International Journal of Applied Engineering Research, ISSN 0973-4562 Vol. 11 No.1 (2016)
© Research India Publications; http/www.ripublication.com/ijaer.htm

(m,N) POLICY FOR REPAIRABLE BULK ARRIVAL QUEUEINGMODELWITH SETUP,SECOND MULTI OPTIONAL SERVICE FACILITY UNDER RESTRICTED NUMBER OF (L) SERVER VACATIONS

FijyJose.P[1], *[1] Research Scholar,*
M.I. Afthab Begum[2] *[2] Professor,*

Department of Mathematics, Avinashilingam University, Tamil Nadu, India
[1] fijyjosep@gmail.com *[2] aftabbegum2013@gmail.com*

Abstract:**This paper analyses a repairable batch arrival queueing model with a second multi optional service(SOS) channels under (m,N) policy in which the server takes restricted number of multiple vacations during his idle period.The server leaves the system for vacation as soon as the system empties and after returning from each vacation if the server finds 'm' or more customers in the system then he immediately starts the setup work. Otherwise he repeats his vacation until he finally finds at least 'm' customers or, returns to the system after taking 'L' vacations.ie At the end of Lth vacation if the server finds less than 'm' customers, he joins the system and stays idle until the queue length reaches at least 'm' to start the setup work. At the end of the setup period, if the queue length is greater than or equal to N, then the server begins to serve the customers one at a time.Otherwise, the server remains dormant in the system, waiting for the queue length to reach atleast N, to start the service. The server may undergo unpredictable breakdowns during the service and sent for repair immediately.As soon as the repair is completed, the server returns to the customer whose service was interrupted. The system size distribution at random epochs and mean system length are calculated and the corresponding results for the classical single and multiple vacation models are obtained as special cases.**

Keywords:(m,N) policy, Batch arrival, Second multi optional Service, Restricted Vacation.

I INTRODUCTION

In classical single server queueing models an idle server will start the service as soon as a customer enters to the empty system. But in many situations it is important to decide when the server should start his service as, frequent setup inevitably make the operating cost too high. The N policy introduced by Yadin and Naor[1] is the most general control policy in which the server does not start his service until there are N customers in the queue. This minimizes server switch overs and avoids excessive frequent use of setups. The N policy of queues has been widely used to provide stochastic modelling of many problems arising in production inventory system, in telephone switching system and in quality control problems. The study of the N policy M/G/1 queueing system was first analysed by Heyman [2] and was developed by several researchers such as Wang and Ke[3], Bell [4], Teghem[5]. Lee and Park [6] modeled a production system(M /G/1) and analysed the N-policy with early setup. This early setup policy is termed as (m,N)-policy which is more general than the single threshold (N, N)-policy.Lee et al. [7] extended the bilevel control policy to a batch arrival queueing system M^X / M / 1 and obtained queue length and waiting time distribution for the model.Lee et al.[8] analyzed a batch arrival systems $M^X / G / 1$ under bilevel threshold with early setup and with/ without server vacation.Later J.C.Ke[9] investigated the (m.N) policy for an unreliable server.In recent years,queues due to server vacations have emerged as an important area of queueing theory and have been studied exhaustively and successfully, in various applications. At present, most of the studies are developed for batch.arrival vacation models under various vacation policies. Ke et al.[10] studied the operating characteristics of an M/G/1 queueing system with a randomized control policy and at most J vacations.Queueing models with server breakdowns are more realistic representation of systems.The server may undergo unpredictable breakdowns during the service and sent for repair immediately.As soon as the repair is completed, the server returns to the customer whose service was interrupted. Discussing queueing systems with N policy and server breakdowns,Wang[11] first proposed Markovian queueing system under N policy and server breakdowns and derived analytical steady-state solutions of the N policy M/M/1 queue.Wang and Ke[12]analysed the control policy for M/G/1 queueing system with an unreliable server.The N policy Markovian system with finite capacity and a non-reliable server was proposed by Wang and Hisch[13].Recently Ke,Huang,Pearn[14] has studied randomized policy of a Poisson input queue with J vacations and then Huang,Pearn[15]analysed performance measures and randomized optimization of unreliable server $M^X/G/1$ vacation system.

In classical single vaction (m,N) policy queueing models, the server after returning from the vacation, joins the system even if the required number of customers are not present.The server stays idle in the system after returning from vacation until atleast 'm' customers accumulate.In multiple vacation (m,N) policy queueing models the server takes repeated unlimited number of vacations until the queue length becomes atleast 'm'. In the present paper a modified multiple vacation policy, where, the server takes atmost L vacations repeatedly until atleast 'm' customers accumulate for services is considered. The model of the present paper is a generalization of the classical single and multiple vacation queueing models with multi Second Optional Services,breakdown,setup and dormant period.

The steady state system size probability distributions of the model are obtained and the results of the classical single and multiple vacation models are obtained as special case of the model of the present paper.

II. MATHEMATICAL ANALYSIS OF THE SYSTEM

Model Description:

Arrival Pattern:

Consider an $M^x/G/1$ queueing system where the arrivals occur in batches according to a compound Poisson process with random batch size X, group arrival rate λ .(Gross and Harris [16]) Arriving customers form a single waiting line and the service is done one by one.

Idle Period and Vacation Policy:

As soon as the system empties, the server is turned off and leaves the system for vacation of random length V. After returning from the vacation,if the server finds 'm' or more customers in the system,then he immediately starts a setup operation of random length D.Otherwise he repeats his vacation until he finds at least 'm'customers or returns to the system after taking 'L' vacations which ever occurs earlier.At the end of L^{th} vacation if the server finds less than 'm' customers he joins the system and stays idle until the queue length reaches at least 'm'.The server can go for vacation again only after completing a subsequent completion period(sum of busy and repair period).The period during which the server remains idle in the system before starting the setup work is called buildup period.At the end of the setup operation,if the queue length is $\geq N$,then the server begins to serve the customers one by one exhaustively.Otherwise,the server remains dormant in the system waiting for the queue length to reach or exceed N.Thus the sum of buildup period,setup period dormant period and vacation period will constitute the idle period.

Busy Period:

During busy period, the server provides single First Essential Service(FES) to all the arriving customers in the first phase and "multi-optional" services in the second phase for selected customers. The multi-optional service consists of 'c'types of heterogeneous services. As soon as the FES is completed,with probability $r_i (1 \le i \le c)$, the customers may either opt for a second service from c ($c \ge 1$) kinds of different services or else with probability 1-r($r = \sum_{i=1}^{c} r_i$). may leave the system.

Breakdown Period

The server may breakdown at any time while serving customers. It is assumed that the server's life times follow exponential distribution with mean(1/a) in FES and the server fails at an exponential rate a_i ($1 \le i \le c$) in second multi optional services. Whenever the breakdowns occur, the server is sent for repair immediately. The customer just being served before server breaks down, waits for the server to return from the repair facility. Immediately after the server is fixed, he starts to serve the customer who is waiting in the service facility to complete the remaining service.It is assumed that the service time for a customer is cumulative and after repair the server is considered as good as new.

