# CHAPTER VIII

# INTUITIONISTIC FUZZY NEUTROSOPHIC SOFT SETS AND INTUITIONISTIC FUZZY NEUTROSOPHIC SOFT TOPOLOGICAL SPACES

**Definition:8.1**

A **fuzzy neutrosophic set** $A$ on the universe of discourse $U$ is defined as $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in U\}$ where $T_A, I_A, F_A : U \to [0,1]$ define respectively the degree of truth membership ,the degree of indeterminacy membership and the degree of falsity membership of the elements to the set $A$ with the condition

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

**Definition:8.2**

Let $U$ be an universe of discourse. Let $X$ be a non empty fixed set. An **intuitionistic fuzzy neutrosophic set** A is an object having the form $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in U\}$ where $T_A(x)$ represents the degree of truth membership, $I_A(x)$ represents the degree of indeterminacy membership and $F_A(x)$ represents the degree of falsity membership function respectively of each element $x \in X$ to the set $A$ where

$\min[T_A(x), I_A(x)] \leq 0.5,\ \min[F_A(x), I_A(x)] \leq 0.5,\ \min[T_A(x), F_A(x)] \leq 0.5, \forall x \in U.$

and the function satisfies the condition

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 2$$

**Note:8.3**

An element $x$ of $U$ is called significant with respect to intuitionistic fuzzy neutrosophic set $A$ if the degree of truth membership $T_A(x)$ , the degree of indeterminacy membership $I_A(x)$ and the degree of falsity membership function $F_A(x) \leq 0.5$ . Otherwise it is called insignificant.

**Definition:8.4**

Let $U$ be an initial universe set and $E$ be a set of parameters and a non empty set $A \subset E$. Let $IFN(U)$ denotes the set of all intuitionistic fuzzy neutrosophic sets of $U$. The pair $(F,A)$ is called **intuitionistic fuzzy neutrosophic soft set** over $U$, where $F$ is a mapping given by $F : A \to IFN(U)$

and an intuitionistic fuzzy neutrosophic soft set is denoted as $\widetilde{F}_A$

**Definition:8.5**

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two intuitionistic fuzzy neutrosophic soft sets (In short IFNSS) over the common universe $U$. Then $\widetilde{F}_A$ is said to be **intuitionistic fuzzy neutrosophic soft subset** of $\widetilde{G}_B$ if and only if,

5. $A \subset B$
6. $\widetilde{F}(e)$ is an intuitionistic fuzzy neutrosophic subset of $\widetilde{G}(e)$

or $T_{\widetilde{F}(e)}(x) \leq T_{\widetilde{G}(e)}(x), I_{\widetilde{F}(e)}(x) \leq I_{\widetilde{G}(e)}(x), F_{\widetilde{F}(e)}(x) \geq F_{\widetilde{G}(e)}(x),\ \forall e \in A, \forall x \in U$

This relationship is denoted by $\widetilde{F}_A \subseteq \widetilde{G}_B$

$\widetilde{F}_A$ is said to be **intuitionistic fuzzy neutrosophic soft super set** of $\widetilde{G}_B$ if $\widetilde{G}_B$ is an intuitionistic fuzzy neutrosophic soft subset of $\widetilde{F}_A$. This relationship is denoted by $\widetilde{F}_A \supseteq \widetilde{G}_B$.

**Definition:8.6**

The **complement of an intuitionistic fuzzy neutrosophic soft set** $\widetilde{F}_A$ is denoted by $\widetilde{F}_A^C$ and is defined as $\widetilde{F}_A^C = (F^C, \neg A)$ where $\widetilde{F}_A^C$ is a mapping given by $\widetilde{F}_A^C : \neg A \to IFN(U)$ and $\widetilde{F}_A^C(e) =$ intuitionistic fuzzy neutrosophic soft complement with $T_{\widetilde{F}^C(e)}(x) = F_{\widetilde{F}(e)}(x)$, $I_{\widetilde{F}^C(e)}(x) = I_{\widetilde{F}(e)}(x)$ and $F_{\widetilde{F}^C(e)}(x) = T_{\widetilde{F}(e)}(x)$ $\forall x \in U, \forall e \in \neg A$.

