CHAPTER III

THE (m, N) POLICY FOR A REPAIRABLE BATCH ARRIVAL SINGLE SERVICE QUEUEING SYSTEM WITH DISTINCT BEHAVIOUR OF SERVICE INTERRUPTED CUSTOMERS UNDER J-VACATION POLICY

INTRODUCTION

In the present chapter, the (m, N) policy for a repairable $M^X/G/1$ queueing system in which the service interrupted customers complete the service either through repeat or resume option or else quit the service forever is analysed. The customers' behaviour, regarding the completion of interrupted service are characterized by a set of probabilities. The breakdowns occur according to the Poisson process and the breakdown server is sent for repair immediately whose repair times follow heterogeneous general distributions.

Depending on the situations there are several possible scenarios to restoring an interrupted service. For example in production system, failures occur during production, may either lead to the complete loss of the items that is in production or cause a temporary production halt. In the former case, the interrupted item has to start all over again. In the later case, the production may continue after the failure is fixed.

The present chapter discusses three types of service disciplines governing the disposition of the service interrupted customers. They include :

(1) Repeat the interrupted service ; i.e., The interrupted customers may be placed at the head of the queue to repeat the service from the beginning, when the server is fixed.

(2) The failed customers may leave the system without completing the service.

(3) And customers may resume the interrupted service, from where they got interrupted.

In the literature of server breakdown queueing models, there exists different rules, for the customers to decide whether to repeat or resume the interrupted service, on completion of interruptions. Krishnamurthy et al. (2009(a), 2009(b), 2011) provide specific rules to take decision on it. Fiems et al. (2008) fix in advance, the probability for repeat or resumption of service. In the present work the author considers that if the server fails, the customer in service may join the head of the queue to repeat the service with probability (q_1) or may leave the system with probability q_2 without completing the service or else stay in the service facility with probability q_3 to complete the remaining service. The repair times of the server follow distinct general distributions of finite moments.

Recently there has been a rapid increase in the literature of queueing systems with negative arrivals, (also known as G-queue). E Gelenb (1989) first introduced the queueing system with negative arrivals to model neural network. The negative customers in many cases assumed to arrive according to Poisson process and remove one or more customers in service or queue which causes server failures. Case 2 of the present model can also be considered as a special case of negative arrival queueing models.

Apart from the applications, the system under study has theoretical interest because the queueing models in which the server operates the (m, N) policy during vacation and the service interrupted customers behave in different ways during the breakdown period are not considered together in the existing literature.

3.1 MATHEMATICAL ANALYSIS OF THE SYSTEM

3.1.1 Model Description

The customers arrive in batches according to the time-homogeneous compound Poisson process with group arrival rate λ (as in Chapter II).

Idle Period and Vacation Policy

The hypothesis regarding arrival pattern and the (m, N) policy with atmost J-vacations are as same as in Chapter II.

Busy Period

During busy period, the server provides only single service to all the arriving customers. The customers are served one at a time according to the order of their arrivals. It is assumed that the service times follow general distribution S(x) with density function s(x), finite moments $E(S^k)$, k = 1, 2 and Laplace-Stieltjes Transform (LST) $S^*(\theta)$.

Breakdowns and Repairs

The server is subjected to breakdowns at any time while serving customers. It is assumed that the life time of the server follows exponential distribution with rate **a** and the breakdown server is immediately sent for repair. If the server fails during the service of a customer then, the customer either joins the head of the queue to repeat the service from the beginning or the customer under service may be removed or quit and hence the service is lost or else stays in the service facility for the server to return from the repair facility to complete the remaining service. It is assumed that during breakdown period the probability that the customer joins the queue to start a new service is q_1, the probability that the customer quits the service and leaves the system is q_2 and probability that the customer stays in the service facility to complete the remaining service is q_3 with $q_1 + q_2 + q_3 = 1$. The corresponding repair times of the server follow heterogeneous general distributions $R_1(y)$, $R_2(y)$ and $R_3(y)$ with density functions $r_1(y)$, $r_2(y)$, $r_3(y)$ and finite moments $E(R_i^k)$, i = 1, 2, 3, k =1, 2. The server returning from repair facility is considered as good as new.

The busy period and breakdown period constitute a completion period. The completion period ends when the system becomes empty and the server is turned off for vacation. The system will be turned on again for setup only when the queue length reaches atleast **m** and server is available in the system after completing vacations. Thus the cycle is made up of completion period and idle period. Finally, various stochastic processes involved in the system are assumed to be independent of each other. The model of the present chapter differs from the model of Chapter II only in completion period. The system is denoted by $M^{X}_{(m,N)}$/G/1/Vac(J)/Breakdown with 3 types of repair facilities.

3.1.2 System Size Distribution at Random Epoch

The states of the system is denoted by Y(t). Y(t) = 0, 1, 2, 3 and 4, if the server is on vacation, buildup, setup, dormant and busy state respectively. Y(t) = 5, 6 and 7 respectively denote that the system is in repair mode with the service interrupted customer is in the head of the queue to repeat the service, quit the service and the customer stays in the service facility to complete the remaining service.

The definitions of the state dependent probabilities such as $QI_{n,j}(x,t)$, $PI_n(t)$, $SE_n(x, t)$, $U_n(t)$, when the server is on vacation, build up state, setup state and dormant state are the same as in Chapter II. The probabilities corresponding to the busy state and breakdown states are explained below :

$P_n(x, t)\, dt = P_r\{N_S(t) = n, x < S^o(t) \le x + dt, Y(t) = 4\}, n \ge 1$ is the joint probability that at time t, there are n customers in the system, the server is busy and the remaining service time $S^o(t)$ lies between x and x + dt, $n \ge 1$.

$BR_{1,n}(y) = P_r\{N_S(t) = n, y < R_1^o(t) \le y + dt, Y(t) = 5\}, n \ge 1$ denotes the case Y(t) = 5, and the customer, whose service is interrupted, joins the head of the queue to start a new service and the remaining repair time $R_1^o(t)$ of the server lies in (y, y + dt).

