Avinashilingam Institute for Home Science and Higher Education for Women
Coimbatore- 641 043
Master's Degree Examination – November 2017
Semester I

Class : I PG **Max. Marks : 60**
Majors : Mathematics **Time : 3 Hours**

17MMAC02 Real Analysis

Part – A **10x½/ = 5**

Choose the Correct Answer

1. If $P \subseteq P'$, then ----------------
   a. $\|P\| \le \|P'\|$ b. $\|P'\| \le \|P\|$
   c. $\|P'\| < \|P\|$ d. $\|P\| < \|P'\|$

2. If a<b, then $\int_b^a f d\alpha$ = -----------------.
   a. $-\int_a^b (f\alpha) d\alpha$ b. $\int_a^b f d\alpha$ c. $-\int_b^a f d\alpha$ d. none of these

3. $\int_a^b (f+g) d\alpha$ -------------- $\int_a^b f d\alpha + \int_a^b g d\alpha$
   a. $\ge$ b. = c. $\le$ d. >

4. $[x]$ is the unique integer satisfying ---------------
   a. $[x]+1 \le x < [x]$ b. $[x] \le x \le [x]+1$
   c. $[x] \le x < [x]+1$ d. $[x]+1 \le x \le [x]$

5. Assume that $\alpha \nearrow$ on [a,b] .Then $\underline{I}(f, \alpha)$ -------------- $\overline{I}(f, \alpha)$.
   a. > b.< c.$\ge$ d. $\le$

6. S (p,f, α) = ---------------
   a. $\sum_{k=1}^{n} f(t_k) \Delta\alpha_k$ b. $\sum_{k=1}^{n} f(\alpha_k)\Delta t_k$ c. $\sum_{k=1}^{n} \alpha_k t_k$ d. $\sum_{k=1}^{n} f(d_k) t_k$

7. The integral of a step function over the subinterval $[x_{k-1}, x_k]$ is defined by $\int_{x_{k-1}}^{x_k} s(x)\,dx$ = ------------------.
   a. $\sum_{k=1}^{n} c_k (x_{k-1} - x_k)$ b. $\sum_{k=1}^{n} c_k (x_k - x_{k-1})$
   c. $\sum_{k=1}^{n} x_k (c_{k-1} - c_k)$ d. $\sum_{k=1}^{n} x_k (c_k - c_{k-1})$

8. The class U(I) is ------------------ the class of Riemann-intergrable functions on I.
   a. Smaller than b. larger than c. the same as d. not related to

9. The positive part f^+, of a real-valued function f, is defined by f^+ = ------------------
   a. Max (-f,o) b. –max (f,o) c. max (f,o) d. none of these

10. The ganna function is defined by Γ (y)= ------------------.

a) $\int_0^\infty e^{-x} x^{y-1} dx$ b. $\int_0^\infty e^{-y} y^{x-1} dy$

c. $\int_0^1 e^{-y} y^{x-1} dy$ d. $\int_0^1 e^{-x} x^{y-1} dx$

Part B **5x4=20**

Answer All Questions

11.a. Assume that c $\in$ (a,b). If two of the three integrals $\int_a^c f d\alpha$, $\int_c^b f d\alpha$ and $\int_a^b f d\alpha$ exist, prove that the third also exists and $\int_a^c f d\alpha + \int_c^b d\alpha = \int_a^b f d\alpha$. (Or)

b. If $f \in R(\alpha)$ and if $g \in R(\alpha)$ on [a,b], prove that $c_1 f + c_2 g \in R(\alpha)$ on [a,b] and

$\int_a^b (c_1 f + c\ g)\, d\alpha = c_1 \int_a^b f d\alpha + c_2 \int_a^b g d\alpha$

12a. Define a step function. (Or)

b.Prove that every finite sum can be written as Riemann-stieltjes integral.

13. a.Assume that $\alpha \nearrow$ on [a,b]. If $f \in R(\alpha)$ and $g \in R(\alpha)$ on [a,b] and if $f(x) \le g(x)$ for all in [a,b], prove that $\int_a^b f(x)\ d(\alpha)(x) \le \int_a^b g(x)\ d\alpha(x)$. (Or)

b.If f is contiuous on [a,b] and if $\alpha$is of bounded variation on [a,b] , prove that $f \in R(\alpha)$ On [a,b].

14. a.Let $\{t_n\}$ be a sequence of step functions on an internal I such that (i) There is a function f such that $t_n \uparrow f$ a.e on I and (ii) $\{\int_I^a t_n\}$ converge.Prove that for any any step function t such that $t(x) \le f(x)$ a.e. on I and $\int_I t \le \lim_{n\to\infty} \int_I tn$. (Or)

b.Define the integral of a stpe function over a general intterval I.

15. a.Define a Lebesgue integrable function on a general interval I. (Or)

b.State and prove Levi theorem for sequences of Lebesgue-integrable functions.

Part-C **5x7=35**

Answer All Questions.

16. a) State and prove the formala for intergration by parts. (Or)

b.Assume $f \in R(\alpha)$ on [a,b] and assume that α has a continuous derivative α^1 on [a,b]. Prove that the Riemenn integral $\int_a^b f(x)\ \alpha'(x)\ dx$ exists and

$\int_a^b f(x)\, d\alpha(x) = \int_a^b f(x)\, \alpha'(x)\, dx.$

17. a. Assume that $\alpha \uparrow$ on [a,b]. Prove that

i) If p' is finer than p, $U(P', f, \alpha) \le U(P, f, \alpha)$ and $L(P', f, \alpha) \ge L(P, f, \alpha)$

ii) For any two partitions P_1 and P_2, $L(P_1, f, \alpha) \le U(P_2, f, \alpha)$.

(Or

b. State and prove Euler's summation formuls.

18. a. Assume that α is of bounded variation on [a,b]. Let $V(x)$ denote the total variation of α on [a,x] if $a < x \le b$, and $V(a)=0$; f be defined and bounded on [a,b]. If $f \in R(\alpha)$ on [a,b], prove that $f \in R(v)$ on [a,b]. (Or)

b. State and prove the first mean-value theorem for Riemann-stieltjes integrals.

19. a. Let $\{s_n\}$ be a decreating sequence of nonnegative step functions such that $s_n \searrow 0$ a.e. on an interval I. Prove that $\lim\limits_{n \to \infty} \int_I s_n = 0$. (Or)

b. Let f be defined and bounded on a compact interval [a,b], and assume that f is continous almost everywhere on [a,b]. Prove that $f \in U([a,b])$ and the intergral of f, as a function in $U([a,b])$, is equal to the Riemann integral $\int_a^b f(x)\,dx$.

20. a. State and prove Levi theorem for step functions. (Or)

c. If f and g are in L (I), prove that the functions f^+, f^-, $|f|$, max (f,g) and min (f,g) are also in L(I). Also prove that $\left|\int_I f\right| \le \int_I |f|$.