Review of Literature

Topology, the study of surfaces, is a branch of mathematics concerned with spatial properties preserved under bi-continuous deformation. It emerged through the development of concept from geometry and set theory. Topology plays an important role in pure and applied mathematics. Topological structures are suitable mathematical models for formulation of both qualitative and quantitative data.

Initially the topological spaces were characterized by open sets. Later Stone (1937) introduced regular open sets which is stronger than open sets. In 1963, Levine introduced the notion of semi-openness which is weaker than the notion of openness.Since then several interesting weaker and stronger forms of open sets have been introduced by many topologists.

Njastad (1965) introduced α-open sets which lies between open sets and semi-open sets. Velicko (1968) introduced the concept of δ-open sets, which are stronger than open sets, in order to investigate the characterization of H-closed spaces in terms of arbitrary filter bases and showed that, the collection ζ_δ of all δ-open sets, is a topology on Y turns into a semi-regular space if $\zeta = \zeta_\delta = \zeta_s$, the family ζ_s is known as semi-regularization of ζ.In a semi regular space,the family of all δ-open sets coincide with that of open sets. In order to extend some important properties of closed sets to a larger family of sets. Jankovic et al (1985), Noiri (1980),continued the work of Velicko.

Levine (1970) initiated the study of generalized closed sets in order to extend many of the important properties of closed sets to a larger family.Moreover, he characterized them as well as determines their behaviour relative to unions, intersections and subspaces. Also it was shown that compactness, normality and completeness in a uniform space are inherited by g-closed subsets.He defined that the complement of g-closed set is a g-open set. Then the images and inverse images of g-closed and g-open sets under continuous closed transformations were explored by him.

Further he used g-closed sets to define the new separation axiom, called $T_{1/2}$-spaces. He defines a $T_{1/2}$ -space to be one in which the closed sets and the g-closed sets coincide. As the notation suggests, $T_{1/2}$ is strictly between T_1 and T_0. Also the study of Cartesian products of g-closed sets, g-open sets and $T_{1/2}$ -spaces were undertaken by him.

Dunham (1977) continued the study of $T_{1/2}$ -spaces and characterized $T_{1/2}$-spaces through singleton set by defining "every singleton of Y is either open or closed if and only if every subset of Y is the intersection of all open sets and all closed sets containing it". Further he gave some characterizations of $T_{1/2}$-spaces which are independent of generalized closed sets. In particular, he investigated their behaviour with respect to subspaces, transformations and products.

Moreover, they proved that "closed" can be replaced by "g-closed" in the statement of Tietze Extension Theorem and hence established "Generalized Tietze Extension Theorem" which states that "A continuous, real valued function defined on a g-closed subset of a normal space has continuous extension to the entire space".

Sundarm and Nagaveni (1998) introduced weakly generalized closed sets. According to them, a subset A of (Y,ζ) is called weakly generalized closed (briefly, weakly g-closed) if $Cl(int(A)) \subseteq G$ whenever $A \subseteq G$ and G is open in Y.

Sundaram and Pushpalatha (2001) introduced the concepts of strongly generalized closed sets and strongly generalized open sets, which are generalizations of closed sets and open sets in topological spaces.

Jin and Jin (2004) introduced and studied the class of mildly generalized closed sets, which is properly placed between the classes of strongly generalized closed sets due to Sundaram and Pushpalatha (2001) and weakly generalized closed sets due to Sundarm and Nagaveni(1998). The relations with other notions directly or indirectly connected with generalized closed were investigated. Moreover, they used it to obtain new characterizations and preservation theorems of almost normal spaces due to Singal and Shashi (1970) and mildly normal spaces due to Singal and Asha (1973) respectively.

Mashhour et al (1982) first studied the notion of pre open sets in topological spaces and obtained various properties. With the aid of pre open sets, they introduced and investigated modified continuous functions called pre continuous functions and weak pre continuous function.Dontchev et al (2000) introduced and investigated p-closed spaces.

