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Construction of Control Chart for Random Queue Length for (M / M / c): (∞ / FCFS) Queueing Model Using Skewness

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ABSTRACT

In this paper, first-come, first served, multiple channel Poisson/exponential queueing system M / M / c with infinite waiting capacity is considered. For this model, we introduce a new procedure for construction of

i) Shewhart's control chart C_1,

ii) Control chart C_3 for random queue length N using the method on skewness suggested by Shore(2000) and the control charts C_1 and C_3 are compared.

Index Terms: Random queue length, Average run length, False alarm rate, Average queue length.

1. INTRODUCTION

Waiting in line for service is one of the most unpleasant experiences of life on this world. Barrer (1957) says, In certain queueing processes a potential customer is considered "lost" if the system is busy at the time service is demanded. If not served during this time, the customer leaves the system and is considered lost. In queueing system the customer satisfaction can be increased by constructing control charts for N and providing control limits for N so as to make effective utilization of time. If control limits are displayed so that customer can have prior idea about control limits.

With this factor, an attempt is made to find control limits for random queue length N for (M/M/c) : (∞/FCFS) queueing model. The pioneering work in this direction was made by Haim Shore in 2000. Two control charts C_1 and C_2 are constructed for random queue length N for (M/M/1) : (∞/FCFS) queueing model by Khaparde and Dhabe (2010) and also C_3 and C_4 are constructed for random queue length N for (M/M/1) : (∞/FCFS) queueing model by Khaparde and Dhabe (2011) where

Control chart C_1 : is the Shewhart control chart,

Control chart C_2 : is the control chart using method of weighted variance,

Control chart C_3 : is the control chart for random queue length N based on skewness and

Control chart C_4 : is the control chart using Nelson's power transformation.

In this paper construction of control chart C_3 for random queue length N for (M/M/c) : (∞/FCFS) queueing model is done using method based on skewness. Performance of control chart C_3 is compared with control chart C_1.

2. (M / M / c) : (∞ / FCFS) QUEUEING MODEL

In this model arrival follows Poisson distribution with parameter λ and service time follows independently and identically distributed exponential distribution with parameter μ. There are c identical servers present in the system. Figure I represent the state-transition diagram for this model.

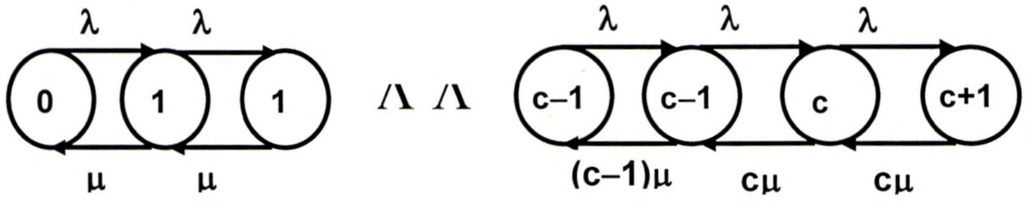

FIGURE I : STATE-TRANSITION RATE DIAGRAM FOR M / M / c MODEL

Let random variable N denote the number of customers in the system both waiting and in service and P_n denote the steady state distribution of random variable N given by

$$P_n = \begin{cases} \left(\dfrac{\lambda}{\mu}\right)^n \dfrac{1}{n!} P_0, & 1 \le n < c \\ \left(\dfrac{\lambda}{\mu}\right)^c \dfrac{1}{c^{n-c}\, c!} P_0, & c \le n < \infty \end{cases} \tag{1}$$

where $P_0 = \left[\sum_{n=0}^{c-1} \left(\dfrac{\lambda}{\mu}\right)^n \dfrac{1}{n!} + \left(\dfrac{\lambda}{\mu}\right)^c \dfrac{1}{c!} \left(\dfrac{c\mu}{c\mu - \lambda}\right) \right]^{-1}$,

λ is the mean arrival rate and μ is the mean service rate.

