CHAPTER – I

CHAPTER I

PRELIMINARY DEFINITIONS AND NOTATIONS

Definition: 1.1

Let X be a non-empty set. A **fuzzy set** in X is a function with domain X and values in the closed interval $I = [0,1]$. I^X denote the set of all fuzzy sets in X.

Definition: 1.2

Let $\mu \in I^X$. The subset of X in which μ assumes non-zero values is known as the **support of μ** .

Definition: 1.3

Let $\mu, \rho \in I^X$. We define the following fuzzy sets.

(1) $\mu \leq \rho \Rightarrow \mu(x) \leq \rho(x) \ \forall x \in X$.

(2) $\mu \wedge \rho \in I^X$ by $(\mu \wedge \rho)(x) = \min \{ \mu(x) , \rho(x) \}$ for every $x \in X$.

(3) $\mu \vee \rho \in I^X$ by $(\mu \vee \rho)(x) = \max \mu(x) , \rho(x) \}$ for every $x \in X$.

Let Λ be an indexing set and $\{ \mu_\lambda / \lambda \in \Lambda \}$ be a family of fuzzy sets in X. Then their **union** and **intersection** are defined as follows:

$(\vee \mu_\lambda)(x) = \sup \{ \mu_\lambda(x) / \lambda \in V \}$

$(\wedge \mu_\lambda)(x) = \inf \{ \mu_\lambda(x) / \lambda \in \Lambda \}$

(4) $\mu^C \in I^X$ by $\mu^C(x) = 1 - \mu(x)$ for every $x \in X$. μ^C is called **complement of μ** and is denoted as μ' or $(1 - \mu)$.

(5) Let $f : X \longrightarrow Y$, $\mu \in I^X$ and $\rho \in I^Y$. Then f(A) is a fuzzy set in Y called **image of A** defined by

$$f(\mu)(y) = \begin{cases} \sup \{ \mu(x) : x \in f^{-1}(y) \} & \text{if } f^{-1}(y) \neq \Phi \\ 0 & \text{if } f^{-1}(y) = \Phi \end{cases}$$

and $f^{-1}(\rho)$ is a fuzzy set in X called **inverse image** of ρ defined by

$f^{-1}(\rho)(x) = \rho (f(x))$,$\forall x \in X$.

(6) By the **constant function α** on X we mean the function $\boldsymbol{\alpha} : X \longrightarrow I$ given by

$\boldsymbol{\alpha}(\mathbf{x}) = \alpha$, for every $x \in X$ and $\alpha \in [0,1]$.

(7) The **constant fuzzy sets** taking on the values 0 and 1 on X are denoted by $\mathbf{0_X}$ and $\mathbf{1_X}$ respectively.

Definition: 1.4 (Chang , C.L., [16])

A **fuzzy topological space** is a pair (X,τ), where X is a set and τ is a **fuzzy topology** on it , that is , a family of fuzzy sets ($\tau \subseteq I^X$) satisfying the following three axioms :

(1) **0, 1** ∈ τ where $\mathbf{0}(x) = 0 \ \forall x \in X$, $\mathbf{1}(x) = 1 \ \forall x \in X$.

(2) If $\mu, \rho \in \tau$., $\mu \wedge \rho \in \tau$.

(3) If $\{ \mu_j : j \in J \} \subseteq \tau$, then $\vee \{ \mu_j : j \in J \} \subseteq \tau$.

The elements of τ are called **fuzzy open sets**. A fuzzy set λ is called a **fuzzy closed set** if $\lambda^C \in \tau$. We denote τ^C the collection of all fuzzy closed sets in this fuzzy topological space.

Definition: 1.5

The **closure cl (μ) or μ^-** and the **interior int (μ) or μ^0** of a fuzzy set μ of X is defined as :

$\text{cl}(\mu) = \inf\{\gamma : \mu \le \gamma, \lambda^C \in \tau\}$,

$\text{int}(\mu) = \sup\{\zeta : \zeta \le \mu, \zeta \in \tau\}$ respectively.

Definition: 1.6

A fuzzy set in X is called a **fuzzy point** if and only if it takes the value 0 for all y $\in$X except one, say x $\in$ X. If its value at x is λ $(0 < \lambda \le 1)$ .We denote the fuzzy point x_λ , where the point x is called its **support.** When the support and the value of a fuzzy point are trivial, we use briefly the symbol **e** to denote the fuzzy point.

Definition: 1.7

A fuzzy point in x_λ is said to be **contained in a** fuzzy set μ or **to belong to** μ denote by $x_\lambda \in \mu$, if and only if $\lambda \le \mu(x)$. Evidently, every fuzzy set μ can be expressed as the union of all the fuzzy points which belong to μ.

Definition: 1.8

A fuzzy set μ in a fuzzy topological space (X,τ) is called a **fuzzy neighbourhood** of a fuzzy point x_λ iff there exists a $\nu \in \tau$ such that $x_\lambda \in \nu \subseteq \mu$.

Definition: 1.9

A function f from a fuzzy topological space X into a fuzzy topological space Y is **fuzzy continuous** if and only if for every fuzzy point x_λ in X and every neighbourhood V of f (x_λ) , there exists a neighborhood U of x_λ such that f (U) $\leq$ V .

Definition: 1.10

A subset S of a topological space X is said to be **semi-open** if there exists an open set U such that $U \subset S \subset cl(U)$.

The complement of a semi-open set is called a **semi-closed** set.

Definition: 1.11

A subset A of a space (X,τ) is called a **generalized closed set (briefly, g-closed)** if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ) .

The complement of a g-closed set is called a **g-open** set.

Definition: 1.12

A subset S of a space (X,τ) is said to be **semi-regular** if it is both

semi-open and semi-closed.

Notation: 1.13

The family of all **semi-open** (respectively **semi-regular**) sets of X is denoted by **SO(X)** (respectively **SR(X)**). For each x ∈ X, the family of all semi-open (respectively semi-regular) sets of X containing x is denoted by SO(X)(respectively SR(X)).

Definition: 1.14

Let X and Y be topological spaces, a function $f : X \longrightarrow Y$ is said to be **pre-semi-open** if and only if for all A ∈ SO(X) , f(A) ∈ SO(Y) .

The complement of a pre-semi-open set is called a **pre-semi-closed** set.