SPECIMEN FORMAT FOR THESES OF MONTH

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Abstract within 300 words:

The present study is focused on second order bipolar fuzzy structures. The concepts such as second order bipolar fuzzy sets and second order bipolar fuzzy topological spaces following both Chang and Lowen sense are introduced. Relations between first and second order bipolar fuzzy topological spaces and relations between crisp topological spaces and second order bipolar fuzzy topological spaces are analysed. Second order bipolar fuzzy continuity is introduced and its properties are discussed. The definitions of first order and second order bipolar fuzzy product topology are introduced. Relations between first and second order bipolar fuzzy product topology and relations between crisp product topology and second order bipolar fuzzy product topology are examined. The concept of first and second order bipolar fuzzy gradation of openness are introduced. A new definition of first order bipolar fuzzy topology induced by first order bipolar fuzzy gradation of openness is given. Relations between first order bipolar fuzzy gradation of openness, second order bipolar fuzzy gradation of openness and first order gradation of openness are discussed. Results related to second order bipolar fuzzy topologies induced by second order bipolar fuzzy gradation of openness are obtained. Five types of second order bipolar fuzzy compactness are introduced. Results related to second order bipolar fuzzy compactness are obtained. Second order bipolar fuzzy matrix is introduced. Operations such as addition, multiplication and complement of second order bipolar fuzzy matrices are given and definitions like transpose, trace and identity of second order bipolar fuzzy matrix are presented. Also properties like associative law and distributive law are verified. The working procedure of second order bipolar fuzzy TOPSIS method is given and an optimal solution for a decision making problem on selecting a best project proposal submitted for project funding is obtained.

i) Major objectives:

- To introduce the notion of second order bipolar fuzzy set, second order bipolar fuzzy topology and second order bipolar fuzzy matrix.
- To obtain the relations between first and second order bipolar fuzzy topological spaces and to construct the relations between second order bipolar fuzzy topological spaces and crisp topological spaces.
- To introduce the concepts of product, separation axioms, gradation of

openness in First Order Bipolar Fuzzy Topological Spaces (FOBPFTS).

- To study the concepts of second order bipolar fuzzy structures such as product, continuity, separation axioms, gradation of openness and compactness.
- To obtain the solution for the decision-making problem using second order bipolar fuzzy TOPSIS method.

ii) Methodology :

The research methods carried out in the present study are as follows:

- Conceptual research method to define SOBPFS, SOBPFT and its structures.
- Analytical method for proving results.
- Relational analysis between FOBPFTS and SOBPFTS.
- Examples and counter examples for implications and nonimplications.

In the present work, the concepts of second order bipolar fuzzy set and second order bipolar fuzzy topology are introduced and various second order bipolar fuzzy structures are studied.

The concepts studied are

(i) Second order bipolar fuzzy continuity

(ii) Second order bipolar fuzzy product topology

(iii) Different versions of Hausdorff separation axioms in second order bipolar fuzzy topological space.

(iv) Second order bipolar fuzzy gradation of openness.

(v) Second order bipolar fuzzy compactness

Chapter 1:

Preliminary definitions of first order fuzzy set, first order fuzzy topological spaces, second order fuzzy set, second order fuzzy topological spaces, first order bipolar fuzzy set, first order bipolar fuzzy topological spaces, first order fuzzy matrix and first order bipolar fuzzy matrix are presented in the first chapter.

Chapter 2:

Fundamental definitions and properties of second order bipolar fuzzy sets and second order bipolar fuzzy topological spaces are introduced and studied in the second chapter. Some important and interesting relations between first order bipolar fuzzy topological spaces and second order bipolar fuzzy topological spaces are analysed.

In the first section, definitions and properties of second order bipolar fuzzy sets and second order bipolar fuzzy topological spaces are introduced and studied.

In the second section, the relations between first order bipolar fuzzy and second order bipolar fuzzy topological spaces are analysed. Examples for second order bipolar fuzzy topological spaces are also constructed.

The Relations are

(i) With every first order bipolar fuzzy topology $\tau_{\mathfrak{B}}$ on X, a second order bipolar fuzzy topology $\hat{\tau}_{\mathfrak{B}}$ on X is associated. The association $\tau_{\mathfrak{B}} \rightarrow \hat{\tau}_{\mathfrak{B}}$ is referred as $\mathbb{C}_1$.

(ii) With every second order bipolar fuzzy topology $\hat{\tau}_{\mathfrak{B}}$ on X and for every $x \in X$, a first order bipolar fuzzy topology $(\hat{\tau}_{\mathfrak{B}})_x$ on I is associated. The association $\hat{\tau}_{\mathfrak{B}} \rightarrow (\hat{\tau}_{\mathfrak{B}})_x$ is referred as $\mathbb{C}_2$.

