CHAPTER 4

CHAPTER – 4

FUZZY SEMI PRE-GENERALIZED CLOSED SETS AND FUZZY SEMI PRE-GENERALIZED CONTINUOUS MAPPINGS

In this chapter, fuzzy semi-pre-generalized closed sets due to Saraf et al. [67] are studied. The concept of fuzzy semi-pre-generalized closed set is a generalization of fuzzy semi-pre-open set in topological spaces.

Section: 4.1

Preliminary Definitions and Results of Fuzzy semi Pre- Generalized Closed Sets.

Definition: 4.1.1

A fuzzy set of (X,τ) is called fuzzy preopen (fp-open) if $A \leq Int(cl(A))$ and a fuzzy preclosed (fp-closed) if $(cl(Int(A))) \leq A$.

Definition: 4.1.2

A fuzzy set of (X,τ) is called fuzzy α-open (fα-open) if $A \leq Int(cl(Int(A)))$ and a fuzzy α-closed (fα-closed) if $cl(Int(cl(A))) \leq A$.

Definition: 4.1.3

A fuzzy set of (X, τ) is called fuzzy semi-preopen (fsp-open) if $A \leq cl(Int(cl(A))$ and a fuzzy semi-preclosed (fsp-closed) if $Int(cl(Int(A))) \leq A$.

Definition: 4.1.4

A fuzzy set A of (X,τ) is called generalized fuzzy semi closed (gfs-closed) if $scl\ (A) \leq H$, whenever $A \leq H$ and H is a fuzzy semi-open (fs-open) set in X.

Definition: 4.1.5

A fuzzy set A of (X, τ) is called fuzzy generalized semi closed (fgs-closed) if $scl\ (A) \leq H$, whenever $A \leq H$ and H is a fuzzy open set in X.

Definition: 4.1.6

A fuzzy set A of (X, τ) is called fuzzy α-generalized closed (fαg-closed) if α cl A $\leq$ H, whenever A $\leq$ H and H is a fuzzy open set in X.

Definition: 4.1.7

A fuzzy set A of (X, τ) is called fuzzy generalized α-closed (fgα-closed) if α cl A $\leq$ H, whenever A $\leq$ H and H is a fα-open set in X.

Definition: 4.1.8

A fuzzy set A of (X,τ) is called fuzzy generalized semi preclosed (fgsp-closed) if spcl (A) $\leq$ H, whenever A $\leq$ H and H is a fuzzy open set in X.

Definition: 4.1.9

A mapping f: (X,τ) $\rightarrow$ (Y,σ) is said to be fuzzy semi-continuous fs-continuous if $f^{-1}(V)$ is fs-open in X, for each fuzzy open set V in Y.

Definition: 4.1.10

A mapping f: (X,τ) $\rightarrow$ (Y,σ) is said to be fuzzy irresolute if $f^{-1}(V)$ is fs-open in X, for each fs-open set V in Y.

Definition: 4.1.11

A mapping f: (X,τ) $\rightarrow$ (Y,σ) is said to be fuzzy pre-continuous if $f^{-1}(V)$ is fp-open in X, for each fuzzy open set V in Y.

Definition: 4.1.12

Let X, Y be two fts. A function f: X $\rightarrow$ Y is called fuzzy pre-continuous (fuzzy pre-continuous) function $f^{-1}(A)$ is pre-open fuzzy set in X, for every open fuzzy set A of Y.

Definition: 4.1.13

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy fα-continuous if $f^{-1}(V)$ is fα-open in X, for each fuzzy open set V in Y.

Definition: 4.1.14

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be generalized fuzzy semi-continuous (gfs-continuous) if $f^{-1}(V)$ is gfs-closed in X, for each fuzzy closed set V in Y.

Definition: 4.1.15

Let X, Y be two fts. A function f: X $\rightarrow$ Y is called fuzzy generalized continuous if $f^{-1}(A)$ is a g-closed fuzzy set in X, for every closed fuzzy set A of Y.

Definition: 4.1.16

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy generalized semi-continuous (fgs-continuous) if $f^{-1}(V)$ is fgs-closed in X, for each fuzzy closed set V in Y.

Definition: 4.1.17

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy semipre-continuous (fsp-continuous) if $f^{-1}(V)$ is fsp-open in X, for each fuzzy open set V in Y.

Definition: 4.1.18

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy M-semiprecontinuous if $f^{-1}(V)$ is fsp-open in X, for each fsp-open set V in Y.