Let random variable W_S denote the waiting time spent in the system by the customer. This includes both the waiting time and service time. The p.d.f. of random variable W_S is given by

$$f(W_S) = (\mu - \lambda)\, e^{-(\mu-\lambda)W_S}, \quad W_S > 0 \tag{2}$$

Thus W_S follows an exponential distribution with mean

$$E(W_S) = \int_0^\infty W_S\, f(W_S)\, dW_S$$

$$= \int_0^\infty W_S\, (\mu-\lambda)\, e^{-(\mu-\lambda)W_S}\, dW_S$$

$$E(W_S) = \frac{1}{(\mu-\lambda)} \tag{3}$$

$$E(W_S^2) = \frac{2}{(\mu-\lambda)^2} \tag{4}$$

Using (2.3) and (2.4), the variance is given by

$$V(W_S) = \frac{1}{(\mu-\lambda)^2} \tag{5}$$

The distribution function of W_S is

$$F(x) = P(W_S \le x) = \int_0^\infty f(W_S)\, dW_S$$

$$= \int_0^\infty (\mu-\lambda)\, e^{-(\mu-\lambda)W_S}\, dW_S$$

$$= 1 - e^{-(\mu-\lambda)x}, \quad x > 0.$$

$$\therefore \quad F(x) = 1 - e^{-\mu(1-\rho)x}, \quad x > 0.$$

The probability distribution of N is a geometric distribution with parameter $(1 - \rho)$. The steady state distribution of the random variable N depends on two parameters μ and λ only through their ratio.

Using (1), the average number of customers and variance of queue length in the system is given by

$$E(N) = \mu_1' = \sum_{n=0}^{c-1} n P_n + \sum_{n=c}^{\infty} n P_n$$

$$= \sum_{n=0}^{c-1} n \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} P_0 + \sum_{n=c}^{\infty} n \left(\frac{\lambda}{\mu}\right)^n \frac{1}{c^{n-c}\, c!} P_0$$

$$= P_0\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]$$

Here $P_0 = \left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}$

$$\therefore E(N) = \mu_1' = \left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}$$

$$\text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right] \qquad (6)$$

$$E(N^2) = \mu_2' = \sum_{n=0}^{\infty}(n(n-1)P_n+nP_n)$$

$$= \sum_{n=0}^{c-1}n(n-1)P_n+\sum_{n=c}^{\infty}c(c-1)P_n+\sum_{n=0}^{c-1}nP_n+\sum_{n=c}^{\infty}cP_n$$

$$= \sum_{n=0}^{c-1}n(n-1)\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}P_0+\sum_{n=c}^{\infty}c(c-1)\left(\frac{\lambda}{\mu}\right)^n\frac{1}{c^{n-c}\,c!}P_0$$

$$+\sum_{n=0}^{c-1}n\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}P_0+\sum_{n=c}^{\infty}c\left(\frac{\lambda}{\mu}\right)^n\frac{1}{c^{n-c}\,c!}P_0$$

$$E(N^2) = \mu_2' = \left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}$$

$$\text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{n^2}{n!}+\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)\frac{c^2}{c!}\right]^{-1} \qquad (7)$$

Using (6) and (7), the variance of queue length in the system is given by

$$V(N) = \Bigg[\Bigg\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}$$

$$\text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{n^2}{n!}+\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)\frac{c^2}{c!}\right]^{-1}\Bigg\}$$

$$-\left\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-2}\right.$$

$$\left.\left.\times\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{2}\right\}\right] \quad (8)$$

Using (8), the standard deviation is calculated as

$$\sigma=\frac{\left(\left\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\times\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{n^{2}}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\left(\frac{c\mu}{c\mu-\lambda}\right)\frac{c^{2}}{c!}\right]\right\}-\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{2}\right)^{1/2}}{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]} \quad (9)$$

$$E(N^{3}) = \mu_3' = \sum_{n=0}^{\infty}(n(n-1)(n-2)+3n(n-1)+n)P_n$$

$$= P_0\left\{\sum_{n=0}^{c-1}n(n-1)(n-2)\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}\right.$$

$$+\sum_{n=c}^{\infty}\frac{c(c-1)(c-2)}{c!}\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c^{n-c}}$$

$$+3\sum_{n=0}^{c-1}n(n-1)\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+3\sum_{n=c}^{\infty}\frac{c(c-1)}{c!}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{c^{n-c}}$$

$$\left.+\sum_{n=0}^{c-1}n\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\sum_{n=c}^{\infty}\frac{c}{c!}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{c^{n-c}}\right\}$$

$$\mu_3' = \left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}$$

$$\times\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{n^{3}}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\left(\frac{c\mu}{c\mu-\lambda}\right)\frac{c^{3}}{c!}\right] \quad (10)$$

