Chapter 6

J-Irresolute Functions in Topological Spaces

§ 6.1. Introduction

Functions and obviously irresolute functions remain among the most significant ideas and most explored focuses in the entire of mathematical science. Crossley and Hildebrand (1972) presented the idea of irresoluteness. Different fascinating issues emerge when one thinks about irresoluteness. In this Chapter, new concepts of irresoluteness namely, J-irresolute functions and contra J-irresolute functions are introduced. Continuities introduced in the last Chapter are compared with irresoluteness here. Likewise the composition of these functions and the properties of irresoluteness are examined.

§ 6.2. J - Irresolute Functions

The following segment explains the launch of J-irresolute functions in topological spaces and analysis of some of their properties.

Definition 6.2.1. A function f : (Y,ζ)→ (Z,σ) is said to be **J-irresolute function** if the inverse image of every J-open set in (Z,σ) is J-open in (Y,ζ).

Example 6.2.2. Let f : (Y,ζ) → (Z,σ) be the identity function. Consider Y = Z = {p,q,r} with ζ = {Y, ϕ, {p}} and σ = {Z, ϕ,{p},{q},{p,q}} = JO(Z,σ).Here JO(Y,ζ)= P(Y) −{q,r}.Then f is a J-irresolute function.

Note 6.2.3. Hereafter we find the relation of J-irresolute functions with other J-continuous functions.

Proposition 6.2.4. A J-irresolute function f: (Y,ζ)→ (Z,σ) is a J-continuous function but the converse is not true.

Proof Given that f : (Y,ζ) →(Z,σ) is a J-irresolute function. Let U be any open set in (Z,σ). By **Theorem 2.3.75.,** U is J-open in (Z,σ). Since f is a J-irresolute function, f^{-1}(U) is J-open in (Y,ζ). Hence f is J-continuous.

Counter Example 6.2.5. Let f : (Y,ζ) → (Z,σ) be the identity function. Consider Y = Z = {p,q,r} with ζ = {Y, ϕ,{p}} and σ = {Z, ϕ,{p,q}}. It is a J-continuous function as JO(Z,σ) = P(Z) but not a J-irresolute function as JO(Y,ζ) = P(Y) –{q,r}.

Proposition 6.2.6. A quasi J-continuous function f: (Y,ζ)→ (Z,σ) is a J-irresolute function but the converse is not true.

Proof Given f : (Y,ζ) →(Z,σ) is a quasi J-continuous function. Let U be any J-open set in (Z,σ). Since f is a quasi J-continuous function, f^{-1}(U) is open in (Y,ζ). By **Theorem 2.3.75.**, f^{-1}(U) is J-open in (Y,ζ). Hence f is J-irresolute.

Counter Example 6.2.7. In **Example 6.2.2.,** it is a J-irresolute function but not a quasi J-continuous function. Since for the J-open set {q} in (Z,σ), the inverse image is not open in (Y,ζ).

Proposition 6.2.8. A strongly J-continuous function f : (Y,ζ)→ (Z,σ) is a J-irresolute function but the converse is not true.

Proof Given f : (Y,ζ) →(Z,σ) is a strongly J-continuous function. Let U be a J-open set in (Z,σ). Since f is a strongly J-continuous function, f^{-1}(U) is J-clopen in (Y,ζ). Then f^{-1}(U) is J-open in (Y,ζ). Hence f is J-irresolute.

Remark 6.2.9. The converse of the above Proposition 6.2.8. is not true from the following Counter Example.

Counter Example 6.2.10. In the above **Example 6.2.2.**, it is J-irresolute but not strongly J-continuous. Because for a subset {q,r} in (Z,σ), its inverse image is {q,r} not J-clopen in (Y,ζ).

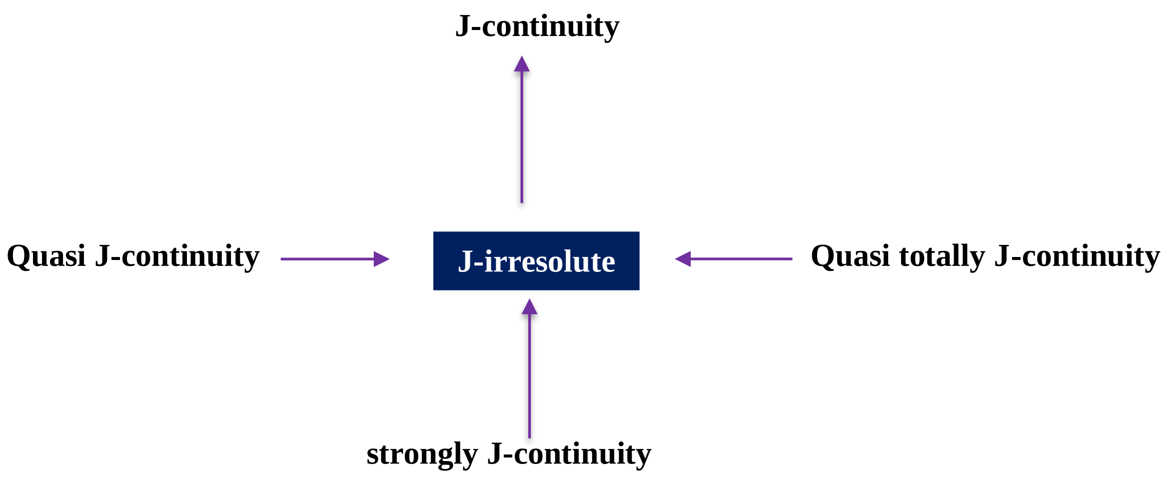
Proposition 6.2.11. A Quasi totally J-continuous function f : (Y,ζ)→ (Z,σ) is a J-irresolute function but the converse is not true.