Definition: 4.1.19

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy generalized semi pre-continuous (fgsp-continuous) if $f^{-1}(V)$ is fgsp-closed in X, for every fuzzy closed set V in Y.

Definition: 4.1.20

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fgsp-irresolute if $f^{-1}(V)$ is a fgsp-closed set in X, for every fgsp-closed set V in Y.

Definition: 4.1.21

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy M-semipreclosed if f (V) is a fsp-closed set in Y, for every fsp-closed set V in X.

Definition: 4.1.22

A fuzzy point $x_p \in A$ is said to be quasi-coincident with the fuzzy set A denoted by x_p q A iff P + A(x) > 1.

A fuzzy set A is said to be quasi-coincident with a fuzzy set B denoted by A q B iff there exists $x \in X$ such that A(x) + B(x) > 1. If A and B are not quasi-coincident then we write $A \bar{q} B$.

Remark: 4.1.23

$A \leq B \Leftrightarrow A \bar{q} (1-B)$.

Definition: 4.1.24

A fuzzy set A of (X, τ) is called fuzzy semi-pre-generalized closed (fspg-closed) if Spcl(A) ≤ H, whenever A ≤ H and H is fs-open in X.

Notation: 4.1.25

The family of all fuzzy semi-pre-generalized closed sets of fts X is denoted by FSPGC (X, τ).

Preposition: 4.1.26

Every fp-closed set is fspg-closed.

Preposition: 4.1.27

Every gfs-closed set is fspg-closed.

Preposition: 4.1.28

Every fsp-closed set is fspg-closed.

Preposition: 4.1.29

Every fspg-closed set is fgsp-closed.

Example: 4.1.30

Let X = {a, b} and Y = {x, y, z} and fuzzy sets A, B, E, H, K and M be defined by:

A(a) =0.3	A(b) = 0.4	B(a) = 0.4	B(b)=0.5
E(a) = 0.3	E(b) = 0.7	H(a) = 0.7	H(b)=0.6
K(x) = 0.1	K(y) = 0.2	K(z) = 0.7	
M(x) = 0.9	M(y) = 0.2,	M(z) = 0.5	

Let τ = {0, A, 1}, σ = {0, E, 1} and γ = {0, K, 1}. Then B is fspg-closed in (X, τ) but not fp-closed. M is fspg-closed in (Y,γ) but not gfs-closed because if we consider the fuzzy set, T(x) = 0.9, T(y) = 0.2 T (z) = 0.7

Clearly Scl(M) $\nleq$ T, whereas M $\leq$ T and T is fs-open in (y, γ) and H is fgsp-closed in (X, σ) but neither fspg-closed because : If we consider a fuzzy set L(a) = 0.8, L(b) = 0.7, then clearly Spcl (H) $\nleq$ L whereas H $\leq$ L and L is fs-open in (X, σ) nor fsp-closed because Int(cl(Int(H))) $\nleq$ H.

Theorem: 4.1.31

If A is fs-open and fspg-closed in (X, τ) then A is a fsp-closed in (X, τ).

Proof

Since A $\leq$ A and A is fs-open and fspg-closed, then Spcl(A) $\leq$ A.

Since A ≤ Spcl(A), we have A = Spcl(A) and thus A is a fsp-closed set in X.Hence the theorem.

Theorem: 4.1.32

A fuzzy set A of (X, τ) is fspg-closed iff $A\bar{q}E \Rightarrow Spcl(A)\bar{q}E$, for every fs-closed set E of X.

Proof

(Necessity) Let A be fspg closed and let E be a fs-closed set of X such that $A\ \bar{q}\ E$. Then A ≤ 1 – E and 1 – E is fs-open in X which implies that Spcl(A) ≤ 1 – E as A is fspg-closed.Hence, $Spcl(A)\bar{q}E$.

(Sufficiency) Let H be a fs-open set of X such that A ≤ H. Then $A\ \bar{q}\ (1 - H)$ and 1 – H is fs-closed in X. By hypothesis, $Spcl(A)\ \bar{q}\ (1 - H) \Rightarrow Spcl(A) \le H$.

Hence, A is fspg-closed in X. Hence the theorem.

Theorem: 4.1.33

Let A be a fspg-closed set of (X, τ) and x_p be a fuzzy point of X such that x_p q Spcl(A) then Spcl(x_p) q A.

Proof

If $Spcl(x_p)\bar{q}A$ then A ≤ 1-Spcl(x_p) and so Spcl(A) ≤ 1 – Spcl(x_p) ≤ 1 – x_p, because 1 – Spcl(x_p) is Fs-open and A is fspg-closed in X. Hence, $x_p\ \bar{q}$ Spcl(A), a contradiction. Hence the theorem.