(iii) With every second order bipolar fuzzy topology $\tau_{\mathfrak{B}}$ on X and for every $\alpha \in I$, a first order bipolar fuzzy topology $(\hat{\tau}_{\mathfrak{B}})_\alpha$ on X is associated. The association $\tau_{\mathfrak{B}} \rightarrow (\hat{\tau}_{\mathfrak{B}})_\alpha$ is referred as $\mathbb{C}_3$.

(iv) With every first order bipolar fuzzy topology $\tau_{\mathfrak{B}}$ on I, a second order bipolar fuzzy topology $(\hat{\tau}_{\mathfrak{B}})_I$ on X is associated. The association $\tau_{\mathfrak{B}} \rightarrow (\hat{\tau}_{\mathfrak{B}})_I$ is referred as $\mathbb{C}_4$.

(v) Complement of second order bipolar fuzzy set $\widehat{A}_{bp}$ on X is defined in two different ways and they are denoted as $\left(\widehat{A}_{bp}\right)_c$ and $\left(\widehat{A}_{bp}\right)^c$. If $\hat{\tau}_{\mathfrak{B}} = \{\left(\widehat{A}_{bp}\right)_\lambda / \left(A_{bp}\right)_\lambda \in \tau_{\mathfrak{B}}\}$ is a second order bipolar fuzzy topology on X, then $(\hat{\tau}_{\mathfrak{B}})_c = \left\{\left(\left(\widehat{A}_{bp}\right)_\lambda\right)_c / \left(\widehat{A}_{bp}\right)_\lambda \in \hat{\tau}_{\mathfrak{B}}\right\}$ is a second order bipolar fuzzy topology on X. The association $\tau_{\mathfrak{B}} \rightarrow (\hat{\tau}_{\mathfrak{B}})_c$ is referred as $\mathbb{C}_5$.

In the third section, relations between crisp topology τ on X and the second order bipolar fuzzy topology on X are discussed. From a crisp topology τ on X, three different second order bipolar fuzzy topologies denoted by $\widehat{\omega(\tau)}$, $\widehat{\omega_*(\tau)}$ and $\widehat{\omega_\varepsilon(\tau)}$ are constructed. Similarly, from a second order bipolar fuzzy topology $\hat{\tau}_{\mathfrak{B}}$ on X, three crisp topologies denoted by $i(\hat{\tau}_{\mathfrak{B}})$, $i^*(\hat{\tau}_{\mathfrak{B}})$ and $i_\varepsilon(\hat{\tau}_{\mathfrak{B}})$ are constructed. Some relations between these structures are also established.

The following results are proved:

1) Let τ_1, τ_2 be two topologies on X such that $\tau_1 \subseteq \tau_2$. Then

i. $\widehat{\omega(\tau_1)} \subseteq \widehat{\omega(\tau_2)}$

ii. $\widehat{\omega_\varepsilon(\tau_1)} \subseteq \widehat{\omega_\varepsilon(\tau_2)}$

iii. $\widehat{\omega_*(\tau_1)} \subseteq \widehat{\omega_*(\tau_2)}$

2) Let (X, τ) be a topological space. Then

i. $\tau \subseteq i^*\left(\widehat{\omega_*(\tau)}\right)$

ii. For $\varepsilon \in (0,1)$, $\tau \subseteq i_\varepsilon\left(\widehat{\omega_\varepsilon(\tau)}\right)$

iii. $\tau \subseteq i\left(\widehat{\omega(\tau)}\right)$

Chapter 3:

Third chapter deals with second order bipolar fuzzy continuity and second order bipolar fuzzy product topology.

In the first section, second order bipolar fuzzy continuity is defined.

"A function $\theta: (X, \hat{\tau}_{\mathfrak{B}_1}) \to (Y, \hat{\tau}_{\mathfrak{B}_2})$ is said to be second order bipolar fuzzy continuous, if the following condition is satisfied: $\theta^{-1}(\widehat{A}_{bp}) \in \hat{\tau}_{\mathfrak{B}_1}$, if $\widehat{A}_{bp} \in \hat{\tau}_{\mathfrak{B}_2}$"

It is proved that the associations $\mathbb{C}_1$, $\mathbb{C}_3$ and $\mathbb{C}_5$ preserve continuity. It is also proved that the associations $\hat{\tau}_{\mathfrak{B}} \to i_\varepsilon(\hat{\tau}_{\mathfrak{B}})$, $\hat{\tau}_{\mathfrak{B}} \to i^*(\hat{\tau}_{\mathfrak{B}})$, $\hat{\tau}_{\mathfrak{B}} \to i(\hat{\tau}_{\mathfrak{B}})$, $\tau \to \widehat{\omega_\varepsilon(\tau)}$, $\tau \to \widehat{\omega_*(\tau)}$ and $\tau \to \widehat{\omega(\tau)}$ are functorial.

