}(V)$ is ψgs - open in (X,τ). Hence f is contra ψgs - continuous.

Example 3.2.11

Let X = Y = {a,b,c}, τ = {ϕ,{a},{a,b},X} and σ = {ϕ,{a},Y}.Let f:(X,τ)→(Y,σ) be a function defined by f(a) = b, f(b) = a,f(c) = c. Then f is contra ψgs– continuous but not totally ψgs – continuous , since for the closed set {b,c} in (Y,σ), f^{-1}({b,c}) = {a,c} is ψgs – open but not ψgs – closed in (X,τ).

Proposition 3.2.12

Every ψgs - totally continuous function is a contra ψgs - continuous function but not conversely.

Proof:

Let V be any open set in (Y, σ).By result 1.10, V is ψgs - open in (Y,σ). Since f is ψgs- totally continuous , $f^{-1}(V)$ is clopen in (X,τ). By result 1.10 , $f^{-1}(V)$ is ψgs - clopen in (X,τ) which implies that $f^{-1}(V)$ is ψgs - closed in (X,τ). Hence f is contra ψgs - continuous.

Example 3.2.13

Let X = Y = {a,b,c}, τ = {ϕ,{a},{a,b},{a,c),X} and σ = {ϕ,{a},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = c,f(b) = b, f(c) = a. Then f is contra ψgs-continuous but not ψgs - totally continuous , since for the ψgs - open set {a} in (Y,σ), f^{-1}({a})= {c} is not clopen in (X,τ).

Remark 3.2.14

The following examples show that contra ψgs - continuous function and continuous function are independent .

Example 3.2.15

Let X = Y = {a,b,c}, τ = {ϕ,{a,b},X} and σ = {ϕ,{a},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = b, f(b) = c, f(c) = a .Then f is contra ψgs - continuous but not continuous , since for the open set {a} in (Y,σ), f^{-1}({a}) = {c} is ψgs - closed but not open in (X,τ).

Example 3.2.16

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\}, \{a,b\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is continuous but not contra ψgs-continuous , since for the open set {a} in (Y,σ), $f^{-1}(\{a\}) = \{a\}$ is open but not ψgs - closed in (X,τ).

Remark 3.2.17

Contra ψgs - continuous function is independent from ψgs - continuous function as seen from the following examples.

Example 3.2.18

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a,b\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by f(a) = c, f(b) = b ,f(c) = a. Then f is contra ψgs- continuous but not ψgs- continuous , since for the closed set {c} in (Y,σ), $f^{-1}(\{c\}) = \{a\}$ is ψgs - open but not ψgs - closed in (X,τ).

Example 3.2.19

Let $X = Y = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},\{a,c\},X\}$ and $\sigma = \{\phi,\{a\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is ψgs - continuous but not contra ψgs - continuous , since for the closed set {b,c} in (Y,σ), $f^{-1}(\{b,c\}) = \{b,c\}$ is ψgs - closed but not ψgs- open in (X,τ).

Proposition 3.2.20

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is a contra ψgs - continuous function and $g:(Y,\sigma)\rightarrow(Z, \eta)$ is a continuous function, then $g \circ f :(X,\tau) \rightarrow(Z,\eta)$ is a contra ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ).Since f is contra ψgs - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - open in (X,τ) . Hence g o f is a contra ψgs - continuous function .

Proposition 3.2.21

If f:(X,τ)→(Y,σ) is a contra semi - continuous function and g:(Y,σ)→(Z, η) is a continuous function, then g o f :(X,τ) →(Z,η) is a contra ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ).Since f is contra semi - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semi open in (X,τ) . By result 1.10, $(g \circ f)^{-1}(V)$ is ψgs - open in (X,τ) . Hence g o f is a contra ψgs- continuous function .

Proposition 3.2.22

If f:(X,τ)→(Y,σ) is a totally ψgs - continuous function and g:(Y,σ)→(Z, η) is a continuous function, then g o f :(X,τ) →(Z,η) is a contra ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ).Since f is totally ψgs - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - clopen in (X,τ) which implies that $(g \circ f)^{-1}(V)$ is ψgs - open in (X,τ) . Hence g o f is a contra ψgs - continuous function .

Proposition 3.2.23

If f:(X,τ)→(Y,σ) is a totally continuous function and g:(Y,σ)→(Z, η) is a continuous function, then g o f :(X,τ) →(Z,η) is a contra ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ).Since f is totally continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ).By result 1.10, $(g \text{ o } f)^{-1}(V)$ is ψgs - clopen in (X,τ) which implies that $(g \text{ o } f)^{-1}(V)$ is ψgs - open in (X,τ) . Hence g o f is a contra ψgs - continuous function.

Proposition 3.2.24

If f:(X,τ)→(Y,σ) is a ψgs - totally continuous function and g:(Y,σ)→(Z, η) is a continuous function, then g o f :(X,τ) →(Z,η) is a contra ψgs - continuous function.