Theorem: 4.1.34

If A is a fspg-closed set of (X, τ) and A ≤ B ≤ Spcl(A), then B is a fspg-closed set of (X, τ).

Proof

Let H be a fs-open set of (X, τ) such that B ≤ H. Then A ≤ H. Since A is fspg-closed, Spcl(A) ≤ H. As B ≤ Spcl(A), Spcl(B) ≤ Spcl (Spcl(A)) = Spcl(A). Thus, Spcl(B) ≤ H. This proves that B is also a fspg-closed set of (X, τ).Hence the theorem.

Definition: 4.1.35

A fuzzy set of (X, τ) is called fspg-open iff (1 – A) is fspg-closed in X. That is, A is fspg-open iff E ≤ sp Int(A) whenever E ≤ A and E is a fs-closed set in X.

Notation: 4.1.36

The family of all fuzzy semi-pre-generalized open sets in a fts x is denoted by FSPGO (X, τ).

Theorem: 4.1.37

FSPO (X,τ) ≤ FSPGO (X,τ).

Proof

Let A be any fsp-open set in X. Then 1 – A is fsp-closed and hence fspg-closed by preposition 3.4.31.This implies that A is fspg-open. Hence, FSPO (X, τ) ≤ FSPGO (X, τ). Hence the theorem.

Theorem: 4.1.38

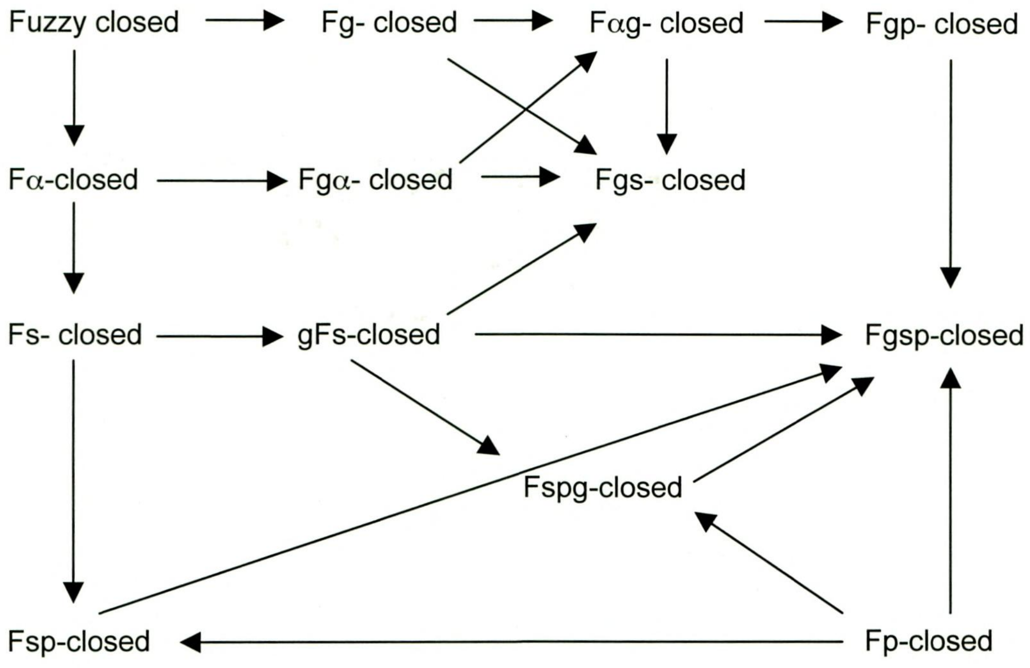
Let A be a fspg-open in X and spInt(A) ≤ B ≤ A. Then B is fspg-open.

Proof

Suppose A is fspg-open in X and spInt A(A) ≤ B ≤ A. Then 1 – A is fspg-closed and 1 – A ≤ 1 – B ≤ spCl(1 – A). Then 1 – B is fspg-closed set by Theorem 3.4.37.Hence, B is a fspg-open set in X.

Hence the theorem.

The following diagram is the enlargement of diagram form Definitions.



Here A → B means "A Implies B" but "B does not imply A".

Section: 4.2

Fuzzy Semi Pre-Generalized Continuous and Fuzzy Semi Pre-Generalized-Irresolute Mappings

Definition: 4.2.1

A mapping f: (X,τ) → (Y,σ) is called fuzzy semi-pre-generalized continuous (fspg-continuous) if $f^{-1}(V)$ is fspg-closed in (X,τ) for every fuzzy closed set V of (Y,σ).