**Definition:8.7**

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two IFNSSs over the common universe $U$. **Intuitionistic fuzzy neutrosophic soft union** of $\widetilde{F}_A$ and $\widetilde{G}_B$ is denoted by '$\widetilde{F}_A \cup \widetilde{G}_B$' and is defined as $\widetilde{F}_A \cup \widetilde{G}_B = \widetilde{K}_D$, where $D = A \cup B$ and the truth membership, indeterminacy membership and falsity membership of $\widetilde{K}_D$ are as follows:

$$T_{\widetilde{K}(e)}(x) = \max(T_{\widetilde{F}(e)}(x), T_{\widetilde{G}(e)}(x)), I_{\widetilde{K}(e)}(x) = \min(I_{\widetilde{F}(e)}(x), I_{\widetilde{G}(e)}(x))$$

$$F_{\widetilde{K}(e)}(x) = \min(F_{\widetilde{F}(e)}(x), F_{\widetilde{G}(e)}(x)), \forall e \in A \cap B, \forall x \in U$$

**Definition:8.8**

Let $\widetilde{F}_A$ and $\widetilde{G}_B$ be two IFNSSs over the common universe $U$. **Intuitionistic fuzzy neutrosophic soft intersection** of $\widetilde{F}_A$ and $\widetilde{G}_B$ is denoted by '$\widetilde{F}_A \cap \widetilde{G}_B$' and is defined as $\widetilde{F}_A \cap \widetilde{G}_B = \widetilde{K}_D$, where $D = A \cap B$ the truth membership, indeterminacy membership and falsity membership of $\widetilde{K}_D$ are as follows:

$$T_{\tilde{K}(e)}(x) = \min(T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)), I_{\tilde{K}(e)}(x) = \min(I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x))$$

$$F_{\tilde{K}(e)}(x) = \max(F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)), \forall e \in A \cap B, \forall x \in U.$$

**Definition:8.9**

An intuitionistic fuzzy neutrosophic soft set $\tilde{F}_A$ over $U$ is said to be **empty intuitionistic fuzzy neutrosophic soft set** with respect to the parameter $A$ if

$T_{\tilde{F}_A(e)}(x) = 0, I_{\tilde{F}_A(e)}(x) = 0, F_{\tilde{F}_A(e)}(x) = 1 \forall e \in A, \forall x \in U$.

It is denoted by $\tilde{0}_A$. $\therefore T_{\tilde{0}_A(e)}(x) = 0, I_{\tilde{0}_A(e)}(x) = 0, F_{\tilde{0}_A(e)}(x) = 1$

**Definition:8.10**

An intuitionistic fuzzy neutrosophic soft set $\tilde{F}_A$ over $U$ is said to be **Universe intuitionistic fuzzy neutrosophic soft set** with respect to the parameter $A$ if

$T_{\tilde{F}_A(e)}(x) = 1, I_{\tilde{F}_A(e)}(x) = 0, F_{\tilde{F}_A(e)}(x) = 0 \forall e \in A, \forall x \in U$

It is denoted by $\tilde{1}_A$. $\therefore T_{\tilde{1}_A(e)}(x) = 1, I_{\tilde{1}_A(e)}(x) = 0, F_{\tilde{1}_A(e)}(x) = 0$.

**Note:8.11**

$(\tilde{0}_A)^C = \tilde{1}_A$ and $(\tilde{1}_A)^C = \tilde{0}_A$

**Definition:8.12**

Let $U$ be an initial universe set. Let $E$ be a set of parameters. An **intuitionistic fuzzy neutrosophic soft topology** (In short IFNST) $\tau$ on $U$ is a family of intuitionistic fuzzy neutrosophic soft sets over $U$ satisfying the following properties:

1. $\tilde{0}_E, \tilde{1}_E \in \tau$
2. If $\tilde{F}_A$ , $\tilde{G}_B \in \tau$ Then $\tilde{F}_A \cap \tilde{G}_B \in \tau$
3. If $\tilde{F}_{A_i} \in \tau, \forall i \in I, I$ be any arbitrary index set, then $\bigcup_{i \in I}(\tilde{F}_{A_i}) \in \tau$

The triplet $(U, E, \tau)$ is said to be **intuitionistic fuzzy neutrosophic soft topological space** (in short IFNSTS).

**Definition:8.13**

If $\tau$ is an intuitionistic fuzzy neutrosophic soft topology on $U$, the triplet $(U,E,\ \tau)$ is said to be intuitionistic fuzzy neutrosophic soft topological space .Each member of $\tau$ is called an **intuitionistic fuzzy neutrosophic soft open set** in $(U,E,\ \tau)$.