Proof Given f : (Y,ζ) →(Z,σ) is a Quasi totally J-continuous function. Let U be any J-open set in (Z,σ). Since f is a Quasi totally J-continuous function, f^{-1}(U) is clopen in (Y,ζ). By **Proposition 2.3.81.**, f^{-1}(U) is J-open in (Y,ζ). Hence f is J-irresolute .

Remark 6.2.12. The converse of the above Proposition 6.2.11. is not true from the following Counter Example.

Counter Example 6.2.13. In the above **Example 6.2.2.**, it is J-irresolute but not Quasi totally J-continuous. Because for the J-open set {p} in (Z,σ), it is open in (Y,ζ) but not closed in (Y,ζ).

Result 6.2.14. From the above discussions to get the following pictorial representation.



Note 6.2.15. In general J-irresolute functions and irresolute functions are independent. This is shown by the following Counter Examples.

Counter Example 6.2.16. Consider the same above **Example 6.2.2.**, sO(Y,ζ)= {Y, ϕ, {p},{p,q},{p,r}} and sO(Z,σ) = {Z, ϕ,{p},{q},{p,q},{p,r},{q,r}}.It is J-irresolute function but not irresolute function. Since for the semi-open set {q} in (Z,σ),the inverse image is not semi-open in (Y,ζ).

Counter Example 6.2.17. Let f : (Y,ζ)→ (Z,σ) be a function and defined by f(p) = q, f(q) = r, f(r) = r and Y = Z ={p,q,r}, ζ = {Y,ϕ,{p},{p,q}}, σ = {Z, ϕ,{p,q}}.Here JO(Y,ζ) = P(Y) –{q,r}, JO(Z,σ) = P(Z), sO(Y,ζ)= {Y, ϕ, {p},{p,q},{p,r}} and sO(Z,σ) = {Z, ϕ,{p,q}}.It is irresolute but not J-irresolute.Since for the J-open set {r},their corresponding inverse image {q,r} is not J-open in (Y,ζ).

Remark 6.2.18. The following Counter Examples show that g-irresolute function and J-irresolute function are independent.

Counter Example 6.2.19. Let f : (Y,ζ)→ (Z,σ) be the function defined by f(p) = q,f(q) = q,f(r) = p.Consider Y = Z = {p,q,r} with ζ = {Y, ϕ,{p},{p,q}} and σ = {Z, ϕ,{p},{p,q},{p,r}}.Here JO(Y,ζ) = P(Y) –{q,r}, gO(Y,ζ) ={Y,ϕ,{p},{q},{p,q}} and JO(Z,σ) = P(Z) , gO(Z,σ) ={Z, ϕ,{p}}. It is a J-irresolute function but not a g-irresolute function. Since for the g-open set {p} in (Z,σ), the inverse image is not g-open in (Y,ζ).

Counter Example 6.2.20. Let f : (Y,ζ)→ (Z,σ) be the function defied by f(p) = r,f(q) = p,f(r) = r. Consider Y = Z = {p,q,r} with ζ ={Y, ϕ,{p},{q},{p,q}} and σ = {Z, ϕ,{p},{q},{p,q},{p,r}}.Here gO(Y,ζ) =JO(Y,ζ) = ζ and JO(Z,σ) = P(Z),gO(Z,σ) = σ. It is a g-irresolute function but not a J-irresolute function. Since for the J-open set {r} in (Z,σ), the inverse image is not J-open in (Y,ζ).

Remark 6.2.21. The following Counter Examples show that δg- irresolute function and J-irresolute function are independent.

Counter Example 6.2.22. Let f : (Y,ζ)→ (Z,σ) be the function defined by f(p) = q,f(q) = q,f(r) = p. Consider Y = Z = {p,q,r} with ζ = {Y, ϕ,{p},{p,q}} and σ = {Z, ϕ,{p},{p,q},{p,r}}.Here JO(Y,ζ) = P(Y) −{q,r}, δgO(Y,ζ) ={Y,ϕ,{p},{q},{p,q}} and JO(Z,σ) = P(Z) , δgO(Z,σ) ={Z, ϕ,{p}}. It is a J-irresolute function but not a δg-irresolute function. Since for the δg-open set {p} in (Z,σ), the inverse image is not δg-open in (Y,ζ).