Definition: 4.2.2

A mapping f: (X,τ) → (Y,σ) is called fuzzy semi-pre-generalized irresolute (fspg-irresolute) if $f^{-1}(V)$ is fspg-closed in (X,τ) for every fspg- closed set V of (Y,σ).

Theorem: 4.2.3

Let f: (X,τ) → (Y,σ) be gfs-continuous. Then f is fspg-continuous.

Proof

Let V be a fuzzy closed set of Y. Since f is gfs-continuous $f^{-1}(V)$ is gfs-closed in X. Since every gFs-closed set is Fspg-closed, $f^{-1}(V)$ is fspg-closed. Thus, f is Fspg-continuous. Hence the theorem.

The converse of the above theorem is not true in general.

Example: 4.2.4

Let X = {a, b, c}, Y = {x, y, z}. Fuzzy set A and B are defined as:

(3.3.1) A (a) = 0.1, A (b) = 0.2, A(c) = 0.7;

(3.3.2) B(x) = 0.1, B(y) = 0.8, B (z) = 0.5.

Let τ = {0, A, 1} and σ = {0, B, 1}. Then the mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ defined by f (a) = x, f (b) = y and f(c) = z is fspg-continuous but not gfs-continuous.

Theorem: 4.2.5

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be fspg-irresolute map. Then f is fspg-continuous.

Proof

As every fuzzy closed set is fspg-closed and f is fspg-irresolute map. $f^{-1}(V)$ is fspg-closed in X every fuzzy closed set V in Y. Hence the theorem.

The converse of the above theorem is not true in general.

Example: 4.2.6

Let X = {a, b}, Y = {x, y}. The fuzzy set A is defined as

A (a) = 0.3, A (b) = 0.7.

Let τ = {0, A, 1} and σ = {0, 1}

The mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ defined by f (a) = x and f (b) = y is fspg-continuous but not fspg-irresolute.

Theorem: 4.2.7

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be fspg-continuous. Then f is Fgsp-continuous but not conversely.

Proof

Let V be a fuzzy closed set of Y.Since f is fspg-continuous, $f^{-1}(V)$ is a fspg-closed set of X.Since every fspg-closed set is fgsp- closed, $f^{-1}(V)$ is also a fgsp-closed set of X.Thus, f is fgsp-continuous.

Example: 4.2.8

Let X = {a, b}, Y = {x, y}. Fuzzy sets A and B are defined as

A (a) = 0.3, A (b) = 0.7;

B(x) = 0.3, B(y) = 0.4.

Let τ = {0, A, 1} and σ = {0, B, 1}

Then the mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ defined by f (a) = x and

f (b) = y is fgsp-continuous but not fspg-continuous.

Theorem: 4.2.9

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be fp-continuous. Then f is fspg-continuous. The converse of the above theorem need to be true.

Example: 4.2.10

Let X = {a, b}, Y = {x, y}. Fuzzy set A and B are defined as

A (a) = 0.3, A (b) = 0.4;

B(x) = 0.6, B(y) = 0.5.

Let $\tau = \{0, A, 1\}$ and $\sigma = \{0, B, 1\}$

Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by f(a) = x and

f (b) = y is fspg-continuous but not fp-continuous.

Theorem: 4.2.11

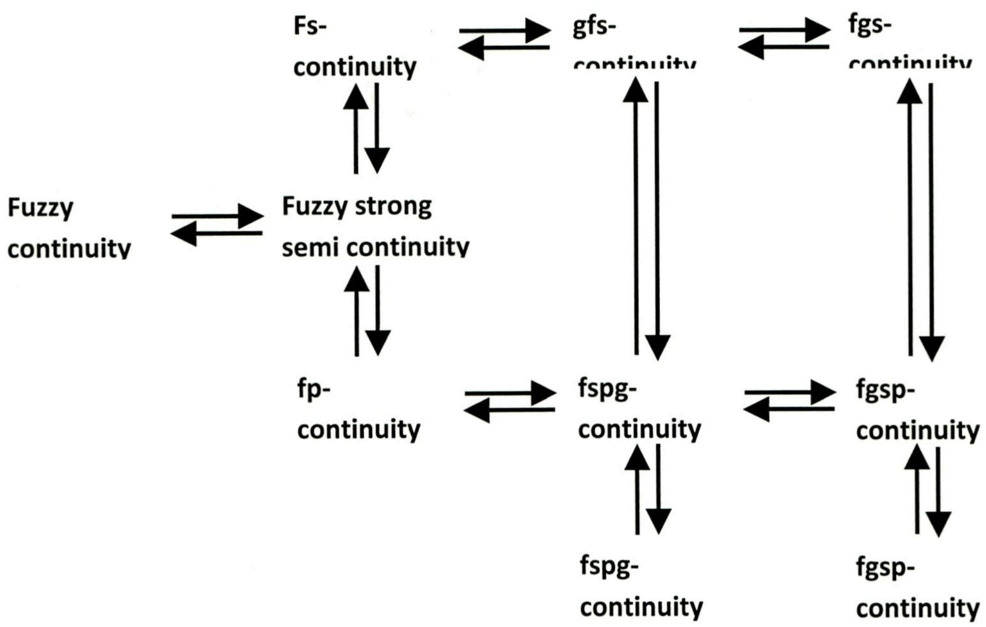
Every fs-continuous function is gfs-continuous but not conversely.

