Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination –August 2020
VI Semester

Class : III UG **Time : 2 Hours**
Major : Special Education and Mathematics **Max. Marks : 50**

15BSMC14 Real Analysis-II

Part A **10 x 1 = 10**
Choose the Correct Answer

1. The image of a closed set under a continuous mapping is
   a.necessarily closed b. necessarily a single point
   c. not necessarily closed d. not necessarily a single point

2. The continuous image of a connected set is
   a. connected b. disconnected
   c. need not be connected d. none of these

3. The function f defined by $f(x) = \frac{x}{|x|}$ if $x \neq 0, f(0) = A$ has …………… at 0.
   a. continuous b. jump discontinuity
   c. removable jump discontinuity d. irremovable discontinuity

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5. The limit, $\lim_{x \to c} \frac{f(x)-f(c)}{x-c}$, denoted by $f'(c)$ is called the …… of f at c.
   a. integrability function b. derivative
   c. continuity d. uniform continuity
6. The function f defined on a closed interval S has a derivative at an interior point $e$iff
   a. $f_+^{'}(c) = f_-^{'}(c) = f'(c)$ b. $f_-^{'}(c) = f'(c)$
   c. $f_+^{'}(c) = f'(c)$ d. $f_+^{'}(c) = f_-^{'}(c) \neq f'(c)$
7. The function $f(x) = |x|$has a minimum at
   a. $x = -\infty$ b. $x = 0$
   c. $x = 1$ d. $x = +\infty$

8. If f and g are continuous on $[a, b]$ and have equal finite derivative in (a, b) then$f$ – g is …….. on$[a, b]$.
   a. constant b. increasing
   c. decreasing d. monotonic

9. If f is $a\ function\ of$ bounded variation on $[a, b]$, then the number V_f*is*
   a. countable b. finite
   c. infinite d. countably infinit

10. $V_f(a, b) = 0$if and only if f is ……….. on $[a, b]$.
    a. increasing b. decreasing
    c. monotonic d. constant

Part B $3 \times 6 = 18$

Answer any **Three** questions

Each answer should not exceed 400 words or two pages

11. Let $f: S \to R^k$ be a function from a metric space S to Euclidean space R^k.
    If f is continuous on a compact subset X of S, then prove that f is bounded on X.

12. State and prove intermediate value theorem for real continuous functions.

13. Let f be strictly increasing on a set S in R. Then prove that f^{-1} exists and is strictly increasing on $f(S)$.

14. Let f be strictly increasing and continuous on a compact interval $[a, b]$.
    Then prove that f^{-1} is continuous and strictly increasing on the interval $[f(a), f(b)]$.

15. If f is defined on (a, b) and differentiable at a point c in (a, b), then prove that there is a function f^* which is continuous at c and which satisfies the equation $f(x) - f(c) = (x - c)f^*(x)$, for all x in (a, b), with $f^*(c) = f'(c)$. Show abo that, if there is a function f^*, continuous at e, which satisfies the above equation, then f is differentiable at c and $f'(c) = f^*(c)$.

16. If f is differentiable at c, then prove that f is continuous at c.

17. Let f be defined on an open interval (a, b) and assume that for some c in (a, b) we have $f'(c) > 0$ or $f'(c) = +\infty$. Prove that there is a 1-ball $B(c) \subseteq (a, b)$ in which $f(x) > f(c)$ if $x > c$, and $f(x) < f(c)$ if $x < c$.

18. State and prove Rolle's theorem.

19. If f is monotonic on $[a, b]$, then prove that the set of discontinuous of f is countable.

20. If f is monotonic on $[a, b]$, then prove that f is of bounded variation on $[a, b]$.

Part C $2 \times 11 = 22$

Answer any **Two** questions

Each answer should not exceed 800 words or four pages

21. Let $S \to T$ be a function from one metric space (S, d_s) to another (T, d_T).
    Then prove that f is continuous on S if and only if, for every open set Y in T, the inverse image $f^{-1}(Y)$ is open in S.

22. State the prove Bolzano's theorem.

23. State and prove Heine theorem for uniform continuity.

24. State and prove fixed point theorem for contractions.

25. Assume f and g are defined on (a, b) and differentiable at c, then prove that $f + g, f - g, f.g$ and f/g are differentiable at c.

26. State and prove chain rule for differentiating composite functions.

27. (i) Let f be defined on an open interval (a, b) and assume that f has a local maximum or a local minimum at an interior point c of (a, b). If f has a derivative at c,then prove that $f'(c)$ must be 0.

(ii) State and prove generalized mean-value theorem.

28. State and prove Intermediate value theorem for derivatives.

29. Let f and g be two functions having finite n^{th} derivatives $f^{(n)}$and $g^{(n)}$ in an open interval (a, b) and continuous $(n - 1)st$ derivatives in the closed interval $[a, b]$. Assume that $c \in [a, b]$. Then prove that, for every $x$in $[a, b], x \neq c$, there exists a point x, interior to the interval joining x and c such that $\left[f(x) - \sum_{k=0}^{n-1} \frac{f^{(k)}(c)}{K!}\right] g^{(n)}(x_i) = f^{(n)}(x_1)\left[g(x) - \sum_{K=0}^{n-1} \frac{g^{(K)}(c)}{K!}(x - c)^K\right]$.

30. State and prove additive property of total variation.

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