An intuitionistic fuzzy neutrosophic soft set is called **intuitionistic fuzzy neutrosophic soft closed** if its complement is intuitionistic fuzzy neutrosophic soft open.

**Definition:8.14**

Let $(U,E,\tau)$ be an intuitionistic fuzzy neutrosophic soft topological space .Let $\widetilde{F}_A$ be an IFNSS over $U$. Then an **intuitionistic fuzzy neutrosophic soft closure** of $\widetilde{F}_A$ denoted by $Cl(\widetilde{F}_A)$ and is defined as the intersection of all intuitionistic fuzzy neutrosophic soft closed sets containing $\widetilde{F}_A$.

$Cl(\widetilde{F}_A) = \bigcap \{\widetilde{G}_B : \widetilde{G}_B$ is intuitionistic fuzzy neutrosophic soft closed and $\widetilde{F}_A \subseteq \widetilde{G}_B\}$.

It is also clear that $Cl(\widetilde{F}_A)$ is intuitionistic fuzzy neutrosophic soft closed and $\widetilde{F}_A \subseteq Cl(\widetilde{F}_A)$.

**Note: 8.15**

$Cl(\widetilde{0}_E) = \widetilde{0}_E$ and $Cl(\widetilde{1}_E) = \widetilde{1}_E$

**Theorem:8.16**

Let $(U,E,\tau)$ be an IFNSTS. Let $\widetilde{F}_A$ be IFNSS over $U$. Then

1. $\widetilde{F}_A \subseteq Cl(\widetilde{F}_A)$
2. $\widetilde{F}_A$ is intuitionistic fuzzy neutrosophic soft closed iff $\widetilde{F}_A = Cl(\widetilde{F}_A)$
3. $Cl(Cl(\widetilde{F}_A)) = Cl(\widetilde{F}_A)$

**Proof:**

1. From the definition of intuitionistic fuzzy neutrosophic soft closure it follows that $\widetilde{F}_A \subseteq Cl(\widetilde{F}_A)$
2. $\widetilde{F}_A$ is intuitionistic fuzzy neutrosophic soft closed set over $U$. Then $\widetilde{F}_A$ is itself an intuitionistic fuzzy neutrosophic soft closed set containing $\widetilde{F}_A$.Also $Cl(\widetilde{F}_A)$ is the smallest intuitionistic fuzzy neutrosophic soft closed set containing $\widetilde{F}_A$.

$$Cl(\widetilde{F}_A) \subseteq \widetilde{F}_A \text{ ----------(i)}$$

From (1) we know $\widetilde{F}_A \subseteq Cl(\widetilde{F}_A)$ ----------(ii)

From (i) and (ii) $\widetilde{F}_A = Cl(\widetilde{F}_A)$

Conversely, let $\widetilde{F}_A = Cl(\widetilde{F}_A)$ .Since $Cl(\widetilde{F}_A)$ is intuitionistic fuzzy neutrosophic soft closed set. So, $\widetilde{F}_A$ is intuitionistic fuzzy neutrosophic soft closed set over $U$.

3. Since $Cl(\widetilde{F}_A)$ is intuitionistic fuzzy neutrosophic soft closed set. Then by (2)

$$Cl(Cl(\widetilde{F}_A)) = Cl(\widetilde{F}_A)$$

**Theorem:8.17**

Let $(U,E,\tau)$ be IFNSTS. Let $\widetilde{F}_A$ and $\widetilde{G}_B$ are two IFNSS over $U$. then, $\widetilde{F}_A \subseteq \widetilde{G}_B \Rightarrow Cl(\widetilde{F}_A) \subseteq Cl(\widetilde{G}_B)$

**Proof:**

Since $\widetilde{F}_A \subseteq \widetilde{G}_B$

From the definition of intuitionistic fuzzy neutrosophic soft closure it follows that $\widetilde{G}_B \subseteq Cl(\widetilde{G}_B)$

$\widetilde{F}_A \subseteq \widetilde{G}_B$ and $\widetilde{G}_B \subseteq Cl(\widetilde{G}_B) \Rightarrow \widetilde{F}_A \subseteq Cl(\widetilde{G}_B)$ ----------(i)

Since $Cl(\widetilde{F}_A)$ is the smallest intuitionistic fuzzy neutrosophic soft closed set containing $\widetilde{F}_A$.

