Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University, Estd. u/s 3 of UGC Act 1956 Category 'A' by MHRD)
Re-accredited with 'A++' Grade by NAAC. Recognised by UGC Under Section 12 B
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination – November 2024
III Semester

Class : II. PG **Time: 3 Hours**
Major : Mathematics **Max. Marks: 100**

23MMAC13 Topology – I

Course Outcomes:

CO1: Understand the properties of various topologies on a general set.
CO2: Construct continuous functions in topological spaces.
CO3: Analyze the relation between metric spaces and topological spaces.
CO4: Relate the concept of continuity to connected space.
CO5: Demonstrate the properties of compact spaces

Part A **10 x 1 = 10**
Choose the Correct Answer

1. If τ is any set the collection of all subsets of X, Then τ is a _____on X CO1K2
   a. countable b. discrete topology c. indiscrete topology d. not a topology

2. Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies τ and τ' on X. Then_____ CO1K2
   a. τ' is finer than τ b. τ' smaller than τ c. τ less than τ' d. τ greater than τ'

3. Let Y be a subspace of X. If U is open in Y and Y is open in X, then U is _____ in X. CO2K2
   a. closed b. basis c. subspace d. open

4. The_____ of A is defined as the union of all open sets contained in A. CO2K1
   a. interior b. closure c. limit point d. closed

5. A_____ is a metrizable space X together with a specific metric d that gives the topology on X. CO3K2
   a. topological space b. bounded metric c. metric space d. open

6. The topologies on $\mathfrak{R}^n$ induced by the Euclidean metric d the square metric ρ are the same as The_____on $\mathfrak{R}^n$. CO3K3
   a. Euclidean metric b. square metric c. product topology d. metric topology.

7. The union of a collection of connected subspaces X that have a point in common is_____ CO4K3
   a. closure b. interior c. connected d. disconnected

8. A simply ordered set L having more than one element is called a _____ CO4K1
   a. linear continum b. path connected c. real line d. locally path connected

9. In the order topology each closed interval in X is_____ CO5K3
   a. finite intersection b. upper bound c. connected d. compact

10. Every closed interval in $\mathfrak{R}$ is_____ CO5K1
    a. compact b. connected c. Euclidean metric d. connected

Part B 5 x 6 = 30

Answer ALL questions

Each answer should not exceed 400 words or two pages

11.a .Define basis for the topology and Let X be a set and $\mathcal{B}$ be basis for the topology τ on X. Then prove that τ equals the collection of all union of elements of B. CO1K2

(or)

11.b. If $\mathcal{B}$ is a basis for the topology of X and C is a basis for the topology of Y, then the collection D={B X C/ B$\in\mathcal{B}$ and C$\in\mathcal{C}$ } is a basis for the topology of X x Y. CO1K3

12.a. Let Y be a subspace of X. If U is open in Y and Y is open in X, then prove that U is open in X CO2K4

(or)

12.b. Let A be a subset of the topological space X. Let A' be the set of all limit points of A. Then prove that $\overline{A} = A \cup A'$. CO2K1

13.a. Let X be a metric space d. Define $\overline{d} : XxX \to R$ by the equation $\overline{d}(x,y) = \min\{d(x,y),1\}$. Then prove that $\overline{d}$ is a metric that induces the same topology as d. CO3K4

(or)

13.b. State and prove the sequence lemma. CO3K4

14.a. Let A be a connected subspace of X. If $A \subset B \subset \overline{A}$ then prove that B is also connected. CO4K2

(or)

14.b. Prove that a space X is locally connected iff for every open set U of X, each component of U is open in X. CO4K5

15.a. Prove that every compact subspace of a Hausdroff space is closed. CO5K6

(or)

15.b. State and prove Uniform continuity theorem. CO5K5

Part C 5 x 12 = 60

Answer ALL questions

Each answer should not exceed 800 words or four pages

16.a. Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and each x in U, there is an element C of $\mathcal{C}$ such that x$\in$C$\subset\mathcal{C}$. Then prove that C is a basis for the topology of X. CO1K3

(or)

16.b. Prove that the topologies of $\mathcal{R}_\ell$ and $\mathcal{R}_k$ are strictly finer than the standard topology on $\mathcal{R}$ but not comparable with one another. CO1K4

17.a. Let X be a topological space. Then prove that the following conditions are hold.
(i) ϕ and X are closed (ii) Arbitrary intersections of closed sets are closed.
(iii) Finite unions of closed sets are closed. CO2K

(or)

17.b. State and Prove the Pasting Lemma. CO2K4

18.a. Prove that the topologies on $\Re^n$ induced by the Euclidean metric d and the square metric ρ are the same as the product topology on $\Re^n$. CO3K2

(or)

18.b. Let $f : X \to Y$. If the function f is continuous, then prove that for every convergent sequence $x_n \to x$ in X, the sequence f(x_n) converges to f(x). CO3K3

19.a. Prove that the image of a connected space under a continuous map is connected. CO4K4

(or)

19.b. State and Prove Intermediate value theorem. CO4K4

20.a.Prove that the image of a compact space under a continuous map is compact. CO5K4

(or)

20.b. State and Prove the Lebesgue number lemma. CO5K2

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