Proof:

` Let V be any closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ).By result 1.10, $g^{-1}(V)$ is ψgs - closed in (Y,σ) . Since f is ψgs - totally continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 1.10, $(g \text{ o } f)^{-1}(V)$ is ψgs - clopen in (X,τ) which implies that $(g \text{ o } f)^{-1}(V)$ is ψgs - open in (X,τ) . Hence g o f is a contra ψgs - continuous function .

Proposition 3.2.25

If f:(X,τ)→(Y,σ) is a contra continuous function and g:(Y,σ)→(Z, η) is a continuous function, then g o f :(X,τ) →(Z,η) is a contra ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ).Since f is contra continuous,$(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in (X,τ). By result 1.10, $(g \text{ o } f)^{-1}(V)$ is ψgs - open in (X,τ).Hence g o f is a contra ψgs - continuous function.

Proposition 3.2.26

If f:(X,τ)→(Y,σ) is a contra ψgs - continuous function and g:(Y,σ)→(Z, η) is a totally continuous function, then g o f :(X,τ) →(Z,η) is a contra ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η). Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies that $g^{-1}(V)$ is closed in (Y,σ) .Since f is contra ψgs- continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - open in (X,τ). Hence g o f is a contra ψgs - continuous function .

Proposition 3.2.27

If f:(X,τ)→(Y,σ) is a contra ψgs - continuous function and g:(Y,σ)→(Z, η) is a ψgs - totally continuous function, then g o f :(X,τ) →(Z,η) is a contra ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η). By result 1.10, V is ψgs - closed in (Y,σ).Since g is ψgs - totally continuous, $g^{-1}(V)$ is clopen in (Y,σ) which implies that $g^{-1}(V)$ is closed in (Y,σ).Since f is contra ψgs - continuous,$(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - open in (X,τ).Hence g o f is a contra ψgs - continuous function.

Proposition 3.2.28

If f:(X,τ)→(Y,σ) is a totally ψgs - continuous function and g:(Y,σ)→(Z, η) is a completely continuous function, then g o f :(X,τ) →(Z,η) is a contra ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η).Since g is completely continuous, $g^{-1}(V)$ is regular closed in (Y,σ). By result 1.10, $g^{-1}(V)$ is closed in (Y,σ).Since f is totally ψgs - continuous, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - clopen in (X,τ) which implies that $(g \text{ o } f)^{-1}(V)$ is ψgs - open in (X,τ). Hence g o f is a contra ψgs - continuous function .

Remark 3.2.29

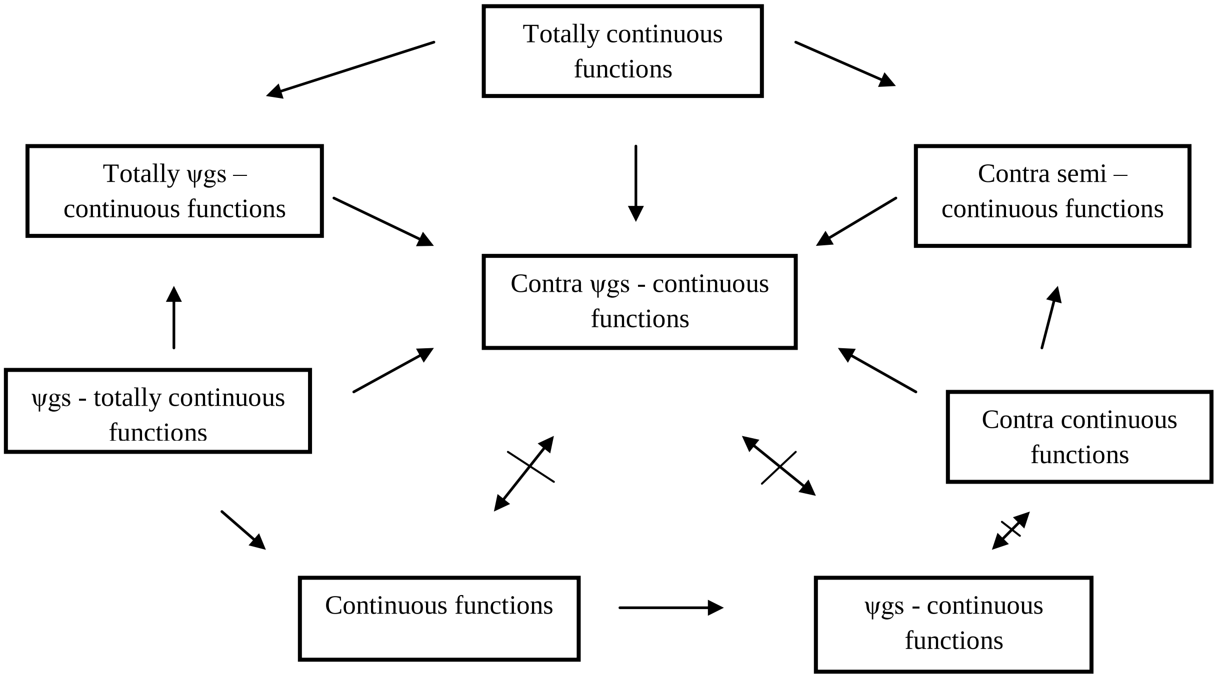
The composition of two contra ψgs - continuous functions need not be a contra ψgs - continuous function as seen from the following example.