$BR_{2,n}(y, t) = P_r\{N_S(t) = n, y < R_2^o(t) \le y + dt, Y(t) = 6\}, n \ge 1$ denotes the case where Y(t) = 6 and the customer whose service is interrupted left the system and remaining repair time of the server $R_2^o(t)$ lies in (y, y + dt).

$BR_{3,n}(x, y) = P_r\{N_S(t) = n, S^o(t) = x, y < R_3^o(t) \le y + dt, Y(t) = 7\}, n \ge 1$ denotes the joint probability that the interrupted customer stays in the service facility to complete the remaining service time.

$P_n(0, t)$, $BR_{1,n}(0, t)$, $BR_{2,n}(0, t)$ and $BR_{3,n}(x, 0, t)$ respectively denote, the service termination epoch and the corresponding repair termination epochs.

Following the argument of Lee et al. (1994a) and observing the changes of states during the interval $(t, t + \Delta t)$ for any time t, the steady-state equations are obtained. The steady-state system size equations corresponding to idle period (vacation, buildup, setup and dormant) are similar to that of Chapter II. The equations regarding completion period are listed below :

The Steady State System Size Equations

Busy State

$$-\frac{d}{dx}P_n(x) = -(\lambda + a)\,P_n(x) + P_{n+1}(0)\,s(x) + BR_{3,n}(x, 0) + (BR_{1,n}(0) + BR_{2,n}(0))\,s(x) + (1 - \delta_{1,n})\,\lambda \sum_{k=1}^{n-1} P_{n-k}(x)\,g_k \qquad 1 \le n \le N-1$$

$$-\frac{d}{dx}P_n(x) = -(\lambda + a)\,P_n(x) + P_{n+1}(0)\,s(x) + BR_{3,n}(x, 0) + (BR_{1,n}(0) + BR_{2,n}(0))\,s(x) + \lambda \sum_{k=1}^{n-1} P_{n-k}(x)\,g_k + \lambda\,s(x) \sum_{k=n-N+1}^{n-m} U_{n-k}\,g_k + SE_n(0)\,s(x) \qquad n \ge N$$

Breakdown State

(If the interrupted customer joins the queue to repeat the service)

$$-\frac{\partial}{\partial y}BR_{1,n}(y) = -\lambda\,BR_{1,n}(y) + a\,q_1\left(\int_0^\infty P_n(w)\,dw\right) r_1(y) + (1 - \delta_{1,n})\,\lambda \sum_{k=1}^{n-1} BR_{1,n-k}(y)\,g_k \qquad n \ge 1$$

(If the interrupted customer leaves the system without completing the service)

$$-\frac{\partial}{\partial y}BR_{2,n}(y) = -\lambda\,BR_{2,n}(y) + a\,q_2\left(\int_0^\infty P_{n+1}(w)\,dw\right) r_2(y) + (1 - \delta_{0,n})\,\lambda \sum_{k=1}^{n} BR_{2,n-k}(y)\,g_k \qquad n \ge 0$$

(When the interrupted customer stays in the service facility to complete the remaining service)

$$-\frac{\partial}{\partial y} BR_{3,n}(x, y) = -\lambda\, BR_{3,n}(x, y) + a\, q_3\, P_n(x)\, r_3(y) + (1-\delta_{1,n})\, \lambda \sum_{k=1}^{n-1} BR_{3,n-k}(x, y)\, g_k \qquad n \geq 1$$

Thus the LST of the steady state equations corresponding to different states of the system are listed below :

During Idle Period (As in Chapter II)

$$\lambda\, PI_n = QI_{n,J}(0) + (1-\delta_{0,n})\, \lambda \sum_{k=1}^{n} PI_{n-k}\, g_k,\; 0 \leq n \leq m-1 \qquad (3.0.0)$$

$$\theta\, SE_n^*(\theta) - SE_n(0) = \lambda\, SE_n^*(\theta) - (1-\delta_{m,n})\, \lambda \sum_{k=1}^{n-m} SE_{n-k}^*(\theta)\, g_k - \sum_{j=1}^{J} QI_{n,j}(0)\, D^*(\theta) - \lambda \sum_{k=n-m+1}^{n} PI_{n-k}\, g_k\, D^*(\theta),\; n \geq m+1 \qquad (3.0)$$

$$\theta\, QI_{0,1}^*(\theta) - QI_{0,1}(0) = \lambda\, QI_{0,1}^*(\theta) - \overline{P}(0)\; VI^*(\theta), \text{ where } \overline{P}(0) = P_1(0) + B_{2,0}(0) \qquad (3.1)$$

$$\theta\, QI_{0,j}^*(\theta) - QI_{0,j}(0) = \lambda\, QI_{0,j}^*(\theta) - QI_{0,j-1}(0)\; VI^*(\theta) \qquad 2 \leq j \leq J \qquad (3.2)$$

$$\theta\, QI_{n,j}^*(\theta) - QI_{n,j}(0) = \lambda\, QI_{n,j}^*(\theta) - (1-\delta_{1,j})\, QI_{n,j-1}(0)\; VI^*(\theta) - \lambda \sum_{k=1}^{n} QI_{n-k,j}^*(\theta)\, g_k, \quad 1 \leq n \leq m-1\; ;\; 1 \leq j \leq J \qquad (3.3)$$

$$\theta\, QI_{n,j}^*(\theta) - QI_{n,j}(0) = \lambda\, QI_{n,j}^*(\theta) - \lambda \sum_{k=1}^{n} QI_{n-k,j}^*(\theta)\, g_k,\; n \geq m,\; 2 \leq j \leq J \qquad (3.4)$$

