CHAPTER - III

INTUITIONISTIC FUZZY SOFT MATRICES

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Definition : 3.1

Let $U = \{c_1, c_2, c_3 \ldots c_m\}$ be the universal set and $E = \{e_1, e_2, e_3 \ldots e_n\}$ be the set of parameters. Let $A \subseteq E$ and (F, A) be a intuitionistic fuzzy soft set in the fuzzy soft class (U, E). Then intuitionistic fuzzy soft set (F, A) can be represented in matrix form as $S_{mxn} = [a_{ij}]_{mxn}$ or $S = [a_{ij}]$ i =1, 2, 3,…,m, j=1, 2, 3,….,n, where

$$a_{ij} = \begin{cases} (\mu_j(c_i), \nu_j(c_i)) & \text{if } e_j \in A \\ (0,1) & \text{if } e_j \notin A \end{cases}$$

Here $\mu_j(c_i)$ represents the membership of c_i in the intuitionistic fuzzy set $F(e_j)$ and $\nu_j(c_i)$ represents the non-membership of c_i in the intuitionistic fuzzy set $F(e_j)$. The matrix $S_{m\times n}$ is called **intuitionistic fuzzy soft matrix**. This matrix S can also be written as $S = (\mu_A, \nu_A)$ or $S = [(\mu_{ij}, \nu_{ij})]$.

Example : 3.2

Suppose that $U = \{s_1, s_2, s_3, s_4\}$ is a set of students and $E = \{e_1, e_2, e_3\}$ is a set of parameters, which stand for result, conduct and sports performances respectively. Consider the mapping from parameters set $A \subseteq E$ to the set of all intuitionistic fuzzy subsets of power set U. Then soft set (F, A) describes the character of the students with respect to the given parameters, for finding the best student of an academic year. Consider $A=\{ e_1, e_2\}$ then intuitionistic fuzzy soft set is

$(F, A) = \{ F(e_1) = \{(s_1,0.8,0.1), (s_2,0.3,0.6), (s_3,0.8,0.2), (s_4,0.9,0.0)\},$

$F(e_2) = \{(s_1,0.8,0.1), (s_2,0.9,0.1), (s_3,0.4,0.5), (s_4,0.3,0.6)\}.$

we would represent this intuitionistic fuzzy soft set in matrix form is

$$\begin{bmatrix} (0.8,0.1) & (0.8,0.1) & (0.0,1.0) \\ (0.3,0.6) & (0.9,0.1) & (0.0,1.0) \\ (0.8,0.2) & (0.4,0.5) & (0.0,1.0) \\ (0.9,0.0) & (0.3,0.6) & (0.0,1.0) \end{bmatrix}$$

Definition : 3.3

Let A = $[a_{ij}] \in IFSM_{mxn}$, B = $[b_{ij}] \in IFSM_{mxn}$. Then A is a intuitionistic fuzzy soft submatrix of B, denoted by $A \subseteq B$ if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$ $\forall$ i, j and A is equal to B, denoted by A = B if $\mu_A = \mu_B$ and $\nu_A = \nu_B$ $\forall$ i, j.

Definition : 3.4

Let A = $[a_{ij}] \in IFSM_{mxn}$. Then A is a

1) **intuitionistic fuzzy soft null(zero) matrix**, if all its elements are (0,0). It is denoted by φ or $0 = [(0,0)]$.
2) **intuitionistic fuzzy soft universal matrix**, if all its elements are (0,1). It is denoted by U.
3) **intuitionistic fuzzy soft rectangular matrix** if $m \neq n$.
4) **intuitionistic fuzzy soft square matrix** if m = n.
5) **intuitionistic fuzzy soft row matrix** if m = 1.
6) **intuitionistic fuzzy soft row matrix** if n = 1.
7) **intuitionistic fuzzy soft diagonal matrix** if m = n and a_{ij} = (0, 1) for all $i \neq j$.
8) **intuitionistic fuzzy soft scalar matrix** if m = n and a_{ij} = (0, 1) for all $i \neq j$ and $a_{ij} = (\alpha, \beta)$, $\alpha \in [0,1]$, $\beta \in [0,1]$ $\forall i = j$.
9) **intuitionistic fuzzy soft upper triangular matrix** if m = n and a_{ij} = (0, 1) for all $i > j$.
10) **intuitionistic fuzzy soft lower triangular matrix** if m = n and a_{ij} = (0, 1) for all $i < j$.

