CHAPTER 3

CHAPTER 3

τ * GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

In this chapter, τ *-generalized closed sets due to Pushpalatha et.al [39], τ *g*- closed sets and $\tau * \alpha$ -generalized closed sets in topological spaces are discussed.

SECTION 3.1

3.1 τ *- generalized closed sets in topological space

In this section, the concept of τ * -g-closed sets in topological spaces to Pushpalatha et.al [39] is studied. Properties and characterizations of τ *-g-closed sets are discussed. The relations between τ *-g-closed (open) sets with various closed (open) sets are analyzed.

Definition 3.1.1

For a subset A of a topological space (X, τ), **the α -closure of A** (briefly cl_{α}(A)) is defined as the intersection of all α -closed sets containing A.

Definition 3.1.2

For a subset A of a topological space (X, τ), the **semi-closure** of A (briefly scl(A)) is defined as the intersection of all semi-closed sets containing A.

Definition 3.1.3

For a subset A of a topological space (X, τ), the **semi - preclosure** of A (briefly spcl(A)) is defined as the intersection of all semi-preclosed sets containing A.

Definition 3.1.4

For a subset A of a topological space (X, τ), the **generalized closure operator cl*(A)** is defined as the intersection of all g-closed sets containing A.

Definition 3.1.5

Let (X, τ) be a topological space, the topology τ^* is defined by $\tau^* = \{A: cl^*(A^c) = A^c\}$, where cl*(A) is the intersection of all g-closed sets containing A.

Definition 3.1.6

A subset A of a topological space X is called a **τ^*-generalized closed** set (briefly τ^*-g-closed) if $cl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is τ^*-open. The complement of a τ^*-generalized closed set is called τ^*-generalized open (τ^*-g-open) in (X, τ).

Theorem 3.1.7

Every closed set in X is τ^*-g-closed.

Proof

Let A be a closed set. Let $A \subseteq G$, where G is τ^*-open. Since A is closed, $cl(A) = A \subseteq G$. But $cl^*(A) \subseteq cl(A)$. Thus $cl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is τ^*-open. Therefore A is τ^*-g-closed.

The converse of the theorem (3.1.7) need not be true as seen from the following example.

Example 3.1.8

Consider the topological space $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}\}$. Then the set $\{a\}$ is τ^*-closed but not closed .

Theorem 3.1.9

Every τ*-closed set in X is τ*-g-closed

Proof

Let A be a τ*-closed set. Let $A \subseteq G$ where G is τ*-open. Since A is τ*-closed, cl*(A) = A $\subseteq$ G .Thus, cl*(A) $\subseteq$G whenever $A \subseteq G$ and G is τ*-open. Therefore A is τ*-g-closed.

Theorem 3.1.10

Every g-closed set in X is a τ*-g-closed set but not conversely.

Proof

Let A be a g - closed set . Assume that A $\subseteq$ G, G is τ*-open in (X, τ).Since A is g-closed A = cl*(A).Therefore cl*(A)$\subseteq$G . Hence A is τ*-g-closed.

The converse of the theorem (3.1.10) need not be true as seen from the following example.

Example 3.1.11 .

Consider the topological space X= {a, b, c} with topology ={X, ϕ, {a}}.Then the set {a} is τ*-g-closed but not g-closed .

Remark 3.1.12

The following example shows that τ*-g-closed sets are independent from sp - closed set, sg - closed set, α - closed set, pre closed set , gs - closed set, gsp-closed set, α g-closed set and gα -closed set.

Example 3.1.13

Let X= {a,b,c} be a topological space

(i) Consider the topology τ={X, ϕ, {a}}.Then the sets {a},{a,b} and {a,c} and τ *-g-closed but not sp-closed.

(ii) Consider the topology τ={X,ϕ,{a,b}}.Then the sets {a} and {b} are sp-closed but not τ *-g-closed.

(iii) Consider the topology τ={X,ϕ }.Then the sets {a} ,{b},{c},{a,b} ,{b,c}and {a,c} are τ *-g-closed but not sg-closed.

(iv) Consider the topology τ={X,ϕ,{a},{b},{a,b}}.Then the sets {a} and {b} are sg-closed but not τ *-g-closed.