The busy period and repair period constitute a completion period. The idle period and completion period . will determine a cycle. The customers continue to arrive and join the system independent of the system states, following the compound Poisson process.Various stochastic processes involved in the queueing system are assumed to be independent to each other. The system is denoted by

$$M^{X}_{(m,N)} / G; G_{i(1 \le i \le c)} / 1 / L_{vacation} / Breakdown$$

Notation

The following notations are used to discuss the model.

N (t) : The system size at time t

λ : Group arrival rate

X : Group size random variable

g_k : Pr(X = k), k = 1, 2, 3, …

$g_n^{(i)}$:i-fold convolution of $\{g_n\}$with itself ; $g_n^{(i)} = \sum_{k=1}^{n-1} g_{n-k}^{(i-1)} g_k$, $g_n^{(0)} = \delta_{n,0}$.

$\alpha_n^{(i)}$ i=1 to c can similarly be defined.

X(z) : Probability generating function of X.

Y(t) = {0,1,2,3,4,5,6,7} according as the system is in vacation, buildup state, setupstate, busywith FES , busy with SOS, repair mode in FES andrepair mode in SOS state respectively.

The vacation time (V),setup time (D),service times (S, S_i) and repair times(R, R_i) i=1 to c follow general distributions and are assumed to be mutually independent of each other.

The notations of Random Variables (RV), Cumulative Distribution Functions (CDF), Probability Density Function (PDF), Laplace-StieltjesTransform (LST) and its k^{th} moments of the RVs are listed below

	RV	**CDF**	**PDF**	**LST**	**k^{th}moments**
Vacation Time	V	V(x)	v(x)	$V^*(\theta)$	$E(V^k)$
Setup time	D	D(x)	d(x)	$D^*(\theta)$	$E(D^k)$
FES	S	S (x)	s (x)	$S^*(\theta)$	$E(S^k)$
SOS	S_i	$S_i(x)$	$s_i(x)$	$S_i^*(\theta)$	$E(S_i^k)$, $1 \le i \le c$
Repair time in FES	R	R(y)	r(y)	$R^{*1}(\theta_1)$	$E(R^k)$
Repair time in SOS	R_i	$R_i(y)$	$r_i(y)$	$R_i^{*1}(\theta_1)$	$E(R_i^k)$, i=1 to c

Let $V^0(t), D^0(t), S^0(t), S_i^0(t), (i=1 \text{ to } c), R^0(t)$ and $R_i^0(t)$,(i=1 to c) denote the remaining vacation time,setuptime,FEStime,SOS time, repair time due to FES and repair time due to SOS)respectively at time t.Then the state space$\{N(t),\delta(t)\}$,where $\delta(t)=(V^0(t),0,D^0(t),0,S^0(t), S_i^0(t), R^0(t), R_i^0(t)$)according as Y(t)= 0 to 7 respectively defines a bivariate Markov process.

Thus $P_{n,0}(t) = \Pr(N(t) = n, Y(t) = 1)$,

$0 \le n \le m-1$ and $U_n(t) = \Pr(N(t) = n, Y(t) = 3)$

,($m \le n \le N-1$) respectively denote the probability that at time t the server is in **buildup** and **dormant** states, and there are n customers in the system.

Let Z(t)=j(j=1,2...L) denote that the server is on jth vacation at time t counting from the idle period initiation point.

$Q_{n,j}(x,t)dt = \Pr(N(t) = n, x \le V^0(t) \le x+dt, Y(t) = 0, Z(t) = j, j \ge 1), n \ge 0$ denotes the joint probability that at time t, there are n customers in the system, the server is in the jth vacation and the remaining **vacation** time of the server is between x and x + dt.

$D_n(x,t)dt = \Pr(N(t) = n, x \le D^0(t) \le x+dt, Y(t) = 2)$ is the joint probability that at time t, there are n customers in the system, and the remaining **setup** of the server is between x and x + dt where $n \ge m$.

$P_n(x,t)dt = \Pr(N(t) = n, x \le S^0(t) \le x+dt, Y(t) = 4)$ is the joint probability that at time t, there are n customers in the system, the server is **busy in FES** with remaining service time between x and x + dt, n≥1.

$P_{n,i}(x,t)dt = \Pr(N(t) = n, x \le S_i^0(t) \le x+dt, Y(t) = 5)$ is the joint probability that at time t, there are n customers in the system, the server is **busy** and a customer being served is ith**(second optional service)** channel with remaining service time between x and x + dt, n≥1and $1 \le i \le c$

$B_n(x,y,t)dt = \Pr(N(t) = n, S^0(t) = x, y \le R^0(t) \le y+dt, Y(t) = 6)$

$(B_{n,i}(x,y,t)dt = \Pr(N(t) = n, S_i^0(t) = x, y \le R_i^0(t) \le y+dt, Y(t) = 7))$

is the joint probability that at time t, there are n customers in the system, the remaining repair time due to FES (due to SOS) for the customer under service is equal to x, and the server is being repaired with the remaining repair time between y and y + dt, $n \ge 1$ and $1 \le i \le c$.

Further $Q_n(0)$, $D_n(0)$, $P_n(0)$,$P_{n,i}(0)$ and $B_{n,i}(x, 0)$, i = 1, 2 denote the probability that there are n customers in the system at the termination of vacation period, setup period, and the corresponding service time and repair time respectively.

System size Distribution at Random Epoch

Following the argument of Cox [17] and observing the changes of states during the interval (t,t+Δt) for any time t,the steady state equations are given by

Vacation State

$$\frac{-d}{dx}Q_{0,1}(x) = -\lambda Q_{0,1}(x) + \overline{P}(0)v(x), \text{ where}$$

$$\overline{P}(0) = P_1(0)(1-r) + \sum_{i=1}^{c} P_{1,i}(0)$$

$$\frac{-d}{dx}Q_{0,j}(x) = -\lambda Q_{0,j}(x) + Q_{0,j-1}(0)v(x)$$

$$2 \le j \le L$$

$$\frac{-d}{dx}Q_{n,1}(x) = -\lambda Q_{n,1}(x) + \lambda \sum_{k=1}^{n} Q_{n-k,1}(x)g_k$$

$$n \ge 1$$

$$\frac{-d}{dx}Q_{n,j}(x) = -\lambda Q_{n,j}(x) + Q_{n,j-1}(0)v(x) + \lambda \sum_{k=1}^{n} Q_{n-k,j}(x)g_k$$

$$1 \le n \le m-1 \quad , \quad 2 \le j \le L$$

$$\frac{-d}{dx}Q_{n,j}(x) = -\lambda Q_{n,j}(x) + \lambda \sum_{k=1}^{n} Q_{n-k,j}(x)g_k$$

$$n \ge m \text{ and } 2 \le j \le L$$

Buildup State

$$\lambda P_{0,0} = Q_{0,L}(0)$$

$$\lambda P_{n,0} = Q_{n,L}(0) + \lambda\sum_{k=1}^{n} P_{n-k,0}g_k \quad 1 \le n \le m-1$$