11) **intuitionistic fuzzy soft triangular matrix** if it is either intuitionistic fuzzy soft lower triangular matrix or intuitionistic fuzzy soft upper triangular matrix.

Definition : 3.5

Let A = $[a_{ij}] \in IFSM_{mxn}$. Then A^T is a **intuitionistic fuzzy soft transpose matrix** of A if $A^T = [a_{ji}]$.

Definition : 3.6

If A = $[a_{ij}] \in IFSM_{mxn}$, B = $[b_{ij}] \in IFSM_{mxn}$, then we define A+B, **addition** of A and B as

$$A+B = [c_{ij}]_{mxn}$$

$$= (\max(\mu_A, \mu_B), \min(\nu_A, \nu_B)) \ \forall \ i, j.$$

Example : 3.7

Consider $A = \begin{bmatrix} (0.8,0.1) & (0.4,0.5) \\ (0.7,0.3) & (0.4,0.6) \end{bmatrix}_{2\times2}$ $B = \begin{bmatrix} (0.6,0.3) & (0.8,0.2) \\ (0.7,0.3) & (0.5,0.5) \end{bmatrix}_{2\times2}$ are two intuitionistic fuzzy soft matrices, then the sum of these two is

$$A + B = \begin{bmatrix} (0.8,0.1) & (0.8,0.2) \\ (0.7,0.3) & (0.5,0.5) \end{bmatrix}_{2\times2}$$

Definiton : 3.8

If A = $[a_{ij}] \in IFSM_{mxn}$, B = $[b_{ij}] \in IFSM_{mxn}$, then we define A-B, **subtraction** of A and B as

$$A-B = [c_{ij}]_{mxn}$$

$$= (\min(\mu_A, \mu_B), \max(\nu_A, \nu_B)) \ \forall \ i, j$$

Example : 3.9

Consider $A = \begin{bmatrix} (0.8,0.1) & (0.4,0.5) \\ (0.7,0.3) & (0.4,0.6) \end{bmatrix}_{2\times2}$ $B = \begin{bmatrix} (0.6,0.3) & (0.8,0.2) \\ (0.7,0.3) & (0.5,0.5) \end{bmatrix}_{2\times2}$ are two intuitionistic fuzzy soft matrices, then the subtraction of these two is

$$A - B = \begin{bmatrix} (0.6,0.3) & (0.4,0.5) \\ (0.7,0.3) & (0.4,0.6) \end{bmatrix}_{2\times2}$$

Definition : 3.10

If $A = [a_{ij}] \in IFSM_{mxn}$, $B = [b_{jk}] \in IFSM_{n\times p}$, then we define $A * B$, **multiplication** of A and B as

$$A * B = [\, c_{ik}]_{mxp}$$

$$= (\max\min(\mu_{Aj}, \mu_{Bj}), \min\max(\nu_{Aj}, \nu_{Bj})) \quad \forall\ i, j.$$

Example : 3.11

Consider $A = \begin{bmatrix} (0.8,0.1) & (0.4,0.5) \\ (0.7,0.3) & (0.4,0.6) \end{bmatrix}_{2\times2}$ and $B = \begin{bmatrix} (0.6,0.3) & (0.8,0.2) \\ (0.7,0.3) & (0.5,0.5) \end{bmatrix}_{2\times2}$ are two intuitionistic fuzzy soft matrices, then the product of these two matrices is

$$A * B = \begin{bmatrix} (0.6,0.3) & (0.8,0.2) \\ (0.6,0.3) & (0.7,0.3) \end{bmatrix}_{2\times2}$$

Remark : 3.12

$$A * B \neq B * A$$

Theorem : 3.13

Let $A = [a_{ij}] \in IFSM_{mxn}$, $B = [b_{ij}] \in IFSM_{mxn}$, $C = [c_{ij}] \in IFSM_{mxn}$ then