(v) Consider the topology τ={X,ϕ,{a}}.Then the sets {a} ,{b},{c},{a,b} and {a,c} are τ *-g-closed but not α -closed.

Theorem 3.1.14

For any two sets A and B cl*(A$\cup$B) = cl*(A) $\cup$ cl*(B)

Proof

Since A $\subseteq$ A$\cup$B , cl*(A) $\subseteq$ cl*(A$\cup$B). Since B $\subseteq$ A $\cup$ B, cl*(B) $\subseteq$ cl*(A$\cup$B) .Therefore cl*(A) $\cup$ cl*(B) $\subseteq$ cl*(A$\cup$B).Again A$\subseteq$ cl*(A) and B $\subseteq$ cl*(B) implies A$\cup$B $\subseteq$ cl*(A) $\cup$ cl*(B). Thus cl*(A) $\cup$ cl*(B) is an intersection of g-closed sets containing A$\cup$B. Since cl*(A$\cup$B) the intersection of all g-closed sets containing A$\cup$B, cl*(A$\cup$B) $\subseteq$ cl*(A) $\cup$ cl*(B). Thus cl* (A$\cup$B) = cl*(A) $\cup$ cl*(B).

Theorem 3.1.15

Union of two τ * -g-closed sets in X is a τ * -g-closed set in X

Proof

Let A and B be two τ * -g-closed sets. Let A$\cup$B$\subseteq$G, where G is τ * -open .Since A and B are τ * -g-closed sets, cl*(A) $\cup$ cl*(B) $\subseteq$ G. But by previous Theorem 3.1.14 cl*(A) $\cup$ cl*(B) = cl* (A$\cup$B).Therefore cl*(A$\cup$B) $\subseteq$ G. Hence A$\cup$B is a τ * -g-closed set

Theorem 3.1.16

A subset A of X is a τ * -g-closed sets if and only if cl*(A) –A contains no nonempty τ * -closed set in X

Proof

Let A be a τ * -g-closed set. Suppose that F is a nonempty τ *-closed subset of cl*(A) –A. Now F $\subseteq$ cl*(A) – A. Thus F $\subseteq$ cl*(A) $\cap A^C$. Therefore F $\subseteq A^C$ .Since F^C is a τ *-open set and A is a τ *-g-closed set. cl*(A) $\subseteq F^C$ .That is F $\subseteq (cl^*(A))^C$. Hence F $\subseteq cl^*(A) \cap (cl^*(A))^C = \phi$ .That is F=ϕ a contradiction. Thus cl*(A) –A contain no nonempty τ *-closed set in X. Conversely assume that cl*(A) –A contains no nonempty τ *-closed set. Let A $\subseteq$ G, where G is τ *-open. Suppose that cl*(A) is not contained in G, then cl*(A)$\cap G^C$ is a nonempty τ *-closed set of cl*(A) –A which is a contradiction. Therefore cl* (A) $\subseteq$ G and hence A is τ *-g-closed set.

Corollary 3.1.17

A subset A of X is a τ *-g-closed set if and only if cl*(A) –A contain no nonempty closed set in X.

Proof

The proof follows from the Theorem (3.1.16) and the fact that every closed set in X is τ *-closed set is τ *-g-closed set in X.

Corollary 3.1.18

A subset A of X is τ *-g-closed set if and only if cl*(A) –A contain no nonempty open set in X.

Proof

The proof follows from the Theorem 3.1.16 and the fact that every open set is τ *-open set.

Theorem 3.1.19

If a subset A of X is τ *-g-closed set and A $\subseteq$B $\subseteq$ cl*(A).Then B is τ *-g-closed

Proof

Let A be a τ *-g-closed set such that A $\subseteq$B $\subseteq$ cl*(A).Let U be a τ *-open set of X such that B $\subseteq$ U. Since A is τ *-g-closed, then cl*(A) $\subseteq$U. Now cl*(A) $\subseteq$ cl*(B) $\subseteq$cl*(cl*(A))= cl*(A) $\subseteq$U .That is cl*(B) $\subseteq$U,U is τ *-open. Therefore B isτ *-g-closed set in X.