Example 3.2.30

Let X = Y = Z = {a,b,c}, τ = {ϕ,{a},{b},{a,b},{a,c},X} , σ = {ϕ,{a},{b,c},Y} and η = {ϕ, {a},Z}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = b, f(b) = a, f(c) = c and g:(Y,σ) →(Z, η) be a function defined by g(a) = b, g(b) = a, g(c) = c . Then f and g are contra ψgs - continuous but their composition g o f :(X,τ)→(Z,η) is not contra ψgs - continuous , since for the closed set {b,c} in (Z,η), $(g \text{ o } f)^{-1}(\{b,c\})$ = {b,c} is not ψgs - open in (X,τ).

Remark 3.2.31 :

The above discussions are depicted in the following diagram.



3.3 Almost contra ψgs - continuous functions

In this section almost contra ψgs - continuous functions in topological spaces is introduced and obtained the interrelations between almost contra ψgs - continuous functions with various contra continuous functions.

Definition 3.3.1

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **almost contra ψgs - continuous** if $f^{-1}(V)$ is ψgs closed in (X,τ) for every regular open set V of (Y,σ).

Example 3.3.2

Let $X = Y = \{a,b,c\}, \tau = \{\phi,\{a\},\{b\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},\{b,c\},Y\}$. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is almost contra ψgs - continuous.

Theorem 3.3.3

A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is almost contra ψgs - continuous if and only if the inverse image of every regular open subset of (Y,σ) is ψgs - closed in (X,τ) .

Proof:

Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a almost contra ψgs - continuous function. Let V be any regular open set in (Y,σ). Then Y -V is regular closed in (Y,σ).Since f is almost contra ψgs - continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is ψgs - open in (X,τ) which implies that $f^{-1}(V)$ is ψgs - closed in (X,τ).

Conversely, assume that U is any regular closed set in (Y,σ). Then Y-U is regular open in (Y,σ). By assumption, $f^{-1}(Y - U) = X - f^{-1}(U)$ is ψgs - closed in (X,τ) which implies that $f^{-1}(U)$ is ψgs - open in (X,τ). Hence f is almost contra ψgs - continuous.

Proposition 3.3.4

Every totally continuous function is a almost contra ψgs - continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ).By result 1.10, V is open in (Y,σ). Since f is totally continuous, $f^{-1}(V)$ is clopen in (X,τ).By result 1.10,$f^{-1}(V)$ is ψgs - clopen in (X,τ) which implies that $f^{-1}(V)$ is ψgs - closed in (X,τ). Hence f is almost contra ψgs - continuous.

Example 3.3.5

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b,c},X} and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is almost contra ψgs - continuous but not totally continuous, since for the open set {a,c} in (Y,σ), $f^{-1}(\{a,c\})$ = {a,c} is not clopen in (X,τ).

Proposition 3.3.6

Every contra continuous function is a almost contra ψgs - continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ).By result 1.10, V is open in (Y,σ). Since f is contra continuous, $f^{-1}(V)$ is closed in (X,τ).By result 1.10,$f^{-1}(V)$ is ψgs - closed in (X,τ).Hence f is almost contra ψgs - continuous.

Example 3.3.7

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X} and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is almost contra ψgs - continuous but not contra continuous, since for the open set {a,b} in (Y,σ), $f^{-1}(\{a,b\})$ = {a,b} is not closed in (X,τ).

Proposition 3.3.8

Every contra semi continuous function is a almost contra ψgs - continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ).By result 1.10, V is open in (Y,σ). Since f is contra semi continuous, $f^{-1}(V)$ is semi closed in (X,τ).By result 1.10,$f^{-1}(V)$ is ψgs - closed in (X,τ). Hence f is almost contra ψgs - continuous.

Example 3.3.9

Let X = Y = {a,b,c},τ = {ϕ,{a},{b,c},X} and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is almost contra ψgs - continuous but not contra semi continuous, since for the open set {b} in (Y,σ), $f^{-1}(\{b\})$ = {b} is not semi closed in (X,τ).

