Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University), Coimbatore – 641 043.
Master's Degree Examination – November 2018
Semester I

Class : I PG — Time : 3 hrs
Major: Mathematics — Max. Marks: 60

17MMAC05 ORDINARY DIFFERENTIAL EQUATIONS

Part A

Choose the correct Answer **10 x ½ = 5**

1. In a homogeneous equation, $L(y) =$ ______

   a) $b(x)$ b) 0 c) $\phi(x)$ d) 1

2. In finding the Wronskian of two solutions of $L(y) =$ 0, Wronskian satisfies______ order linear equation.

   a) third b) second c) any d) first

3. The roots of the characteristic polynomial $p(r) = r^3 + r^2 + r + 1$ are ______

   a) $i, -i, -1$ b) $i, -i, 1$ c) $i, 1, -1$ d) $-i, 1, -1$

4. To find a particular solution of a non-homogeneous equation of order *n*, the method of ______ is used

   a) variation of variables b) variation of constants
   c) variation of parameters d) separation of variables

5. The analyticity of a function at a point needs to the power series with ______ radius of convergence

   a) positive b) positive or negative c) positive d) any

6. In a non-homogeneous linear differential equation with variable coefficients, it is assumed that the coefficient and right side functions

   a) are analytic b) are continuous c) need not be continuous
   d) are both analytic and continuous

7. An equation of the type $L(y) = x^2y'' + axy' + by = 0$ is known as ______ equation

   a) Bessel b) Legendre c) Euler d) Wronskian

8. $\int_0^\infty e^{-x}x^{t-1}dx =$ ______

   a) $\Gamma(x)$ b) $\Gamma(z)$ c) $\Gamma(e)$ d) $\Gamma(t)$

9. A solution ϕ of the equation $y' = y^2$ satisfying the initial condition $\phi(1) = -1$ is $\phi(x) =$ ______

   a) x b) $-x$ c) $-\frac{1}{x}$ d) $\frac{1}{x}$

10. The set $R = \{(x, y) : |x - x_0| \le a, |y - y_0| \le a\}$ in the real (*x*, *y*) plane represents a______

    a) square b) rectangle c) circle d) sphere

Part B

Answer ALL questions. **5 X 4 = 20 marks**

Each question should not exceed 200 words or one page.

11.a) Give an example for an ordinary differential equation whose solution is oscillatory.

(or)

b) What do you mean by an initial value problem?

12. a) Let $\phi_1, \phi_2, \cdots, \phi_n$ be n linearly independent solutions of $L(y) = 0$ on an interval I. Prove that, if $c_1, c_2, \cdots, c_n$ are any constants $\phi = c_1\phi_1 + c_2\phi_2 + \cdots + c_n\phi_n$ is a solution, and every solution may be represented in this form.

(or)

b) Solve $y^{(4)} + y = 0$.

13. a) Discuss briefly about homogeneous and non-homogeneous equations with variable coefficients.

(or)

b) State existence theorem for analytic coefficients.

14. a) Differentiate singular points and regular singular points.

(or)

b) Write Bessel function of order α of the first kind and hence write $J_0(x)$.

15. a) Solve $y' = 3y^{\frac{2}{3}}$ by separating variables.

(or)

b) Derive the conditions for an equation to be exact.

Part C

Answer ALL questions. **5 X 7 = 35 marks**

Each question should not exceed 600 words or three pages.

16.a) State and prove the existence theorem of initial value problems for second order equations.

(or)

b) Solve $L(y) = b(x)$ assuming that $p(r) = r^2 + a_1 r + a_2$ has two distinct roots r_1 and r_2.

17. a) Prove that the n solutions of $L(y) = 0$ given by $e^{r_1(x)}, xe^{r_1(x)}, \cdots, x^{m_1-1}e^{r_1(x)}$; $e^{r_2(x)}, xe^{r_2(x)}, \cdots, x^{m_2-1}e^{r_2(x)}$; ...., $e^{r_s(x)}, xe^{r_s(x)}, \cdots, x^{m_s-1}e^{r_s(x)}$ are linearly independent on any interval I.

(or)

b) Solve $y''' + y'' + y' + y = 1$, given $\psi(0) = 0$, $\psi'(0) = 1$, $\psi''(0) = 0$.

18. a) If $\phi_1, \phi_2, \cdots, \phi_n$ are n solutions of $L(y) = 0$ on an interval I, they are linearly independent if and only if $W(\phi_1, \phi_2, \cdots, \phi_n)(x) \neq 0$ for all x in I. Prove!

(or)

b) Solve the Legendre equation and find a basis for all the solutions.

19. a) Solve $x^2y'' + xy' + y = 0$.

(or)

b) Derive Bessel function of zero order of the first kind.

20. a) Let M, N be two real-valued functions which have continuous first partial derivatives on some rectangle R. Then, prove that the equation $M(x,y) + N(x,y)y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

(or)

b) Use the method of successive approximations to solve $y' = xy$, given $y(0) = 1$.

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