Counter Example 6.2.23. Let f : (Y,ζ)→ (Z,σ) be the function defied by f(p) = r,f(q) = p,f(r) = r. Consider Y = Z = {p,q,r} with ζ ={Y, ϕ,{p},{q},{p,q}} and σ = {Z, ϕ,{p},{q},{p,q},{p,r}}.Here δgO(Y,ζ) =JO(Y,ζ) = ζ and JO(Z,σ) = P(Z),δgO(Z,σ) = σ. It is a δg-irresolute function but not a J-irresolute function. Since for the J-open set {r} in (Z,σ), the inverse image is not J-open set in (Y,ζ).

Remark 6.2.24. The following Counter Examples show that δg*-irresolute function and J-irresolute function are independent.

Counter Example 6.2.25. Let f : (Y,ζ)→ (Z,σ) be the identity function. Consider Y = Z = {p,q,r} with ζ = {Y, ϕ,{p}} and σ = {Z, ϕ,{p},{q},{p,q}}.Here JO(Y,ζ) = P(Y) −{q,r}, δg*O(Y,ζ) ={Y,ϕ,{p}} and JO(Z,σ) = σ = δg*O(Z,σ). It is a J-irresolute function but not a δg*-irresolute function. Since for the δg*-open set {q} in (Z,σ), the inverse image is not δg*-open in (Y,ζ).

Counter Example 6.2.26. Let f : (Y,ζ)→ (Z,σ) be the function defined by f(p) = r,f(q) = p,f(r) = r. Consider Y = Z = {p,q,r} with ζ ={Y,ϕ,{p},{q},{p,q}} and σ = {Z,

ϕ,{p},{q},{p,q},{p,r}}.Here δg*O(Y,ζ) =JO(Y,ζ) = ζ and JO(Z,σ) = P(Z), δg*O(Z,σ) = σ. It is a δg*-irresolute function but not a J-irresolute function. Since for the J-open set {r} in (Z,σ), the inverse image is not J-open in (Y,ζ).

Remark 6.2.27. In general J-irresolute functions and gδ-irresolute functions are independent. This is shown by the following Counter Examples.

Counter Example 6.2.28. Let f : (Y,ζ) → (Z,σ) be the function defined by f(p) = r,f(q) = p,f(r) = q. Consider Y = Z = {p,q,r} with ζ = {Y, ϕ,{p}} and σ = {Z, ϕ,{p},{q},{p,q}}.Here JO(Y,ζ) = P(Y) –{q,r}, gδO(Y,ζ) = P(Y) and JO(Z,σ) = σ = gδO(Z,σ). It is a gδ-irresolute function but not a J-irresolute function. Since for the J-open set {p,q} in (Z,σ), the inverse image is not J-open in (Y,ζ).

Counter Example 6.2.29. Let f : (Y,ζ) → (Z,σ) be an identity function. Consider Y = Z = {p,q,r,s} with ζ = {Y, ϕ,{p},{r},{p,q},{p,r},{p,q,r},{p,r,s}} and σ = {Z, ϕ,{p,q}}.Here JO(Y,ζ) = {Y,ϕ,{p},{q},{r},{s},{p,q},{p,r},{p,s},{q,r},{q,s},{p,q,r},{p,r,s},{p,q,s}}, gδO(Y,ζ) = {Y, ϕ,{p},{q},{r},{p,q},{p,r},{q,r},{p,q,r},{p,r,s}} and JO(Z,σ) = {Z, ϕ,{p},{q},{r},{s},{p,q},{p,r},{p,s},{q,r},{q,s},{p,q,r},{p,q,s}}, gδO(Z,σ) = P(Z). It is a J-irresolute function but not a gδ-irresolute function. Since for the gδ-open set {r,s} in (Z,σ), the inverse image {r,s}is not gδ-open in (Y,ζ).

Remark 6.2.30. In general J-irresolute functions and gs-irresolute functions are independent. This is shown by the following Counter Examples.

Counter Example 6.2.31. Let f : (Y,ζ) → (Z,σ) be the identity function. Consider Y = Z = {p,q,r} with ζ = {Y, ϕ,{p},{p,q}} and σ = {Z, ϕ,{p,q}}.Here JO(Y,ζ) = P(Y) –{q,r}, gsO(Y,ζ) = {Y,ϕ,{p},{q},{p,q},{p,r}} and JO(Z,σ) = P(Z), gsO(Z,σ) = {Z, ϕ,{p},{q},{p,q}}. It is a gs-irresolute function but not J-irresolute function. Since for the J-open set {q,r} in (Z,σ),the inverse image is not J-open in (Y,ζ).

Counter Example 6.2.32. Let f : (Y,ζ) → (Z,σ) be the identity function. Consider Y = Z = {p,q,r} with ζ = {Y, ϕ,{p,q}} and σ = {Z, ϕ,{p},{p,q},{p,r}}. Here JO(Y,ζ) = P(Y) , gsO(Y,ζ) = {Y,ϕ,{p},{q},{p,q}} and JO(Z,σ) = P(Z), gsO(Z,σ) = {Z, ϕ,{p},{p,q},{p,r}}.