Semi pre-open sets, also known as β-sets, were brought out by Abd El-Monsef et al. (1983), Dontchev (1995) presented gsp-closed sets and discussed their properties. Gnanambal (1998) developed the concepts of generalized pre-regular closed sets and the continuity concepts with application as pre-regular $T_{1/2}$-spaces and studied their properties.

Intensive research on the field of generalized closed sets was carried out which can be visualized by the theory developed by way of

- Arya et al (1990) - gs-closed sets
- Palaniappan et al (1993) - rg-closed sets
- Maki et al (1994) - αg-closed sets
- Nagaveni (1999) - rwg -closed sets
- Dontchev (2000) - πg-closed sets
- Veera Kumar (2000) - g*-closed sets
- Veera Kumar (2003) - ĝ -closed sets
- Veera Kumar (2005) - #gs -closed sets
- Veera Kumar (2006) - *g -closed sets
- Park (2006) - πgp-closed sets
- Aslim (2006) - πgs-closed sets
- Janaki (2009) - πgα -closed sets
- Sarsak (2010) - πgsp-closed sets and gspr-closed sets
- Pushpalatha (2011) - g*s-closed sets

Since the number of research papers published on various closed sets in topological spaces is numerous, a brief review of literature on some of the important articles published on this topic is given here.

- **Title : A New Closure Operator for Non T_1-Topologies**
- **Author : Dunham,W. (1982)**
- **Inference observed :** Dunham has established a generalized closure operator and

used it to define new topology ζ^* using Levine's generalized closed sets as Cl* and examined some properties of generalized closure operator with emphasis on the transfer of regularity conditions on (Y,ζ) to separation conditions on (Y,ζ^*).

- **Title : Semi-preopen sets**
- **Author : Andrijevic,D. (1986)**
- **Inference observed :** Andrijevic (1986) considers a new class of so called nearly

open sets together with its corresponding operators semi-pre closure and semi-pre interior. A few relations between these operators and the operators defined before are established.

- **Title : Semi-generalized closed sets in topology**
- **Author : Bhattacharyya,P. and Lahiri,B.K. (1987)**
- **Inference observed :** Bhattacharyya generalised the concept of closed sets to

semi generalised-closed sets with the help of semi-openness. They has no connection with the generalised closed sets as considered by Levine although both generalise the concept of closed sets, this notions are in general independent.

- **Title : On δ-Generalized Closed Sets and $T_{3/4}$ -Spaces**
- **Authors : Dontchev,J. and Ganster,M. (1996)**
- **Inference observed :** Through the semi regularization of a given topology and the

associated δ-closure operator, a stronger form of g-closedness, properly placed between δ-closedness and g-closedness called δ-generalised closed sets, briefly denoted by δg-closed sets was introduced in this article. Also a new separation axiom, namely, $T_{3/4}$,which is properly placed between $T_{1/2}$ and T_1-spaces was defined and proved that every δg-closed set coincides with δ-closed set in this space. Further, the concept of δg-continuous and δg-irresolute functions were introduced and investigated in this paper.

- **Title : A Study on Some Generalizations of δ-closed sets in Topological Spaces**
- **Authors : Sudha,R. and Sivakamasundari,K. (2014)**
- **Inference observed :** In 2014, Sudha studied the notion of δg^*-closed sets and

wδg^*-closed sets. The concepts of δg^*-closed sets and wδg^*-closed sets are both stronger than δg-closed sets but weaker than δ-closed sets. i.e., δ-closed $\rightarrow \delta g^*$-closed $\rightarrow$ wδg^*-closed $\rightarrow \delta$g-closed $\rightarrow$ g-closed.

- **Title : Between Regular Open sets and Open sets**
- **Authors : Annalakshmi, M. Pious missier, S. (2016)**
- **Inference Observed :** The author has introduced regular*-open sets using Cl*.The family of regular*-open sets lies between the family of regular open sets and the family of open sets. Also the author has studied its fundamental properties and examine it with some different kinds of sets.

Separation Axioms

Separation axioms is one of the most important and interesting concepts in topological spaces.One of the most well known low separation axioms is the one which requires that singletons are closed. i.e. T_1.