Proposition 3.3.10

Every almost contra continuous function is a almost contra ψgs - continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ). Since f is almost contra continuous, $f^{-1}(V)$ is closed in (X,τ).By result 1.10,$f^{-1}(V)$ is ψgs - closed in (X,τ). Hence f is almost contra ψgs - continuous.

Example 3.3.11

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X} and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is almost contra ψgs - continuous but not almost contra continuous, since for the regular open set {b} in (Y,σ),$f^{-1}(\{b\})$ = {b} is ψgs - closed but not closed in (X,τ).

Proposition 3.3.12

Every totally ψgs - continuous function is a almost contra ψgs - continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ).By result 1.10,V is open in (Y,σ). Since f is totally ψgs - continuous, $f^{-1}(V)$ is ψgs - clopen in (X,τ) which implies that $f^{-1}(V)$ is ψgs - closed in (X,τ). Hence f is almost contra ψgs - continuous.

Example 3.3.13

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X} and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is almost contra ψgs - continuous but not totally ψgs - continuous, since for the open set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not ψgs - clopen in (X,τ).

Proposition 3.3.14

Every ψgs - totally continuous function is a almost contra ψgs - continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ).By result 1.10, V is open in (Y,σ) and hence V is ψgs - open in (Y,σ). Since f is ψgs - totally continuous, $f^{-1}(V)$ is clopen in (X,τ).By result 1.10,$f^{-1}(V)$ is ψgs - clopen in (X,τ) which implies that $f^{-1}(V)$ is ψgs - closed in (X,τ). Hence f is almost contra ψgs - continuous.

Example 3.3.15

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},X} and σ = {ϕ,{a},{b,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is almost contra ψgs - continuous but not ψgs - totally continuous, since for the ψgs - open set {b} in (Y,σ), f^{-1}({b}) = {b} is not clopen in (X,τ).

Proposition 3.3.16

Every contra ψgs - continuous function is a almost contra ψgs - continuous function but not conversely.

Proof:

Let V be any regular open set in (Y,σ). By result 1.10,V is open in (Y,σ). Since f is contra ψgs - continuous, f^{-1}(V) is ψgs - closed in (X,τ). Hence f is almost contra ψgs - continuous.

Example 3.3.17

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X} and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is almost contra ψgs - continuous but not contra ψgs - continuous, since for the open set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not ψgs - closed in (X,τ).

Proposition 3.3.18

If f:(X,τ)→(Y,σ) is a almost contra ψgs - continuous function and g:(Y,σ)→(Z, η) is a completely continuous function , then g o f :(X,τ) →(Z,η) is a almost contra ψgs - continuous function.

Proof:

Let V be any regular open set in (Z, η).By result 1.10, V is open in (Z, η).Since g is completely continuous, $g^{-1}(V)$ is regular open in (Y,σ).Since f is almost contra ψgs- continuous,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - closed in (X,τ). Hence g o f is a almost contra ψgs - continuous function .

Proposition 3.3.19

If f:(X,τ)→(Y,σ) is a contra ψgs - continuous function and g:(Y,σ)→(Z, η) is a completely continuous function, then g o f :(X,τ) →(Z,η) is a almost contra ψgs - continuous function.

Proof:

Let V be any regular open set in (Z, η).By result 1.10, V is open in (Z, η).Since g is completely continuous, $g^{-1}(V)$ is regular open in (Y,σ).By result 1.10, $g^{-1}(V)$ is open in (Y,σ).Since f is contra ψgs continuous,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - closed in (X,τ). Hence g o f is a almost contra ψgs - continuous function .