From (i), $Cl(\widetilde{G}_B)$ contains $\widetilde{F}_A$ $\therefore Cl(\widetilde{F}_A) \subseteq Cl(\widetilde{G}_B)$

**Definition:8.18**

Let $(U,E,\tau)$ be an intuitionistic fuzzy neutrosophic soft topological space. Let $\widetilde{F}_A$ be an IFNSS over $U$. Then the **intuitionistic fuzzy neutrosophic soft interior** of $\widetilde{F}_A$ denoted by $\text{int}(\widetilde{F}_A)$ and is defined as the union of all intuitionistic fuzzy neutrosophic soft open sets contained in $\widetilde{F}_A$.

$\text{int}(\widetilde{F}_A) = \bigcup\{\widetilde{G}_B : \widetilde{G}_B$ is intuitionistic fuzzy neutrosophic soft open and $\widetilde{G}_B \subseteq \widetilde{F}_A\}$. It is also clear that $\text{int}(\widetilde{F}_A)$ is intuitionistic fuzzy neutrosophic soft open and $\text{int}(\widetilde{F}_A) \subseteq \widetilde{F}_A$

**Theorem:8.19**

Let $(U,E,\tau)$ be an IFNSTS. Let $\widetilde{F}_A$ be IFNSS over $U$.Then

1. $\text{int}(\widetilde{F}_A) \subseteq \widetilde{F}_A$
2. $\widetilde{F}_A$ is intuitionistic fuzzy neutrosophic soft open iff $\widetilde{F}_A = \text{int}(\widetilde{F}_A)$
3. $\text{int}(\text{int}(\widetilde{F}_A)) = \text{int}(\widetilde{F}_A)$

**Proof:**

1. From the definition of intuitionistic fuzzy neutrosophic soft interior it follows that $\text{int}(\widetilde{F}_A) \subseteq \widetilde{F}_A$
2. $\widetilde{F}_A$ is intuitionistic fuzzy neutrosophic soft open set over $U$. Then $\widetilde{F}_A$ is itself an intuitionistic fuzzy neutrosophic open set contained in $\widetilde{F}_A$. $\text{int}(\widetilde{F}_A)$ is the largest intuitionistic fuzzy neutrosophic open set contained in $\widetilde{F}_A$.

$$\widetilde{F}_A \subseteq \text{int}(\widetilde{F}_A) \qquad \text{----------(i)}$$

From (1) , $\text{int}(\widetilde{F}_A) \subseteq \widetilde{F}_A$ ----------(ii)

From (i) and (ii) $\widetilde{F}_A = \text{int}(\widetilde{F}_A)$

Conversely, let $\widetilde{F}_A = \text{int}\,\widetilde{F}_A$ .Since $\text{int}(\widetilde{F}_A)$ is intuitionistic fuzzy neutrosophic soft open set. So, $\widetilde{F}_A$ is intuitionistic fuzzy neutrosophic soft open set over $U$.

3. Since $\widetilde{F}_A$ is intuitionistic fuzzy neutrosophic soft open set. From (2)

$$\text{int}(\text{int}(\widetilde{F}_A)) = \text{int}(\widetilde{F}_A)$$

**Note: 8.20**

$\text{int}(\widetilde{0}_E) = \widetilde{0}_E$ and $\text{int}(\widetilde{1}_E) = \widetilde{1}_E$

**Theorem:8.21**

Let $(U,E,\tau)$ be an IFNSTS. Let $\widetilde{F}_A$ be IFNSS over $U$. Then

1. $[Cl(\widetilde{F}_A)]^C = \text{int}(\widetilde{F}_A{}^C)$
2. $[\text{int}(\widetilde{F}_A)]^C = Cl(\widetilde{F}_A{}^C)$

**Proof:**

1. $Cl(\widetilde{F}_A) = \bigcap\{\widetilde{G}_B : \widetilde{G}_B$ is IFNS closed and $\widetilde{F}_A \subseteq \widetilde{G}_B\}$

$[Cl(\widetilde{F}_A)]^C = [\bigcap\{\widetilde{G}_B : \widetilde{G}_B$ is IFNS closed and $\widetilde{F}_A \subseteq \widetilde{G}_B\}]^C$

$= \bigcup\{\widetilde{G}_B{}^C : \widetilde{G}_B{}^C$ is IFNS open and $\widetilde{G}_B{}^C \subseteq \widetilde{F}_A^C\}$