Generalised closed sets are a strong tool in the characterisation of topological spaces satisfying weak separation axioms. Recently the class of almost weakly Hausdorff spaces was introduced by Dontchev and Ganster (1996) as the spaces whose semi-regularisation are $T_{1/2}$.Every weakly Hausdorff space is almost weakly Hausdorff and every almost weakly Hausdorff space is a $T_{3/4}$ -space,i.e. its singletons are regular open or closed (Dontchev and Ganster,1996),but not vice versa.The digital line or the so called Khalimsky line (Khalimsky et.al,1990 and Kovalevsky et.al,1994),i.e.the set of all integers $\mathcal{Z}$,equipped with the topology $\mathcal{K}$,generated by $\mathcal{G}_{\mathcal{K}} = \{ \{ 2n-1, 2n, 2n+1\}: n \in \mathcal{Z}\}$is an example of an almost weakly Hausdorff space,which is not weakly Hausdorff. On the other hand,the real line with the cofinite topology is an example of a $T_{3/4}$ -space,which is not almost weakly Hausdorff.

Bhattacharyya,P. and Lahiri,B.K. (1987) defined the concept of as new class of topological spaces called Semi-$T_{1/2}$ (i.e., the spaces where the class of semi-closed sets and the sg-closed sets coincide), and they proved that every Semi-T_1 space is Semi-$T_{1/2}$ and every Semi-$T_{1/2}$ space is Semi-T_0,although none of these applications is reversible.In 1993, Devi introduced T_b -space and T_d -space. Also Devi et al (1998) introduced αT_b -spaces.

Veerakumar (2000) introduced the following spaces namely $*T_{1/2}$-space ,$T_{1/2}*$-space and T_c -space. Furthermore, a stronger form of semi regularity, called T_δ-spaces, was introduced by Dontchev (2000) and it is shown that, it is equal to semi-regularity plus almost weak Hausdroffness. Sudha (2014) initiated the following six spaces namely $_{gs}T_{\delta g*}$ -space, $_{\delta g*}T_\delta$ -space, $_{\delta g}T_{\delta g*}$-space, $_{g\delta}T_{\delta g*}$-space, $_{g}T_{\delta g*}$-space and $_{g*}T_{\delta g*}$-space respectively.

- **Title : An Overview of Separation Axioms in Recent Research**
- **Author : Narasimhan,D.(2012)**
- **Inference observed :** The aim of this paper is to exhibit the recent research on

separation axioms, T_S-space, pairwise T_S-space, semi star generalized W-$T_{1/2}$ space,pairwise semi star generalized W- $T_{1/2}$ spaces and pairwise complemented spaces and its properties.

- **Title : α-Generalized & α*-Separation Axioms for topological spaces**
- **Author : Raman, C. K., Vidyottama Kumari, Sharma,M. K. (2014)**
- **Inference observed :** The present paper introduces a new class of separation

axioms called α-generalized separation axioms using α-generalized open sets and also includes the study of the connections between these separation axioms and the existing α-separation axioms. Also, here, the concept of α*- closed set has been coined and then α*- separation axioms have been framed with respect to α*-open sets.

Continuous Functions

Continuity of functions is one of the core concepts of topology. Continuity in almost any other context can be reduced to this definition by an appropriate choice of topology.

Norman Levine (1960) introduced the concept of strongly continuous functions. Noiri (1980) introduced a new class of functions called a δ-continuous and investigate the concepts of continuity and δ-continuity are independent of each other. Totally continuous functions and super continuous functions in topological spaces are introduced by Jain (1980) and Munshi (1982) respectively. Balachandran et al (1991) introduced the concept of generalized continuous maps in topological spaces.In successive years Palaniappan et.al (1993), and Devi et.al (1995),have been found new continuous functions namely rg-continuous functions and gs-continuous functions respectively. Further they introduced the stronger form of connectedness, called GO-connectedness in 1991 by using g-closed sets. A topological space Y is said to be GO-connected if Y cannot be written as a disjoint union of two non-empty g-open sets. A subset of Y is GO-connected if it is GO-connected as a subspace.

