Avinashilingam Institute for Home Science and Higher Education for Women
Coimbatore – 641 043
Bachelor's Degree Examination – November 2017

V Semester

Class : III UG | **Time : 3 Hours**
Major : Special Education & Reh. and Mathematics | **Max. Marks : 100**

15BSMC11 Real Analysis - I
Part – A **10 x 1 = 10**
Choose the Correct Answer

1. The cardinal number of the empty set is
   a. 1 b. 0
   c. infinite d. ϕ

2. The set of algebraic numbers is
   a. finite b. infinite
   c. countable d. non – denumerable

3. A set S is open if and only if
   a. SCS b. SC int S
   c. int S C S d. S = int S

4. The accumulation point of the set of rational numbers are
   a. rational numbers b. irrational numbers
   c. integers d. real numbers

5. A set S in R^n is said to be --------- , if it lies entirely within an n – ball B(a;r), r > 0 and $a \epsilon R^n$
   a. unbounded b. bounded
   c. closed d. open

6. The collection of all intervals of the form $\frac{1}{n} < x < \frac{2}{n}$, (n = 2, 3 ...) is an open covering of the interval
   a. $0 \leq x < 1$ b. $0 < x \leq 1$
   c. $0 < x < 1$ d. $0 < \frac{1}{x} < 1$

7. Let M be a non – empty set and $d(x,y) = \begin{cases} 0, \text{ if } x = y \\ 1, \text{ if } x \neq y \end{cases}$ then the metric space (M,d) is called
   a. discrete b. indiscrete
   c. Euclidean d. relative

8. The boundary of the set of rational numbers S in R is all of
   a. R' b. [0,1]
   c. (0,1) d. R x R

9. Every Euclidean space R^K is
   a. compact b. closed
   c. open d. complete

10. In C, Let $Z_n = 1 + n^{-2} + (2-\frac{1}{n})\, i$, then $\underset{n \to \infty}{lt}$ Zn =
   a. 1 b. 1 + 2i
   c. 0 d. 2i

Part – B

5 x 6 = 30

Answer the following

Answer should not exceed 400 words or two pages

11.a. Define the following sets and give examples
i. similar ii. infinite iii. Countable iv. uncountable

(Or)

11.b. Let z^+ denote the set of all positive integers then prove that $z^+ \times z^+$ is countable.

12.a. State and prove Cauchy – Schwarz inequality

(Or)

12.b. If x is an accumulation point of S, then prove that every n – ball B(x) contains infinitely many points of S.

13.a. State and prove Lindelof covering theorem.

(Or)

13.b. Prove that the set of open disks in the xy – plane with center at (x,x) and Radius x > 0, x rational, is a countable covering of the set $\{ (x, y) : x > 0, y > 0\}$

14.a. Prove that (M, d) where $M = R^n$ and $d(x,y) = || x - y ||$ is a metric space.

(Or)

14.b. Prove that (M,d), where M = C and $d (z_1, z_2) = | z_1 - z_2|$ is a metric space.

15.a. Prove that a sequence $\{ x_n\}$ in a metric space (S,d) can converge to at most one point in S.

(Or)

15.b. In any metric space (S,d) prove that every compact subset T is complete.

Part – C

5 x 12 = 60

Answer the following

Answer should not exceed 800 words or four pages

16.a. Prove that every subset of a denumerable set is denumerable.

(Or)

16.b. Prove that the set of all real numbers is uncountable.

17.a. State and prove representation theorem for open sets on the real line in R^n.

(Or)

17.b.i. Prove that the union of any collection of open sets is an open set.
ii. Prove that the intersection of a finite collection of open sets is open.

18.a. State and prove the Bolzano – Weierstrass theorem.

(Or)

18.b. State and prove the Cantor intersection theorem.

19.a. State and prove the Heine – Borel theorem.

(Or)

19.b. Let S be a subset of R^n. Then prove the following statements are equivalent:
i. S is compact (ii) S is closed and bounded.iii. Every infinite subset of S has an accumulation point in S.

20.a. Prove that in a Euclidean space R^k, every Cauchy sequence is convergent.

(Or)

20.b. Assume p is an accumulation point of A and assume $b \epsilon T$. Then $\underset{x \to p}{lt} f(x)=b$ if and only if $\underset{x \to \infty}{lt} f(x_n) = b$, for every sequence $\{x_n\}$ of points in A - $\{p\}$ which converges to p. Prove it.

****