$= \text{int}(\widetilde{F}_A{}^C)$

2. $\text{int}(\widetilde{F}_A) = \bigcup\{\widetilde{G}_B : \widetilde{G}_B$ is IFNS open and $\widetilde{G}_B \subseteq \widetilde{F}_A\}$

$[\text{int}(\widetilde{F}_A)]^C = [\bigcup\{\widetilde{G}_B : \widetilde{G}_B$ is IFNS open and $\widetilde{G}_B \subseteq \widetilde{F}_A\}]^C$

$= \bigcap\{\widetilde{G}_B^C : \widetilde{G}_B^C$ is IFNS closed and $\widetilde{F}_A{}^C \subseteq \widetilde{G}_B{}^C\}$

$= Cl(\widetilde{F}_A^C)$

**Theorem:8.22**

Let $(U,E,\tau)$ be IFNSTS. Let $\widetilde{F}_A$ and $\widetilde{G}_B$ are two IFNSS over $U$. Then

1. $Cl(\widetilde{F}_A \cup \widetilde{G}_B) = Cl(\widetilde{F}_A) \cup Cl(\widetilde{G}_B)$
2. $Cl(\widetilde{F}_A \cap \widetilde{G}_B) \subseteq Cl(\widetilde{F}_A) \cap Cl(\widetilde{G}_B)$

**Proof:**

1. Since $\widetilde{F}_A \subseteq \widetilde{F}_A \cup \widetilde{G}_B, \widetilde{G}_B \subseteq \widetilde{F}_A \cup \widetilde{G}_B$

$Cl(\widetilde{F}_A) \subseteq Cl(\widetilde{F}_A \cup \widetilde{G}_B),\ Cl(\widetilde{G}_B) \subseteq Cl(\widetilde{F}_A \cup \widetilde{G}_B)$

So, $Cl(\widetilde{F}_A) \cup Cl(\widetilde{G}_B) \subseteq Cl(\widetilde{F}_A \cup \widetilde{G}_B)$ ----------(i)

$\widetilde{F}_A \subseteq \text{C}\,l(\widetilde{F}_A), \widetilde{G}_B \subseteq \text{C}\,l(\widetilde{G}_B)$

$\widetilde{F}_A \cup \widetilde{G}_B \subseteq Cl(\widetilde{F}_A) \cup Cl(\widetilde{G}_B)$

$Cl(\widetilde{F}_A) \cup Cl(\widetilde{G}_B)$ is IFNS closed set containing $\widetilde{F}_A \cup \widetilde{G}_B$ over $U$. Thus

$Cl(\widetilde{F}_A \cup \widetilde{G}_B) \subseteq Cl(\widetilde{F}_A) \cup Cl(\widetilde{G}_B)$ ----------(ii)

from (i) & (ii)

$Cl(\widetilde{F}_A \cup \widetilde{G}_B) = Cl(\widetilde{F}_A) \cup Cl(\widetilde{G}_B)$

2. Since $\widetilde{F}_A \cap \widetilde{G}_B \subseteq \widetilde{F}_A, \widetilde{F}_A \cap \widetilde{G}_B \subseteq \widetilde{G}_B$

$Cl(\widetilde{F}_A \cap \widetilde{G}_B) \subseteq Cl(\widetilde{F}_A)$

$Cl(\widetilde{F}_A \cap \widetilde{G}_B) \subseteq Cl(\widetilde{G}_B)$

$\therefore Cl(\widetilde{F}_A \cap \widetilde{G}_B) \subseteq Cl(\widetilde{F}_A) \cap Cl(\widetilde{G}_B)$

**Definition:8.23**

Let $(U,E,\tau)$ be an intuitionistic fuzzy neutrosophic soft topological space. $\widetilde{F}_A$ be an intuitionistic fuzzy neutrosophic soft set over $U$ and $x \in U$ .Then $\widetilde{F}_A$ is called an **intuitionistic fuzzy neutrosophic soft neighbourhood** of $x$ if there exists an intuitionistic fuzzy neutrosophic soft open set $\widetilde{G}_B$ such that $x \in \widetilde{G}_B \subseteq \widetilde{F}_A$ .An intuitionistic fuzzy neutrosophic soft neighbourhood system of a point $x$ denoted by $N_\tau(x)$, is the family of all its neighbourhoods.