Caldas (1993) improved the Theorem 6.3 of Levine (1970). The following is the slight improvement of that theorem. If B is a g-closed (g-open) set in Y and if f : $Y \longrightarrow Z$ is g-continuous and closed, then $f^{-1}(B)$ is g-closed (g-open) in Y.Also it is proved if Y is Hausdorff,$A \subseteq Y$ and r: $Y \longrightarrow A$ is g-continuous retraction, then A is a g-closed set in Y.

Baker (1996) introduced new forms of continuity and closure, called a-continuity and a-closure and used to strengthen several results concerning the preservation of g-closed sets. Chawalit (2003a, b) continued the study of g-continuous mappings and proved some relationships between continuous functions and g-continuous functions and also studied g-continuous functions from any topological spaces into product spaces.

Meanwhile, Popa (1979) introduced the notion of rare continuity as a generalization of weak continuity (1961) which had been further investigated by Long and Herrington (1982) and Jafari (1995, 1997). In this sequel, Caldas and Jafari (2005) introduced the concept of rare g-continuity in topological spaces as a generalization of rare continuity and weak continuity.

In 1996, the concepts of δg-continuous functions have been introduced and investigated by Dontchev and he introduced contra continuity. Successively, Gnanambal (1998) and Nagaveni (1999) both researchers has introduced gpr-continuous and rwg-continuous functions respectively.In 2000, gδ-continuous functions had been introduced and studied by Dontchev. Further many authors Veerakumar (2003),Park(2006),Aslim(2006), Ekici and Baker (2007), Sarsak(2010), Navalagi(2010), Pushpalatha(2011), Sudha(2014) have contributed their research towards $\hat{g}$ -continuous , πgp-continuous , πgs-continuous , πgα-continuous and πg-continuous , πgsp-continuous , gspr-continuous , g^*s-continuous, δg^*-continuity in topological spaces respectively.

- ♣ **Title : On pre continuous and weak pre continuous functions**
- ♣ **Author : Mashhour, A.S. et.al., (1982)**

- **Inference observed :** Mashhour first studied the notion of pre-open sets in

topological spaces and obtained various properties.With the aid of pre-open sets, they introduced and investigated modified continuous functions called pre continuous functions and weak pre continuous function.

- **Title : δsg*-Continuity in Topological Spaces**
- **Author : Geethagnanaselvi,B. and Sivakamasundari, K. (2017)**
- **Inference observed :** In this paper, a new class of continuous functions namely

δsg* -continuous function using δsg* -closed sets is introduced. Some relationships like dependency, independency and composition of functions regarding δsg* -continuous functions are analysed.Additionally separation axioms are used to modify some properties of δsg* -continuous functions.

Irresolute Functions

Many different forms of irresolute functions have been introduced over the course of years. Certainly, it is hard to say whether one form is more or less important than another.Functions and of course irresolute functions stand among the most important and most researched points in the whole of mathematical science.

The first kind of function to be considered is one for which the inverses of semi-open sets are semi-open investigated by Crossley and Hildebrand (1972). They presented a new generalization of irresoluteness called contra-irresolute. In 2000,Miguel Caldas Cueva defined this class of function by the requirement that the inverse image of each semi-open set in the codomain is semi-closed in the domain.Later many mathematicians have been extended their contributions towards irresoluteness. In 2009, Ekici introduced a new class of generalized open sets called e*-open sets and studied several fundamental and interesting properties of e*- open sets and introduced a new class of continuous functions called e*-continuous functions into the field of topology. Recently, in 2012, Rajesh introduced two new types of irresolute functions via b-open sets. The purpose of this paper is to introduce and investigate other new types of irresolute functions via e*-open sets called completely e*-irresolute functions and completely weakly e*-irresolute functions. Some characterizations and several properties concerning such these functions are obtained.

- **Title : Δ*- Irresolute Maps in Topological Spaces**
- **Author : Meena,K and Sivakamasundari, K.(2016)**
- **Inference observed :** In this paper a new class of irresolute map called

Δ*-irresolute map via Δ*-open set is introduced and some of its characteristics are proved. Also the independent relationship of this map with various irresolute maps are investigated. Moreover the association of Δ*-irresolute map with δ-open and δg*-open surjective maps under separation axioms are analysed. Furthermore some of its properties under composition mapping are derived in this paper.