Using (10), the third central moment is given by

$$\mu_3 = \Bigg(\Bigg\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{n^3}{n!}+\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\frac{c^3}{c!}\right]\Bigg\}$$

$$-3\Bigg\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-2}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{n^2}{n!}+\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\frac{c^2}{c!}\right]$$

$$\text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\frac{1}{(c-1)!}\right]\Bigg\}\Bigg)$$

$$+2\Bigg\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\frac{1}{c!}\right]^{-3}$$

$$\mu_3 = \frac{\begin{gathered}\Bigg\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^2\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{n^3}{n!}+\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\frac{c^3}{c!}\right]\Bigg\} \\ -3\Bigg\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right] \\ \text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\frac{1}{(c-1)!}\right] \\ \text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{n^2}{n!}+\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\frac{c^2}{c!}\right]\Bigg\}\end{gathered}}{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\frac{1}{c!}\right]^3} \qquad (11)$$

Let $S_k(N)$ denote the traditional measure of skewness for random queue length namely,

$$S_k(N) = \frac{\mu_3}{\sigma^3}.$$

where μ_3 is the third central moment.

Using (9) and (11), the skewness for random queue length is given by $S_k(N)$ =

$$\frac{\left\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^2\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{n^3}{n!}+\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)\frac{c^3}{c!}\right]\right\} -3\left\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right] \times\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)\frac{1}{(c-1)!}\right] \times\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{n^2}{n!}+\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)\frac{c^2}{c!}\right]\right\} +2\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^3}{\left(\left\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{n^2}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{c^2}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\right\} -\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^2\right)^{3/2}} \tag{12}$$

1.1 SHEWHART'S CONTROL CHART C_1

When it is possible to specify standard values for the process mean and standard deviation, we may use these standards to establish the control chart. Then the parameters of the chart are

UCL = $\mu + A\,\sigma$

CL = μ

LCL = $\mu - A\,\sigma$

where the quantity $\frac{3}{\sqrt{n}}$ = A, say, is a constant that depends on n.

Using expressions (6) and (9), the control limits for (M / M / c) : (∞ / FCFS) model are calculated as follows.

$$\text{UCL} = \frac{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\frac{1}{(c-1)!}\right] + A\left(\left\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right] \times \left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{n^2}{n!} + \left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\frac{c^2}{c!}\right]\right\} - \left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^2\right)^{1/2}}{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]}$$

$$\text{CL} = \frac{\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)}{\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)}$$

$$\text{LCL} = \frac{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right] - A\left(\left\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right] \times \left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{n^2}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{c^2}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\right\} - \left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^2\right)^{1/2}}{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]}$$

TABLE I

MEAN, STANDARD DEVIATION, UPPER CONTROL LIMIT FOR THE RANDOM VARIABLE N FOR DIFFERENT VALUES OF ρ USING SHEWHART'S CONTROL CHART C_1 WITH A = 1 AND C = 2

S.No.	λ	μ	ρ	σ	CL	UCL	α_u	ARL
1.	1	10	0.05	0.3148	0.090	0.4048	0.3937	2.54
2.	2	10	0.10	0.4430	0.200	0.6430	0.3553	2.81
3.	3	10	0.15	0.5719	0.226	0.7979	0.3826	2.61
4.	4	10	0.20	0.6110	0.400	1.011	0.3960	2.52
5.	5	10	0.25	0.6705	0.500	1.1705	0.4443	2.25
6.	6	10	0.30	0.7190	0.600	1.3190	0.5098	1.96
7.	7	10	0.35	0.7569	0.700	1.4569	0.5947	1.68
8.	8	10	0.40	0.7857	0.800	1.4857	0.7178	1.39
9.	9	10	0.45	0.8052	0.900	1.7052	0.8355	1.20

1.2 CONSTRUCTION OF CONTROL CHART C_3 FOR RANDOM VARIABLE N

Before constructing the chart, it is essential to notice that the probability function of a geometric distribution is a monotonously decreasing function of n and since control limits for queueing system are to be obtained, there is no meaning in defining lower control limit (LCL) for this distribution. Also upper control limit (UCL) obviously makes sense since it will enable monitoring improbable high value of N.