During Completion Period

$$\theta\, P_n^*(\theta) - P_n(0) = (\lambda + a)\, P_1^*(\theta) - P_{n+1}(0)\, S^*(\theta) - BR_{3,n}^*(\theta, 0) - (BR_{1,n}(0) + BR_{2,n}(0))\, S^*(\theta) - (1-\delta_{1,n})\, \lambda \sum_{k=1}^{n-1} P_{n-k}^*(\theta)\, g_k, \quad 1 \leq n \leq N-1 \qquad (3.5)$$

$$\theta\, P_n^*(\theta) - P_n(0) = (\lambda + a)\, P_n^*(\theta) - P_{n+1}(0)\, S^*(\theta) - BR_{3,n}^*(\theta, 0) - (BR_{1,n}(0) + BR_{2,n}(0))\, S^*(\theta) - \lambda \sum_{k=1}^{n-1} P_{n-k}^*(\theta) g_k - \lambda S^*(\theta) \sum_{k=n-N+1}^{n-m} U_{n-k}\, g_k - SE_n(0)\, S^*(\theta), \quad n \geq N \qquad (3.6)$$

$$-\frac{\partial}{\partial y} BR_{1,n}(y) = -\lambda\, BR_{1,n}(y) + aq_1 \int_0^\infty P_n(w)\,dw\, r_1(y) + (1 - \delta_{1,n})\, \lambda \sum_{k=1}^{n-1} BR_{1,n-k}(y) g_k \quad n \geq 1 \qquad (3.7)$$

$$-\frac{\partial}{\partial y} BR_{2,n}(y) = -\lambda\, BR_{2,n}(y) + aq_2 \int_0^\infty P_{n+1}(w)\,dw\, r_2(y) + (1 - \delta_{0,n})\, \lambda \sum_{k=1}^{n} BR_{2,n-k}(y) g_k \quad n \geq 0 \qquad (3.8)$$

$$-\frac{\partial}{\partial y} BR_{3,n}^*(\theta, y) = -\lambda\, BR_{3,n}^*(\theta, y) + aq_3\, P_n^*(\theta)\, r_3(y) + (1 - \delta_{1,n})\, \lambda \sum_{k=1}^{n-1} BR_{3,n-k}^*(\theta, y)\, g_k \quad n \geq 1 \qquad (3.9)$$

Taking the LST w.r.t. to y, equations (3.7) to (3.9) imply

$$\theta_1\, BR_{1,n}^{*_1}(\theta_1) - BR_{1,n}(0) = \lambda\, BR_{1,n}^{*_1}(\theta_1) - a\, q_1 \int_0^\infty P_n(w)\,dw\, R_1^{*_1}(\theta_1) - (1 - \delta_{1,n})\, \lambda \sum_{k=1}^{n-1} BR_{1,n-k}^{*_1}(\theta_1) g_k \quad n \geq 1 \qquad (3.10)$$

$$\theta_1\, BR_{2,n}^{*_1}(\theta_1) - BR_{2,n}(0) = \lambda\, BR_{2,n}^{*_1}(\theta_1) - a\, q_2 \int_0^\infty P_{n+1}(w)\,dw\, R_2^{*_1}(\theta_1) - (1 - \delta_{0,n})\, \lambda \sum_{k=1}^{n} BR_{2,n-k}^{*_1}(\theta_1) g_k \quad n \geq 0 \qquad (3.11)$$

$$\theta_1\, BR_{3,n}^{**_1}(\theta, \theta_1) - BR_{3,n}^*(\theta, 0) = \lambda\, BR_{3,n}^{**_1}(\theta, \theta_1) - a\, q_3\, P_n^*(\theta)\, R_3^{*_1}(\theta_1) - (1 - \delta_{1,n})\, \lambda \sum_{k=1}^{n-1} BR_{3,n-k}^{**_1}(\theta, \theta_1) g_k \quad n \geq 1 \qquad (3.12)$$

3.1.3 Probability Generating Functions

Since the steady state equations for idle states are similar to the corresponding equations of Chapter II, the partial generating functions corresponding to the buildup state, setup state, dormant state and vacation state are obtained similarly and are listed below :

$$PI(z) = \bar{P}(0)\ \psi(z) \tag{3.13}$$

where $\psi(z) = \left(\frac{1}{\lambda}\right) \sum_{n=0}^{m-1} \psi_n z^n$, $\psi_0 = \alpha I_0^J$ and $\psi_n = \sum_{k=0}^{n} \alpha I_k^{(J)} \pi_{n-k}$ $1 \le n \le m-1$

(3.13.1)

$$\pi_0 = 1 \text{ and } \pi_n = \sum_{k=1}^{n} \pi_{n-k} g_k \tag{3.13.2}$$

and $\alpha I_n^{(i)} = \sum_{k=0}^{n} \alpha I_{n-k}^{(i-1)} \alpha I_k$, $1 \le i \le J$, $n \ge 0$ and $\alpha I_0^{(i)} = \alpha I_0^i$ (3.13.3)

where αI_n denotes the probability that n customers arrive during a vacation time (VI).

$$SE(z, 0) = \bar{P}(0)\ D^*(w_X(z))\ ((VI^*(w_X(z)) - 1)\ \beta(z) + 1 - \psi(z)\ w_X(z)) \tag{3.14}$$

where $\beta(z) = \sum_{n=0}^{m-1} \frac{\beta_n z^n}{1-\alpha I_0}$, $\beta_0 = 1 - \alpha I_0^J$ and

$$\beta_n = \sum_{j=1}^{n} \frac{\alpha I_j\ \beta_{n-j}}{1-\alpha I_0} - \alpha I_n^{(J)} \text{ for } 1 \le n \le m-1 \tag{3.14.1}$$

$$SE^*(z,\theta) = \frac{\bar{P}(0)\,(D^*(w_X(z)) - D^*(\theta))}{(\theta - w_X(z))}((VI^*(w_X(z)) - 1)\ \beta(z) + 1 - \psi(z)\ w_X(z))$$

(3.15)

$$U(z) = \bar{P}(0)\ \phi(z) \tag{3.16}$$

$$\text{where } \phi(z) = \left(\frac{1}{\lambda}\right) \sum_{n=m}^{N-1} \phi_n z^n \tag{3.16.1}$$

$$\phi_n = \sum_{r=m}^{n} \xi_r \sum_{k=0}^{n-r} h_k\ \pi_{n-r-k} \tag{3.16.2}$$

and h_k denotes the probability that k customers arrive during setup period.