Theorem: 4.2.12

Let $f: (X,\tau) \to (Y,\sigma)$ be fgs-continuous. Then f is fgsp-continuous but not conversely.

Bin Shahna [17] introduced the concept of fuzzy strongly semi continuity and showed that the class of fuzzy strongly semi continuous functions properly contains the class of fuzzy continuous function and is properly contained in the class of fs-continuous function as well as the class of fp-continuous functions.

The following diagram summarizes the above discussions:



Theorem: 4.2.12

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is fspg-continuous iff inverse image of each fuzzy open set of Y is fspg-open in X.

Proof

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be fspg-continuous. Let H be an open set in Y, then 1 – H is closed in Y. As f is fspg-continuous. $f^{-1}(1 - H)$ is fspg closed in X. As $f^{-1}(1 - H) = 1 - f^{-1}(H)$, $f^{-1}(H)$ is fspg open in X. Hence the theorem.

Theorem: 4.2.13

If f : $(X, \tau) \rightarrow (Y, \sigma)$ is fspg-continuous, then for each fuzzy point x_p of X and each $A \in \sigma$ such that $f(x_p) \in A$, there exists a fspg-open set B of X such that $x_p \in B$ and $f(B) \leq A$.

Proof

Let x_p be a fuzzy point of X and $A \in \sigma$ such that $f(x_p) \in A$. Let $B = f^{-1}(A)$. As f is fspg continuous, B is a fspg-open set of X such that $x_p \in B$ and $f(B) = f(f^{-1}(A)) \leq A$. Hence the theorem.

Theorem: 4.2.14

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ is fspg-continuous then for each fuzzy point x_p of X and each $A \in \sigma$ such that $f(x_p)\ q\ A$, there exists a fspg-open set B of X such that $x_p\ q\ B$ and $f(B) \leq A$.

Proof

Let $x_p \in X$ and $A \in \sigma$ such that $f(x_p)\ q\ A$. Let $B = f^{-1}(A)$.
Then by hypothesis B is a fspg-open set of X such that $x_p\ q\ B$ and

$f(B) = f(f^{-1}(A)) \leq A$. Hence the theorem.

Definition: 4.2.15

A fts (X,τ) is called a fuzzy $T_{1/2}$ space if every g-closed fuzzy set in X is a closed fuzzy set in X.

Theorem: 4.2.16

If f: $(X,\tau) \rightarrow (Y,\sigma)$ is fspg-continuous and g: $(Y,\sigma) \rightarrow (Z,\gamma)$ is fg-continuous and Y is a fuzzy $T_{1/2}$ -space. Then g o f: $(X,\tau) \rightarrow (Z,\gamma)$ will be fspg-continuous.

Proof

Let A be a fuzzy closed set in Z. Then $g^{-1}(A)$ is fg-closed in Y.
Since Y is a fuzzy $T_{1/2}$ -space, $g^{-1}(A)$ is f-closed in Y hence $f^{-1}(g^{-1}(A))$ is a fspg-closed set in X. Thus, g o f is fspg-continuous. Hence the theorem.

Definition: 4.2.17

If every fspg-closed set in X is fsp-closed in X, then the space is said to be as fsp $T_{1/2}$ -space.

Theorem: 4.2.18

A fuzzy topological space (X,τ) is fsp $T_{1/2}$-space iff FSPO (X,τ) = FSPGO (X,τ).