1.3 CONTROL CHART C_3

In the queueing model (M / M / c) : (∞ / FCFS), the random variable N denote the number of customers in the system both waiting and in service. The steady state distribution of random variable N is given by expression (1). In order to obtain control limits using Shore's method (2000), the first three moments of the random variable N given by expressions (6), (8) and (12) respectively, are needed.

The expression (12) shows that skewness of the distribution of N depends on λ, μ, n and c. Taking into account the mean, standard deviation and skewness of the underlying geometric distribution of N the expression for the new control limits for the number of customers in (M / M / c) : (∞ / FCFS) system using Shore's method (2000) are as follows :

Control Limits for Control Chart C_3 when $S_k < 0.5$

$$UCL = E(N) + 3\sqrt{V(N)} + 1.324\sqrt{V(N)}\ S_k(N) - \frac{1}{2}.$$

$$CL = E(N)$$

$$LCL = E(N) - 3\sqrt{V(N)} + 1.324\sqrt{V(N)}\ S_k(N) + \frac{1}{2}.$$

Control Limits for Control Chart C_3 when $S_k > 0.5$

$$UCL = E(N) + 3.642\sqrt{V(N)} + 0.9146\sqrt{V(N)}\ S_k(N) - \frac{1}{2}.$$

$$CL = E(N)$$

$$LCL = E(N) - 3.642\sqrt{V(N)} + (1.40)\,(0.9146)\sqrt{V(N)}\ S_k(N) + \frac{1}{2}.$$

Control Limits for Control Chart C_3 for (M / M / c) : (∞ / FCFS) Model when $S_k < 0.5$

$$UCL = \left\{\left(\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]\right.\right.$$

$$\left. x\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]^{-1}\right)$$

$$+3\left\{\left[\left(\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]\right.\right.\right.$$

$$\left. x\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{n^2}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{c^2}{c!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]\right)$$

$$\left. -\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]^2\right]$$

$$\left. x\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]^{-1}\right\}$$

$$+ 1.324 \left\{ \left[\left(\left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu - \lambda}\right) \right] \right. \right. \right.$$

$$\left. \left. x \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{n^2}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{c^2}{c!} \left(\frac{c\mu}{c\mu - \lambda}\right) \right] \right] \right)$$

$$- \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!} \left(\frac{c\mu}{c\mu - \lambda}\right) \right]^2 \Bigg]$$

$$\left. x \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu - \lambda}\right) \right]^{-1} \right\}^{-1}$$

$$x \left(\left\{ \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu - \lambda}\right) \right]^2 \right. \right.$$

$$\left. x \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{n^3}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{c^3}{c!} \left(\frac{c\mu}{c\mu - \lambda}\right) \right] \right\}$$

$$- 3 \left\{ \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu - \lambda}\right) \right] \right.$$

$$x \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!} \left(\frac{c\mu}{c\mu - \lambda}\right) \right]$$

$$\left. \left. x \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{n^2}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{c^2}{c!} \left(\frac{c\mu}{c\mu - \lambda}\right) \right] \right\} \right)$$

$$\left. \left. + 2 \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!} \left(\frac{c\mu}{c\mu - \lambda}\right) \right]^3 \right) - \frac{1}{2} \right\}$$

$$CL = \left\{ \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!} \left(\frac{c\mu}{c\mu - \lambda}\right) \right] \right.$$

$$\text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}\Bigg\}$$

$$\text{LCL} = \Bigg\{\Bigg(\Bigg[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\Bigg]$$

$$\text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}\Bigg)$$

$$-3\Bigg\{\Bigg[\Bigg(\Bigg[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\Bigg]$$

$$\text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{n^2}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{c^2}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\Bigg)$$

$$-\left[\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^2\right]$$

$$\text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}\Bigg\}$$

$$+1.324\Bigg(\Bigg\{\Bigg[\Bigg(\Bigg[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\Bigg]$$

$$\text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\Bigg)$$

$$-\left[\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^2\right]$$

$$\text{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^c\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}\Bigg\}^{-1}$$

$$\mathrm{x}\left(\left\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{2}\right.\right.$$

$$\left.\mathrm{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{n^{3}}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{c^{3}}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\right\}$$

$$-3\left\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\right.$$

$$\mathrm{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]$$

$$\left.\left.\mathrm{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{n^{2}}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{c^{2}}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\right\}\right)$$

$$\left.\left.+2\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{3}\right)+\frac{1}{2}\right\}$$