$$\xi_r = \sum_{k=0}^{m-1} \left(\frac{\alpha I_{r-k}\ \beta_k}{1-\alpha I_0} + \psi_k\ g_{r-k} \right) \tag{3.16.3}$$

$$QI(z, 0) = VI^*(w_X(z)) \sum_{n=0}^{m-1} \frac{\beta_n z^n}{1-\alpha I_0}\ \bar{P}(0) \tag{3.17}$$

$$QI^*(z,\theta) = \frac{(VI^*(w_X(z)) - VI^*(\theta))}{(\theta - w_X(z))}\ \beta(z)\ \bar{P}(0) \tag{3.18}$$

where $y_V(z) = \sum_{n=0}^{m-1} \frac{\beta_n z^n}{1-\alpha I_0}\ \bar{P}(0)$ with $\bar{P}(0) = P_1(0) + B_{2,0}(0)$ (3.18.1)

To obtain the partial probability generating functions of the number of customers in the system corresponding to completion period, the following probability generating functions are defined.

$$P^*(z,\theta) = \sum_{n=1}^{\infty} P_n^*(\theta)\, z^n, \qquad P(z,0) = \sum_{n=1}^{\infty} P_n(0)\, z^n$$

$$BR_i^{*1}(z,\theta_1) = \sum_{n=1}^{\infty} BR_{i,n}^{*1}(\theta_1)\, z^n, \qquad BR_i(z,0) = \sum_{n=1}^{\infty} BR_{i,n}(0)\, z^n,\ i=1,2$$

$$BR_3^{**1}(z,\theta,\theta_1) = \sum_{n=1}^{\infty} BR_{3,n}^{**1}(\theta,\theta_1)\, z^n, \qquad BR_3^{*}(z,\theta,0) = \sum_{n=1}^{\infty} BR_{3,n}^{**1}(\theta,0)\, z^n$$

Since $\int_0^{\infty} P_n(w)\,dw = \left(\int_0^{\infty} e^{-\theta w} P_n(w)\,dw\right)_{\theta=0} = P_n^*(0)$, equation (3.10) implies

$$\theta_1\, BR_{1,n}^{*1}(\theta_1) - BR_{1,n}(0) = \lambda\, BR_{1,n}^{*1}(\theta_1) - a\, q_1\, P_n^*(0)\, R_1^{*1}(\theta_1) - (1-\delta_{1,n})\,\lambda \sum_{k=1}^{n-1} BR_{1,n-k}^{*1}(\theta_1)\, g_k, \qquad n \geq 1$$

Multiplying the above equation by z^n and adding over n = 1 to ∞,

$$(\theta_1 - w_X(z))\, BR_1^{*1}(z,\theta_1) = BR_1(z,0) - a\, q_1\, P^*(z,0)\, R_1^{*1}(\theta_1)$$

At $\theta_1 = w_X(z)$,

$$BR_1(z,0) = a\, q_1\, R_1^{*1}(w_X(z))\, P^*(z,0), \text{ and hence,} \tag{3.19}$$

$$BR_1^{*1}(z,\theta_1) = a\, q_1\, P^*(z,0)\, \frac{(R_1^{*1}(w_X(z)) - R_1^{*1}(\theta_1))}{(\theta_1 - w_X(z))} \tag{3.20}$$

The generating functions of the system size when the server is in breakdown state and the customer joins the head of the queue for a new service are calculated by using the equation (3.11) and are found to be

$$BR_2(z,0) = \frac{a\, q_2}{z}\, R_2^{*1}(w_X(z))\, P^*(z,0) \tag{3.21}$$

$$BR_2^{*1}(z,\theta_1) = \frac{a\, q_2}{z}\, P^*(z,0)\, \frac{(R_2^{*1}(w_X(z)) - R_2^{*1}(\theta_1))}{(\theta_1 - w_X(z))} \tag{3.22}$$

The probability generating functions of the system size when the server is in breakdown state and the customer waits in the service facility for the server to return, to complete the reaming service are calculated by using the equation (3.12) and given by

$$BR_3^*(z,\theta,0) = a\, q_3\, R_3^{*1}(w_X(z))\, P^*(z,\theta) \tag{3.23}$$

$$BR_3^{**1}(z,\theta,\theta_1) = a\, q_3\, P^*(z,\theta)\, \frac{(R_3^{*1}(w_X(z)) - R_3^{*1}(\theta_1))}{(\theta_1 - w_X(z))} \tag{3.24}$$

To derive the generating functions of the system size at arbitrary epoch and at service completion epoch, when the server is busy, the equations (3.5) and (3.6) are used and resulted in

$$\begin{aligned}(\theta - h_{a,3}(w_X(z))\, P^*(z,\theta) = {} & \frac{P(z,0)}{z}(z - S^*(\theta)) - S^*(\theta)\,(SE(z,0) - U(z)\, w_X(z) \\ & - \bar{P}(0)) - \frac{S^*(\theta)\, a}{z}(z\, q_1\, R_1^{*1}(w_X(z)) \\ & + q_2\, R_2^{*1}(w_X(z)))\, P^*(z,0)\end{aligned} \tag{3.25}$$

where $h_{a,3}(w_X(z)) = w_X(z) + a\,(1 - q_3\, R_3^{*1}(w_X(z)))$ (3.25.1)

At $\theta = 0$,

$$P^*(z,0) = \frac{P(z,0)(z-1) - z\,(SE(z,0) - U(z)\, w_X(z) - \bar{P}(0))}{a\,(q_1\, z\, R_1^{*1}(w_X(z)) + q_2\, R_2^{*1}(w_X(z))) - z\, h_{a,3}(w_X(z))} \tag{3.26}$$

Substituting for $P^*(z, 0)$ in equation (3.25) and evaluating $\frac{P(z,0)}{z}$, at $\theta = h_{a,3}(w_X(z))$, we get

$$\frac{P(z,0)}{z} = \frac{h_{a,3}(w_X(z))\, S^*(h_{a,3}(w_X(z)))\,(\bar{P}(0) + U(z)\, w_X(z) - SE(z,0))}{-D(z)} \tag{3.27}$$

where $D(z)$

$$\begin{aligned} = {} & (1 - S^*(h_{a,3}(w_X(z))))\,[h_{a,3}(w_X(z)) - (z\, a\, q_1\, R_1^{*1}(w_X(z)) + a\, q_2\, R_2^{*1}(w_X(z)))] \\ & + h_{a,3}(w_X(z))\,(z-1)\end{aligned} \tag{3.27.1}$$

