Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University) Coimbatore-641 043

Master's Degree Examination – November 2018
III Semester

Class : II PG **Time : 3 Hrs**
Major : Mathematics **Max. Marks : 60**

17MMAC15 COMBINATORICS
Part-A **10 x 1/2=5**
Choose the correct answer

1. If one event can occur in m ways and another event can occur in n ways, then the number of ways in which one of these two events can occur is equal to _____

a. m x n b. m/n c. m + n d. m - n

2. The number of ways to arrange r objects when they are selected out of distinct objects with unlimited repetitions is _____

a. n^r b. r^n c. $n+r$ d. $n-r$

3. An exponential generating function that gives the number of combinations or permutations is called an_____

a. enumerator b. exponential enumerator c. ordinary enumerator d. identifier

4. The number of ways of placing r distinct objects into n non-distinct cells with no cell left empty is equal to _____

a. $P(n,r)$ b. $C(n,r)$ c. $S(n,r)$ d. $P(n)$

5. The recurrence relation for the Fibonacci sequence starting with $a_0 = 1, a_1 = 1$ is given by

a. $a_n = a_{n-1} + a_{n-2}$ b. $a_n = a_{n-1} - a_{n-2}$ c. $a_n = a_{n-1} + a_{n-2} + 1$ d. $a_n = a_{n-1}a_{n-2}$

6. The recurrence relation representing the sequence $1,5,5^2,5^3,....,5^n,....$ is _____

a. $a_n = 5a_{n-1}, a_0 = 5$ b. $a_n = 5a_{n-1}, a_0 = 1$
c. $a_{n-1} = 5a_n, a_0 = 5$ d. $a_{n-1} = 5a_n, a_0 = 1$

7. The expansion formula of the rook polynomial is given by _____

a. $R(x,C) = x^2 R(x,C_i) + R(x,C_e)$ b. $R(x,C) = xR(x,C_i) - R(x,C_e)$
c. $R(x,C) = xR(x,C_i) \times R(x,C_e)$ d. $R(x,C) = xR(x,C_i) + R(x,C_e)$

8. The number of objects that have neither the property a_i and nor the property a_j is denoted by _____

a. $N(a_i a_j)$ b. $N(a_i' a_j')$ c. $N(a_i' a_j)$ d. $N(a_i a_j')$

9. The composition of permutation is _____

a. commutative b. not commutative c. not associative d. not defined

10. If $\pi = \begin{pmatrix} a & b & c & d \\ b & a & c & d \end{pmatrix}$ is a permutation of the set $\{a,b,c,d\}$, then $\psi(\pi) =$ ____

a. 0 b. 1 c. 2 d. 4

Part B 5 X 4=20

Answer the following

Answer should not exceed 200 words or one page

11.a. In how many ways can n people stand to form a ring?

(Or)

11.b. Five distinct letters are to be transmitted through a communications channel. A total of 15 blanks are to be inserted between the letters with at least three blanks between every two letters. In how many ways can the letters and the blanks be arranged?

12.a. Prove the identity: $\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots + \binom{n}{r}^2 + \ldots + \binom{n}{n}^2 = \binom{2n}{n}$.

(Or)

12.b. Show by an example that the transposition of the Ferrers graph of a partition having exactly m parts is the Ferrers graph of a partition having m as the largest part.

13.a. Solve the recurrence relation $a_n + 5a_{n-1} = 0,\ a_0 = 6$.

(Or)

13.b. Explain the tower of Hanoi problem.

14.a. Find the number of permutations of the letters a, b, c, d, e and f in which neither the pattern ace nor the pattern fd appears.

(Or)

b. In how many ways can the letters a, a, a, a, b, b, b, c and c be arranged so that all the letters of the same kind are not in a single block?

15.a. Prove that the binary relation on a set induced by a permutation group of the set is an equivalence relation.

(Or)

b. With usual notations, prove that $(\pi_1\pi_2)^{(i)} = \pi_1^{(i)}\pi_2^{(i)}$.

Part C 5 x 7=35

Answer the following

Answer should not exceed 600 words or three pages

16.a. (i) What is the number of n-digit binary sequences that contain an even number of 0's (zero is considered as an even number) ? .

(ii) Among 10 billion numbers between 1 and 10,000,000,000, how many of them contain the digit 1 ?

(Or)

b. State and prove Stirling's formula.

17.a. Show that the ordinary generating function of the sequence $\binom{0}{0}, \binom{2}{1}, \binom{4}{2}, \binom{6}{3}, \binom{2r}{r}, \ldots$ is $(1-4x)^{-\frac{1}{2}}$

(Or)

b. Evaluate the sum $\sum_{i=0}^{r} \frac{r!}{(r-i+1)!(i+1)!}$

18.a. Solve the recurrence relation $a_n + 2a_{n-1} = n+3$ with $a_0 = 3$.

(Or)

b. Find the number of n-digit binary sequences that have the pattern 010 occuring at the n th digit.

19.a. Prove the formula $e_m = s_m - \binom{m+1}{1} s_{m+1} + \binom{m+2}{2} s_{m+2} - \ldots + (-1)^{r-m} \binom{r}{r-m} s_r$ for the number of objects that have exactly m of the r properties for $m = 0, 1, \ldots, r$.

(Or)

b. What are rook polynomials? Describe in detail with illustrations.

20.a. State and prove Polya's fundamental theorem.

(Or)

b. Find the distinct ways of painting the eight vertices of a cube with two colors x and y.