In the second section, first order bipolar fuzzy product topology is introduced and its properties are analysed.

"The **product** of first order bipolar fuzzy set $A_{bp} \times B_{bp}$ on $X \times Y$ is defined as follows: $A_{bp} \times B_{bp} = \left({A_{bp}}^+ \times {B_{bp}}^+, {A_{bp}}^- \times {B_{bp}}^-\right)$ where

$$\left(A_{bp}{}^{+} \times B_{bp}{}^{+}\right)(x,y) = \min\{ A_{bp}{}^{+}(x), B_{bp}{}^{+}(y)\} \text{ and}$$

$$\left(A_{bp}{}^{-} \times B_{bp}{}^{-}\right)(x,y) = \max\left\{A_{bp}{}^{-}(x), B_{bp}{}^{-}(y)\right\}, \text{ for every } (x,y) \in X \times Y.$$"

Some results are obtained.

In the third section, second order bipolar fuzzy product topology is defined. It is proved that the associations $\mathbb{C}_1$, $\mathbb{C}_3$ and $\mathbb{C}_5$ preserve product. Further the following results are obtained.

(i) $i_\varepsilon(\hat{\tau}_{\mathfrak{B}_1}) \times i_\varepsilon(\hat{\tau}_{\mathfrak{B}_2}) \subseteq i_\varepsilon(\hat{\tau}_{\mathfrak{B}_1} \times \hat{\tau}_{\mathfrak{B}_2})$

(ii) $i^*\left(\hat{\tau}_{\mathfrak{B}_1}\right) \times i^*\left(\hat{\tau}_{\mathfrak{B}_2}\right) \subseteq i^*\left(\hat{\tau}_{\mathfrak{B}_1} \times \hat{\tau}_{\mathfrak{B}_2}\right)$

(iii) $\widehat{\omega_\varepsilon(\tau)} \times \widehat{\omega_\varepsilon(\tau')} \subseteq \omega_\varepsilon(\widehat{\tau \times \tau'})$

(iv) $\widehat{\omega_*(\tau)} \times \widehat{\omega_*(\tau')} \subseteq \omega_*(\widehat{\tau \times \tau'})$

(v) $S_2\left(\hat{\tau}_{\mathfrak{B}_1}\right) \times S_2\left(\hat{\tau}_{\mathfrak{B}_2}\right) \subseteq S_2\left(\hat{\tau}_{\mathfrak{B}_1} \times \hat{\tau}_{\mathfrak{B}_2}\right)$

Chapter 4:

In chapter four, different versions of Hausdorff separation axioms in first order and second order bipolar fuzzy topological spaces are studied. Different versions of Hausdorff separation axioms have been defined and studied by many authors in the first order fuzzy topological spaces. Three different Hausdorff separation axioms in fuzzy topological spaces introduced by Gantner, Steinlage and Warren (1978), Katsaras (1981) and Srivastava et al. (1981) were extended to second order fuzzy topological spaces by Kalaichelvi (2011, 2013) and were denoted as W – Hausdorff, K – Hausdorff and S – Hausdorff axioms respectively.

In the first section, W – Hausdorff, K – Hausdorff and S – Hausdorff separation axioms of Warren, Katsaras and Srivastava are extended to first order bipolar fuzzy topological spaces.

"A first order bipolar fuzzy topological space $(X, \tau_{\mathfrak{B}})$ is said to be a first order bipolar fuzzy W-Hausdorff or bipolar fuzzy W-T_2, if for any two distinct points $x, y \in X$, there exists two bipolar fuzzy open sets $A_{bp}, B_{bp} \in \tau_{\mathfrak{B}}$ such that $A_{bp}{}^{+}(x) = 1, A_{bp}{}^{-}(x) = -1$, $B_{bp}{}^{+}(y) = 1, B_{bp}{}^{-}(y) = -1$ and $A_{bp} \cap B_{bp} = 0_{bp}$."

Chapter 5:

The fifth chapter is devoted to the study of first order bipolar fuzzy gradation of openness, second order bipolar fuzzy gradation of openness and second order bipolar fuzzy compactness.