It is J-irresolute but not gs-irresolute. Since for the gs-open set {p,r},their corresponding inverse image is {p,r}not gs-open in (Y,ζ).

Remark 6.2.33. In general J-irresolute functions and g*s-irresolute functions are independent. This is shown by the following Counter Examples.

Counter Example 6.2.34. Let f : (Y,ζ)→ (Z,σ) be defined by f(p) = q,f(q) = p,f(r) = r. Consider Y = Z = {p,q,r} with ζ = {Y, ϕ,{p},{p,q},{p,r}} and σ = {Z, ϕ,{p},{q},{p,q}}.Here JO(Y,ζ) = P(Y), g*sO(Y,ζ) = ζ and JO(Z,σ) = σ, g*sO(Z,σ) = P(Z)−{r}. It is a J-irresolute function but not a g*s-irresolute function. Since for the g*s-open set {p} in (Z,σ), the inverse image is not g*s-open in (Y,ζ).

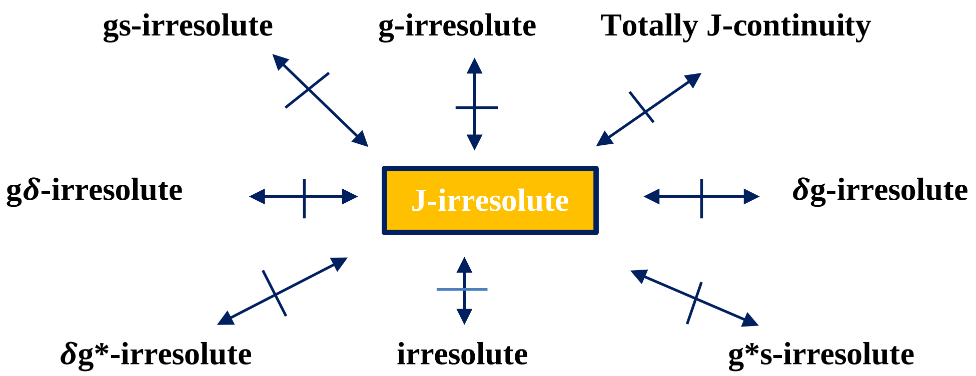
Counter Example 6.2.35. Let f : (Y,ζ)→ (Z,σ) be defined by f(p) = r,f(q) = q,f(r) = r. Consider Y = Z = {p,q,r} with ζ = {Y, ϕ,{p},{p,q}} and σ = {Z, ϕ,{p},{p,q},{p,r}}.Here JO(Y,ζ) = ζ, g*sO(Y,ζ) = P(Y) –{r} and JO(Z,σ) = P(Z), g*sO(Z,σ) = σ. It is g*s-irresolute function but not J-irresolute function. Since for the J-open set {p,r} in (Z,σ),the inverse image is not J-open in (Y,ζ).

Remark 6.2.36. In general J-irresolute functions and totally J-continuous functions are independent. This is shown by the following Counter Examples.

Counter Example 6.2.37. In **Example 6.2.2.**, it is J-irresolute but not totally J-continuous. Because for an open set {p} in (Z,σ), its inverse image is {p} not a J-clopen set in (Y,ζ).

Counter Example 6.2.38. Let f : (Y,ζ) → (Z,σ) be an identity function. Consider Y = Z = {p,q,r} with ζ = {Y, ϕ,{p},{p,q}} and σ = {Z, ϕ,{q}}.Here JO(Y,ζ) = P(Y) –{q,r}, JO(Z,σ) = P(Z) −{p,r}, σ^c = {Z, ϕ,{p,r}}.It is totally J-continuous function but not a J-irresolute function. Since for the J-open set {q,r} in (Z,σ), the inverse image is {q,r} not J-open in (Y,ζ).

Result 6.2.39. From the above discussions we get the following pictorial representation.



Theorem 6.2.40. Let f : (Y,ζ)→ (Z,σ) be a function where (Z,σ) is a JTC-space. Then the following are equivalent :(a) J-irresolute function (b) J-continuous function

Proof (a) ⟹ (b) From **Proposition 6.2.4.,**a J-irresolute function f is a J-continuous function. **(b) ⟹ (a)** Given f : (Y,ζ) →(Z,σ) is a J-continuous function. Let U be any J-closed set in (Z,σ).Since (Z,σ) is a JTC-space, U is a closed set in (Z,σ). Since f is a J-continuous function, f^{-1}(U) is J-closed in (Y,ζ). Hence f is J-irresolute.

Composition of J-Irresolute Functions

Proposition 6.2.41. If f : (Y,ζ)→ (Z,σ) and g : (Z,σ) →(P,μ) are J-irresolute functions, then g ◦ f : (Y,ζ)→ (P, μ) is a J-irresolute function.

Proof Direct proof.