Setup state

$$\frac{-d}{dx}D_m(x) = -\lambda D_m(x) + \sum_{j=1}^{L} Q_{m,j}(0)d(x) + \lambda\sum_{k=1}^{m} P_{m-k,0}g_k d(x)$$

$$\frac{-d}{dx}D_n(x) = -\lambda D_n(x) + \sum_{j=1}^{L} Q_{n,j}(0)d(x) + \lambda\sum_{k=1}^{n-m} D_{n-k}(x)g_k$$

$$+ \lambda \sum_{k=n-m+1}^{n} P_{n-k,0}g_k d(x) \quad n \ge m+1$$

Dormant state

$$\lambda U_m = D_m(0)$$

$$\lambda U_n = D_n(0) + \lambda\sum_{k=1}^{n-m} U_{n-k}g_k \quad m+1 \le n \le N-1$$

Busy with FES

$$\frac{-d}{dx}P_1(x) = -(\lambda + a)P_1(x) + (1-r)P_2(0)s(x)$$

$$+ \sum_{i=1}^{c} P_{2,i}(0)s(x) + B_1(x,0)$$

$$-\frac{d}{dx}P_n(x) = -(\lambda + a)P_n(x) + (1-r)P_{n+1}(0)s(x)$$

$$+ \lambda\sum_{k=1}^{n-1} P_{n-k}(x)g_k$$

$$+ \sum_{i=1}^{c} P_{n+1,i}(0)s(x) + B_n(x,0) \quad 2 \le n \le N-1$$

$$\frac{-d}{dx}P_n(x) = -(\lambda + a)P_n(x) + (1-r)P_{n+1}(0)s(x)$$

$$+ \lambda\sum_{k=1}^{n-1} P_{n-k}(x)g_k + \sum_{i=1}^{c} P_{n+1,i}(0)s(x)$$

$$+ B_n(x,0) + D_n(0)s(x) + \lambda \sum_{k=n-N+1}^{n-m} U_{n-k}g_k s(x)$$

$$n \ge N$$

Busy with ithtype of SOS

$$-\frac{d}{dx}P_{1,i}(x) = -(\lambda + a_i)P_{1,i}(x) + r_i P_1(0)s_i(x) + B_{1,i}(x,0)$$

$$\frac{-d}{dx}P_{n,i}(x) = -(\lambda + a_i)P_{n,i}(x) + r_i P_n(0)s_i(x) + \lambda\sum_{k=1}^{n-1} P_{n-k,i}(x)g_k + B_{n,i}(x$$

$$n \ge 2 \;,\; 1 \le i \le c$$

Breakdown during FES

$$\frac{-\partial}{\partial y}B_1(x,y) = -\lambda B_1(x,y) + aP_1(x)r(y)$$

$$\frac{-\partial}{\partial y}B_n(x,y) = -\lambda B_n(x,y) + aP_n(x)r(y) + \lambda\sum_{k=1}^{n-1} B_{n-k}(x,y)g_k$$

$$n \ge 2$$

Breakdown during ithtype of SOS

$$\frac{-\partial}{\partial y}B_{1,i}(x,y) = -\lambda B_{1,i}(x,y) + a_i P_{1,i}(x)r_i(y)$$

$$\frac{-\partial}{\partial y}B_{n,i}(x,y) = -\lambda B_{n,i}(x,y) + a_i P_{n,i}(x)r_i(y) + \lambda\sum_{k=1}^{n-1} B_{n-k,i}(x,y)g_k,$$

$$n \ge 2$$

The LST of the steady-state equations w.r.to x, lead to

$$\theta Q_{0,1}^{\ *}(\theta) - Q_{0,1}(0) = \lambda Q_{0,1}^{\ *}(\theta) - \overline{P}(0)V^*(\theta) \tag{1}$$

$$\theta Q_{0,j}^{\ *}(\theta) - Q_{0,j}(0) = \lambda Q_{0,j}^{\ *}(\theta) - Q_{0,j-1}(0)V^*(\theta)$$

$$2 \le j \le L \tag{2}$$

$$\theta Q_{n,1}^{\ *}(\theta) - Q_{n,1}(0) = \lambda Q_{n,1}^{\ *}(\theta) - \lambda\sum_{k=1}^{n} Q_{n-k,1}^{\ *}(\theta)g_k$$

$$n \ge 1 \tag{3}$$

$$\theta Q_{n,j}^{\ *}(\theta) - Q_{n,j}(0) = \lambda Q_{n,j}^{\ *}(\theta) - Q_{n,j-1}(0)V^*(\theta)$$

$$-\lambda\sum_{k=1}^{n} Q^*_{n-k,j}(\theta)g_k \quad 1 \le n \le m-1, \; 2 \le j \le L \tag{4}$$

$$\theta Q_{n,j}^{\ *}(\theta) - Q_{n,j}(0) = \lambda Q_{n,j}^{\ *}(\theta) - \lambda\sum_{k=1}^{n} Q^*_{n-k,j}(\theta)g_k$$

$$n \ge m \text{ and } 2 \le j \le L \tag{5}$$

$$\lambda P_{0,0} = Q_{0,L}(0) \tag{6}$$

$$\lambda P_{n,0} = Q_{n,L}(0) + \lambda \sum_{k=1}^{n} P_{n-k,0} g_k \quad 1 \le n \le m-1 \qquad (7)$$

$$\theta D_m^*(\theta) - D_m(0) = \lambda D_m^*(\theta) - \sum_{j=1}^{L} Q_{m,j}(0) D^*(\theta)$$

$$- \lambda D^*(\theta) \sum_{k=1}^{m} P_{m-k,0} g_k \qquad (8)$$

$$\theta D_n^*(\theta) - D_n(0) = \lambda D_n^*(\theta) - \sum_{j=1}^{L} Q_{n,j}(0) D^*(\theta) - \lambda \sum_{k=1}^{n-m} D^*{}_{n-k}(\theta) g_k$$

$$- \lambda D^*(\theta) \sum_{k=n-m+1}^{n} P_{n-k,0} g_k \quad n \ge m+1 \qquad (9)$$

$$\lambda U_m = D_m(0) \qquad (10)$$

$$\lambda U_n = D_n(0) + \lambda \sum_{k=1}^{n-m} U_{n-k} g_k$$

$$m+1 \le n \le N-1 \qquad (11)$$

$$\theta P_1^*(\theta) - P_1(0) = (\lambda + a) P_1^*(\theta) - (1-r) P_2(0) S^*(\theta))$$

$$- \sum_{i=1}^{c} P_{2,i}(0) S^*(\theta) - B_1^*(\theta,0) \qquad (12)$$

$$\theta P_n^*(\theta) - P_n(0) = (\lambda + a) P_n^*(\theta) - (1-r) P_{n+1}(0) S^*(\theta)$$

$$- \sum_{i=1}^{c} P_{n+1,i}(0) S^*(\theta)$$

$$- \lambda \sum_{k=1}^{n-1} P_{n-k}^*(\theta) g_k - B_n^*(\theta,0)$$

$$2 \le n \le N-1 \qquad (13)$$

$$\theta P_n^*(\theta) - P_n(0) = (\lambda + a) P_n^*(\theta) - (1-r) P_{n+1}(0) S^*(\theta)$$