Substituting the value of P(z, 0) in equation (3.26),

$$P^*(z,0) = \frac{z\,(1 - S^*(h_{a,3}(w_X(z))))\,(SE(z,0) - U(z)\, w_X(z) - \bar{P}(0))}{D(z)} \tag{3.28}$$

By proceeding as in Chapter II, it is found that

$$SE(z, 0) - U(z)\, w_X(z) - \overline{P}(0) = -\, \overline{P}(0)\, w_X(z)\, I_{(m,N)}(z) \qquad (3.29)$$

where $$I_{(m,N)}(z) = \frac{(1 - VI^*(w_X(z)))}{w_X(z)} D^*(w_X(z))\,\beta(z) + \frac{1 - D^*(w_X(z))}{w_X(z)} + \phi(z)$$

$$+ \psi(z)\, D^*(w_X(z)) \qquad (3.29.1)$$

Thus the equation (3.28) implies

$$P^*(z, 0) = \frac{z\,(1 - S^*(h_{a,3}(w_X(z))))\,(-\overline{P}(0)\, w_X(z)\, I_{(m,N)}(z))}{D(z)} \qquad (3.30)$$

Hence the partial PGFs of the system size of the present model are listed by

$$PI(z) = \overline{P}(0)\ \psi(z) \qquad (3.31.1)$$

$$QI^*(z, 0) = \frac{\overline{P}(0)\,(1 - VI^*(w_X(z)))}{w_X(z)}\, \beta(z) \qquad (3.31.2)$$

$$SE^*(z, 0) = \frac{\overline{P}(0)\,(1 - D^*(w_X(z)))}{w_X(z)}\, ((VI^*(w_X(z)) - 1)\, \beta(z) + 1 - \psi(z)\, w_X(z)) \qquad (3.31.3)$$

$$U(z) = \overline{P}(0) \left(\frac{1}{\lambda}\right) \sum_{n=m}^{N-1} \phi_n\, z^n = \overline{P}(0)\ \phi(z) \qquad (3.31.4)$$

$$P^*(z, 0) = \frac{z\,(1 - S^*(h_{a,3}(w_X(z))))\,(-\overline{P}(0)\, w_X(z)\, I_{(m,N)}(z))}{D(z)} \qquad (3.31.5)$$

$$BR_1^{*1}(z, 0) = \frac{a\, q_1\, P^*(z, 0)\,(1 - R_1^{*1}(w_X(z))}{w_X(z)} \qquad (3.31.6)$$

$$BR_2^{*1}(z, 0) = \frac{a\, q_2\, P^*(z, 0)\,(1 - R_2^{*1}(w_X(z))}{z\,(w_X(z))} \qquad (3.31.7)$$

$$BR_3^{**1}(z, 0, 0) = \frac{a\, q_3\, P^*(z, 0)\,(1 - R_3^{*1}(w_X(z))}{w_X(z)} \qquad (3.31.8)$$

To derive the total PGF of the system size distribution, the following generating functions are considered.

$P_{Idle}(z)$ = Probability generating function of the system size when the server is idle (vacation + setup + dormant + buildup) state

$$= QI^*(z, 0) + SE^*(z, 0) + U(z) + PI(z) = \overline{P}(0)\, I_{(m,N)}(z) \qquad (3.32)$$

$P_{Comp}(z)$ = The PGF of the system size when server is busy (or) in breakdown state

$$= P^*(z,0) + BR_1^{*1}(z,0) + BR_2^{*1}(z,0) + BR_3^{**1}(z,0,0)$$

$$= \frac{\{\overline{P}(0)\, z\, I_{(m,N)}(z)\,(S^*(h_{a,3}(w_X(z))) - 1)\,[a\,(q_1\,(1-R_1^{*1}(w_X(z))) + \frac{q_2}{z}(1-R_2^{*1}(w_X(z))) + q_3 - 1) + h_{a,3}(w_X(z))]\}}{D(z)} \tag{3.33}$$

Thus the total PGF of the system size distribution is given by

$$P_{(m,N)}^{3R}(z) = P_{Idle}(z) + P_{Comp}(z) = \frac{\overline{P}(0)\,(z-1)\, I_{(m,N)}(z)\, S_{3R}(w_X(z))}{D(z)} \tag{3.34}$$

where $S_{3R}(w_X(z)) = h_{a,3}(w_X(z))\, S^*(h_{a,3}(w_X(z))) + (1 - S^*(h_{a,3}(w_X(z))))\, a\, q_2$ (3.34.1)

and D(z) is given by the equation (3.27.1)

The constant $\overline{P}(0)$ can be calculated by using the normalizing condition $P_{m,N}^{3R}(1) = 1$ and found to be $\overline{P}(0) = \dfrac{1-\rho_{3R}}{I_{(m,N)}(1)}$ (3.35)

where

$$\rho_{3R} = \lambda\, E(X)\, eH \tag{3.36}$$

$$eH = \frac{(1 - S^*(a\,(1-q_3)))}{a\,(q_2 + q_1\, S^*(a\,(1-q_3)))}(1 + a\,(q_1\, E(R_1) + q_2\, E(R_2) + q_3\, E(R_3)))$$

$$I_{(m,N)}(1) = E(D) + E(V)\sum_{n=0}^{m-1}\frac{\beta_n}{1-\alpha\, I_0} + \left(\frac{1}{\lambda}\right)\sum_{n=m}^{N-1}\phi_n + \left(\frac{1}{\lambda}\right)\sum_{n=0}^{m-1}\psi_n \tag{3.37}$$

Hence

$$P_{(m,N)}^{3R}(z) = \frac{(1-\rho_{3R})\,(z-1)\, S_{3R}(w_X(z))}{D(z)}\;\frac{I_{(m,N)}(z)}{I_{(m,N)}(1)} \tag{3.38}$$

Equation (3.38) justifies the decomposition property.