Theorem 3.1.20

Let A be a τ *-g-closed in X. Then A is τ *-closed if and only if cl*(A) –A is τ *-open.

Proof

Suppose A is τ * -closed in X. Then cl*(A) =A and so cl*(A) –A=ϕ, which is τ * -open in X. Conversely, suppose that cl*(A) –A is τ *-open in X. Since A is τ *-g-closed. By Theorem 3.1.16 cl*(A) –A no nonempty contains τ *-g-closed sets in X. Then cl*(A) –A= ϕ.Hence A isτ *-g-closed.

Theorem 3.1.21

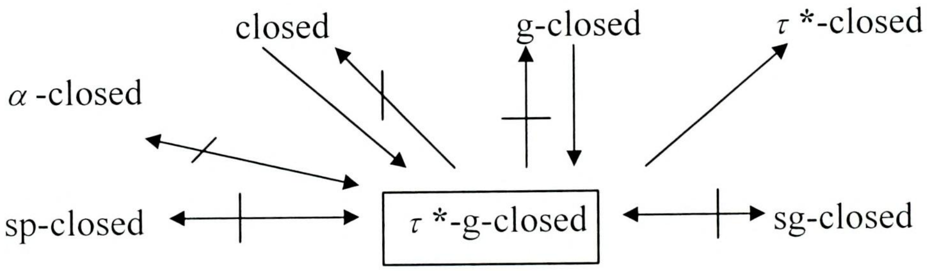
For x$\in$X, the set X-{x} is τ *-g-closed or τ *-open

Proof

Suppose X-{x} is not τ *-open. Then X is the only τ *-open set containing X-{x}.This implies cl*(X-{x}) $\subseteq$X. Hence X-{x} is a τ *-g-closed in X.

Remark 3.1.2.15

From the above discussion, the following implications are obtained.



SECTION 3.2

τ *g* -CLOSED SETS IN TOPOLOGICAL SPACES

In this section, the concept of τ *g* -closed sets in topological spaces and its basic properties are studied.

Definition 3.2.1

A subset A of a topological space (X,τ) is said to be τ *g* -closed set, if cl* (A)$\subseteq$G whenever A$\subseteq$G and G is τ *g – open .

Notation 3.2.2

The collection of all τ *g*-closed sets of (X, τ) is denoted by τ *G*C(X, τ) .

Theorem 3.2.3

Every closed set is a τ *g*-closed set.

Proof

Let A be a closed set .Let A$\subseteq$G, where G is τ *g – open .Since A is closed, cl(A) =A$\subseteq$G. But cl*(A) $\subseteq$ cl(A).Thus cl*(A) $\subseteq$G whenever A$\subseteq$ G and G is τ *g- open .Hence A is τ *g*- closed set.

The converse of the theorem (3.2.3) need not be as seen from the following example.

Example 3.2.4

Let X= {a, b,c} and τ={ϕ,X,{a},{ a,c}}. Let A= {a,b}. A is a τ *g*-closed set but not a closed set.

Theorem 3.2.5

Every τ *g*-closed set is a τ *g-closed set.

Proof:

Let A be a τ *g*-closed set .Let A$\subseteq$G, where G is τ * - open. As every τ * open set is τ *g – open and since A is τ *g*-closed. cl*(A) =A$\subseteq$G. Thus cl*(A) $\subseteq$G whenever A$\subseteq$ G and G is τ *- open .Hence A is τ *g- closed.

The converse of the theorem (3.2.5) need not be true as seen from the following example.

Example 3.2.6

Let X= {a, b, c} , τ={ϕ, X ,{a} , { b ,c}} and B = {b}. B is not a τ *g*-closed set. Since {b} is a g- open set of (X, τ) such that B$\subseteq${b} but cl(B)=cl ({b}) = {b,c} $\not\subset${b}.However B is a g- closed set.