Proposition 6.2.42. If f: (Y,ζ)→ (Z,σ) is J-irresolute and g : (Z,σ) →(P,μ) is a J-continuous function, then g ◦ f : (Y,ζ)→ (P, μ) is a J-continuous function.

Proof Given g: (Z,σ)→(P,μ) is a J-continuous function. Let U be any open set in (P,μ).Theng^{-1}(U) is J-open in (Z,σ) and since f is J-irresolute, we get $f^{-1}(g^{-1}(U))$ = $(g∘f)^{-1}$(U) is J-open in (Y,ζ). Hence g ◦ f : (Y,ζ)→ (P, μ) is a J-continuous function.

Proposition 6.2.43. If a function f :(Y,ζ) →(Z,σ) is a δ-open surjective, J-irresolute function and (Y,ζ) is a JTδ-space, then (Z,σ) is a JTδ-space.

Proof Given f : (Y,ζ) →(Z,σ) is a J-irresolute function. Let U be any J-open set in (Z,σ).Then f^{-1}(U) is J-open in (Y,ζ).When (Y,ζ) is a JTδ-space, f^{-1}(U) is δ-open in (Y,ζ) which is open in (Y,ζ) . By the given condition f is δ-open surjective, $f(f^{-1}(U))$ = U is δ-open in (Z,σ). Therefore (Z,σ) is a JTδ-space.

Proposition 6.2.44. If f: $(Y,\zeta)\rightarrow(Z,\sigma)$ is J-irresolute and g : $(Z,\sigma)\rightarrow(P,\mu)$ is a contra J-continuous function, then $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a contra J-continuous function.

Proof Given g: $(Z,\sigma)\rightarrow(P,\mu)$ is a contra J-continuous function. Let U be any closed set in (P,μ).Then $g^{-1}(U)$ is J-open in (Z,σ) and since f is J-irresolute, we get $f^{-1}(g^{-1}(U)) = (g\circ f)^{-1}(U)$ is J-open in (Y,ζ). Hence $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a contra J-continuous function.

Proposition 6.2.45. If f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is J-irresolute and g : $(Z,\sigma)\rightarrow(P,\mu)$ is a totally J-continuous function, then $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a totally J-continuous function.

Proof Given g: $(Z,\sigma)\rightarrow(P,\mu)$ is a totally J-continuous function. Let U be any closed set in (P,μ).Then $g^{-1}(U)$ is J-closed and J-open in (Z,σ) and since f is J-irresolute, we get $f^{-1}(g^{-1}(U)) = (g\circ f)^{-1}(U)$ is J-closed and J-open in (Y,ζ).Hence $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a totally J-continuous function.

Proposition 6.2.46. If f: $(Y,\zeta)\rightarrow(Z,\sigma)$ is J-irresolute and g : $(Z,\sigma)\rightarrow(P,\mu)$ is a strongly J-continuous function, then $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a strongly J-continuous function.

Proof Given g: $(Z,\sigma)\rightarrow(P,\mu)$ is a strongly J-continuous function. Let U be any subset in (P,μ).Then $g^{-1}(U)$ is J-closed and J-open in (Z,σ) and since f is J-irresolute, we get $f^{-1}(g^{-1}(U)) = (g\circ f)^{-1}(U)$ is J-closed and J-open in (Y,ζ). Hence $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a strongly J-continuous function.

Property 6.2.47. A function f :$(Y,\zeta)\rightarrow(Z,\sigma)$ is a J-irresolute function if and only if $f^{-1}(U)$ is J-closed in (Y,ζ) for every J-closed set U in (Z,σ).

Proof The proof is obvious from **Definition 6.2.1.**

Proposition 6.2.48. If a function f :$(Y,\zeta)\rightarrow(Z,\sigma)$ is a closed surjective, J-irresolute function and (Y,ζ) is a JTC-space, then (Z,σ) is a JTC-space.

Proof Given f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is a J-irresolute function. Let U be any J-closed set in (Z,σ).Since f is J-irresolute , $f^{-1}(U)$ is J-closed in (Y,ζ).When (Y,ζ) is a JTC-space,

f^{-1}(U) is closed in (Y,ζ). By the given condition f is closed surjective, f(f^{-1}(U)) = U is closed in (Z,σ). Therefore (Z,σ) is a JTC-space.

Proposition 6.2.49. Let f : (Y,ζ)→ (Z,σ) be a J-irresolute function. Then for each subset D of (Y,ζ), f(JCl(D))⊆ Cl(f(D)).

Proof Let D be any subset of (Y,ζ).Then Cl(f(D)) is closed in (Z,σ).By **Proposition 2.3.2.**, Cl(f(D)) is J-closed in (Z,σ).Since f is irresolute, f^{-1}(Cl(f(D))) is J-closed in (Y,ζ) ---(1).Since f(D) ⊆ Cl(f(D)) implies D ⊆ f^{-1}(Cl(f(D))).Then JCl(D) ⊆ JCl**(** f^{-1}(Cl(f(D))) = f^{-1}(Cl(f(D))) by (1).Hence the result.