3.1.4 Decomposition Property

Equation (3.38) shows that the PGF $P_{(m,N)}^{3R}(z)$ of the present model is decomposed into the product of two generating functions of which $\dfrac{(1-\rho_{3R})\,(z-1)\, S_{3R}(w_X(z))}{D(z)}$, gives the PGF of the system size for the corresponding $M^X/G/1$ queueing model (with three types of behaviour of the service interrupted customers) without (m, N) policy and without vacation and

$\frac{P_{Idle}(z)}{P_{Idle}(1)} = \frac{I_{(m,N)}(z)}{I_{(m,N)}(1)}$ gives the PGF of the conditional system size distribution during the idle period under the steady state condition $\rho_{3R} < 1$.

3.1.5 Queue Size Distribution at Departure Epoch

A departing customer will see n customers in the system just after the departure if and only if there are (n+1) customers in the system just before the departure (Lee et al., 1994a). Thus, if π_n^+ denotes the probability that there are n customers in the system at departure epoch, then

$$\pi_n^+ = D_1 \left[P_{n+1}(0) + a\, q_2 \int_0^\infty P_{n+1}(x)\,dx\right]$$

$$= D_1 \left[P_{n+1}(0) + a\, q_2\, P_{n+1}^*(0)\right], \qquad n \geq 0$$

where D_1 is the normalizing constant.

Therefore the PGF $\pi^+(z)$ of $\{\pi_n^+ \,;\, n \geq 0\}$ is given by

$$\pi^+(z) = \sum_{n=0}^{\infty} \pi_n z^n = \frac{D_1}{z}\left[P(z,0) + a\, q_2\, P^*(z,0)\right]$$

using equations (3.27) and (3.31.5), $\pi^+(z)$ is simplified as,

$$\pi^+(z) = \frac{D_1 \lambda (X(z)-1) P_{(m,N)}^{3R}(z)}{z-1}$$; evaluating the normalizing constant D,

$$\pi^+(z) = \frac{1-X(z)}{E(X)(1-z)} P_{(m,N)}^{3R}(z)$$

3.1.6 Performance Measures

In this section, the steady-state system size probabilities and the expected number of customers in the system, when the server is in different states are calculated.

The Server in Idle State

Let P_{VI}, P_{build}, P_{set} and P_{dor} denote the steady-state system size probabilities and L_{VI}, L_{build}, L_{set} and L_{dor} denote the mean number of customers, when the system is in vacation during idle period, buildup, setup and dormant states respectively. Then using equations (3.31.1) to (3.31.4), we have

$$P_{build} = \lim_{z \to 1} PI(z) = \overline{P}(0) \sum_{n=0}^{m-1} \frac{\psi_n}{\lambda} \tag{3.39}$$

$$L_{build} = \left[\frac{d}{dz} PI(z)\right]_{z=1} = \bar{P}(0) \sum_{n=0}^{m-1} \frac{n\,\psi_n}{\lambda} \tag{3.39.1}$$

$$P_{VI} = \lim_{z\to 1} QI^*(z,0) = \bar{P}(0) E(VI) \sum_{n=0}^{m-1} \frac{\beta_n}{1-\alpha I_0} \tag{3.40}$$

$$L_{VI} = \left[\frac{d}{dz} QI^*(z,0)\right]_{z=1} = \bar{P}(0)\left(E(VI)\sum_{n=0}^{m-1}\frac{n\,\beta_n}{1-\alpha I_0} + \lambda E(X)\frac{E(VI^2)}{2}\sum_{n=0}^{m-1}\frac{\beta_n}{1-\alpha I_0}\right) \tag{3.40.1}$$

$$P_{set} = \lim_{z\to 1} SE^*(z,0) = \bar{P}(0)\, E(D) \tag{3.41}$$

$$L_{set} = \left[\frac{d}{dz} SE^*(z,0)\right]_{z=1} = \bar{P}(0)\,\lambda\, E(X)\left(E(D)\left(\sum_{n=0}^{m-1}\frac{\beta_n}{1-\alpha I_0}E(V) + \sum_{n=0}^{m-1}\frac{\psi_n}{\lambda}\right) + \frac{E(D^2)}{2}\right) \tag{3.41.1}$$

$$P_{dor} = \lim_{z\to 1} U(z) = \bar{P}(0) \sum_{n=m}^{N-1} \frac{\phi_n}{\lambda} \tag{3.42}$$

$$L_{dor} = \left[\frac{d}{dz} U(z)\right]_{z=1} = \bar{P}(0) \sum_{n=m}^{N-1} \frac{n\,\phi_n}{\lambda} \tag{3.42.1}$$

Thus, the probability that the server is idle (P_I) and the mean system size (L_I) when the server is idle are given by

$$P_I = P_{VI} + P_{build} + P_{set} + P_{dor} = (1-\rho_{3R}) \tag{3.43}$$

$$L_I = L_{VI} + L_{build} + L_{set} + L_{dor} = (1-\rho_{3R})\,\frac{I'_{(m,N)}(1)}{I_{(m,N)}(1)} \tag{3.43.1}$$

where

$$I'_{(m,N)}(1) = \left[\frac{d}{dz} I_{(m,N)}(z)\right]_{z=1}$$

$$= \lambda E(X)\left(\left(\frac{E(VI^2)}{2} + E(VI)E(D)\right)\sum_{n=0}^{m-1}\frac{\beta_n}{1-\alpha I_0} + \frac{E(D^2)}{2} + E(D)\sum_{n=0}^{m-1}\frac{\psi_n}{\lambda}\right)$$

$$+ E(VI)\sum_{n=0}^{m-1}\frac{n\,\beta_n}{1-\alpha I_0} + \sum_{n=m}^{N-1}\frac{n\,\phi_n}{\lambda} + \sum_{n=0}^{m-1}\frac{n\,\psi_n}{\lambda} \tag{3.43.2}$$

$I_{m,N}(1)$ is given by equation (3.37).