**Theorem:8.24**

Let $(U,E,\tau)$ be IFNSTS. Let $\widetilde{F}_A$ and $\widetilde{G}_B$ are two IFNSS over $U$. Then

$\widetilde{F}_A \subseteq \widetilde{G}_B \Rightarrow \text{int}(\widetilde{F}_A) \subseteq \text{int}(\widetilde{G}_B)$

**Proof:**

Since $\widetilde{F}_A \subseteq \widetilde{G}_B$ .Let $x \in \text{int}(\widetilde{F}_A)$ .Then there is an intuitionistic fuzzy neutrosophic soft open set $\widetilde{H}$ in $U$ such that $x \in \widetilde{H} \subseteq \widetilde{F}_A$ .Since $\widetilde{F}_A \subseteq \widetilde{G}_B$ , $x \in \widetilde{H} \subseteq \widetilde{G}_B$ and $\widetilde{G}_B$ is an intuitionistic fuzzy neutrosophic soft neighbourhood of $x$ .Thus $x \in \text{int}(\widetilde{F}_A)$ .

$\therefore \text{int}(\widetilde{F}_A) \subseteq \text{int}(\widetilde{G}_B)$

**Theorem:8.25**

Let $(U,E,\tau)$ be IFNSTS. Let $\widetilde{F}_A$ and $\widetilde{G}_B$ are two IFNSS over $U$. Then

1. $\text{int}(\widetilde{F}_A \cap \widetilde{G}_B) = \text{int}(\widetilde{F}_A) \cap \text{int}(\widetilde{G}_B)$
2. $\text{int}(\widetilde{F}_A \cup \widetilde{G}_B) \subseteq \text{int}(\widetilde{F}_A) \cup \text{int}(\widetilde{G}_B)$

**Proof:**

1. since $\widetilde{F}_A \cap \widetilde{G}_B \subseteq \widetilde{F}_A, \widetilde{F}_A \cap \widetilde{G}_B \subseteq \widetilde{G}_B$

$\text{int}(\widetilde{F}_A \cap \widetilde{G}_B) \subseteq \text{int}(\widetilde{F}_A)$

$\text{int}(\widetilde{F}_A \cap \widetilde{G}_B) \subseteq \text{int}(\widetilde{G}_B)$

$\text{int}(\widetilde{F}_A \cap \widetilde{G}_B) \subseteq \text{int}(\widetilde{F}_A) \cap \text{int}(\widetilde{G}_B)$ ----------(i)

By definition, $\text{int}(\widetilde{F}_A) \subseteq \widetilde{F}_A$, $\text{int}(\widetilde{G}_B) \subseteq \widetilde{G}_B$

$\text{int}(\widetilde{F}_A) \cap \text{int}(\widetilde{G}_B) \subseteq \widetilde{F}_A \cap \widetilde{G}_B$

$\text{int}(\widetilde{F}_A \cap \widetilde{G}_B)$ is the largest IFNS open set contained in $(\widetilde{F}_A \cap \widetilde{G}_B)$

$\text{int}(\widetilde{F}_A) \cap \text{int}(\widetilde{G}_B) \subseteq \text{int}(\widetilde{F}_A \cap \widetilde{G}_B)$ ----------(ii)

from (i) & (ii) $\therefore \text{int}(\widetilde{F}_A \cap \widetilde{G}_B) = \text{int}(\widetilde{F}_A) \cap \text{int}(\widetilde{G}_B)$

2. Since $\text{int}(\widetilde{F}_A) \subseteq \widetilde{F}_A$ and $\text{int}(\widetilde{G}_B) \subseteq \widetilde{G}_B$

$\text{int}(\widetilde{F}_A) \cup \text{int}(\widetilde{G}_B) \subseteq \widetilde{F}_A \cup \widetilde{G}_B$

$\text{int}(\widetilde{F}_A \cup \widetilde{G}_B)$ is the largest IFNS open set contained in $(\widetilde{F}_A \cup \widetilde{G}_B)$

$\therefore \text{int}(\widetilde{F}_A) \cup \text{int}(\widetilde{G}_B) \subseteq \text{int}(\widetilde{F}_A \cup \widetilde{G}_B)$

**Definition:8.26**

Let $(U,E,\tau)$ be an intuitionistic fuzzy neutrosophic soft topological space. Let $\widetilde{F}_A$ be an intuitionistic fuzzy soft set over $U$. Then an **intuitionistic fuzzy neutrosophic soft exterior** of $\widetilde{F}_A$ denoted by $\text{ext}(\widetilde{F}_A)$ and is defined as