Theorem 3.2.7

For any two sets A and B, cl*(A $\cup$B) = cl* (A) $\cup$cl*(B)

Proof

Since A $\subseteq$ A$\cup$B, cl*(A) $\subseteq$ cl*(A$\cup$B). Since B$\subseteq$ A$\cup$B, cl*(B) $\subseteq$ cl*(A$\cup$B) .Therefore cl*(A) $\cup$ cl*(B) $\subseteq$ cl*(A$\cup$B).Again A$\subseteq$ cl*(A) and B $\subseteq$ cl*(B) implies A$\cup$B $\subseteq$ cl*(A) $\cup$ cl*(B). Thus cl*(A) $\cup$ cl*(B) is an intersection of g-closed sets containing A$\cup$B. Since

$cl^*(A\cup B)$ is the intersection of all g-closed sets containing $A\cup B$, $cl^*(A\cup B) \subseteq cl^*(A) \cup cl^*(B)$. Thus $cl^*(A\cup B) = cl^*(A) \cup cl^*(B)$.

Theorem 3.2.8

Union of two τ *g*-closed sets in X is a τ *g*-closed set in X

Proof

Let A and B be two τ *g*-closed sets. Let $A\cup B\subseteq G$, where G is τ *g-open. Since A and B are τ *g*-closed sets, we have $cl^*(A)\cup cl^*(B) \subseteq G$. But by theorem 3.2.7 $cl^*(A\cup B) = cl^*(A) \cup cl^*(B)$.Therefore $cl^*(A\cup B)\subseteq G$. Hence $A \cup B$ is a τ *g*- closed sets.

Theorem 3.2.9

A subset A of X is a τ *-g-closed if and only if cl*(A)-A contains no nonempty τ *g*- closed set in X

Proof

Let A be a τ *-g-closed set. Suppose that F is a non empty τ *-g*- closed subset of cl*(A)-A. Then $F\subseteq$ cl*(A)-A and hence $F\subseteq cl^*(A)\cap A^C$. Therefore $F\subseteq A^C$ .Since FC is a τ *g-open set and A is a τ *g-closed, $cl^*(A) \subseteq F^C$ .That is $F \subseteq (cl^*(A))^C$. Hence $F\subseteq cl^*(A) \cap (cl^*(A))^C$ .That is $F=\phi$, a contradiction. Thus cl*(A)-A contains no nonempty τ *g*-closed set in X.

Conversely, assume that cl*(A)-A contains no nonempty τ *g*- closed set .Let $A \subseteq G$, G is τ *-open .Suppose that cl*(A) is not contained in G. Then $cl^*(A) \cap G^C$ is a nonempty τ *g*- closed set of cl*(A)-A. This is a contradiction. Therefore $cl^*(A) \subseteq G$ and hence A is τ *-g- closed.

Corollary 3.2.10

A subset A of X is τ *g- closed if and only if cl*(A)-A contains no nonempty closed set in X.

Proof:

The proof follows from the theorem 3.2.9 and the fact that every closed set is τ *g*- closed set in X.

Theorem 3.2.11

If a subset A of X is τ *g *- closed and A$\subseteq$B$\subseteq$cl*(A), then B is τ *g * - closed set is X.

Proof

Let A be a τ *-g* - closed set such that A$\subseteq$B$\subseteq$cl*(A). Let U be a τ *g-open set of X such that B$\subseteq$U. Since A is τ *-g* - closed set, cl*(A) $\subseteq$ U .Now cl*(A) $\subseteq$ cl*(B) $\subseteq$ cl*(cl*(A)) = cl*(A) $\subseteq$U .That is cl*(B) $\subseteq$U, U is τ *g-open. Therefore B is τ *-g * - closed set in X.

The converse of the above theorem need not be true as seen from the following example.

Example 3.2.12

Consider the topological space (X,τ), where X= {a,b,c} and the topology τ ={X,ϕ,{a},{a,b}} .

Let A={c} and B= {a,c}. Then A and B are τ *g- closed sets is (X,τ) .But A $\subseteq$ B is not a subset of cl*(A).

Theorem 3.2.13

Let A be aτ *-g*-closed in (X,τ).Then A is g-closed if and only if cl*(A)-A is τ *g*-open.