The Server in Busy State

The probability that the server is busy (P_{busy}) and the expected number of customers in the system in busy state (L_{busy}) are calculated by using the equation (3.31.5).

$$P_{busy} = \lim_{z \to 1} P^*(z,0) = \lambda\, E(X) \frac{(1 - S^*(a(1-q_3))}{a(q_2 + q_1 S^*(a(1-q_3)))} \qquad (3.44)$$

$$L_{busy} = \left[\frac{d}{dz} P^*(z,0)\right]_{z=1}$$

$$L_{busy} = P_{busy} + \overline{P}(0)\{(\lambda\, E(X))^2 (1 + a\, q_3\, E(R_3))\, S^{*'}(a(1-q_3))\, \frac{I_{(m,N)}(1)}{D'(1)}$$

$$+ (1 - S^*(a(1-q_3)))\, I'_{(m,N)}(1)\, \frac{\lambda\, E(X)}{D'(1)}$$

$$+ (1 - S^*(a(1-q_3)))\, I_{(m,N)}(1)\, \left(\frac{\lambda\, E(X(X-1))}{2D'(1)} + \frac{\lambda\, E(X)(-D''(1))}{2(D'(1))^2}\right)\}$$

(3.44.1)

$$D'(1) = a(q_2 + q_1 S^*(a(1-q_3)))(1 - \rho_{3R}) \qquad (3.44.2)$$

$$-D''(1) = \lambda\, E(X(X-1))\, e\, H_1 + (\lambda\, E(X))^2\, e\, H_2$$

$$+ 2\lambda\, E(X)\, ((1 - S^*(a(1-q_3)))\, a\, q_1\, E(R_1)$$

$$+ (1 + a\, q_3\, E(R_3))\, (a\, q_1\, S^{*'}(a(1-q_3)) + 1)) \qquad (3.44.3)$$

$$e\, H_1 = (1 - S^*(a(1-q_3)))\, (1 + \sum_{i=1}^{3} a\, q_i\, E(R_i)) \qquad (3.44.4)$$

$$e\, H_2 = (1 - S^*(a(1-q_3)))\, (\sum_{i=1}^{3} a\, q_i\, E(R_i^2))$$

$$+ 2\, S^{*'}(a(1-q_3))\, (1 + a\, q_3\, E(R_3))\, (1 + \sum_{i=1}^{3} a\, q_i\, E(R_i)) \qquad (3.44.5)$$

The Server in Breakdown States

The probabilities that the server is in breakdown states P_{br_1}, P_{br_2} and P_{br_3} and the corresponding expected number of customers in the system L_{br_1}, L_{br_2} and L_{br_3} are calculated by using the equations (3.31.6) to (3.31.8). They are given by

$$P_{br_i} = P_{busy}\, a\, q_i\, E(R_i) \qquad i = 1 \text{ to } 3 \qquad (3.45)$$

$$L_{br_1} = a\, q_1\, (P_{busy}\, \lambda\, E(X)\, \frac{E(R_1^2)}{2} + L_{busy}\, E(R_1)) \qquad (3.46)$$

$$L_{br_2} = a\, q_2\, (L_{busy}\, E(R_2) + P_{busy}\, \lambda\, E(X)\, \frac{E(R_2^2)}{2} - P_{busy}\, E(R_2)) \qquad (3.46.1)$$

$$L_{br_3} = a\, q_3\, (L_{busy}\, E(R_3) + P_{busy}\, \lambda\, E(X)\, \frac{E(R_3^2)}{2} \qquad (3.46.2)$$

$$L_{br} = L_{br_1} + L_{br_2} + L_{br_3}$$

Mean System Size

The expected system size $L^{3R}_{(m,N)}$ of the model is given by

$$L^{3R}_{(m,N)} = L_I + L_{busy} + \sum_{i=1}^{3} L_{br_i}$$

$$= \frac{I_{(m,N)}'(1)}{I_{(m,N)}(1)} + L_1 \quad (3.47)$$

where $L_1 = \dfrac{1}{a\,(q_2 + q_1\,S^*(a\,(1-q_3)))\,(1-\rho_{3R})}$

$$\left[\frac{\lambda\,E(X(X-1)\,e\,H_1 + (\lambda\,E(X))^2\,eH_2}{2} + \lambda\,E(X)\,eH_3\right] \quad (3.47.1)$$

$$e\,H_3 = \frac{(1-S^*(a\,(1-q_3)))\,(1+a\,q_1\,E(R_1)+a\,q_3\,E(R_3))}{\rho_{3R}\,(1+a\,q_3\,E(R_3))\,(a\,q_1\,S^{*'}(a\,(1-q_3))+S^*(a\,(1-q_3)))}$$

and e H_1, e H_2 are given by the equations (3.44.4) and (3.44.5) respectively.

3.1.7 Other System Characteristics

Let E(Cycle), E(Busy), E(I) and E(w_S) denote the expected length of the cycle, busy period, idle period and expected waiting time of the system. Then

(i) E(Cycle) $= \dfrac{I_{(m,N)}(1)}{1-\rho_{3R}}$

(ii) E(I) $= I_{(m,N)}(1)$

(iii) E(Busy) $= \dfrac{\lambda\,E(X)\,(1-S^*(a\,(1-q_3)))}{a\,(q_2 + q_1\,S^*(a\,(1-q_3)))}\,\dfrac{I_{(m,N)}(1)}{(1-\rho_{3R})}$

(iv) E(w_S) $= \dfrac{L^{3R}_{(m,N)}}{\lambda\,E(X)}$ (Using Little's formula)

3.2 OPTIMAL MANAGEMENT POLICY

By following the procedure of Chapter II, the optimal threshold values (m^*, N^*) can be obtained.

Recalling the cost structures C_y (startup cost per cycle), C_{set} (setup cost), C_{build} (buildup cost), C_{dor} (standby cost), C_{br} (breakdown cost), C_{busy}

