Avinashilingam Institute for Home Science and Higher Education for Women
Coimbatore-641 043
Bachelor's Degree Examination – November 2017

V Semester

Class : III UG
Major : Mathematics

Time : 3 Hrs
Max. Marks : 100

15BMAC15 Complex Analysis-I
Part-A
Choose the correct answer

10 x 1=10

1. A continuous function is ________.
   a. nowhere differentiable
   b. not necessarily differentiable
   c. everywhere differentiable
   d. a constant function

2. Cauchy – Riemann equations are given by
   a. $u_x = v_y, u_y = v_x$
   b. $u_x = v_x, u_y = v_y$
   c. $u_x = v_y, u_y = -v_x$
   d. $u_x = -v_y, u_y = -v_x$

3. The function $f(z) = \bar{z}$ is ___________
   a. nowhere differentiable
   b. everywhere differentiable
   c. an entire function
   d. an analytic function

4. An analytic function with constant modulus is ________
   a. 0
   b. 1
   c. not constant
   d. constant

5. The equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is called _______ equation.
   a. Poisson
   b. Cauchy – Riemann
   c. Laplace
   d. Euler

6. If $f(z)$ is analytic, then $\nabla^2 |\text{Re}\, f(z)|^2 =$ _________
   a. $|f'(z)|^2$
   b. $2|f'(z)|^2$
   c. $z|f'(z)|^2$
   d. $z^2|f'(z)|^2$

7. The bilinear transformation $w = \frac{az+b}{cz+d}$ is conformal if ______
   a. $ab - cd = 0$
   b. $ab - cd \neq 0$
   c. $ad - bc = 0$
   d. $ad - bc \neq 0$

8. If the two values of Z are equal, then the bilinear transformation is called _______
   a. elliptic
   b. parabolic
   c. hyperbolic
   d. fixed

9. If C is the circle $|Z| = 1$, then $\int_C e^z dz =$ ______.
   a. 0
   b. 1
   c. $2\pi i$
   d. $-2\pi i$

10. If Γ is a positively oriented circle with radius r and centre a, then $\int_\Gamma \frac{dz}{z-a} =$ _____
    a. 0
    b. πi
    c. $2\pi i$
    d. $4\pi i$

Part B

5 X 6=30

Answer the following

Answer should not exceed 400 words or two pages

11.a. Show that the function $f(z)=|z|^2$ is differentiable only at the origin.

(or)

11.b. Show that the function $f(z)=\sin z$ is analytic.

12.a. Show that an analytic function with constant real part is constant.

(or)

12.b. Prove that the real and imaginary parts of an analytic function satisfies Laplace equation.

13.a. Find an analytic function whose imaginary part is $3x^2y-y^3$.

(or)

13.b. If $u+iv=z^3$, then show that u, v are harmonic.

14.a. Find the bilinear transformation that maps the points $0, 1, \infty$ onto $i, -1, -i$ respectively.

(or)

b. Find the invariant points of the transformation $w=\dfrac{2z+4i}{iz+1}$.

15.a. Evaluate $\int_C \dfrac{e^{2z}}{(z-1)(z-2)}dz$ where C is the circle $|Z|=3$ using Cauchy's integral formula.

(or)

b. Evaluate $\int_C \dfrac{zdz}{z^2-1}$ where C is the positively oriented circle $|Z|=2$.

Part C

5 x 12=60

Answer the following

Answer should not exceed 800 words or four pages

16.a. Derive Cauchy – Riemann equations in polar form.

(or)

b. Derive the sufficient condition for differentiability of $f(z)$.

17.a. If $w=\log z$, find $\dfrac{dw}{dt}$ and determine where w is not analytic.

(or)

b. Show that the function $f(z)=\sqrt{|xy|}$ is not analytic at the origin, although C-R equations are satisfied at origin.

18.a. Find the analytic function $f(z)=u+iv$ if $u+v=\dfrac{\sin 2x}{\cosh 2y-\cos 2x}$.

(or)

b. Show that $u=\log\sqrt{x^2+y^2}$ is harmonic and determine its conjugate and hence find the corresponding analytic function.

19.a. Prove that any bilinear transformation which maps the unit circle $|w|=1$ onto unit circle $|w|=1$ can be written in the form $w=e^{i\lambda}\left(\dfrac{z-z_1}{z\bar{z}_1-1}\right)$ where λ is real.

(or)

b. Prove that any bilinear transformation which maps the real axis onto unit circle $|w|=1$ can be written in the form $w=e^{i\lambda}\left(\dfrac{z-z_1}{z-\bar{z}_1}\right)$ where λ is real.

20.a. State and prove Cauchy's integral formula.

(or)

b. State and prove Morera's theorem.

****