1) $\varphi \subseteq A$

2) $A \subseteq U$

3) $A \subseteq A$

4) $A \subseteq B, B \subseteq C \Rightarrow A \subseteq C$

5) $A + \varphi = A$

6) $A + U = U$

7) $A + B = B + A$

8) $(A + B) + C = A + (B + C)$

Definition : 3.14

Let $A = [a_{ij}] = [(\mu_A, \nu_A)] = [(\mu_{ij}', \nu_{ij}')]$, $B = [b_{ij}] = [(\mu_B, \nu_B)] = [(\mu_{ij}'', \nu_{ij}'')] \in IFSM_{m\times n}$. Then the IFSM $C = [c_{ij}] = [(\mu_C, \nu_C)] = [(\mu_{ij}, \nu_{ij})]$ is called

1) **union** of A and B, denoted by $A \cup B$ if $\mu_C = \max\{\mu_A, \mu_B\} = \max\{\mu_{ij}', \mu_{ij}''\}$ and $\nu_C = \min\{\nu_A, \nu_B\} = \min\{\nu_{ij}', \nu_{ij}''\}$ for all i and j.
2) **intersection** A and B, denoted by $A \cap B$ if $\mu_C = \min\{\mu_A, \mu_B\} = \min\{\mu_{ij}', \mu_{ij}''\}$ and $\nu_C = \max\{\nu_A, \nu_B\} = \max\{\nu_{ij}', \nu_{ij}''\}$ for all i and j.
3) **complement** of $A = [(\mu_A, \nu_A)] = [(\mu_{ij}', \nu_{ij}')]$, denoted by $A^C = [(\nu_A, \mu_A)] = [(\nu_{ij}', \mu_{ij}')]$ for all i and j.

Example : 3.15

Let $A = \begin{bmatrix} (.1,.2) & (.5,.4) & (.3,.6) \\ (.4,.4) & (.2,.3) & (.5,.1) \\ (.5,.2) & (.3,.4) & (.6,.2) \\ (.7,.2) & (.6,.1) & (.5,.3) \end{bmatrix}$ and $B = \begin{bmatrix} (.5,.3) & (.1,.6) & (.7,.1) \\ (.8,.1) & (.4,.3) & (.5,.2) \\ (.2,.5) & (.3,.6) & (.4,.5) \\ (.1,.7) & (.2,.5) & (.5,.1) \end{bmatrix}$

Then $A \cup B = \begin{bmatrix} (.5,.2) & (.5,.4) & (.7,.1) \\ (.8,.1) & (.4,.3) & (.5,.1) \\ (.5,.2) & (.3,.4) & (.6,.2) \\ (.7,.2) & (.6,.1) & (.5,.1) \end{bmatrix}$ $A \cap B = \begin{bmatrix} (.1,.3) & (.1,.6) & (.3,.6) \\ (.4,.4) & (.2,.3) & (.5,.2) \\ (.2,.5) & (.3,.6) & (.4,.5) \\ (.1,.7) & (.2,.5) & (.5,.3) \end{bmatrix}$

$$A^C = \begin{bmatrix} (.2,.1) & (.4,.5) & (.6,.3) \\ (.4,.4) & (.3,.2) & (.1,.5) \\ (.2,.5) & (.4,.3) & (.2,.6) \\ (.2,.7) & (.1,.6) & (.3,.5) \end{bmatrix}$$

Theorem : 3.16

Let $A = [(\mu_A, \nu_A)]$, $B = [(\mu_B, \nu_B)] \in IFSM_{mxn}$. Then

1) $(A \cup B)^C = A^C \cap B^C$
2) $(A \cap B)^C = A^C \cup B^C$

Proof :

1) $(A \cup B)^C = ([(\mu_A, \nu_A)] \cup [(\mu_B, \nu_B)])^C$

$= [(\max\{\mu_A, \mu_B\}, \min\{\nu_A, \nu_B\})]^C$

$= [(\min\{\nu_A, \nu_B\}, \max\{\mu_A, \mu_B\})]$

$= [(\nu_A, \mu_A)] \cap [(\nu_B, \mu_B)]$

$= A^C \cap B^C$

2) Similar to (1).