Proof

Suppose A is g-closed in X. Then cl*(A)=A and so cl*(A)-A=ϕ which is τ *g*-open in X. Conversely, suppose cl*(A)-A is τ *g*-open in X. Since A is τ *-g*-closed, by the Theorem 3.2.9 cl*(A)-A contains no nonempty τ *-g*-closed set in X. Then cl*(A)-A=ϕ .Hence A is g-closed.

Theorem 3.2.14

For x$\in$ X, the set X-{x} is τ *-g*- closed or τ *g*-open

Proof

Suppose X-{x} is not τ *g*-open .Then X is the only τ *g*-open containing X-{x}.This implies cl*(X-{x})$\subseteq$X. Hence X-{x} is a τ *-g*-closed in X.

Theorem 3.2.15

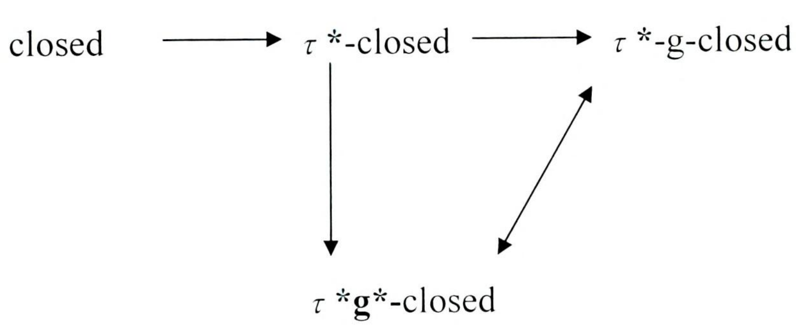
If A is τ *g*- closed set of (X,τ) such that A$\subseteq$B$\subseteq$cl*(A), then B is also a τ *g*- closed set of (X, τ).

Proof

Let U be a τ *g*- open set of (X, τ) such that B$\subseteq$ U. Then A$\subseteq$U. Since A is τ *g*-closed cl*(A) $\subseteq$U. Now cl*(B) $\subseteq$ cl*(cl*(A)) = cl*(A) $\subseteq$U. Therefore B is also a τ *g*-closed set of (X, τ).

Remark 3.2.16

From the above discussions, we obtain the following implications.



SECTION 3.3

$\tau*\alpha$ - generalized closed sets in topological space

In this section, the concept of $\tau*\alpha$ -g-closed sets in topological spaces is studied. Properties and characterizations of $\tau*\alpha$ -g-closed sets are discussed. The relations between $\tau*\alpha$ -g-closed (open) sets with various closed (open) sets are analyzed.

Definition 3.3.1

For a subset A of a topological space (X,τ), the generalized α -closure operator $cl_\alpha*(A)$ is defined as the intersection of all $g\alpha$ -closed sets containing A.

Definition 3.3.4

A subset A of a topological space X is called $\tau*\alpha$ - generalized closed set(briefly $\tau*\alpha$ -g-closed) if $cl_\alpha*(A) \subseteq G$ whenever $A\subseteq G$ and G is $\tau*\alpha$ -open. The complement of a $\tau*\alpha$ -generalized closed set is called $\tau*\alpha$ -generalized open ($\tau*\alpha$ -g-open) in$(X, \tau*)$.

Theorem 3.3.5

Every closed set in X is $\tau*\alpha$ -g-closed.

Proof

Let A be a closed set. Let $A\subseteq G$, where G is $\tau*\alpha$ -open . Since A is closed $cl(A) = A\subseteq G$ .But $cl_\alpha*(A) \subseteq cl*(A) \subseteq cl(A)$. Thus $cl_\alpha*(A) \subseteq G$ whenever $A\subseteq G$ and G is $\tau*\alpha$ -open. Therefore A is $\tau*\alpha$ -g-closed.

The converse of the above theorem need not be true as seen from the following example.

Example 3.3.6

Consider the topological space X= {a, b, c} with topology τ ={X, ϕ ,{a}}.Then the set {a} is τ *-closed but not closed .