Proof

(Necessity) Let (X, τ) be fsp $T_{1/2}$-space. Let $A \in$ FSPGO (X, τ). Then $1 - A$ is a fspg-closed. By hypothesis, $1 - A$ is a fsp-closed set and thus $A \in$ FSPO (X, τ).Hence FSPO (X, τ) = FSPGO (X, τ).

(Sufficiency) Let FSPO (X, τ) = FSPGO (X, τ).Let A be a fspg-closed set. Then $1 - A$ is a fspg-open set. Hence, $1 - A \in$ FSPO(X,τ). Thus, A is a fsp-closed set. Therefore, (X, τ) is a fsp $T_{1/2}$ -space. Hence the theorem.

Theorem: 4.2.19

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ and g: $(Y,\sigma) \rightarrow (Z,\gamma)$ be any two functions. Then

(i) g o f: $(X,\tau) \rightarrow (Z,\gamma)$ is fspg-continuous if g is fuzzy continuous and f is fspg-continuous.

(ii) g o f is fspg-irresolute, if f and g both are fspg-irresolute

(iii) g o f is fspg-continuous, if g is fspg-continuous and f is Fspg-irresolute.

(iv) Let Y be a fsp $T_{½}$-space. Then g o f is fspg-continuous, if g is fspg-continuous and f is fuzzy M-semi-pre-continuous.

Proof

Let f: $(X,\tau) \to (Y,\sigma)$ and g: $(Y,\sigma) \to (Z,\gamma)$ be any two functions. (i) g o f: $(X,\tau) \to (X,\gamma)$ is fspg-continuous if g is fuzzy continuous and fspg-continuous.

Let f: $(X,\tau) \to (Y,\sigma)$ be fspg continuous and let g: $(Y,\sigma) \to (z,\gamma)$ is fuzzy continuous. Let A be a fuzzy closed set in Z when $g^{-1}(A)$ is f-closed in Y. Since $g^{-1}(A)$ is f-closed in Y implies $f^{-1}(g^{-1}(A))$ is a fspg-closed set in X. Thus g o f is fspg-continuous.

(ii) Let f: $(X,\tau) \to (Y,\sigma)$ be fspg-irresolute and g: $(Y,\sigma) \to (Z,\gamma)$ be fspg-irresolute. Let V be a fuzzy closed subset of Y. As f,g are fspg-irresolute, $g^{-1}(V)$ is fspg-closed in (Y,σ). As f is fspg-irresolute, $f^{-1}(g^{-1}(V))$ is fspg-closed in (X,τ), thus g o f is fspg-irresolute.

(iii) g o f is fspg-continues. Let V be fuzzy closed set of Z. As g is fspg-continuous, $g^{-1}(V)$ is fspg-closed in (Y,σ). As f is fspg-irresolute, $f^{-1}(g^{-1}(V))$ is fspg-closed in (X,σ). Thus g o f is fspg-continues.

(iv) Let Y be a fsp-$T_{1/2}$-space. Let V be a fuzzy closed set of Z.
As g if fspg-continuous, $g^{-1}(V)$ is fspg-closed in Y as y is fsp $T_{½}$. As f is fuzzy M-semi-pre-continues, $f^{-1}(g^{-1}(V))$ is fsp-open in X. Thus g o f is fspg-continuous.
Hence the theorem.

Theorem: 4.2. 20

Let f: $(X,\tau) \to (Y,\sigma)$ be fspg-continuous. Then f is fuzzy semi-pre continuous if (X,τ) is fsp $T_{½}$ -space.

Proof

Let V be a fuzzy closed set of Y. Since f is fspg-continuous, $f^{-1}(V)$ is a fspg-closed set of X. As, X is fsp $T_{½}$ -space, $f^{-1}(V)$ is a fsp-closed set of X. This implies that f is fuzzy semi-pre continuous.

Hence the theorem.

Theorem: 4.2.21

Let f: (X,τ) → (Y,σ) be fuzzy irresolute and fuzzy M-semi-pre-closed.Then for every fspg-closed set A of X, f(A) is a fspg-closed in Y.

Proof

Let A be a fspg-closed set of X. Let V be a fs-open set of Y containing f(A).Since f is fuzzy irresolute, $f^{-1}(V)$ is a fs-open set of X. As A ≤ $f^{-1}(V)$ and A is a fspg-closed in X, Spcl(A) ≤ $f^{-1}(V)$.Hence that f(Spcl(A)) ≤ V. Since f is fuzzy M-semi-pre closed, f(Spcl(A))= Spcl(f(Spcl(A))). Then, Spcl(f(A)) ≤ Spcl(f(Spcl(A))) = f(Spcl(A) ≤ V. Therefore, f(A) is a fspg-closed set in Y. Hence the theorem.