Theorem : 3.17

Let $A = [a_{ij}]$, $B = [b_{ij}]$, and $C = [c_{ij}] \in IFSM_{mxn}$. Then

1) $A \cap B = B \cap A$
2) $A \cup B = B \cup A$
3) $(A \cap B) \cap C = A \cap (B \cap C)$
4) $(A \cup B) \cup C = A \cup (B \cup C)$
5) $A \cup (B \cap C) = (A \cap B) \cap (A \cup C)$

6) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Example : 3.18

Let $A, B \in IFSM_{4\times3}$ as in the example 3.16. Then

$$(A \cup B)^C = A^C \cap B^C = \begin{bmatrix} (.2,.5) & (.4,.5) & (.1,.7) \\ (.1,.8) & (.3,.4) & (.1,.5) \\ (.2,.5) & (.4,.3) & (.2,.6) \\ (.2,.7) & (.1,.6) & (.1,.5) \end{bmatrix}$$

$$(A \cap B)^C = A^C \cup B^C = \begin{bmatrix} (.3,.1) & (.6,.1) & (.6,.3) \\ (.4,.4) & (.3,.2) & (.2,.5) \\ (.5,.2) & (.6,.3) & (.5,.4) \\ (.7,.1) & (.5,.2) & (.3,.5) \end{bmatrix}$$

Theorem : 3.19

Let $A = [a_{ij}] \in IFSM_{mxn}$, $B = [b_{ij}] \in IFSM_{mxn}$ then

1) $(A^C)^C = A$
2) $\varphi^C = U$
3) $(A + U)^C = \varphi$
4) $(A + B)^C = (B + A)^C$

Definition : 3.20

If $A = [a_{ij}]$, $B = [b_{ij}] \in IFSM_{mxn}$. Then the IFSM $C = [c_{ij}]$ is called

1) the '$\cdot$' operation of A and B, denoted by $C = A \cdot B$ if $\mu_C = \mu_A \cdot \mu_B$ and $\nu_C = \nu_A + \nu_B - \nu_A \cdot \nu_B$ for all i and j.
2) the '+' operation of A and B, denoted by $C = A + B$ if $\mu_C = \mu_A + \mu_B - \mu_A \cdot \mu_B$ and $\nu_C = \nu_A \cdot \nu_B$ for all i and j.

3) the '@' operation of A and B, denoted by C = A@B

if $\mu_C = \frac{\mu_A + \mu_B}{2}$, $\nu_C = \frac{\nu_A + \nu_B}{2}$ for all i and j.

4) the '\$' operation of A and B, denoted by C = A\$B

if $\mu_C = \sqrt{\mu_A \cdot \mu_B}$, $\nu_C = \sqrt{\nu_A \cdot \nu_B}$ for all i and j.

Theorem : 3.21

Let $A = [a_{ij}]$, $B = [b_{ij}]$ and $C = [c_{ij}] \in IFSM_{m\times n}$. Then

1) $A \cdot B = B \cdot A$
2) $A + B = B + A$
3) $A@B = B@A$
4) $A\$B = B\$A$
5) $(A+B) + C = A+(B+C)$
6) $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
7) $(A \cap B) + C = (A + C) \cap (B + C)$
8) $(A \cap B) \cdot C = (A \cdot C) \cap (B \cdot C)$
9) $(A \cap B)@C = (A@C) \cap (B@C)$
10) $(A \cup B) + C = (A + C) \cup (B + C)$
11) $(A \cup B) \cdot C = (A \cdot C) \cup (B \cdot C)$

Definition : 3.22

Let $A = [(\mu_{ij}', \nu_{ij}')]$, $B = [(\mu_{ik}'', \nu_{ik}'')] \in IFSM_{mxn}$. Then '$X_1$' product of A and B is defined by $X_1 : IFSM_{m\times n} \times IFSM_{m\times n} \rightarrow IFSM_{m\times n^2}$ such that $A X_1 B = [(\mu_{ij}', \nu_{ij}')] X_1 [(\mu_{ik}'', \nu_{ik}'')] = [(\mu_{ip}, \nu_{ip})]$ where $\mu_{ip} = \mu_{ij}' \cdot \mu_{ik}''$ and $\nu_{ip} = \nu_{ij}' \cdot \nu_{ik}''$ such that $p = n(j - i) + k$.

Definition : 3.23

Let $A = [(\mu_{ij}', \nu_{ij}')]$, $B = [(\mu_{ik}'', \nu_{ik}'')] \in IFSM_{mxn}$. Then '$X_2$' product of A and B is defined by $X_2 : IFSM_{m\times n} \times IFSM_{m\times n} \rightarrow IFSM_{m\times n^2}$ such that $A X_2 B =$

$[(\mu_{ij}', \nu_{ij}')]\ X_2\ [(\mu_{ik}'', \nu_{ik}'')] = [(\mu_{ip}, \nu_{ip})]$ where $\mu_{ip} = \mu_{ij}' + \mu_{ik}'' - \mu_{ij}' \cdot \mu_{ik}''$ and $\nu_{ip} = \nu_{ij}' \cdot \nu_{ik}''$ such that $p = n(j - i) - k$.