$\text{ext}(\widetilde{F}_A) = \text{int}(\widetilde{F}_A^C)$

**Theorem :8.27**

Let $(U,E,\tau)$ be an intuitionistic fuzzy neutrosophic soft topological space. Let $\widetilde{F}_A$ and $\widetilde{G}_B$ are intuitionistic fuzzy neutrosophic soft sets over $U$. Then

1. $ext(\widetilde{F}_A \cup \widetilde{G}_B) = ext(\widetilde{F}_A) \cap ext(\widetilde{G}_B)$
2. $ext(\widetilde{F}_A) \cup ext(\widetilde{G}_B) \subseteq ext(\widetilde{F}_A \cap \widetilde{G}_B)$

**Proof:**

i) $\text{ext}(\widetilde{F}_A \cup \widetilde{G}_B) = \text{int}((\widetilde{F}_A \cup \widetilde{G}_B)^C)$

$= \text{int}(\widetilde{F}_A^C \cap \widetilde{G}_B^C)$

$= \text{int}(\widetilde{F}_A^C) \cap \text{int}(\widetilde{G}_B^C)$

$$= ext(\widetilde{F}_A) \cap ext(\widetilde{G}_B)$$

ii ) $ext(\widetilde{F}_A) \cup ext(\widetilde{G}_B) = \text{int}(\widetilde{F}_A^C) \cup \text{int}(\widetilde{G}_B^C)$

$$\subseteq \text{int}(\widetilde{F}_A^C \cup \widetilde{G}_B^C)$$

$$= \text{int}(\widetilde{F}_A \cap \widetilde{G}_B)^C$$

$$= ext\left(\widetilde{F}_A \cap \widetilde{G}_B\right)$$

$$ext(\widetilde{F}_A) \cup ext(\widetilde{G}_B) \subseteq ext(\widetilde{F}_A \cap \widetilde{G}_B)$$

**Definition:8.28**

Let $(U,E,\tau)$ be an intuitionistic fuzzy neutrosophic soft topological space. Let $\widetilde{F}_A$ be an intuitionistic fuzzy neutrosophic soft set over $U$. Then an **intuitionistic fuzzy neutrosophic soft boundary** of $\widetilde{F}_A$ denoted by bou($\widetilde{F}_A$) and is defined as,

$$\text{bou}(\widetilde{F}_A) = Cl(\widetilde{F}_A) \cap Cl(\widetilde{F}_A{}^C)$$

**Theorem:8.29**

Let $(U,E,\tau)$ be an intuitionistic fuzzy neutrosophic soft topological space. Let $\widetilde{F}_A$ be intuitionistic fuzzy neutrosophic soft set over $U$.Then

1) $[\text{bou}(\widetilde{F}_A)]^C = \text{int}(\widetilde{F}_A) \cup \text{int}(\widetilde{F}_A{}^C) = \text{int}(\widetilde{F}_A) \cup \text{ext}(\widetilde{F}_A)$

**Proof:**

1) $[\text{bou}(\widetilde{F}_A)]^C = [Cl(\widetilde{F}_A) \cap Cl(\widetilde{F}_A{}^C)]^C$

$$= [Cl(\widetilde{F}_A)]^C \cup [Cl(\widetilde{F}_A{}^C)]^C$$

$$= \text{int}(\widetilde{F}_A) \cup \text{int}(\widetilde{F}_A{}^C)$$

$$= \text{int}(\widetilde{F}_A) \cup \text{ext}(\widetilde{F}_A)$$

**Theorem:8.30**

Let $(U,E,\tau)$ be a IFNSTS. Let $\widetilde{F}_A$ be an intuitionistic fuzzy neutrosophic soft set over $U$. Then