Proposition 3.3.20

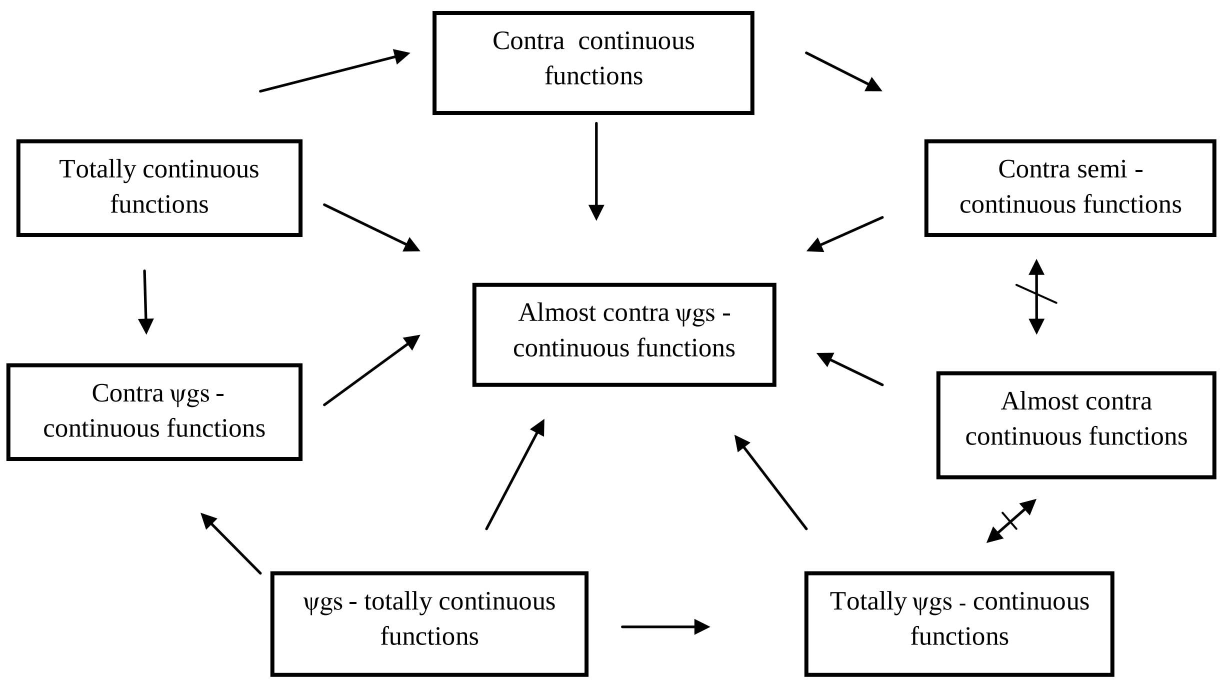
If f:(X,τ)→(Y,σ) is a contra ψgs - continuous function and g:(Y,σ)→(Z, η) is a almost continuous function, then g o f :(X,τ) →(Z,η) is a almost contra ψgs - continuous function.

Proof:

Let V be any regular open set in (Z, η).Since g is almost continuous, $g^{-1}(V)$ is open in (Y,σ).Since f is contra ψgs - continuous,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - closed in (X,τ). Hence g o f is a almost contra ψgs - continuous function.

Remark 3.3.21

The following relations exist for almost contra ψgs - continuous functions



CHAPTER 4

ψgs- Irresolute Functions in Topological Spaces

4.1 Introduction

Crossley and Hildebrand (1972) introduced irresolute functions in topological spaces. Bhattacharyya and Lahiri (1987) introduced sg - irresolute functions in topological spaces. Devi et al.(1995) introduced gs - irresolute functions in topological spaces. Ramya and parvathi (2013) introduced ψg- irresolute functions in topological spaces. In this chapter we introduce ψgs - irresolute functions in topological spaces.

4.2 ψgs - irresolute functions

In this section a stronger form of ψgs - continuous functions called ψgs-irresolute functions is introduced and studied their characterizations.

Definition 4.2.1

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is called **ψgs - irresolute** if $f^{-1}(V)$ is ψgs - closed in (X,τ) for every ψgs - closed V of (Y,σ).

Example 4.2.2

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},X} and σ = {ϕ,{a},{a,b}{a,c},Y}. Let f:$(X,\tau)\rightarrow(Y,\sigma)$ be the identity function. Then f is ψgs - irresolute.

Theorem 4.2.3

A function f: $(X,\tau) \rightarrow (Y,\sigma)$ is ψgs - irresolute if and only if $f^{-1}(V)$ is ψgs - open in (X,τ) for every ψgs - open set V of (Y,σ).

Proof:

Let V be any ψgs - open set in (Y,σ). Then Y -V is ψgs - closed in (Y,σ).Since f is ψgs - irresolute, $f^{-1}(Y - V) = X - f^{-1}(V)$ is ψgs - closed in (X,τ) which implies that $f^{-1}(V)$ is ψgs - open in (X,τ).

Conversely, assume that U is any ψgs - closed set in (Y,σ). Then Y-U is ψgs - open in (Y,σ). By assumption, $f^{-1}(Y - U) = X - f^{-1}(U)$ is ψgs - open in (X,τ) which implies that $f^{-1}(U)$ is ψgs-closed in (X,τ). Hence f is ψgs - irresolute.

Proposition 4.2.4

If f: (X,τ) →(Y,σ) is a ψgs - irresolute function, then it is a ψgs - continuous function but not conversely.

Proof:

Let V be any closed set in (Y,σ). By result 1.10, V is ψgs - closed in (Y,σ).Since f is ψgs - irresolute, $f^{-1}(V)$ is ψgs - closed in (X,τ).Hence f is ψgs-continuous.

Example 4.2.5

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},{a,c},X} and σ = {ϕ,{a},{b,c},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = b,f(b) = a,f(c) = c. Then f is ψgs - continuous but not ψgs - irresolute, since for the ψgs - closed set {a,b} in (Y,σ), $f^{-1}(\{a,b\}) = \{a,b\}$ is not ψgs - closed in (X,τ).

Proposition 4.2.6

Every ψgs - totally continuous function is a ψgs - irresolute function but not conversely.