Definition : 3.24

Let $A = [(\mu_{ij}', \nu_{ij}')]$, $B = [(\mu_{ik}'', \nu_{ik}'')] \in IFSM_{mxn}$. Then '$X_3$' product of A and B is defined by $X_3 : IFSM_{m\times n} \times IFSM_{m\times n} \rightarrow IFSM_{m\times n^2}$ such that $A\ X_3\ B = [(\mu_{ij}', \nu_{ij}')]\ X_3\ [(\mu_{ik}'', \nu_{ik}'')] = [(\mu_{ip}, \nu_{ip})]$ where $\mu_{ip} = \mu_{ij}' \cdot \mu_{ik}''$ and $\nu_{ip} = \nu_{ij}' + \nu_{ij}'' - \nu_{ij}' \cdot \nu_{ij}''$ such that $p = n(j - i) + k$.

Definition : 3.25

Let $A = [(\mu_{ij}', \nu_{ij}')]$, $B = [(\mu_{ik}'', \nu_{ik}'')] \in IFSM_{mxn}$. Then '$X_4$' product of A and B is defined by $X_4 : IFSM_{m\times n} \times IFSM_{m\times n} \rightarrow IFSM_{m\times n^2}$ such that $A\ X_4\ B = [(\mu_{ij}', \nu_{ij}')]\ X_4\ [(\mu_{ik}'', \nu_{ik}'')] = [(\mu_{ip}, \nu_{ip})]$ where $\mu_{ip} = \min\{\mu_{ij}', \mu_{ik}''\}$ and $\nu_{ip} = \max\{\nu_{ij}', \nu_{ik}''\}$ such that $p = n(j - i) + k$.

Definition : 3.26

Let $A = [(\mu_{ij}', \nu_{ij}')]$, $B = [(\mu_{ik}'', \nu_{ik}'')] \in IFSM_{mxn}$. Then '$X_5$' product of A and B is defined by $X_5 : IFSM_{m\times n} \times IFSM_{m\times n} \rightarrow IFSM_{m\times n^2}$ such that $A\ X_5\ B = [(\mu_{ij}', \nu_{ij}')]\ X_5\ [(\mu_{ik}'', \nu_{ik}'')] = [(\mu_{ip}, \nu_{ip})]$ where $\mu_{ip} = \max\{\mu_{ij}', \mu_{ik}''\}$ and $\nu_{ip} = \min\{\nu_{ij}' \cdot \nu_{ik}''\}$ such that $p = n(j - i) + k$.

Theorem : 3.27

Let $A = [(\mu_{ij}, \nu_{ij})]$, $B = [(\mu_{ij}', \nu_{ij}')]$, $C = [(\mu_{ik}'', \nu_{ik}'')] \in IFSM_{mxn}$. Then

1) $(A \times B) \times C = A \times (B \times C)$
2) $(A \cup B) \times C = (A \times C) \cup (B \times C)$
3) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

Example : 3.28

Let A , B $\in$ $IFSM_{4\times3}$ as in the example 3.16. Then

$$A\,X_1\,B = \begin{bmatrix} (.05,.06) & (.01,.12) & (.07,.02) & (.25,.12) & (.05,.24) & (.35,.04) & (.15,.18) & (.03,.36) & (.21,.06) \\ (.32,.04) & (.12,.16) & (.20,.08) & (.16,.03) & (.08,.09) & (.10,.06) & (.40,.01) & (.20,.03) & (.25,.02) \\ (.10,.10) & (.15,.12) & (.20,.10) & (.06,.20) & (.09.24) & (.12,.20) & (.12,.10) & (.18,.12) & (.24,.10) \\ (.07,.14) & (.14,.10) & (.35,.02) & (.06,.07) & (.12,.05) & (.30,.01) & (.05,.21) & (.10,.15) & (.25,.03) \end{bmatrix}$$