$$- \sum_{i=1}^{c} P_{n+1,i}(0) S^*(\theta) - \lambda \sum_{k=1}^{n-1} P_{n-k}^*(\theta) g_k$$

$$- D_n(0) S^*(\theta) - B_n^*(\theta,0) - \lambda \sum_{k=n-N+1}^{n-m} U_{n-k} g_k S^*(\theta)$$

$$\text{n} \ge \text{N} \qquad (14)$$

$$\theta P_{1,i}^*(\theta) - P_{1,i}^*(0) = (\lambda + a_i) P_{1,i}^*(\theta) - r_i P_1(0) S_i^*(\theta) - B^*{}_{1,i}(\theta,0) \qquad (15)$$

$$\theta P_{n,i}^*(\theta) - P_{n,i}(0) = (\lambda + a_i) P_{n,i}^*(\theta) - r_i P_n(0) S_i^*(\theta)$$

$$- \lambda \sum_{k=1}^{n-1} P_{n-k,i}^*(\theta) g_k - B_{n,i}^*(\theta,0)$$

$$1 \le i \le c, \ n \ge 2 \qquad (16)$$

$$\frac{-\partial}{\partial y} B_1^*(\theta, y) = -\lambda B_1^*(\theta, y) + a P_1^*(\theta) r(y) \qquad (17)$$

$$\frac{-\partial}{\partial y} B_n^*(\theta, y) = -\lambda B_n^*(\theta, y) + a P_n^*(\theta) r(y) + \lambda \sum_{k=1}^{n-1} B_{n-k}^*(\theta,$$

$$\text{n} \ge 2 \qquad (18)$$

$$\frac{-\partial}{\partial y} B_{1,i}^*(\theta, y) = -\lambda B_{1,i}^*(\theta, y) + a_i P_{1,i}^*(\theta) r_i(y) \qquad (19)$$

$$\frac{-\partial}{\partial y} B_{n,i}^*(\theta, y) = -\lambda B_{n,i}^*(\theta, y) + a_i P_{n,i}^*(\theta) r_i(y)$$

$$+ \lambda \sum_{k=1}^{n-1} B_{n-k,i}^*(\theta, y) g_k \quad \text{n} \ge 2 \qquad (20)$$

Taking the LST w.r.to y , (17) to (20), imply

$$\theta_1 B_1^{**1}(\theta, \theta_1) - B_1^*(\theta,0) = \lambda B_1^{**1}(\theta, \theta_1)$$

$$- a P_1^*(\theta) R^{*1}(\theta_1) \qquad (21)$$

$$\theta_1 B_n^{**1}(\theta, \theta_1) - B_n^*(\theta,0) = \lambda B_n^{**1}(\theta, \theta_1) - a P_n^*(\theta) R^{*1}(\theta_1)$$

$$- \lambda \sum_{k=1}^{n-1} B_{n-k}^{**1}(\theta, \theta_1) g_k \quad n \ge 2 \qquad (22)$$

$$\theta_1 B_{1,i}^{**1}(\theta, \theta_1) - B_{1,i}^*(\theta,0) = \lambda B_{1,i}^{**1}(\theta, \theta_1)$$

$$- a_i P_{1,i}^*(\theta) R_i^{*1}(\theta_1) \qquad (23)$$

$$\theta_1 B_{n,i}^{**1}(\theta, \theta_1) - B_{n,i}^*(\theta,0) = \lambda B_{n,i}^{**1}(\theta, \theta_1) - a_i P_{n,i}^*(\theta) R_i^{*1}(\theta_1)$$

$$- \lambda \sum_{k=1}^{n-1} B_{n-k,i}^{**1}(\theta, \theta_1) g_k$$

$$n \ge 2, \ 1 \le i \le c \qquad (24)$$

Probability Generating Functions:

The partial PGFs of the number of customers in the system, are defined to analyse the model.

$$P_0(z)=\sum_{n=0}^{m-1}P_{n,0}z^n \qquad U(z)=\sum_{n=m}^{N-1}U_n z^n$$

$$Q_j^*(z,\theta)=\sum_{n=0}^{\infty}Q_{n,j}^*(\theta)z^n$$

$$Q_j(z,0)=\sum_{n=0}^{\infty}Q_{n,j}(0)z^n \qquad 1\le j\le L$$

$$D^*(z,\theta)=\sum_{n=m}^{\infty}D_n^*(\theta)z^n \quad , \quad D(z,0)=\sum_{n=m}^{\infty}D_n(0)z^n$$

$$P^*(z,\theta)=\sum_{n=1}^{\infty}P_n^*(\theta)z^n \quad , \quad P(z,0)=\sum_{n=1}^{\infty}P_n(0)z^n$$

$$B^{**1}(z,\theta,\theta_1)=\sum_{n=1}^{\infty}B_n^{**1}(\theta,\theta_1)z^n$$

$$B^*(z,\theta,0)=\sum_{n=1}^{\infty}B_n^*(\theta,0)z^n$$

$$P_i^*(z,\theta)=\sum_{n=1}^{\infty}P_{n,i}^*(\theta)z^n \quad , \quad P_i(z,0)=\sum_{n=1}^{\infty}P_{n,i}(0)z^n$$

$$B_i^{**1}(z,\theta,\theta_1)=\sum_{n=1}^{\infty}B_{n,i}^{**1}(\theta,\theta_1)z^n$$

$$B_i^*(z,\theta,0)=\sum_{n=1}^{\infty}B_{n,i}^*(\theta,0)z^n\text{, i=1 to c}$$

By solving the system of equations, the partial probability generating functions of the system size probabilities at arbitrary epoch when the server is in different states are obtained through some algebraic manipulations and are listed below.

$$Q*(z,0)=\sum_{j=1}^{L}Q_j^*(z,0)=$$

$$=\overline{P}(0)\frac{(1-V^*(w_x(z))}{w_x(z)}\beta(z) \tag{25}$$

$$P_0(z)=\overline{P}(0)\psi(z) \tag{26}$$

$$U(z)=\overline{P}(0)\phi^{SR}(z) \tag{27}$$

$$D^*(z,0)=\frac{\overline{P}(0)(1-D^*(w_x(z))}{w_x(z)}[(V^*(w_x(z))-1)\beta(z)+1-\psi(z)w_x(z)] \tag{28}$$

$$P^*(z,0)=\frac{\overline{P}(0)z(S^*(h_a(w_x(z)))-1)(w_x(z))I_{SR}^{(m,N)}(z)]}{h_a(w_x(z))(z-H^*(w_x(z)))} \tag{29}$$

$$P_i^*(z,0)=$$

$$\frac{\overline{P}(0)r_i zS^*(h_a(w_x(z)))I_{SR}^{(m,N)}(z)w_x(z))(S_i^*(h_{ai}(w_x(z)))-1)}{ha_i(w_x(z))(z-H^*(w_x(z)))} \tag{30}$$

$$B^{**1}(z,0,0)=\frac{aP^*(z,0)(1-R^{*1}(w_x(z)))}{w_x(z))} \tag{31}$$

$$B_i^{**1}(z,0,0)=\frac{a_iP_i^*(z,0)(1-R_i^{*1}(w_x(z))}{w_x(z)} \tag{32}$$

(1≤i≤c)

where

(a) α_n denotes the probability that n customers arrive during vacation time V,and given by

$$\alpha_n=\int_0^{\infty}\sum_{k=0}^{n}\frac{e^{-\lambda t}(\lambda t)^k}{k!}g_n^{(k)}dV(t)$$

$$\sum_{n=0}^{\infty}\alpha_n z^n=V^*(w_x(z))$$ where $w_x(z)=\lambda(1-x(z))$ (Grossand Harris, [16]).