- **Title : On Strongly αg*p-Irresolute Functions in Topological Spaces**
- **Author : Sathyapriya,S. Kousalya,M.and Kamali, S. (2019)**
- **Inference observed :** In this paper, we introduce and investigate the notion of

strongly αg*p-irresolute functions. We obtain fundamental properties and characterization of strongly αg*p -irresolute functions and discuss the relationships between strongly αg*p -irresolute functions and other related functions.

Quotient Maps and Homeomorphisms

Subasree (2013) has introduced ĝ-quotient map in topological spaces. Various interesting problems arise when one considers irresoluteness. Its importance is significant in various areas of mathematics and related sciences. In the mathematical field of topology, a homeomorphism or topological isomorphism or bi continuous function is a continuous function between topological spaces that has a continuous inverse function. Homeomorphisms are the isomorphisms in the category of topological spaces - that is, they are the mappings that preserve all the topological properties of a given space. Two spaces with a homeomorphism between them are called homeomorphic and from a topological view point they are the same. Moreover, Balachandran et.al. (1991) defined generalized homeomorphisms via generalized closed sets and gc-homeomorphisms in terms of preserving generalized closed sets. Then it was proved that every homeomorphism is a generalized homeomorphism but not vice versa. These two concepts coincide when both the domain and the range satisfy the weak separation axiom $T_{1/2}$. They showed that the class of gc-homeomorphisms is properly placed between the classes of homeomorphisms and g-homeomorphisms.

Homeomorphisms play the same role in topology that linear isomorphisms play in linear algebra, or that biholomorphic functions play in function theory, or group isomorphisms in group theory, or isometries in Riemannian geometry. In the course of generalizations of the notion of homeomorphism, Maki, et al (1991) introduced g-homeomorphisms. Palaniappan et.al., (1993) introduced rg-homeomorphisms.Moreover Devi et al (1995) introduced gs-homeomorphisms. rwg-homeomorphism by Nagaveni (1999). Dontchev (2000) have been identified gδ-homeomorphism and πg-homeomorphism**.** πgs-homeomorphism, πgp-homeomorphism introduced by Aslim et al.,(2006).Further many researchers Janaki (2009),Sarsak (2010),Devamanoharan (2013) and Sudha (2014) have been analysed πgα-homeomorphism, πgsp-homeomorphism, gpr-homeomorphism and δg*-homeomorphism respectively.

- **Title : On Topological $\tilde{g}\alpha$-WG Quotient Mappings**
- **Author : Anitha,G. and Mariasingam,M. (2013)**
- **Inference observed :** The aim of this paper is to introduce $\tilde{g}_\alpha$wg-quotient map

using $\tilde{g}_\alpha$wg-closed sets and study their basic properties. We also study the relation between weak and strong form of $\tilde{g}_\alpha$wg-quotient maps. We also derive the relation between $\tilde{g}_\alpha$wg-quotient maps and $\tilde{g}_\alpha$ -quotient maps and also derive the relation between the $\tilde{g}_\alpha$wg-continuous map and $\tilde{g}_\alpha$wg-quotient maps. Examples are given to illustrate the results.

- **Title : On bĝ-Quotient Mappings in Topological Spaces**
- **Author : Subasree,R.(2016)**
- **Inference observed :** In this paper, we define new type of quotient mappings

namely bĝ-quotient mappings and we prove some of their basic properties. Also, we introduce strongly bĝ-open map, strongly bĝ-quotient map, bĝ*-quotient map and we discuss some of their relationship among other mappings.

- **Title : #RG - Homeomorphisms in Topological Spaces**
- **Author : Syed Ali Fathima,S. and Mariasingam,M.(2012)**
- **Inference observed :** A bijection f : (Y,ζ) → (Z, σ) is called #regular

generalized -homeomorphism if f and f $^{-1}$are #rg-continuous. Also we introduce new class of maps, namely #rgc-homeomorphisms which form a subclass of #rg-homeomorphisms. This class of maps is closed under composition of maps. We prove that the set of all #rgc homeomorphisms forms a group under the operation composition of maps.