1. $bou(\widetilde{F}_A) \cap \text{int}(\widetilde{F}_A) = \widetilde{0}_E$

2. $bou(\widetilde{F}_A) \cap ext(\widetilde{F}_A) = \widetilde{0}_E$

**Proof:**

1. $$\begin{aligned} bou(\widetilde{F}_A) \cap \text{int}(\widetilde{F}_A) &= (Cl(\widetilde{F}_A) \cap Cl(\widetilde{F}_A^C)) \cap \text{int}(\widetilde{F}_A) \\ &= [(Cl(\widetilde{F}_A) \cap \text{int}(\widetilde{F}_A)] \cap [(Cl(\widetilde{F}_A^C) \cap \text{int}(\widetilde{F}_A)] \\ &= [(Cl(\widetilde{F}_A) \cap \text{int}(\widetilde{F}_A)] \cap [(\text{int}(\widetilde{F}_A)]^C \cap [\text{int}(\widetilde{F}_A)] \\ &= [(Cl(\widetilde{F}_A) \cap \text{int}(\widetilde{F}_A)] \cap \widetilde{0}_E \\ &= \widetilde{0}_E \end{aligned}$$

2. $$\begin{aligned} bou(\widetilde{F}_A) \cap ext(\widetilde{F}_A) &= (Cl(\widetilde{F}_A) \cap Cl(\widetilde{F}_A^C)) \cap ext(\widetilde{F}_A) \\ &= (Cl(\widetilde{F}_A) \cap Cl(\widetilde{F}_A^C)) \cap [\text{int}(\widetilde{F}_A{}^C)] \\ &= (Cl(\widetilde{F}_A) \cap Cl(\widetilde{F}_A^C)) \cap [Cl(\widetilde{F}_A)]^C \\ &= \{(Cl(\widetilde{F}_A) \cap [Cl(\widetilde{F}_A)]^C\} \cap \{Cl(\widetilde{F}_A{}^C) \cap [Cl(\widetilde{F}_A)]^C\} \\ &= \widetilde{0}_E \cap \{Cl(\widetilde{F}_A{}^C) \cap [Cl(\widetilde{F}_A)]^C\} \\ &= \widetilde{0}_E \end{aligned}$$

## SUMMARY AND CONCLUSION

Neutrosophic sets, Neutrosophic soft sets and fuzzy Neutrosophic soft sets are now become the most useful mathematical tools to deal with the problems which involves the indeterminacy and inconsistent information.

Combining Neutrosophic models with other mathematical models has attracted the attention of many researchers. By combining the Soft set theory with Neutrosophic set theory, Maji (2012) introduced the notion of "Neutrosophic Soft set".

This thesis is devoted to the study of

1. Neutrosophic soft sets

2. Generalized Neutrosophic soft sets

3. Interval valued Neutrosophic soft sets

4. Generalized interval valued Neutrosophic soft sets

5. Neutrosophic parameterized soft sets

6. Possibility Neutrosophic soft sets

7. Intuitionistic Neutrosophic soft sets

8.Intuitionistic fuzzy Neutrosophic soft sets and Intuitionistic fuzzy Neutrosophic soft topological spaces

In the first chapter, preliminary definitions regarding Neutrosophic sets, Soft sets and Neutrosophic soft sets are studied with interesting examples. Some operations and relations on Neutrosophic soft sets are studied with interesting properties.

The second chapter deals with preliminary definitions, operations and properties regarding generalized Neutrosophic soft sets.

In chapter III, the concept of interval valued Neutrosophic soft sets is defined and some operations on interval valued Neutrosophic soft sets are introduced. Some properties of interval valued Neutrosophic soft sets which are connected to the operations are established.

The concept of generalized interval valued Neutrosophic soft sets and their properties are presented in chapter IV.

In chapter V, the definition of Neutrosophic parameterized soft set is presented with some operations. NP-aggregation operator is defined to form NP-soft decision making method which allows constructing more efficient decision process. Finally, an example is given which shows that they can be successfully applied to problems that contain indeterminacy.

Chapter VI deals with possibility Neutrosophic soft sets. The definition of possibility Neutrosophic soft sets and some operations on possibility Neutrosophic soft sets are given. Properties related to these operations are investigated.

In chapter VII, new operations on intuitionistic Neutrosophic soft sets are introduced. Some results relating to the properties of these operations have been established.

The author of this thesis introduced the concepts of intuitionistic fuzzy Neutrosophic soft sets and intuitionistic fuzzy Neutrosophic soft topological spaces (2015). In chapter VIII intuitionistic fuzzy Neutrosophic soft sets and intuitionistic fuzzy Neutrosophic soft topological spaces are studied and some important theorems are established.

This new class of Neutrosophic soft sets and fuzzy Neutrosophic soft sets widens the scope to do further research in the areas like Smooth topological spaces, Bitopological spaces, Supra topological spaces, Rough topological spaces.