(0.1)

(b) $\beta(z)=\sum_{n=0}^{m-1}\frac{\beta_n z^n}{(1-\alpha_0)}$, $\beta_0=1-\alpha_0^{(L)}$

and

$$\beta_n=\sum_{j=1}^{n}\frac{\alpha_j\beta_{n-j}}{(1-\alpha_0)}-\alpha_n^{(L)}\, for 1\le n\le m-1 \tag{0.2}$$

(c) $\psi(z) = \sum_{n=0}^{m-1} \frac{\psi_n z^n}{\lambda} \qquad \psi_0 = \alpha_0^{(I)},$

$$\psi_n = \sum_{k=0}^{n} \alpha_k^{(I)} \pi_{n-k} \; (1 \le n \le m-1)$$

(0.3)

(d) $\pi_0 = 1$ and $\pi_n = \sum_{k=1}^{n} \pi_{n-k} g_k$

(0.4)

(e) $\phi^{SR}(z) = \sum_{n=m}^{N-1} \frac{\phi_n^{SR} z^n}{\lambda}$,

$\phi_n^{SR} = \sum_{r=m}^{n} S_r \sum_{k=0}^{n-r} h_k \pi_{n-r-k}$ where h_k denotes the probability that k customers arrive during setup period and

$$S_n = \sum_{k=0}^{m-1} \left[\frac{\alpha_{n-k} \beta_k}{1-\alpha_0} + \psi_k g_{n-k} \right]$$

(0.5)

(f) $ha\,(w_x(z)) = w_x(z) + a\,(1 - R^{*1}(w_x(z))$

(0.6)

(g) $h_{ai}\,(w_x(z)) = w_x(z) + a_i(1 - R_i^{*1}(w_x(z))$

(0.7)

(h) $I_{SR}^{(m,N)}(z) = D^*(w_x(z)) \left[\frac{1 - V^*(w_x(z))}{w_x(z)} \beta(z) \right]$

$$+ \frac{1 - D^*(w_x(z))}{w_x(z)} + \psi(z) D^*(w_x(z)) + \phi^{SR}(z)$$

(0.8)

(i) $H^*(w_x(z) =$

$$S^*(h_a(w_x(z))) \left[1 - r + \sum_{i=1}^{c} r_i S_i^*(h_{ai}(w_x(z))) \right]$$

(0.9)

To derive the total PGF of the system size system distribution,partial generating function of the size when the server is in idle state

$$P_I(z) = Q^*(z,0) + P_o(z) + D^*(z,0) + U(z)$$
$$= \overline{P}(0) I_{SR}^{(m,N)}(z)$$ and

Probability generating function of the system size when the server is in busy state(or) in breakdown state

$\mathrm{P_{comp}(z)} =$

$$= P^*(z,0) + \sum_{i=1}^{c} P_i^*(z,0) + B^*(z,0,0)$$

$$+ \sum_{i=1}^{c} B_i^{**1}(z,0,0)$$

$$P_{comp}(z) = \frac{z I_{SR}^{(m,N)}(z) \overline{P}(0) (H^*(w_x(z)) - 1)}{z - H^*(w_x(z))}$$ are obtained.

Thus the total PGF of the system size distribution is given by

$$P_{(m,N)}^{SR}(z) = P_I(z) + P_{comp}(z)$$

$$= \overline{P}(0) I_{SR}^{(m,N)}(z) \frac{(z-1) H^*(w_x(z))}{z - H^*(w_x(z))} \qquad (33)$$

The constant $\overline{\mathrm{P}}(0)$ can be calculated by using the normalizing condition $P_{(m,N)}^{SR}(1) = 1$ and found to be

$$\overline{P}(0) = \frac{1 - \rho_H}{I_{SR}^{(m,N)}(1)} \quad \text{, where } \rho_H = \lambda E(X) E(H)$$

(33.1)

$$I_{SR}^{(m,N)}(1) = E(D) + E(V) \sum_{n=0}^{m-1} \frac{\beta_n}{1-\alpha_0} + \sum_{n=0}^{m-1} \frac{\psi_n}{\lambda} + \sum_{n=m}^{N-1} \frac{\phi_n^{SR}}{\lambda}$$

(33.2)

and

$$E(H) = E(S)(1 + aE(R)) + \sum_{i=1}^{c} r_i E(S_i)[1 + a_i E(R_i)]$$

By substituting $\overline{\mathrm{P}}(0)$, the total PGF of the model can be written as

$$P^{SR}_{(m,N)}(z)$$

$$= \frac{(1-\rho_H)(z-1)H^*(w_x(z))}{z-H^*(w_x(z))}\frac{I^{(m,N)}_{SR}(z)}{I^{(m,N)}_{SR}(1)} \tag{33.3}$$

This verifies the stochastic decomposition property and it is shown that the system size distribution of the model get entwined into the distributions of two or more independent random variables.

III PERFORMANCE MEASURES

Steady State System Size Probabilities

The probabilities P_v, P_{set}, P_{build}, P_{dor}, P_I that the server is on vacation,setup state ,build up state, dormant state,idle state respectively are obtained by considering (25) to (28) at z=1,Then,

(i) $$P_v = \underset{z\to 1}{lt}\, Q^*(z,0) = \overline{P}(0)E(V)\sum_{n=0}^{m-1}\frac{\beta_n}{(1-\alpha_0)} \tag{34}$$

(ii) $$P_{set} = \underset{z\to 1}{lt}\, D^*(z,0) = \overline{P}(0)E(D) \tag{35}$$

(iii) $$P_{build} = \overline{P}(0)\sum_{n=0}^{m-1}\frac{\psi_n}{\lambda} \tag{36}$$

(iv) $$P_{dor} = \overline{P}(0)\sum_{n=m}^{N-1}\frac{\phi^{SR}_n}{\lambda} \tag{37}$$

(v) P_{Idle} = Probability that the server is idle in the system = $P_v + P_{set} + P_{build} + P_{dor} = \overline{P}(0)I^{(m,N)}_{SR}(1)$

Busy state Probabilities:

Let P_{FES} and P_{SOSi} denote the probability that the server is busy with FES and busy with i^th^ stage SOS. Then (29) and (30) give

(iv) $$P_{FES} = \underset{z\to 1}{lt}\, P^*(z,0)$$

$$= \overline{P}(0)I^{(m,N)}_{SR}(1)E(S)\frac{\lambda E(X)}{1-\rho_H} = \lambda E(X)E(S) \tag{38.1}$$