- **Title : β* Homeomorphisms in Topological Spaces**
- **Author : Palanimani, P.G. and Parimelazhagan, R. (2013)**
- **Inference observed :** In this paper the authors define β* homeomorphisms

which are generalization of homeomorphisms and investigate some of their basic properties and also investigate generalized β* closed maps.

Soft topology

Soft systems provide a widespread framework with the involvement of parameters. In current years the development in the discipline of soft set theory and its application has been taking place in a rapid pace.

Some articles on soft topology are given here which I reviewed for my work.

- **Title : Soft Set Theory-First Results**
- **Authors : Molodstov,D. (1999)**
- **Inference Observed :** Molodtsov initiated the idea of soft set theory as a new

mathematical tool for dealing with uncertainty problems. He has introduced the fundamental notions of the theory of soft sets, to current the first consequences of the theory, and to discuss some problems of the future.

- **Title : On soft topological spaces**
- **Authors : Shabir,M. and Naz,M. (2011)**
- **Inference Observed :** Muhammad Shabir and Munazza Naz introduced the

concept of soft topological spaces which are described over an initial universe with a fixed set of parameters. The concept of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms are introduced and their primary properties are investigated. It is proven that a soft topological space gives a parametrized family of topological spaces. Furthermore, with the help of an example it is established that the converse does not hold. The soft subspaces of a soft topological space are defined and inherent concepts as well as the characterization of soft open and soft closed sets in soft subspaces are investigated.

- **Title : Soft generalized closed sets in Soft topological spaces**
- **Authors : Kannan,K. (2012)**
- **Inference Observed :** Kannan introduced soft g-closed sets in soft topological

spaces.Intensive research on the field of soft g-closed sets was done as the idea developed by way of many mathematicians in soft topological spaces. A sufficient condition for a soft g-closed set to be a soft closed is also presented. Moreover, the union and intersection of two soft g-closed sets are discussed.Finally, the new soft separation axiom, namely soft $T_{1/2}$ - space is introduced and its basic properties are investigated.

- **Title : Soft ĝ-Closed Sets in Soft Topological Spaces**
- **Authors : Nandhini,T. (2014)**
- **Inference Observed :** The author has initiated a new class of soft sets called Soft

ĝ-closed sets in Soft topological spaces. This new class is defined over an initial universe and with a fixed set of parameters. Some basic properties of this new class of soft sets are investigated. This new class of Soft ĝ-Closed sets contributes to widening the scope of Soft topological spaces and its applications.

- **Title : On Maximal Soft δ-open (Minimal soft δ-closed) Sets in Soft Topological Spaces**
- **Author : Bishnupada Debnath (2017)**
- **Inference Observed :** In soft topological space there are some existing related

concepts such as soft open, soft closed, soft subspace, soft separation axioms, soft connectedness, soft locally connectedness. In this paper, a new class of soft sets called maximal soft δ -open sets and minimal soft δ -closed sets which are fundamental results for further research are defined on soft topological space and continued in investigating the properties of these new notions of open sets with example and counter examples.

Arockiarani,I. et.al.,(2013) has initiated soft pre-open sets in soft topological spcaes. In 2013, Chen,B. introduced the soft semi-open and soft semi-closed sets in soft topological spaces. He has also analysed the properties of soft semi-open, soft semi-closed, soft semi-interior and soft semi-closure.Janaki,C.(2013) has noted about soft clopen sets in soft topological spaces.

Soft regular open sets are introduced and its properties are studied by Yuksel,S. et.al,.(2014),Akdag and Ozkan (2014) has defined and studied the properties of soft α-open and α-closed sets. Guzel,Z.E. et.al.,(2014) has introduced the concept of soft gpr-closed sets in soft topological spaces.In the same year,Janaki,C. et.al., has been introduced the concept of soft g*-closed sets and soft rwg-closed sets.