In 1992, Hazra et al. introduced the concept of gradation of openness and gave a new definition of fuzzy topology. Using this concept, Kalaichelvi (2000) introduced the definition

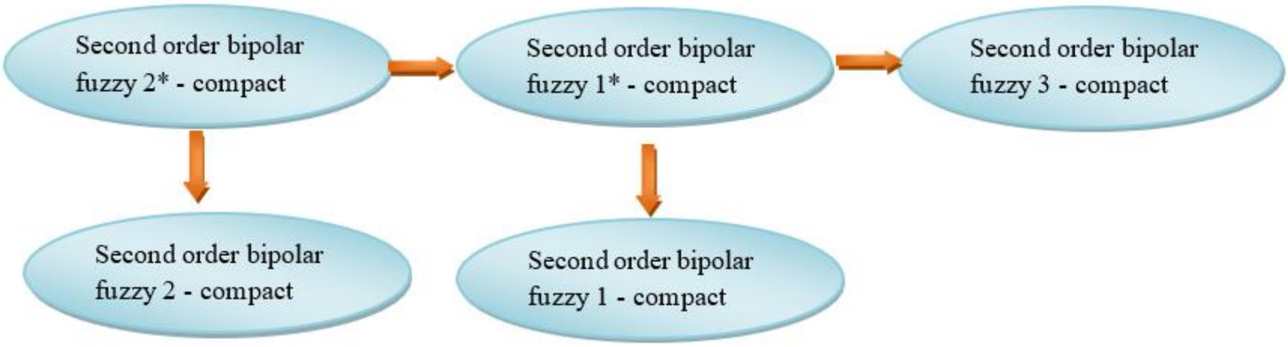
of second order gradation of openness and gave results related to second order fuzzy topologies induced by second order gradation of openness.

In the first section, first order bipolar fuzzy gradation of openness is introduced and a new definition of first order bipolar fuzzy topology induced by first order bipolar fuzzy gradation of openness is given.

In the second section, second order bipolar fuzzy gradation of openness is introduced. Five interesting relations between first order bipolar fuzzy gradation of openness, second order bipolar fuzzy gradation of openness and first order gradation of openness are established. These relations are denoted as D_1, D_2, D_3, D_4 and D_5. It is proved that the relations D_1, D_3 and D_5 are functorial with respect to gradation preserving maps. Propositions related to the second order bipolar fuzzy topologies induced by the second order bipolar fuzzy gradation of openness are proved. Further it is proved that if $\hat{\tau}_{\mathfrak{B}}(\mathcal{G})$ is the second order bipolar fuzzy topology induced by the second order bipolar fuzzy gradation of openness $\mathcal{G}$, then $\mathcal{G} \rightarrow \hat{\tau}_{\mathfrak{B}}(\mathcal{G})$ is functorial.

Third section is devoted to the study of compactness in the second order bipolar fuzzy topological spaces. Five different types of compactness, namely, second order bipolar fuzzy 1- compact, second order bipolar fuzzy 1*- compact, second order bipolar fuzzy 2-compact, second order bipolar fuzzy 2*- compact, second order bipolar fuzzy 3- compact are introduced by extending the definition of bipolar fuzzy compactness.

Relations between five second order bipolar fuzzy compactness:



The relations between these concepts are analysed with special reference to R_1, R_2, R_3, R_4 and R_5. It is proved that all the five concepts of second order bipolar fuzzy compact spaces are preserved under second order bipolar fuzzy continuous function.

Chapter 6:

The sixth chapter is devoted to the study of second order bipolar fuzzy matrix and application of second order bipolar fuzzy TOPSIS method in Multi-Criteria Decision Making (MCDM) problem.

In 2019, M. Pal and Sanjib Mondal introduced the concept of bipolar fuzzy matrix. MCDM is a process to make an optimal choice that has the highest degree of achievement from a set of alternatives which are characterized in terms of multiple conflicting criteria. Hwang and Yoon (1981) devised the Technique for the Order of Preference by Similarity to Ideal Solution (TOPSIS) method, which is one of the most advantageous and successful approaches for resolving MCDM problems. In classical MCDM methods, the attribute values and weights are determined precisely. In 2000, Chen introduced the fuzzy version of TOPSIS method to deal with problems consisting of incomplete and vague information. In 2002, Chung and Chu presented fuzzy TOPSIS method under group decision for facility location selection problem. In 2014, Hadi et al. proposed the fuzzy inferior ratio method for multiple attribute decision making problems. In 2016, Dey et al. considered TOPSIS method for solving the decision making problem under bipolar neutrosophic fuzzy environment. In 2018, Alghamdi et al. studied multi-criteria decision-making methods in bipolar fuzzy environment.

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