(v) $$P_{SOSi} = \underset{z\to 1}{lt}\, P_i^*(z,0)$$

$$= r_i E(S_i)\frac{\lambda E(X)}{1-\rho_H}\overline{P}(0)I^{SR}_{(m,N)}(1)$$

$$P_{SOS} = \sum_{i=1}^{c} P_{SOSi} = \lambda E(X)\sum_{i=1}^{c} r_i E(S_i) \tag{38.2}$$

(vi) $$P_{busy} = P_{FES} + P_{SOS} = \rho_{SOS} \tag{39}$$

where $\rho_{SOS} = \lambda E(X)\left(E(S) + \sum_{i=1}^{c} r_i E(S_i)\right)$

Breakdown State Probabilities

Equations (31) and (32) imply

(vii) P^{FES}_{break} = The probability that the server is in breakdown state and the customer is waiting in FES state= $\underset{z\to 1}{lt}\, B^{**}(z,0,0)$

$$= aE(R)P_{FES} \tag{40.1}$$

(viii) P^{SOS}_{break} = The probability that the server is in breakdown state and the customer isin SOS state = $\underset{z\to 1}{lt}\, B_i^{**}(z,0,0)$

$$= \sum_{i=1}^{c} a_i E(R_i)P_{SOSi} \tag{40.2}$$

Expected System Size

Let L_v,L_{set},L_{dor},L_{build} and L_{busy} denote the expected system size when the server is in vacation state, setup state, dormant state,build up state and in busy state respectively. Let L denote the mean system size of the model then

$L^{SR}_{(m,N)} = \frac{d}{dz}(P^{SR}_{(m,N)}(z))$ at z=1

$$= L_1^{SR} + \frac{I_{SR}^{(m,N)^1}(1)}{I_{SR}^{(m,N)}(1)} \text{ from}(33.3) \qquad (41)$$

where .

$$L_1^{SR} = \frac{\lambda E(X(X-1)E(H) + (\lambda E(X))^2 E(H^2)}{2(1-\rho_H)} + \rho_H$$

(41.1)

and

$$E(H^2) =$$

$$E(S^2)(1+aE(R))^2 + 2E(S)(1+aE(R))(\sum_{i=1}^{c} r_i E(S_i)(1+a_i E(R_i)))$$

$$+\sum_{i=1}^{c} r_i E(S_i^2)(1+a_i E(R_i))^2 + \sum_{i=1}^{c} r_i E(S_i) a_i E(R_i^2) + aE(S)E(R^2)$$

and ρ_H is given by (33.1)

and $I_{SR}^{(m.N)^1}(1) =$

$$\lambda E(X)\left[(\frac{E(V^2)}{2} + E(V)E(D))\sum_{n=0}^{m-1}\frac{\beta_n}{(1-\alpha_0)} + \frac{E(D^2)}{2} + \sum_{n=0}^{m-1}\frac{\psi_n}{\lambda}E(D)\right]$$

$$+E(V)\sum_{n=0}^{m-1}\frac{n\beta_n}{(1-\alpha_0)} + \sum_{n=0}^{m-1}\frac{n\psi_n}{\lambda} + \sum_{n=m}^{N-1}\frac{n\phi_n^{SR}}{\lambda}$$

(41.2)

By differentiating the suitable partial generating functions at z=1the mean system size when the system is in different states (i) to (vii)can be obtained respectively from (25,28,27,26,(29,30),31,32).

(i) $L_v = \left[\frac{d}{dz}Q^*(z,0)\right]_{z=1}$

$$= \left[(\frac{\lambda E(X)E(V^2)}{2})\sum_{n=0}^{m-1}\frac{\beta_n}{(1-\alpha_0)} + \sum_{n=0}^{m-1}\frac{n\beta_n}{(1-\alpha_0)}E(V)\right]\overline{P}(0)$$

(42)

(ii) $L_{set} = \left[\frac{d}{dz}D^*(z,0)\right]_{z=1}$

$$= \overline{P}(0)\lambda E(X)\left[\frac{E(D^2)}{2} + E(D)(E(V)\sum_{n=0}^{m-1}\frac{\beta_n}{(1-\alpha_0)} + \sum_{n=0}^{m-1}\frac{\psi_n}{\lambda})\right]$$

(43)

(iii) $\mathrm{L}_{\mathrm{dor}} = \overline{P}(0)\sum_{n=m}^{N-1}\frac{n\phi_n^{SR}}{\lambda}$ (44)

(iv) $L_{build} = \left[\frac{d}{dz}P_0(z)\right]_{z=1} = \overline{P}(0)\sum_{n=0}^{m-1}\frac{n\psi_n}{\lambda}$ (45)

(v) $\mathrm{L}_{\mathrm{busy}} = \frac{d}{dz}\left[P^*(z,0) + P_i^*(z,0)\right]_{z=1}$

$$= P_{busy} + \frac{\lambda E(X(X-1) + (\lambda E(X)^3 E(H^2)}{2(1-\rho_H)}\left[E(s) + \sum_{I=1}^{C} r_i E(S_i)\right]$$

$$+ \lambda E(X)\frac{I_{SR}^{(m,N)^1}(1)}{I_{SR}^{(m,N)}(1)}\left[E(s) + \sum_{I=1}^{C} r_i E(S_i)\right]$$

$$+(\lambda E(X))^2\left[\frac{E(S^2)}{2}(1+aE(R) + \sum_{i=1}^{c} r_i\frac{E(S_i^2)}{2}(1+a_i E(R_i)\right]$$

$$+(\lambda E(X))^2 E(S)(1+aE(R))\sum_{i=1}^{c} r_i E(S_i) \qquad (46)$$

Expected number of customers in the system corresponding to the breakdown states($\mathrm{L_{BR1}}$) and ($\mathrm{L_{BR2}}$) are calculated by using (31) and (32) as

(vi)$\mathrm{L_{BR1}} = \left[\frac{d}{dz}B^{**1}(z,0,0)\right]_{z=1}$

$$= P_{FES}\, a\lambda E(X)\frac{E(R^2)}{2} + aE(R)L_{FES}$$

(vii) $\mathrm{L_{BR2}} = \left[\frac{\mathrm{d}}{\mathrm{dz}}\mathrm{B}_i^{**1}(\mathrm{z},0,0)\right]_{\mathrm{z}=1}$

$$= P_{SOSi}\, a_i\lambda E(X)\frac{E(R_i^2)}{2} + a_i E(R_i)L_{iSOS}$$

where

$$L_{FES} = P_{FES} + \frac{\lambda E(X(X-1) + (\lambda E(X)^3 E(H^2)}{2(1-\rho_H)}[E(s)]$$

$$+\lambda E(X)\left[\frac{I_{SR}^{(m,N)^{1}}(1)}{I_{SR}^{(m,N)}(1)}E(s)+\lambda E(X)\left(\frac{E(S^2)}{2}(1+aE(R))\right)\right]$$

$$L_{iSOS}=$$

$$P_{SOSi}+\frac{\lambda E(X(X-1)+(\lambda E(X))^3E(H^2)}{2(1-\rho_H)}[r_iE(S_i)]$$

$$+\lambda E(X)\left[\frac{I_{SR}^{(m,N)^{1}}(1)}{I_{SR}^{(m,N)}(1)}r_iE(S_i)\right]$$

$$+(\lambda E(X))^2r_i\left[(1+aE(R)E(S)E(S_i)+\frac{E(S_i^2)}{2}(1+a_iE(R_i))\right]$$