Theorem: 4.2.22

Let f: (X,τ) → (Y,σ) be onto fspg-irresolute and fuzzy M-semi-pre closed. If X is fsp $T_{½}$ -space then (Y,σ) is also fsp $T_{½}$ -space.

Proof

Let A be a fspg-closed of Y. Since f is fspg-irresolute, $f^{-1}(A)$ is fspg-closed set in X. As X is a fsp T ½ -space $f^{-1}(A)$ is fsp-closed in X. As, f is a fuzzy M-semi-pre closed map, $f(f^{-1}(A))$ is a fsp-closed set in Y. Since f is onto $f(f^{-1}(A))$ = A, and hence (Y, σ) is a fsp $T_{1/2}$-space. Hence the theorem.

Theorem: 4.2.23

If the bijective mapping f: (X,τ) → (Y,σ) is fuzzy pre-semi-open and fuzzy M-semi-pre continuous, then f is fspg-irresolute.

Proof

Let V be a fspg-closed set in Y and let $f^{-1}(V) \leq H$ where H is a fs-open set in X. Then $V \leq f(H)$. Since f is a fpsp-open map, f(H) is a fs-open set in Y and V is a fspg-closed set in Y and hence $Spcl(V) \leq f(H)$ and thus $f^{-1}(Spcl(V)) \leq H$. As f is a fuzzy M-semi-pre continuous map and Spcl(V) is fsp-closed set in Y, $f^{-1}(Spcl(V))$ is a fsp-closed set in X. Thus, $Spcl(f^{-1}(V)) \leq Spcl(f^{-1}(Spcl(V))) = f^{-1}(Spcl(V)) \leq H$. So $f^{-1}(V)$ is a fspg-closed set in X. Hence, f is a fspg-irresolute map. Hence the theorem.

Section: 4.3

Fuzzy Semi Pre-Generalized Connectedness

Definition: 4.3.1

A fts (X,τ) is said to be fuzzy semi-pre-generalized connected (fspg-connected) iff the only fuzzy set which are both fspg-open and fspg-closed are 0_X and 1_X.

Example: 4.3.2

Let X = {a, b, c} and a fuzzy topology $\tau = \{0, 1, A\}$, where

$A: X \to [0, 1]$ is such that A(a)=1, A(b) = A(c) = 0. Then it is clearly that (X, τ) is a fspg-connected.

Theorem: 4.3.3

Let (X,τ) be a fts. If X is a fspg-connected space, then it is fs-connected.

Proof

Let X be fspg-connected and X is not fs-connected. Then there exists a proper fuzzy set E such that $E \neq 0_X$, $E \neq 1_X$ and E is both fs-open and fs-closed which implies that E is fspg-open and fspg-closed set.Then X is not fspg-connected, a contradiction. Hence the theorem.

The converse of the above theorem need not be true in general.

Example: 4.3.4

Let τ be the indiscrete fuzzy topology on X. Then (X,τ) is fs-connected space, but not fspg-connected.

Theorem: 4.3.5

A fts (X,τ) is fspg-connected iff X has no non-zero fspg-open sets A and B such that $A + B = 1_X$.

Proof

(Necessity) suppose (X,τ) is fspg-connected. Let X has two non-zero fspg-open sets A and B such that $A + B = 1_X$. Then A is a proper fspg-open and fspg-closed set of X. Hence X is not fspg-connected, a contradiction.

(Sufficiency) If (X,τ) is not fspg-connected, then it has a proper fuzzy set A of X which is both fspg-open and fspg-closed. So $B = 1 - A$, is a fspg-open set of X such that $A + B = 1_X$, which is a contradiction.

Hence the theorem.

Theorem: 4.3.6

If f: $(X,\tau) \rightarrow (Y,\sigma)$ is fspg-continuous surjection and (X,τ) is fspg-connected, then (Y,σ) is fuzzy connected.

Proof

Let X be a fspg-connected space and Y is not fuzzy connected.

As Y is not fuzzy connected, there exists a proper fuzzy set V of Y such that $V \neq 0_Y$ $V \neq 1_Y$ and V is both fuzzy open and fuzzy closed set. Since f is fspg-continuous, $f^{-1}(V)$ is both fspg-open and fspg-closed set in X such that $f^{-1}(V) \neq 0_X$ and $f^{-1}(V) \neq 1_X$. Hence, X is not fspg-connected, a contradiction. Hence the theorem.

