Mmmm.

Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD
Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore-641 043, Tamil Nadu, India
Continuous Internal Assessment Test I – August 2024

Semester - III

Class: II PG | **Time: 2 Hours**

Branch: Mathematics | **Max. Marks: 60**

23MMAC15 - Differential Geometry

Course Outcomes:

CO1: Calculate the curvature and torsion of a curve.
CO2: Find the osculating surface and osculating curve at any point of a given curve.
CO3: Calculate the first and the second fundamental forms of surface.
CO4: Solve the problems related to Gaussian curvature, the mean curvature, the curvature lines, the asymptotic lines.
CO5: Identify the appropriate approach to solve the problems on geodesics of a surface.

PART-A

Choose the correct answer **6x1=6**

1. What is a space curve? CO1K1
A) A curve that exists only in a two-dimensional plane
B) A curve that can be represented by a single equation in Cartesian coordinates
C) A curve that exists in three-dimensional space and is represented parametrically
D) A curve that cannot be represented parametrically

2. What is the arc-length of a space curve? CO1K2
A) The distance between two points on the curve measured along the curve
B) The distance between two points on the curve measured in a straight line
C) The length of the curve as seen from a fixed point
D) The area under the curve

3. What is the osculating plane of a space curve? CO2K1
A) The plane that contains the normal vector and the binormal vector
B) The plane that contains the tangent vector and the normal vector
C) The plane that is perpendicular to the curve at the point
D) The plane that is parallel to the curve

4. The arc rate at which the tangent changes direction as the point P moves along the curve is called ____________ CO2K2
A) normal vector of the curve
B) perpendicular vector of the curve
C) curvature vector of the curve
D) tangent vector of the curve

5. A necessary and sufficient condition that a curve is a plane is ____________ CO2K1
A) $[\dot{r},\ddot{r},\dddot{r}]=\kappa$ B) $[\dot{r},\ddot{r},\dddot{r}]=\tau$ C) $[\dot{r},\ddot{r},\dddot{r}]=0$ D) $[\dot{r},\ddot{r},\dddot{r}]=\kappa^2\tau$

6. The osculating plane at any point P has _______ point contact with the curve at P CO2K1
A) one B) zero C) three D) two

PART-B

Answer ALL questions **3x6=18**

7. a. Prove that the following are the two equivalent representations of a circular helix. CO1K3

(i) $R_1(u)=(a\cos u, a\sin u, bu), u\in I_1=[0,\pi)$

(ii) $R_2(v)=\left(a\,\frac{1-v^2}{1+v^2},\ \frac{2av}{1+v^2}\right), v\in I_2=[0,\infty)$

(or)

7. b. Find the arc length of one complete turn of the circular helix CO1K1

$$r=(a\cos u, a\sin u, bu), -\infty<u<\infty$$

where $a>0$ and obtain the equation of the helix with s as parameter.

8. a. Find the equation of the osculating plane at a point u of the circular helix CO2K3

$$r=(a\cos u, a\sin u, bu)$$

(or)

8. b. Prove that a necessary and sufficient condition for a curve to be a straight line is that $\kappa=0$ at all points of the curve. CO2K3

9. a. Let the curve be of class $m\geq 4$. At a point P on the curve, let the coordinate axes, *ox, oy, oz* be taken along *t, n, b*. If X, Y, Z are the coordinates of the neighbouring point Q on the curve, then prove that CO2K1

$$X=s-\frac{\kappa^2 s^3}{6}-\frac{\kappa\kappa'}{8}s^4+o(s^4)$$

$$Y=\frac{\kappa}{2}s^2+\frac{\kappa'}{6}s^3+\frac{\kappa''-\kappa\tau^2-\kappa^3}{24}s^4+o(s^4)$$

$$Z=\frac{\kappa\tau}{6}s^3+\frac{2\kappa'\tau+\kappa\tau'}{24}s^4+o(s^4)\ as\ s\to 0$$

(or)

9. b. Prove that the radius of the osculating circle at P is the reciprocal of curvature of the curve at P and the position vector of its centre of the osculating circle is $c=r+\rho n$ where $\rho=\frac{1}{\kappa}$. CO2K4

PART-C

Answer ALL questions **3x12=36**

10. a. If $R=R(u)$ is the parametric representation of a curve where $u\in[a,b]$, then show that the length of the curve

$$s=S(u)=\int_a^u|\dot{R}(u)|\,du$$ CO1K2

(or)

10. b. If γ is a curve of class $m\geq 2$ with arc length s as parameter. Prove the following:

If the point P on γ has parameter zero, then equation of the osculating plane is CO1K3

$[R-r(0), r'(0), r''(0)]=0$ where $r''\neq 0$

If $r''(0)=0$, and if it is assumed that the curve γ is analytic, then the equation of the plane at an inflexion point is $[R-r(0), r'(0), r^{(k)}(0)]=0$. Prove it.

11. a. If (t, n, b) is the moving orthogonal triad of unit vectors at a point P on a space curve γ, then derive

(i) $\frac{dt}{ds}=\kappa n$ (ii) $\frac{dn}{ds}=\tau b-\kappa t$ (iii) $\frac{db}{ds}=-\tau n$. CO2K3

(or)

11\. b. If $r=r(u)$ is the equation of the curve with parameter u, then prove that CO2K3

(i) $\kappa=\frac{|\dot{r}\times\ddot{r}|}{|\dot{r}|^3}$ (ii) $\tau=\frac{[\dot{r},\ddot{r},\dddot{r}]}{|\dot{r}\times\ddot{r}|^2}$

12\. a. Find the curvature and torsion of the curve of intersection of the quadratic surfaces

$$ax^2+by^2+cz^2=1, \quad a'x^2+b'y^2+c'z^2=1$$ CO2K4

(or)

12\. b. If $r = r(s)$ is the given curve γ, then show that the centre C and radius R of spherical curvature at a point P on γ are given by CO3K4

$$C=r+\rho n+\sigma\rho' b\ , \ R=\sqrt{\rho^2+\sigma^2\rho'^2}$$

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