IV OPTIMAL MANAGEMENT POLICY

Using a procedure followed in the Ph.D. theses of Jemila Parveen.M.[18] and Julia Rose Mary,K[19] the optimam values (m*,N*) of (m,N) are derived so that the system can be run at minimum cost. The cost elements Cy (startup cost per cycle), Cset (setupcost), Cdor (standby cost), Cbusy (operating cost), Cbr (breakdown cost),Cv(reward cost),Cbuild (buildup cost) and Ch (holding cost per customer) per unit time are introduced to get the minimum total average cost of the model

$$T_c^{(S,R)}(m,N)=$$

$$\frac{C_y}{E(cycle)}+C_hL_{(m,N)}^{SR}+C_{build}P_{build}+C_{set}P_{set}+C_{dor}P_{dor}+C_{br}P_{br}+C_{busy}P_{busy}-C_vP_v \tag{47}$$

which can be re-written using (34) to (41) as

$$T_c^{(S,R)}(m,N)=\overline{A}^{SR}+\frac{1}{I_{(m,N)}^{SR}(1)}\left[A^{SR}+Z^{SR}(m)+C_{dor}(1-\rho_H)\frac{1}{\lambda}\left(\sum_{n=m}^{N-1}\phi_n^{SR}\right)+\frac{C_h}{\lambda}\left(\sum_{n=m}^{N-1}n\phi_n^{SR}\right)\right]$$

where

$$\overline{A}^{SR}=C_{busy}P_{busy}+C_hL_1^{SR}+C_{br}P_{br}$$

$$A^{SR}=(1-\rho_H)(C_y+C_{set}E(D))+C_h\frac{\lambda E(X)}{2}E(D^2)$$

$$Z^{SR}(m)=\frac{C_h}{\lambda}\sum_{n=0}^{m-1}n\psi_n+\frac{1}{\lambda}\sum_{n=0}^{m-1}\psi_n[C_h\lambda E(X)E(D)+C_{build}(1-\rho_H)]$$

$$+\sum_{n=0}^{m-1}\frac{\beta_n}{1-\alpha_0}\left[C_hE(V)+C_h\lambda E(X)\left[\frac{E(V^2)}{2}+E(V)E(D)\right]-C_vE(V)(1-\rho_H)\right]$$

and $I_{(m,N)}^{SR}(1)$ and L_1^{SR} are as given in (33.2) and (41.1) respectively.

By calculation,

$$T_c^{(S,R)}(m,k+1)-T_c^{(S,R)}(m,k)=$$

$$\frac{\phi_k^{SR}}{\lambda I_{(m,k+1)}^{SR}(1)I_{(m,k)}^{SR}(1)}H^{SR}(m,k) \tag{47.1}$$

where

$$H^{SR}(m,k)=C_h\left[kI_{(m,k)}^{SR}(1)+\sum_{n=m}^{k-1}\frac{(k-n)}{\lambda}\phi_n^{SR}\right]$$

$$+\left[C_{dor}(1-\rho_H)I_{(m,k)}^{SR}(1)\right]-\left[A^{SR}+Z^{SR}(m)\right] \tag{47.2}$$

Thus the value of first k,for which $H^{SR}(m,k)>0$ decides the conditional optimal policy and hence the optimum value (m*,N*) of the model.

V. PARTICULAR CASES:

The steady state results of some other (m,N) policy queueing models are deduced from the existing model as special cases.The expressions other than that described in(i) to (iv) below are as same as that of the proposed model.

(i) Single vacation model

When L=1 , the PGF $P_{(m,N)}^S(z)$ for the single vacation model is given by,

$$P_{(m,N)}^S(z)=\frac{(1-\rho_H)(z-1)H^*(w_x(z))}{z-H^*(w_x(z))}\frac{I_S^{(m,N)}(z)}{I_S^{(m,N)}(1)}$$

where

$$I_S^{(m,N)}(z)=\frac{1-V^*(w_x(z))D^*(w_x(z))}{w_x(z)}+\psi(z)D^*(w_x(z))+\sum_{n=m}^{N-1}\frac{\phi_n^sz^n}{\lambda}$$

$$\phi_n^s=\sum_{k=m}^{n}\xi_k\sum_{i=0}^{n-k}h_i\pi_{n-k-i},\quad \xi_k=\alpha_n+\sum_{i=0}^{m-1}\psi_ig_{n-i}$$

(ii) Multiple vacation model

When $L \to \infty$, the PGF $P^{R}_{(m,N)}(z)$ for the multiple vacation model is given by,

$$P^{R}_{(m,N)}(z) = \frac{(1-\rho_H)(z-1)H^{*}(w_x(z))}{z-H^{*}(w_x(z))}\frac{I_R^{(m,N)}(z)}{I_R^{(m,N)}(1)}$$

where $S_k = \sum_{j=0}^{m-1}\frac{\alpha_{k-j}\beta_j}{1-\alpha_0}; \phi_n^R = \sum_{k=m}^{n} S_k \sum_{i=0}^{n-k} h_i \pi_{n-k-i};$

$$I_R^{(m,N)}(z) = \left[D^{*}(w_x(z))\frac{1-V^{*}(w_x(z))}{w_x(z)}\beta(z) + \frac{1-D^{*}(w_x(z))}{w_x(z)} + \sum_{n=m}^{N-1}\frac{\phi_n^R z^n}{\lambda} \right]$$

(iii) When c=1 the corresponding results of the SOS model can be derived.

(iv) The corresponding results of the model in which each customer undergoes both services one after the other are obtained when c=1 and r_1=1.

(v) If r=0, the corresponding results of the single service queueing model can be obtained.

(vi) The results for the corresponding non-vacation(without setup) models,can be obtained by taking E(V)=E(D)=0 .

VI. NUMERICAL ANALYSIS

In this section the numerical values of the system size probabilities and mean queue length are analyzed through numerical values and the effects of various system parameters on different system performance measures are studied. For numerical computations,it is assumed that the second multi optional service consists of three types of (c=3)service facility.The distributions considered for the random variables are given by:the service time of FES (S),setup time (D), and repair time in FES (R) are assumed to follow Erlang (2,3 and 2) types respectively with parameters $(\mu, \gamma \; and \beta)$ .The SOS service times S_i,i=1,2,3 follow two stage hyper exponential and the vacation time Repair time R_i,i=1,2,3 and the life time of the server in FES(SOS) follow Gamma 3 type, deterministic and exponential distributions with parameters $(\mu_i, \eta, \beta_i, a(a_i))$ respectively. Batch size assumed to follow Geometric distribution $(1-p)p^{k-1}$.

Table I gives the mean queue lengths for the present model , for different number of vacations J and ρ .The table values and the graphical representation(Fig.1.) show that as J or ρ increases the system size L also increases.The queue length corresponding to J=1(J tends to ∞) give the queue length of the corresponding single (multiple) vacation model.The parametric values used to obtain tables (I),(II) and Figures (1),(2)and (3) are same as mentioned in table(I).

TABLE I.