Proof:

Let V be any ψgs - closed set in (Y,σ).Since f is ψgs - totally continuous,$f^{-1}(V)$ is clopen in (X,τ).By result 1.10,$f^{-1}(V)$ is ψgs - clopen in (X,τ) which implies that $f^{-1}(V)$ is ψgs - closed in (X,τ).Hence f is ψgs - irresolute.

Example 4.2.7

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b},{a,b},{a,c},X} and σ = {ϕ,{a},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is ψgs - irresolute but not ψgs - totally continuous, since for the ψgs - closed set {c} in (Y,σ), $f^{-1}(\{c\})$ = {c} is not clopen in (X,τ).

Remark 4.2.8

Irresolute function is independent from ψgs - irresolute function as seen from the following examples.

Example 4.2.9

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},{a,c},X} and σ = {ϕ,{a},{b,c},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = b,f(b) = a,f(c) = c. Then f is irresolute but not ψgs -irresolute, since for the ψgs - closed set {b} in (Y,σ), $f^{-1}(\{b\})$ = {a} is not ψgs - closed in (X,τ).

Example 4.2.10

Let X = Y = {a,b,c}, τ = {ϕ,{a},{b,c},X} and σ = {ϕ,{a,b},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is ψgs - irresolute but not irresolute, since for the semi - closed set {c} in (Y,σ), $f^{-1}(\{c\})$ = {c} is not semi - closed in (X,τ).

Remark 4.2.11

The following examples show that gs - irresolute function and ψgs - irresolute function are independent.

Example 4.2.12

Let X = Y = {a,b,c},τ = {ϕ,{a},{a,b},X} and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is gs - irresolute but not ψgs - irresolute, since for the ψgs - closed set {a,c} in (Y,σ), f^{-1}({a,c}) = {a,c} is not ψgs - closed in (X,τ).

Example 4.2.13

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},{a,c},X} and σ = {ϕ,{a},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is ψgs - irresolute but not gs - irresolute, since for the gs - closed set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not gs - closed in (X,τ).

Remark 4.2.14

ψg - irresolute function is independent from ψgs - irresolute function as seen from the following examples.

Example 4.2.15

Let X = Y = {a,b,c},τ = {ϕ,{a},X} and σ = {ϕ,{a},{b},{a,b},{a,c},Y}.Let f:(X,τ)→(Y,σ) be the identity function. Then f is ψg - irresolute but not ψgs -

irresolute, since for the ψgs - closed set {a,c} in (Y,σ), f^{-1}({a,c}) = {a,c} is not ψgs - closed in (X,τ).

Example 4.2.16

Let X = Y = {a,b,c}, τ = {ϕ,{a},{a,b},X} and σ = {ϕ,{a},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is ψgs - irresolute but not ψg - irresolute, since for the ψg - closed set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not ψg - closed in (X,τ).

Remark 4.2.17

The following examples show that totally ψgs - continuous function and ψgs - irresolute function are independent.

Example 4.2.18

Let X = Y = {a,b,c},τ = {ϕ,{a},{b},{a,b},{a,c},X} and σ = {ϕ,{a},{b,c},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = b,f(b) = a,f(c) = c. Then f is totally ψgs -continuous but not ψgs - irresolute, since for the ψgs - closed set {a,b} in (Y,σ), f^{-1}({a,b}) = {a,b} is not ψgs - closed in (X,τ).

Example 4.2.19

Let X = Y = {a,b,c},τ = {ϕ,{a},{a,b},X} and σ = {ϕ,{a},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be the identity function. Then f is ψgs - irresolute but not totally ψgs-continuous, since for the closed set {b,c} in (Y,σ), f^{-1}({b,c}) = {b,c} is not ψgs - clopen in (X,τ).

Remark 4.2.20

Contra ψgs - continuous function is independent from ψgs - irresolute function as seen from the following examples.

Example 4.2.21

Let $X = Y = \{a,b,c\}, \tau = \{\phi,\{a\},\{b\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},\{b,c\},Y\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be a function defined by $f(a) = b, f(b) = a, f(c) = c$. Then f is contra ψgs-continuous but not ψgs - irresolute, since for the ψgs - closed set $\{a,b\}$ in (Y,σ), $f^{-1}(\{a,b\}) = \{a,b\}$ is not ψgs - closed in (X,τ).

Example 4.2.22

Let $X = Y = \{a,b,c\}, \tau = \{\phi,\{a\},\{b\},\{a,b\},X\}$ and $\sigma = \{\phi,\{a\},\{a,b\},\{a,c\},Y\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity function. Then f is ψgs - irresolute but not contra ψgs - continuous, since for the open set $\{a,b\}$ in (Y,σ), $f^{-1}(\{a,b\}) = \{a,b\}$ is not ψgs - closed in (X,τ).