$$A\,X_2\,B = \begin{bmatrix} (.55,.06) & (.19,.12) & (.73,.02) & (.75,.12) & (.55,.24) & (.85,.04) & (.65,.18) & (.37,.36) & (.79,.06) \\ (.88,.04) & (.64,.12) & (.7,.08) & (.84,.03) & (.52,.09) & (.06,.06) & (.90,.01) & (.7,.03) & (.75,.02) \\ (.06,.10) & (.65,.12) & (.7,.10) & (.44,.20) & (.51.24) & (.58,.20) & (.68,.10) & (.72,.12) & (.76,.10) \\ (.73,.14) & (.76,.10) & (.85,.02) & (.64,.07) & (.68,.05) & (.80,.01) & (.55,.21) & (.6,.15) & (.75,.03) \end{bmatrix}$$

$$A\,X_3\,B = \begin{bmatrix} (.05,.44) & (.01,.68) & (.07,.28) & (.25,.58) & (.05,.76) & (.35,.46) & (.15,.72) & (.03,.84) & (.21,.64) \\ (.32,.46) & (.16,.58) & (.20,.52) & (.16,.37) & (.08,.51) & (.10,.44) & (.45,.19) & (.20,.37) & (.25,.28) \\ (.01,.06) & (.15,.68) & (.20,.06) & (.06,.07) & (.09.76) & (.12,.07) & (.12,.06) & (.18,.68) & (.24,.06) \\ (.07,.76) & (.14,.06) & (.35,.28) & (.06,.73) & (.12,.55) & (.30,.37) & (.05,.79) & (.10,.65) & (.25,.37) \end{bmatrix}$$

$$A\,X_4\,B = \begin{bmatrix} (.1,.3) & (.1,.6) & (.1,.2) & (.1,.6) & (.1,.6) & (.5,.4) & (.3,.6) & (.1,.6) & (.3,.6) \\ (.4,.4) & (.4,.4) & (.4,.4) & (.4,.4) & (.2,.3) & (.2,.3) & (.5,.1) & (.4,.3) & (.5,.2) \\ (.2,.5) & (.3,.6) & (.4,.5) & (.3,.6) & (.3,.6) & (.3,.5) & (.2,.5) & (.3,.6) & (.4,.5) \\ (.1,.7) & (.2,.5) & (.5,.2) & (.2,.5) & (.2,.5) & (.5,.1) & (.1,.7) & (.2,.5) & (.5,.3) \end{bmatrix}$$

$$A\,X_5\,B = \begin{bmatrix} (.5,.2) & (.1,.2) & (.7,.1) & (.5,.3) & (.5,.4) & (.7,.1) & (.5,.3) & (.3,.6) & (.7,.1) \\ (.8,.1) & (.4,.3) & (.5,.2) & (.8,.1) & (.4,.3) & (.5,.2) & (.8,.1) & (.5,.1) & (.5,.1) \\ (.5,.2) & (.5,.2) & (.5,.2) & (.3,.4) & (.3,.4) & (.4,.4) & (.6,.2) & (.6,.2) & (.6,.2) \\ (.7,.2) & (.7,.2) & (.7,.1) & (.6,.1) & (.6,.1) & (.6,.1) & (.5,.3) & (.5,.3) & (.5,.1) \end{bmatrix}$$

Definition : 3.29

Let A = $[a_{ij}]$ $\in$ $IFSM_{mxn}$, where a_{ij} = $(\mu_j(c_i), \nu_j(c_i))$. Then **scalar multiple** of intuitionistic fuzzy soft matrix A by a scalar k is defined by kA = $[ka_{ij}]$ where $0 \le k \le 1$.

Example : 3.30

Let A = $\begin{bmatrix} (0.8,0.1) & (0.4,0.5) \\ (0.7,0.3) & (0.4,0.6) \end{bmatrix}_{2\times2}$ be intuitionistic fuzzy soft matrix, then the scalar multiple of this matrix by k = 0.5 is $kA = \begin{bmatrix} (0.40,0.05) & (0.2,0.25) \\ (0.35,0.15) & (0.2,0.30) \end{bmatrix}_{2\times 2}$

Theorem : 3.31

Let $A = [a_{ij}] \in IFSM_{mxn}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$, if m, n are two scalars such that $0 \le m, n \le 1$, then

1) $m(nA) = (mn)A$
2) $m \le n \Rightarrow mA \le nA$
3) $A \subseteq B \Rightarrow mA \subseteq mB$

Definition : 3.32

Let $A = [a_{ij}] \in IFSM_{mxn}$, where m = n, $a_{ij} = (\mu_j(c_i),\nu_j(c_i))$. Then **trace** of intuitionistic fuzzy soft matrix A is $\operatorname{tr} A = \sum_{i=1}^{m} a_{ii} = \sum_{i=1}^{m} \mu_{ii} - \nu_{ii}$.