Proposition 6.2.50. Let f : (Y,ζ)→ (Z,σ) be a J-irresolute function. Then for arbitrary subset D of (Y,ζ), JCl(f^{-1}(D))⊆ f^{-1}(Cl(D)).

Proof Let D be any subset of (Y,ζ).Then Cl(D) is closed in (Y,ζ).By **Proposition 2.3.2.**, Cl(D) is J-closed in (Y,ζ).Since f is J-irresolute,f^{-1}(Cl(D)) is J-closed in (Y,ζ) ---(1).Since D ⊆ Cl(D) implies f^{-1}(D) ⊆ f^{-1}(Cl(D)).Then JCl(f^{-1}(D)) ⊆ JCl**(** f^{-1}(Cl(D)) = f^{-1}(Cl(D)) by (1).Hence the result.

Proposition 6.2.51. If a function f :(Y,ζ) →(Z,σ) is a δg-open surjective, J-irresolute function and (Y,ζ) is a JTC-space, then (Z,σ) is a JTδg-space.

Proof Given f : (Y,ζ) →(Z,σ) is a J-irresolute function. Let U be any J-open set in (Z,σ).Then f^{-1}(U) is J-open in (Y,ζ).When (Y,ζ) is a JTC-space, f^{-1}(U) is open in (Y,ζ). By the given condition f is δg-open surjective, f(f^{-1}(U)) = U is δg-open in (Z,σ).Therefore (Z,σ) is a JTδg-space.

Proposition 6.2.52. If a function f :(Y,ζ) →(Z,σ) is a δg*-open surjective, J-irresolute function and (Y,ζ) is a JTC-space, then (Z,σ) is a JTδg*-space.

Proof Given f : (Y,ζ) →(Z,σ) is a J-irresolute function. Let U be any J-open set in (Z,σ).Since f is J-irresolute , f^{-1}(U) is J-open in (Y,ζ).When (Y,ζ) is a JTC-space, f^{-1}(U) is open in (Y,ζ). By the given condition f is δg*-open surjective, f(f^{-1}(U)) = U is δg*-open in (Z,σ).Therefore (Z,σ) is a JTδg*-space.

Proposition 6.2.53. If a function f :(Y,ζ) →(Z,σ) is a g-closed surjective, J-irresolute function and (Y,ζ) is a JTC-space, then (Z,σ) is a JTg-space.

Proof Given f : (Y,ζ) →(Z,σ) is a J-irresolute function. Let U be any J-closed set in (Z,σ).Since f is J-irresolute , f^{-1}(U) is J-closed in (Y,ζ).When (Y,ζ) is a JTC-space, f^{-1}(U) is closed in (Y,ζ). By the given condition f is g-closed surjective, f(f^{-1}(U)) = U is g-closed in (Z,σ).Therefore (Z,σ) is a JTg-space.

§ 6.3. Contra J-Irresolute Functions

Definition 6.3.1. A function f : (Y,ζ)→ (Z,σ) is said to be **contra J-irresolute function** if the inverse image of every J-closed set in (Z,σ) is a J-open set in (Y,ζ).

Example 6.3.2. Let f : (Y,ζ)→ (Z,σ) be the function defined by f(p) = p, f(q) = r, f(r) = p,f(s) = r. Consider Y ={p,q,r,s},Z = {p,q,r} with ζ = {Y, ϕ, {p,q}} and σ = {Z, ϕ,{p}}.Here JC(Z,σ)= P(Y) −{p},JO(Y,ζ) ={Y, ϕ,{p},{q},{r},{s},{p,q},{q,r}, {p,s},{q,s},{p,r},{p,q,r},{p,q,s}}.Then f is a contra J-irresolute function.

Proposition 6.3.3.(a) A contra J-irresolute function implies a contra J-continuous function but the converse is not true.

(b) A strongly J-continuous function implies a contra J-irresolute function but the converse is not true.

Proof (a) Given f : (Y,ζ) →(Z,σ) is a contra J-irresolute function. Let U be a closed set in (Z,σ). By **Proposition 2.3.2.**, U is a J-closed set in (Z,σ). Since f is a contra J-irresolute function, f^{-1}(U) is J-open in (Y,ζ). Hence f is a contra J-continuous function.

(b) Given f : (Y,ζ) →(Z,σ) is a strongly J-continuous function. Let U be a J-closed set in (Z,σ). Since f is a strongly J-continuous function, f^{-1}(U) is J-clopen in (Y,ζ) which implies f^{-1}(U) is J-open in (Y,ζ). Hence f is a contra J-irresolute function.

Note 6.3.4. From the following Counter Example shows that the converse of the above Proposition is not true.

Counter Example 6.3.5. Let f : (Y,ζ)→ (Z,σ) be an identity function. Consider Y = Z = {p,q,r} with ζ = {Y, ϕ,{p}} and σ = {Z, ϕ,{p,q}}.Here JO(Y,ζ) = P(Y) –{q,r} and

JC(Z,σ) = P(Z),σ^c = {Z, ϕ,{r}}. It is contra J-continuous function but not contra J-irresolute function. Since for the J-closed set {q,r} in (Z,σ), the inverse image is not J-open in (Y,ζ).