Theorem 3.3.7

Every $\tau * \alpha$ -closed set in X is $\tau * \alpha$ -g-closed

Proof

Let A be a $\tau * \alpha$ -closed set. Let A$\subseteq$G where G is $\tau * \alpha$ -open. Since A is $\tau * \alpha$ -closed, cl_α*(A) = A $\subseteq$ G .Thus, cl_α*(A) $\subseteq$G whenever A$\subseteq$G and G is $\tau * \alpha$ -open. Therefore A is $\tau * \alpha$ -g-closed.

Theorem 3.3.8

Every τ *-g-closed set in X is a $\tau * \alpha$ -g-closed set but not conversely.

Proof

Let A be a τ *-g-closed set .Assume that A$\subseteq$G, G is $\tau * \alpha$ -open in (X, τ *).Then cl*(A) $\subseteq$G, since A is$\tau * \alpha$ -g-closed .But cl_α*(A) $\subseteq$cl*(A) .Therefore cl_α*(A) $\subseteq$G. Hence A is $\tau * \alpha$ -g-closed.

The converse of the above theorem need not be true as seen from the following example.

Example 3.3.9

Consider the topological space X= {a, b, c} with topology ={X, ϕ ,{a}}.Then the set {a} is τ *-g-closed but not $\tau * \alpha$ -g-closed .

Theorem 3.3.10

For any two sets A and B cl_α*(A$\cup$B) = cl_α*(A) $\cup$ cl_α*(B)

Proof

Since A $\subseteq$ A$\cup$B, cl*$_\alpha$ (A) $\subseteq$cl*$_\alpha$ (A$\cup$B). Since B$\subseteq$ A$\cup$B, cl*$_\alpha$ (B) $\subseteq$ cl*$_\alpha$ (A$\cup$B) .Therefore cl*$_\alpha$ (A) $\cup$ cl*$_\alpha$ (B) $\subseteq$cl*$_\alpha$(A$\cup$B). Again A$\subseteq$ cl*$_\alpha$ (A) and B $\subseteq$ cl*$_\alpha$ (B) implies A$\cup$B $\subseteq$ cl*$_\alpha$ (A) $\cup$ cl*$_\alpha$ (B). Thus cl*$_\alpha$ (A) $\cup$ cl*$_\alpha$ (B) is an intersection of gα-closed sets containing A$\cup$B. Since cl*$_\alpha$ (A$\cup$B) is the intersection of all gα-closed sets containing A$\cup$B, cl*$_\alpha$ (A$\cup$B) $\subseteq$ cl*$_\alpha$ (A) $\cup$ cl*$_\alpha$ (B) . Thus cl*$_\alpha$ (A$\cup$B) = cl*$_\alpha$ (A) $\cup$ cl*$_\alpha$ (B).

Theorem 3.3.11

Union of two $\tau * \alpha$-g-closed sets in X is a $\tau * \alpha$-g-closed set in X

Proof

Let A and B be two $\tau * \alpha$-g-closed sets. Let A$\cup$B$\subseteq$G, where G is $\tau * \alpha$-open .Since A and B are $\tau * \alpha$-g-closed sets, cl$_\alpha$*(A) $\cup$ cl$_\alpha$*(B) $\subseteq$ G. But by previous Theorem 3.3.10 cl$_\alpha$ (A)$\cup$cl$_\alpha$*(B) = cl$_\alpha$* (A$\cup$B).Therefore cl$_\alpha$*(A$\cup$B) $\subseteq$ G . Hence A$\cup$B is a $\tau * \alpha$-g-closed set.

Theorem 3.3.12

A subset A of X is $\tau * \alpha$-g-closed sets if and only if cl$_\alpha$*(A) –A contains no nonempty $\tau * \alpha$-closed set in X