(m,N,a,a_iE(S),E(S_i),E(D),E(V),E(X),E(R), E(Ri))i=1,2,3(2,5,3.3,2,.7,.5,1,4,2.5,1.7,1.4)

J ρ_H	1	5	10	15	20
0.2	1.35	1.38	1.41	1.44	1.46
0.4	2.55	2.75	2.87	2.92	2.95
0.6	4.86	5.26	5.41	5.46	5.47
0.8	13.64	14.25	14.40	14.43	14.44
0.9	24.64	25.31	25.46	25.48	25.49

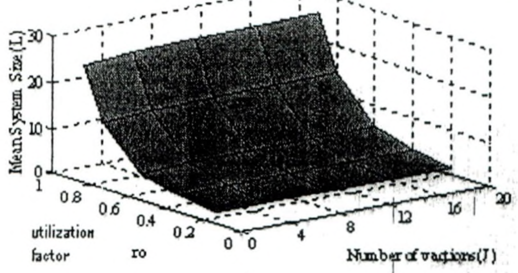

Fig.1.

Table(II) gives the system size probabilities when the system is in different states for the set of parameters in Table(I). The table values show that as ρ increases the probability that the server is busy increases and the probability that the server is in idle state and build up state decreases.

TABLE II:(J=5)

ρ_H	**P_{idle}**	**P_{busy}**	**P_{bu}**
0.2	.806	.123	.344
0.4	.613	.247	.153

0.6	.419	.370	.064
0.8	.187	.518	.016

The optimal value set (m*,N*,T_c(m*,N*)) corresponding to the cost structure(C_h,C_{set},C_{dor},C_{busy},C_y,C_v,C_{br})(10,50, 20,1500,1000,8,20)with respect to the parametric values of Table I with J=1 is given in Fig.2.

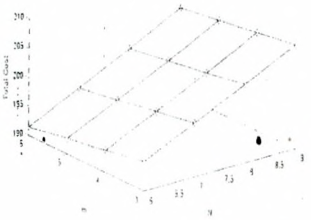

Fig.2.

The graphical representation of mean queue size of the model when the server is busy, is depicted in Fig.3. for J= 5 .The graph shows that as ρ increases L_{idle},$L_{buildup}$ decrease and $L_{completion}$ and mean system length (L)increase.

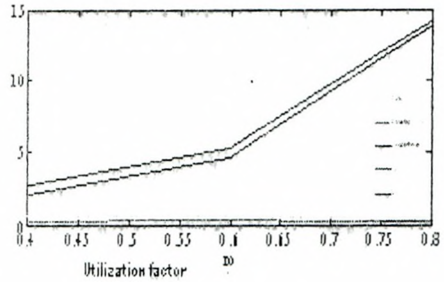

Fig.3.

VII. CONCLUSION:

The author has made an attempt to analyse the (m,N) policy of more general batch arrival queueing model with server vacation form which the results of different types queueingmodels(more than 6 models)can be deduced.In particular the results of the corresponding classical single and multiple queueing models are deduced as a special case by letting L=1 and allowing L→∞ respectively. It is verified that the PGF($\pi(z)$) of the system size at departure epoch and at arbitrary epoch ($P^{SR}_{(m,N)}(z)$)is given by

$$\pi(z) = \frac{1 - X(z)}{E(X)(1 - z)} \left(P^{SR}_{(m,N)}(z) \right).$$

Stochastic decomposition property is verified.With the derived results given in the present paper the authors have also derived a procedure to obtain the optimal values of m,N so that the system can be run at a minimum cost.

REFERENCES:

[1].Yadin, M. and Naor, P. (1963), "Queueing system with a removable service station", Oper. Res. Q., 14, 393-405.

[2].Heyman, D.P. (1968), "Optimal operating policies for M / G / 1 queueing system", Oper. Res., 16, 362-382.

[3]. Wang, K.H. and Ke, J.C. (2000), "A recursive method to the optimal control of an M / G / 1 queueing system with finite capacity and infinite capacity", Appl. Math. Modell., 24, 899-914.

[4] Bell, C.G. (1971), "Characterization and computation of optimal policies for operating an M / G / 1 queueing system with removable server", Operat. Res., 19, 208-218.

[5] Teghem, J. (1987), "Optimal control of a removable server in an M / G / 1 queue with finite capacity", Eur. J. Opl. Res., 31, 358-367.

[6].Lee, H.W. and Park, J.O. (1997), "Optimal Strategy in N-policy production system with early setup", J. Oper. Res. Soc., 48, 306-313.

[7].Lee, H.W. Park, J.O. Kim, B.K., Yoon, S.H. Ahn, B.Y. and Park, N.I. (1998), "Queue length and waiting time analysis of batch arrival queue with bilevel control", Computers Oper. Res., 25(3), 191-205.

[8].Lee, H.W., Park, J.O. and Jeon, J.W. (2003), "Queue length analysis of batch arrival queues under bilevel threshold control with early set-up", Information Journal of Systems Science, 34(3), 195-204.

[9]Ke, J.C.(2004a), "Bilevel control for batch arrival queues with an early startup and unreliable server", Appl. Math. Modelling, 28, 469-485.

[10].Ke, J.C., Wu, C.H. and Zhang, G. (2010), "Recent developments in vacation queueingmodels :

A short survey", Int. Journal of Operations Research, 7 (4), 3-8.

[11].Wang, K.H. (1995), "Optimal operation of a Markovianqueueing system with a removable and non reliable server", Microelectronics Reliability, 35, 1131- 1136.

[12]. Wang, K.H. and Ke, J.C. (2002), "Control policies of an M / G / 1 queueing system with a removable and non reliable server", Int. Trans. Oper.Res., 9, 195-212.

[13].Wang, K.H. and Hisch, W.F. (1995), "Optimal control of a removable and non-reliable server in a Markovianqueueing system with finite capacity", Microelectronics Reliability, 35 (2), 189-196.

[14].Jau-Chuan KE Kai-Bin Huang and Wen Lea Pearn.(2010)." Randomized policy of a poisson input queue with J vacations", JSystSciSystEngg ,19(1):050-071

[15].Jau-Chuan KE Kai-Bin Huang and Wen Lea Pearn.(2011)." A performance measures and randomized optimization and an unreliable server MX/G/1 vacation system", Appl. MathComputation , 217, 8277-8290.

[16].Gross, D. and Harris, C.M. (1998), "Fundamentals of queueing theory", John Wiley, New York, (Second edition).

[17].Cox, D.R. (1955), "The analysis of Non-Markovian stochastic processes by the inclusion of supplementary variables", Proceedings of the Cambridge Philosophical Society, 51, 433-441.

[18].JemilaParveen.M.(2012)," Analysis of some Non-Markovian bulk queueing models under double threshold policy, different heterogeneous services, server vacations ,breakdowns and repair facilities",Ph.D. Thesis awarded by Avinashilingam Deemed University, Coimbatore.

[19] Julia Rose Mary, K. (2011), "Batch arrival queueing systems with bilevel control policy, vacations, breakdowns and heterogeneous service facilities", Ph.D. Thesis awarded by Avinashilingam Deemed University, Coimbatore.