Example : 3.33

Let $A = \begin{bmatrix} (0.8,0.1) & (0.4,0.5) \\ (0.7,0.3) & (0.4,0.6) \end{bmatrix}_{2\times 2}$ be intuitionistic fuzzy soft matrix, then trace of this matrix is tr A = 0.8-0.1+0.4 -0.6 = 0.5

Theorem : 3.34

Let $A = [a_{ij}] \in IFSM_{mxn}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$, if m is two scalar such that $0 \le m \le 1$, then

1) $tr(kA) = k \operatorname{tr} A$
2) $(kA)^T = k A^T$

Definition : 3.35

Let $A = [a_{ij}] \in IFSM_{mxn}$, where $a_{ij} = (\mu_j(c_i), \nu_j(c_i))$. Then we define the **value matrix** of intuitionistic fuzzy soft matrix A is $V(A) = [a_{ij}] = [\mu_{ij} - \nu_{ij}]$ $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$.

Definition : 3.36

If $A = [a_{ij}] \in IFSM_{mxn}$, $B = [b_{ij}] \in IFSM_{mxn}$, then we define **score matrix** of A and B as $S_{(A,B)} = [d_{ij}]_{mxn}$ where $[d_{ij}] = V(A) - V(B)$.

Definition : 3.37

Let $A = [a_{ij}] \in IFSM_{mxn}$, $B = [b_{ij}] \in IFSM_{mxn}$. Let the corresponding value matrices be V(A), V(B) and their score matrix is $S_{(A,B)} = [d_{ij}]_{mxn}$ then we define **total score** for each c_i in U is $S_i = \sum_{j=1}^{n} d_{ij}$.

Methodology

Suppose U is a set of candidates appearing in an interview for appointment in Manager post in a company. Let E is a set of parameters related to managerial level of candidates. We construct IFSS (F,E) over U represent the selection of candidate by field expert X, where F is a mapping $F : E \rightarrow IF^U$, IF^U is the collection of all intuitionistic Fuzzy subsets of U. We further construct another IFSS (G,E) over U represent the selection of candidate by field expert Y, where G is a mapping $G : E \rightarrow IF^U$, IF^U is the collection of all intuitionistic fuzzy subsets of U. The matrices A and B corresponding to the intuitionistic fuzzy soft sets (F,E) and (G,E) are constructed, we compute the complements $(F,E)^c$ and $(G,E)^c$ and their matrices A^c and B^c corresponding to $(F,E)^c$ and $(G,E)^c$ respectively. Compute A+B which is the maximum membership of selection of candidates by the judges. Compute A^c+B^c which is the maximum membership of non selection of candidates by the judges. Using definition(3.35, 3.36 and 3.37), compute V(A+B), V(A^c+B^c), $S_{((A+B),(A^c+B^c))}$ and the total score S_i for each candidate in U. Finally find S_j = max(S_i), then conclude that the candidate c_j has selected by the judges. If S_j has more than one value the process is repeated by reassessing the parameters.

Algorithm for decision making method by using Intuitionistic Fuzzy soft matrix theory.

Step 1 : Input the intuitionistic fuzzy soft set (F,E), (G,E) and obtain the intuitionistic fuzzy soft matrices A, B corresponding to (F,E) and (G,E) respectively .

Step 2 : Write the intuitionistic fuzzy soft complement set $(F,E)^c$, $(G,E)^c$ and obtain the intuitionistic fuzzy soft matrices A^c,B^c corresponding to $(F,E)^c$ and $(G,E)^c$ respectively .Step 3 : Compute (A+B), (A^c+B^c), V(A+B), V(A^c+B^c) and $S_{((A+B),(A^c+B^c))}$

Step 4 : Compute the total score S_i for each c_i in U.

Step 5 : Find c for which max (S_i) .

Then we conclude that the candidate c_i is selected for the post .

Incase max S_i occurs for more than one value, then repeat the process by reassessing the parameters.