Proof

Let A be a $\tau * \alpha$-g-closed set. Suppose that F is a nonempty $\tau * \alpha$ -closed subset of cl$_\alpha$*(A) –A. Now F $\subseteq$ cl$_\alpha$*(A) – A. Thus F $\subseteq$ cl$_\alpha$*(A) $\cap$A^c. Therefore F $\subseteq$cl$_\alpha$*(A) and F $\subseteq$ A^c.Since F^c is a $\tau * \alpha$-open set and A is a $\tau *$ α-g-closed set. cl$_\alpha$*(A) $\subseteq$ F^c.That is F $\subseteq$ (cl$_\alpha$*(A))^c. Hence F $\subseteq$cl$_\alpha$*(A)$\cap$(cl$_\alpha$*(A))^c=ϕ.That is F=ϕ a contradiction. Thus cl$_\alpha$*(A) –A contain no nonempty $\tau * \alpha$-closed set in X. Conversely assume that cl$_\alpha$*(A) –A contains no nonempty $\tau*\alpha$-closed set. Let A $\subseteq$ G, where G is $\tau*\alpha$-open. Suppose that $cl_\alpha*(A)$ is not contained in G, then $cl_\alpha*(A)\cap G^c$ is a nonempty $\tau*\alpha$-closed set of $cl_\alpha*(A)$ –A which is a contradiction. Therefore $cl_\alpha*(A) \subseteq$ G and hence A is. $\tau*\alpha$-g-closed set.

Corollary 3.3.13

A subset A of X is $\tau*\alpha$-g-closed set if and only if $cl_\alpha*(A)$ –A contain no nonempty closed set in X.

Proof

The proof follows from the Theorem 3.3.10 and the fact that every closed set in X is $\tau*\alpha$-closed set is $\tau*\alpha$-g-closed set in X.

Corollary 3.3.14

A subset A of X is $\tau*\alpha$-g-closed set if and only if $cl_\alpha*(A)$ –A contain no nonempty open set in X.

Proof

The proof follows from the Theorem 3.3.10 and the fact that every open set is $\tau*\alpha$-open set.

Theorem 3.3.15

If a subset A of X is $\tau*\alpha$-g-closed set and A $\subseteq$B $\subseteq$ $cl_\alpha*(A)$.Then B is $\tau*\alpha$-g-closed

Proof

Let A be a $\tau*\alpha$-g-closed set such that A $\subseteq$B $\subseteq$ $cl_\alpha*(A)$.Let U be a $\tau*\alpha$-open set of X such that B $\subseteq$ U. Since A is $\tau*\alpha$-g-closed, then $cl_\alpha*(A)$ $\subseteq$B. Now $cl_\alpha*(A) \subseteq cl_\alpha*(B) \subseteq cl_\alpha*(cl_\alpha*(A))= cl_v*(A) \subseteq$U .That is $cl_\alpha*(B)$ $\subseteq$U,U is $\tau*\alpha$-open. Therefore B is$\tau*\alpha$-g-closed set in X.

Theorem 3.3.16

Let A be a $\tau*\alpha$-g-closed in X. Then A is $\tau*\alpha$-closed if and only if $cl_\alpha*(A)-A$ is $\tau*\alpha$-open.

Proof

Suppose A is $\tau*\alpha$-closed in X. Then $cl_\alpha*(A)=A$ and so $cl_v*(A)-A=\phi$, which is $\tau*\alpha$-open in X. Conversely, suppose that $cl_\alpha*(A)-A$ is $\tau*\alpha$-open in X. Since A is $\tau*\alpha$-g-closed. By the previous Theorem 3.3.10 $cl_\alpha*(A)-A$ no nonempty contains $\tau*\alpha$-g-closed sets in X. Then $cl_\alpha*(A)-A=\phi$.Hence A is $\tau*\alpha$-g-closed.

Theorem 3.3.17

For $x\in X$, the set X-{x} is $\tau*\alpha$-g-closed or $\tau*\alpha$-open

Proof

Suppose X-{x} is not $\tau*\alpha$-open. Then X is the only $\tau*\alpha$-open set containing X-{x}.This implies $cl_\alpha*(X-\{x\})\subseteq X$. Hence X-{x} is a $\tau*\alpha$-g-closed in X.

Remark 3.3.18

From the above discussion, the following implications are obtained.

$$\begin{array}{ccc} \text{closed} \Rightarrow \tau^*\text{-closed} & \Rightarrow & \tau^*\text{-g-closed} \\ \Downarrow & & \Downarrow \\ \tau^*\alpha\text{-closed} & \Rightarrow & \tau^*\alpha\text{-g-closed} \end{array}$$