Proposition 4.2.23

If a function $f:(X,\tau)\to(Y,\sigma)$ is ψgs – irresolute, then for every subset A of (X,τ) such that f(A) is ψgs - closed in (Y,σ), $f(\psi gscl(A)) \subseteq \psi gscl(f(A))$.

Proof:

Let A be a subset of (X,τ) such that f(A) is ψgs - closed in (Y,σ). Then $\psi gscl(f(A))$ is ψgs - closed in (Y,σ). Since f is ψgs - irresolute, $f^{-1}(\psi gscl(f(A))$ is ψgs– closed in (X,τ). Now $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}((\psi gscl(f(A)))$. Therefore $\psi gscl(A) \subseteq f^{-1}((\psi gscl(f(A)))$ and hence $f(\psi gscl(A)) \subseteq f(f^{-1}(\psi gscl(f(A))) \subseteq \psi gscl(f(A))$.

Proposition 4.2.24

If a function $f:(X,\tau)\to(Y,\sigma)$ is ψgs – irresolute, then for every ψgs - closed set $B \subseteq Y$, $\psi gscl(f^{-1}(B)) \subseteq f^{-1}(\psi gscl(B))$.

Proof:

Let B be a ψgs - closed set in (Y,σ). Then ψgscl(B) is ψgs - closed in (Y,σ). Since f is ψgs - irresolute, $f^{-1}(\psi gscl(B))$ is ψgs - closed in (X,τ). Since $B \subseteq \psi gscl(B)$, $f^{-1}(B) \subseteq f^{-1}((\psi gscl(B))$. Therefore by definition of ψgs closure, $\psi gscl(f^{-1}(B)) \subseteq f^{-1}((\psi gscl(B))$.

Proposition 4.2.25

If $f:(X,\tau)\to(Y,\sigma)$ is a ψgs - irresolute function and $g:(Y,\sigma)\to(Z,\eta)$ is a ψgs-irresolute function, then $g \circ f :(X,\tau) \to (Z,\eta)$ is a ψgs - irresolute function.

Proof:

Let V be any ψgs - closed set in (Z,η). Since g is ψgs - irresolute, $g^{-1}(V)$ is ψgs-closed in (Y,σ). Since f is ψgs - irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs- closed in (X,τ). Hence $g \circ f$ is a ψgs - irresolute function .

Proposition 4.2.26

If $f:(X,\tau)\to(Y,\sigma)$ is a ψgs - irresolute function and $g:(Y,\sigma)\to(Z,\eta)$ is a ψgs - continuous function, then $g \circ f :(X,\tau) \to (Z,\eta)$ is a ψgs - continuous function.

Proof:

Let V be any closed set in (Z,η). Since g is ψgs - continuous, $g^{-1}(V)$ is ψgs - closed in (Y,σ). Since f is ψgs - irresolute, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - closed in (X,τ). Hence $g \circ f$ is a ψgs - continuous function .

Proposition 4.2.27

If f:(X,τ)→(Y,σ) is a ψgs - irresolute function and g:(Y,σ)→(Z, η) is a contra ψgs - continuous function, then g o f :(X,τ) →(Z,η) is a contra ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η).Since g is contra ψgs - continuous, $g^{-1}(V)$ is ψgs - open in (Y,σ).Since f is ψgs - irresolute,$(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - open in (X,τ). Hence g o f is a contra ψgs - continuous function .

Proposition 4.2.28

If f:(X,τ)→(Y,σ) is a ψgs - irresolute function and g:(Y,σ)→(Z, η) is a continuous function, then g o f :(X,τ) →(Z,η) is a ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η).Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ). By result 1.10, $g^{-1}(V)$ is ψgs - closed in (Y,σ).Since f is ψgs - irresolute, $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - closed in (X,τ). Hence g o f is a ψgs - continuous function .

Proposition 4.2.29

If f:(X,τ)→(Y,σ) is a ψgs - irresolute function and g:(Y,σ)→(Z, η) is a totally ψgs - continuous function, then g o f :(X,τ) →(Z,η) is a ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η).Since g is totally ψgs - continuous, $g^{-1}(V)$ is ψgs - clopen in (Y,σ) which implies that $g^{-1}(V)$ is ψgs - closed in (Y,σ).Since f is ψgs -

irresolute,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - closed in (X,τ). Hence g o f is a ψgs - continuous function.

Proposition 4.2.30

If f:(X,τ)→(Y,σ) is a ψgs - irresolute function and g:(Y,σ)→(Z, η) is a totally continuous function, then g o f :(X,τ) →(Z,η) is a ψgs - continuous function.

Proof:

Let V be any closed set in (Z, η).Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y,σ).By result 1.10, $g^{-1}(V)$ is ψgs - clopen in (Y,σ) which implies that $g^{-1}(V)$ is ψgs - closed in (Y,σ).Since f is ψgs - irresolute,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - closed in (X,τ). Hence g o f is a ψgs - continuous function .