Theorem: 4.3.7

If f: $(X,\tau) \rightarrow (Y,\sigma)$ is fspg-irresolute surjection and X is fspg-connected, then Y is so.

Definition: 4.3.8

A fts (X,τ) is said to be fspg-connected between fuzzy sets A and B if there is no fspg-closed and fspg-open set E in X such that $A \leq E$ and $E \bar{q} B$.

Preposition: 4.3.9

If a fts (X,τ) is fspg-connected between fuzzy sets A and B then it is fuzzy connected between A and B but the converse may not be true.

Example: 4.3.10

Let X = {a, b}. Fuzzy sets A, B and H on X are defined as:

A (a) = 0.4, A (b) = 0.5;

B (a) = 0.5, B (b) = 0.3;

H (a) = 0.4, H (b) = 0.3.

Let τ = {0, H, 1} be fuzzy topology on X. Then (X,τ) is fuzzy connected between A and B but not fspg-connected between A and B.

Theorem: 4.3.11

A fts (X,τ) is fspg-connected between A and B iff there is no fspg-closed, fspg-open set E in X such that $A \leq E \leq 1 - B$.

Theorem: 4.3.12

A fts (X,τ) is fspg-connected between fuzzy sets A and B then A and B are non-zero.

Proof

If A = 0, then A is fspg-closed, fspg-open in X such that $A \leq A$ and $A \bar{q} B$. Hence X cannot be fspg-connected, which is contradiction.

Hence the theorem.

Theorem: 4.3.13

If a fts (X,τ) is fspg-connected between fuzzy sets A and B and $A \leq A_1$ and $B \leq B_1$, then (X,τ) is fspg-connected between A_1 and B_1.

Proof

Suppose (X,τ) is not fspg-connected between A_1 and B_1. Then, there is a fspg-closed, fspg-open sets E in X such that $A_1 \leq E$ and

$E \bar{q} B_1$. Clearly, $A \leq E$. Now, we claim that $E \bar{q} B$. If E q B, then there exists a point $x \in X$ such that $E(x) + B(x) > 1$. Therefore, $E(x) + B_1(x) > 1$ and E q B_1, then a contradiction.

Hence the theorem.

Theorem: 4.3.14

Let (X,τ) be a fts, A and B are fuzzy sets in X. If A q B, then (X,τ) is fspg-connected between A and B.

Proof

If E is any fspg-closed, fspg-open sets in X such that $A \leq E$, then A q B $\Rightarrow$ Eq B. The converse of the above theorem is not true in general.

Example: 4.3.15

Let X = {a, b}. Fuzzy sets A, B and H on X are defined as

A (a) = 0.3, A (b) = 0.5;

B (a) = 0.5, B (b) = 0.4;

H (a) = 0.5, H (b) = 0.7.

Let τ = {0, H, 1} be fuzzy topology on X. Then (X,τ) is fspg-connected between A and B but A $\bar{q}$ B.

Theorem: 4.3.16

A fts (X,τ) is fspg-connected iff it is fspg-connected between every pair of its non-zero fuzzy sets.

Proof

(Necessity) Let A and B be any pair of non-zero fuzzy sets of X.
Suppose, (X, τ) is not fspg-connected between A and B, then there is a fspg-closed, fspg-open set E in X such that A ≤ E and E $\bar{q}$ B. Since A and B are non-zero, it E is a proper fspg-closed, fspg-open set of X. This implies that (X, τ) is not fspg-connected.

(Sufficiency) suppose (X, τ) is not fspg-connected. Then there exists a proper fuzzy set E of X which is both fspg-closed and fspg-open. Consequently, X is not fspg-connected between E and 1 – E, a contradiction.

Preposition: 4.3.17

If a fts (X,τ) is fspg-connected between a pair of its subsets, then it is not necessarily that (X, τ) is fspg-connected between every pair of fuzzy set and so is not necessarily fspg-connected.

Example: 4.3.17

Let X = {a, b}. Fuzzy sets A, B, C and H on X are defined as:

A (a) = 0.2, A (b) = 0.7;
B (a) = 0.5, B (b) = 0.4;
C (a) = 0.3, C (b) = 0.5;
H (a) = 0.3, H (b) = 0.4.

Let τ = {0, H, 1} be the fuzzy topology on X. Then (X,τ) is fspg-connected between A and B but it is not fspg-connected B and C. Also, (X,τ) is not fspg-connected.