Counter Example 6.3.6. Let f : (Y,ζ)→ (Z,σ) be the function defined by f(p) = q, f(q) = r, f(r) = q, f(s) = p. Consider Y = {p,q,r,s}, Z = {p,q,r} with ζ = {Y, ϕ,{p,q}} and σ = {Z, ϕ,{p},{q},{p,q}}.Here JO(Y,ζ) = {Y, ϕ,{p},{q},{r},{s},{p,q},{q,r},{p,s},{q,s},{p,r},{p,q,r},{p,q,s}},and JC(Y,ζ) = {Y, ϕ,{r},{s},{r,s},{q,r},{p,s},{q,s},{p,r},{p,q,r},{p,q,s},{p,r,s},{q,r,s}}, JC(Z,σ) = σ^c. It is contra J-irresolute function but not a strongly J-continuous function. Since for the subset {r} in (Z,σ), the inverse image {q} is not J-clopen in (Y,ζ).

Remark 6.3.7. J-irresolute functions and contra J-irresolute functions are independent.

Counter Example 6.3.8. The **Example 6.2.2.** is a J-irresolute function but not a contra J-irresolute function. Because for a J-closed set {q,r} in (Z,σ),the inverse image {q,r} is not J-open in (Y,ζ).

Counter Example 6.3.9. The **Counter Example 6.3.6.** is a contra J-irresolute function but not a J-irresolute function. Because for a J-open set {p,q} in (Z,σ),the inverse image {p,r,s} is not J-open in (Y,ζ).

Proposition 6.3.10. A function f : (Y,ζ)→ (Z,σ) is a contra J-irresolute function iff the inverse image of each J-open set in (Z,σ) is J-closed in (Y,ζ).

Proof The proof is obvious from the definitions.

Remark 6.3.11. Totally J-continuous functions and contra J-irresolute functions are independent.

Counter Example 6.3.12. The **Counter Example 6.3.6.** is a contra J-irresolute function but not a totally J-continuous function. Since for the subset {r} in (Z,σ), the inverse image {q} is not J-closed in (Y,ζ).

Counter Example 6.3.13. The **Counter Example 6.3.5.** is a totally J-continuous function but not a contra J-irresolute function.

Proposition 6.3.14. A Quasi totally J-continuous function f : (Y,ζ) →(Z,σ) is a contra J-irresolute function but the converse is not true.

Proof Given f : (Y,ζ) →(Z,σ) is a Quasi totally J-continuous function. Let U be a J-open set in (Z,σ). Since f is a Quasi totally J-continuous function, f^{-1}(U) is clopen in (Y,ζ). By **Proposition 2.3.2.,** f^{-1}(U) is J-closed in (Y,ζ). Hence f is a contra J-irresolute function.

Remark 6.3.15. The converse of above Proposition is shown by Counter Example.

Counter Example 6.3.16. The **Example 6.3.2.** is a contra J-irresolute function but not a Quasi totally J-continuous function. Since for the J-open set {p} in (Z,σ),the inverse image {p,r} is not clopen in (Y,ζ).

Remark 6.3.17. J-continuous functions and contra J-irresolute functions are independent.

Counter Example 6.3.18. Let f : (Y,ζ)→ (Z,σ) be an identity function. Consider Y =Z = {p,q,r,s} with ζ = {Y, ϕ,{p},{q},{p,q,{p,q,r},{p,q,s}} and σ = {Z, ϕ,{r},{p,q},{p,q,r}}.Here JC(Y,ζ) = {Y, ϕ, {r},{s},{p,s},{q,s},{r,s},{q,r}, {p,r},{p,q,r},{p,q,s},{q,r,s},{p,r,s}}, JO(Y,ζ) = {Y, ϕ, {p},{q},{r},{s},{p,q},{p,s}, {q,s},{q,r},{p,r},{p,q,r},{p,q,s}}and σ^c = { Z, ϕ,{p,q,s},{r,s},{s}}, JC(Z,σ) = { Z, ϕ,{s},{p,s},{q,s},{r,s},{p,q,s},{q,r,s},{p,r,s}. It is a J-continuous function but not a contra J-irresolute function. Since for the J-open set {r,s} in (Z,σ),the inverse image {r,s} is not J-closed in (Y,ζ).