Proposition 4.2.31

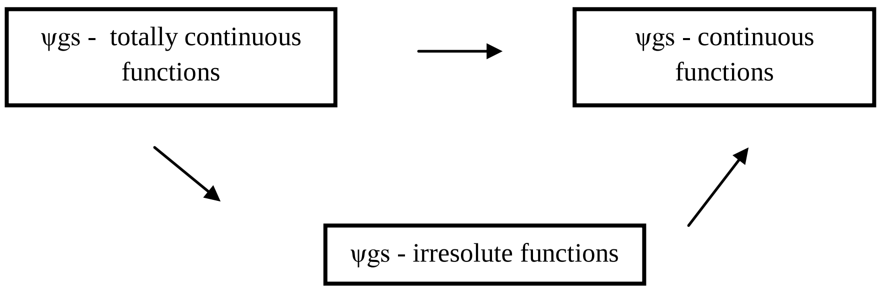
If f:(X,τ)→(Y,σ) is a ψgs – irresolute function and g:(Y,σ)→(Z, η) is a ψgs - totally continuous function, then g o f :(X,τ) →(Z,η) is a ψgs - continuous function.

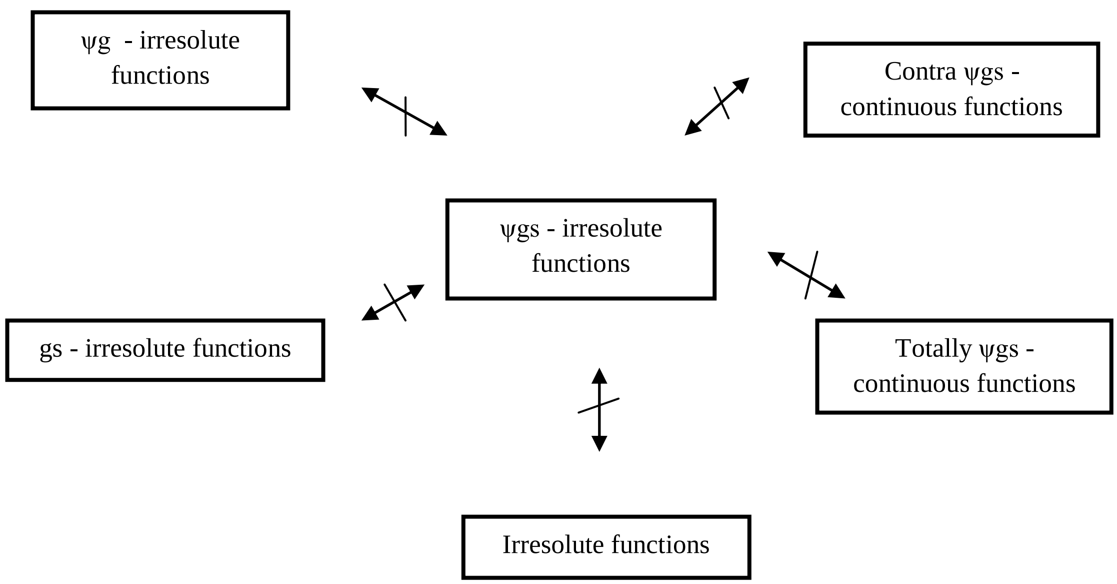
Proof:

Let V be any closed set in (Z, η).By result 1.10, V is ψgs - closed in (Z, η).Since g is ψgs - totally continuous, $g^{-1}(V)$ is clopen in (Y,σ).By result 1.10, $g^{-1}(V)$ is ψgs - clopen in (Y,σ) which implies that $g^{-1}(V)$ is ψgs - closed in (Y,σ).Since f is ψgs - irresolute,$(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψgs - closed in (X,τ).Hence g o f is a ψgs- continuous function .

Remark 4.2.32

The dependency and independency relations between various types of continuous functions with ψgs - irresolute functions are given in the following diagram.


ψgs - totally continuous functions
ψgs - continuous functions
ψgs - irresolute functions


ψg - irresolute functions
Contra ψgs - continuous functions
ψgs - irresolute functions
gs - irresolute functions
Totally ψgs - continuous functions
Irresolute functions

SUMMARY AND CONCLUSION

In chapter 1, preliminary definitions are listed.

Totally ψgs - continuous functions and ψgs - totally continuous functions are introduced and some of their properties are discussed in chapter 2.

In chapter 3, two new kinds of contra continuous functions namely, contra ψgs- continuous functions and almost contra ψgs - continuous functions are introduced and some of their characterizations are analyzed.

ψgs - irresolute functions in topological spaces are introduced and the interrelations between ψgs - irresolute functions with already existing various continuous functions are studied in chapter 4.

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