Control Limits for Control Chart C_3 for (M / M / c) : (∞ / FCFS) Model when $S_k > 0.5$

$$\mathrm{UCL} \;=\; \left\{\left(\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{(c-1)!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\right.\right.$$

$$\left.\mathrm{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}\right)$$

$$+3.642\left\{\left[\left(\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\right.\right.\right.$$

$$\left.\mathrm{x}\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{n^{2}}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{c^{2}}{c!}\left(\frac{c\mu}{c\mu-\lambda}\right)\right]\right)$$

$$- \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right]^2 \Bigg]$$

$$\text{x} \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right]^{-1} \Bigg\}$$

$$+ (0.9146) \left(\left\{ \left[\left(\left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right] \right.\right.\right.\right.$$

$$\text{x} \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{n^2}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{c^2}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right] \Bigg)$$

$$- \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right]^2 \Bigg]$$

$$\text{x} \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right]^{-1} \Bigg\}^{-1}$$

$$\text{x} \left(\left\{ \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right]^2 \right.\right.$$

$$\text{x} \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{n^3}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{c^3}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right] \Bigg\}$$

$$- 3 \left\{ \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right] \right.$$

$$\text{x} \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right]$$

$$\text{x} \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{n^2}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{c^2}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right] \Bigg\} \Bigg)$$

$$+2\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{(c-1)!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]^{3}\Bigg)-\frac{1}{2}\Bigg\}$$

$$CL = \left\{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{(c-1)!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]\right.$$

$$\left. x\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]\right\}$$

$$LCL = \left\{\left(\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{(c-1)!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]\right.\right.$$

$$\left. x\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]^{-1}\right)$$

$$-3.642\left\{\left[\left(\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]\right.\right.\right.$$

$$\left. x\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{n^{2}}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{c^{2}}{c!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]\right)$$

$$-\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{(n-1)!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{(c-1)!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]^{2}\Bigg]$$

$$\left. x\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]^{-1}\right\}$$

$$+(1.40)(0.9146)\left(\left\{\left[\left(\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{1}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{1}{c!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]\right.\right.\right.\right.$$

$$\left. x\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^{n}\frac{n^{2}}{n!}+\left(\frac{\lambda}{\mu}\right)^{c}\frac{c^{2}}{c!}\left(\frac{c\,\mu}{c\,\mu-\lambda}\right)\right]\right)$$

$$- \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right]^2 \Bigg]$$

$$\text{x} \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right]^{-1} \Bigg\}^{-1}$$

$$\text{x} \left(\left\{ \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right]^2 \right.\right.$$

$$\text{x} \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{n^3}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{c^3}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right] \Bigg\}$$

$$-3 \left\{ \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right] \right.$$

$$\text{x} \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right]$$

$$\text{x} \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{n^2}{n!} + \left(\frac{\lambda}{\mu}\right)^c \frac{c^2}{c!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right] \Bigg\} \Bigg)$$

$$+2 \left[\sum_{n=0}^{c-1} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{(n-1)!} + \left(\frac{\lambda}{\mu}\right)^c \frac{1}{(c-1)!} \left(\frac{c\mu}{c\mu-\lambda}\right) \right]^3 \Bigg) + \frac{1}{2} \Bigg\}$$

Table 2.2 shows that the minimum value of skewness is nearly 0.2 for high values of ρ.

TABLE II

SKEWNESS, CONTROL LIMITS FOR THE RANDOM VARIABLE N FOR DIFFERENT VALUES OF ρ USING CONTROL CHART C_3 WITH L = 3 and C = 2

S.No.	λ	μ	ρ	Skewness	CL	UCL	α_u	ARL
1.	1	10	0.05	3.1200	0.090	2.045	0.0091	109.80
2.	2	10	0.1	2.6570	0.200	2.870	0.0099	101.40