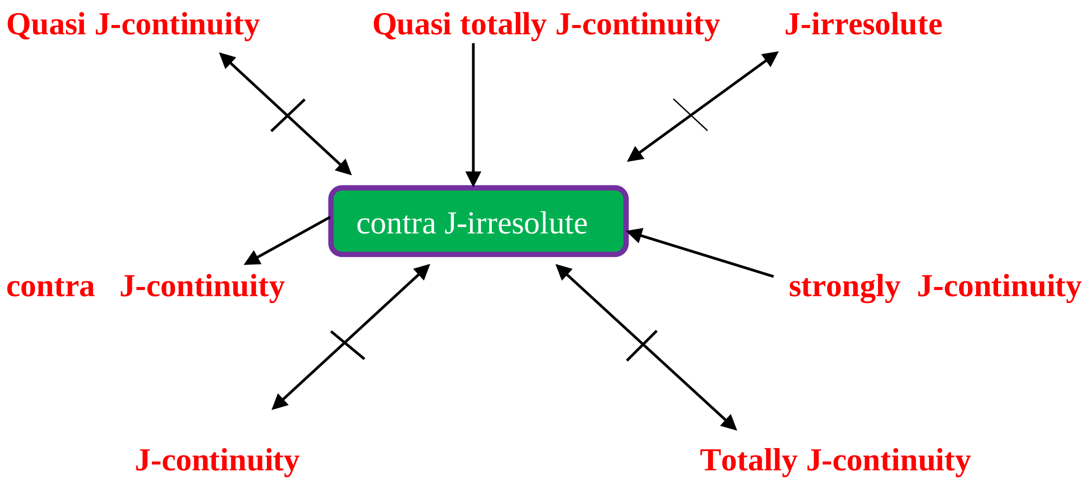
Counter Example 6.3.19. The **Counter Example 6.3.6.** is a contra J-irresolute function but not a J-continuous function.Since for an open set {p,q} in (Z,σ),the inverse image {p,r,s} is not J-open in (Y,ζ).

Remark 6.3.20. Quasi J-continuous functions and contra J-irresolute functions are independent.

Counter Example 6.3.21. Let f : (Y,ζ)→ (Z,σ) be the function defined by f(p) = p,f(q) = p,f (r) = q, f(s) = r . Consider Y = {p,q,r,s}, Z = {p,q,r} with ζ = {Y, ϕ,{r},{p,q},{p,q,r}} and σ = {Z, ϕ,{p},{q},{p,q}}.Here JO(Y,ζ) = {Y, ϕ, {p},{q},{r},{p,q},{q,r},{p,r},{p,q,r}}and ζ^c = { Y, ϕ,{p,q,s},{r,s},{s}}, JC(Z,σ) = { Z, ϕ,{r},{p,r},{q,r}}. It is a quasi J-continuous function but not a contra J-irresolute function. Since for the J-closed set {r} in (Z,σ), the inverse image {s} is not J-open in (Y,ζ).

Counter Example 6.3.22. The **Counter Example 6.3.6.** is a contra J-irresolute function but not a quasi J-continuous function. Because for the J-closed set {r} in (Z,σ),the inverse image {q} is not closed in (Y,ζ).

From the above discussions we get the pictorial representation.



Composition of Contra J-Irresolute Functions

Proposition 6.3.23. The composition g ∘ f : (Y,ζ)→ (P, μ) of two contra J-irresolute functions f: (Y,ζ)→ (Z,σ) and g : (Z,σ) →(P,μ) is a J-irresolute function.

Proof Let U be any J-closed set in (P,μ). Since g is contra J-irresolute, g^{-1}(U) is J-open in (Z,σ).Since f is contra J-irresolute, $f^{-1}(g^{-1}(U))$ is J-closed in (Y,ζ). Hence $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is J-closed in (Y,ζ). Therefore g ∘ f is a J-irresolute function.

Proposition 6.3.24. If f: (Y,ζ)→ (Z,σ) is a J-irresolute function and g : (Z,σ) →(P,μ) is a contra J-irresolute function,then their composition g ∘ f is a contra J-irresolute function.

Proof Let U be any J-closed set in (P,μ). Since g is contra J-irresolute, g^{-1}(U) is J-open in (Z,σ).Since f is J-irresolute , $f^{-1}(g^{-1}(U))$ is J-open in (Y,ζ). Hence $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is J-open in (Y,ζ). Therefore g ∘ f is a contra J-irresolute function.

Proposition 6.3.25. If f: (Y,ζ)→ (Z,σ) is a contra J-irresolute function and g : (Z,σ) →(P,μ) is a J-irresolute function, then their composition g ∘ f is a contra J-irresolute function.

Proof Let U be any J-closed set in (P,μ). Since g is J-irresolute, g^{-1}(U) is J-closed in (Z,σ).Since f is contra J-irresolute, $f^{-1}(g^{-1}(U))$ is J-open in (Y,ζ). Hence $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is J-open in (Y,ζ). Therefore g ∘ f is a contra J-irresolute function.

Proposition 6.3.26. If f: (Y,ζ)→ (Z,σ) is a Quasi totally J-continuous function and g : (Z,σ) →(P,μ) is a contra J-irresolute function, then their composition g ∘ f is a Quasi totally J-continuous function.

Proof Let U be any J-open set in (P,μ). Since g is contra J-irresolute, g^{-1}(U) is J-closed in (Z,σ).Since f is Quasi totally J-continuous, $f^{-1}(g^{-1}(U))$ is clopen in (Y,ζ). Hence $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is clopen in (Y,ζ).Therefore g ∘ f is a Quasi totally J-continuous function.