(operating cost), C_h (holding cost per customer) and C_{VI} (reward cost) per unit time, the total average cost per unit time of the system is given by

$$T_C(m, N) = \frac{C_y}{E(Cycle)} + C_{set} P_{set} + C_{dor} P_{dor} + C_{busy} P_{busy} + C_{build} P_{build}$$

$$+ C_{br} P_{br} + C_h L^{3R}_{(m,N)} - C_{VI} P_{VI}$$

By substituting the corresponding measures,

$$T_C(m, N) = \overline{A} + \frac{1}{I_{(m,N)}(1)} \left(A_1 + z(m) + C_{dor} (1 - \rho_{3R}) \sum_{n=m}^{N-1} \frac{\phi_n}{\lambda} + C_h \sum_{n=m}^{N-1} \frac{n\,\phi_n}{\lambda}\right)$$

where $$\overline{A} = C_h L_1 + P_{busy} \left(C_{br} \sum_{i=1}^{3} a\, q_i\, E(R_i) + C_{busy}\right)$$

$$A_1 = (1 - \rho_{3R}) (C_y + C_{set} E(D)) + C_h \lambda E(X) \frac{E(D^2)}{2}$$

$$z(m) = C_h \left\{\lambda E(X) \left(\left(\frac{E(VI^2)}{2} + E(VI)E(D)\right) \sum_{n=0}^{m-1} \frac{\beta_n}{1-\alpha I_0} + \sum_{n=0}^{m-1} \frac{\psi_n}{\lambda} E(D)\right)\right.$$

$$\left. + E(VI) \sum_{n=0}^{m-1} \frac{n\,\beta_n}{1-\alpha I_0} + \sum_{n=0}^{m-1} \frac{\psi_n}{\lambda}\right\}$$

$$+ C_{build} (1 - \rho_{3R}) \sum_{n=0}^{m-1} \frac{\psi_n}{\lambda} - C_{VI} (1 - \rho_{3R}) E(VI) \sum_{n=0}^{m-1} \frac{\beta_n}{1-\alpha I_0}$$

and L_1 is given by the equation (3.47.1).

By calculation,

$$T_C(m, k+1) - T_C(m, k) = \frac{\phi_k}{\lambda\, I_{(m,k+1)}(1)\, I_{(m,k)}(1)} H(m, k)$$

where

$$H_{(m,k)} = C_h \left[k\, d_m + \sum_{n=m}^{k-1} \frac{(k-n)}{\lambda} \phi_n\right] + C_{dor}\, d_m (1 - \rho_{3R}) - (A_1 + z(m))$$

with $$d_m = E(D) + E(VI) \sum_{n=0}^{m-1} \frac{\beta_n}{1-\alpha I_0} + \left(\frac{1}{\lambda}\right) \sum_{n=0}^{m-1} \psi_n$$

and ϕ_k is given by the equation (3.16.2).

By proceeding as in Chapter II, it is found that the condition under which $T_C(m, k+1) - T_C(m, k) > 0$ for the first value of k is given by

$N^*(m) = \min \{k \,/\, H_{(m,k)} > 0\}$. The optimal threshold values $(m^*, N^*(m))$ can be obtained by following the algorithm given in Chapter II.

3.3 PARTICULAR CASES

In this section, we list some particular cases that can be deduced from the model of the present chapter. The suitable selection of (q_1, q_2, q_3) gives the steady state results of different breakdown models.

Case 1

$q_1 = q_3 = 0$ and $q_2 = 1$ in equation (3.38) give, the PGF of the system size for the negative arrival queueing model under (m, N) policy with atmost J vacations.

$$P(z) = \bar{P}(z)\frac{I_{(m,N)}(z)}{I_{(m,N)}(1)}, \text{ where}$$

$$\bar{P}(z) = \frac{(1-\rho)(z-1)[a + w_X(z) S^*(g_a(w_X(z)))]}{[g_a(w_X(z))(z - S^*(g_a(w_X(z)))) + a R_2^{*1}(w_X(z))(S^*(g_a(w_X(z))) - 1)]}$$

$$\text{and } \rho = \frac{\lambda E(X)}{a}(1 - S^*(a))(1 + a E(R_2))$$

$\bar{P}(z)$ coincides with the PGF of the system size for the classical batch arrival queueing model with negative arrivals and breakdown (Afthab et al., 2015) without vacation.

Case 2

$q_1 = q_2 = 0$ and $q_3 = 1$ in equation (3.38) give the PGF of the system size of Chapter II with $r_i = 0\ \forall\ i$ in equation (2.62).

$$P(z) = (1-\rho)\frac{(z-1) S^*(h(w_X(z)))}{z - S^*(h(w_X(z)))}\frac{I_{(m,N)}(z)}{I_{(m,N)}(1)}$$

where $h(w_X(z)) = w_X(z) + a(1 - R^{*1}(w_X(z)))$

Case 3

$q_2 = q_3 = 0$ and $q_1 = 1$ gives the PGF of the (m, N) policy J vacation queueing model in which the customers repeat their service from the beginning when the server is fixed.

$$\text{i.e., } P(z) = P_1(z)\frac{I_{(m,N)}(z)}{I_{(m,N)}(1)}, \text{ where}$$

$$P_1(z) = \frac{(1-\rho)(z-1)\, g_a(w_X(z))\; S^*(g_a(w_X(z)))}{\{z[g_a(w_X(z)) - a\, R_1^{*1}(w_X(z))(1-S^*(g_a(w_X(z))))] - S^*(g_a(w_X(z)))\, g_a(w_X(z))\}}$$

and

$$\rho = \lambda\, E(X)\, \frac{(1-S^*(a))}{a\, S^*(a)}(1 + a\, E(R_1))$$

Case 4

Since $I_{m,N}(z)$ for the Chapters II and III are the same, the PGFs of the system size for the $M^X/G/1$ queueing model with three types of customers' behaviour during breakdown period corresponding to the classical single and multiple vacations under (m,N)-policy and N-policy can be obtained with the suitable selection of J, m and N as in cases (i), (ii) and (vi) of Chapter 2.

3.4 NUMERICAL ANALYSIS

In this section, we study the influence of the system parameters on some significant performances measures through numerical results for the model $M^X_{(m,N)}$ /G/1/Breakdown/Vac(J) with three types of customer behaviour during the breakdown period. The distributions considered for different random variables are listed in the following table.

Random variables (Y)		**Distribution F(Y)**	**Mean E(Y)**	**Second order moments E(Y^2)**
Service time (S)		Two-stage hyper-exponential	$\frac{a_1}{\mu_1} + \frac{1-a_1}{\mu_2}$ $0 \le a_1 \le 1$	$2\left(\frac{a_1}{\mu_1^2} + \frac{1-a_1}{\mu_2^2}\right)$
Setup time (D)		Erlang-3 type	$1/\gamma$	$4/3\gamma^2$
Vacation time (VI)		Gamma-2 type	$2/\eta$	$6/\eta^2$
Repair time	R_1	Deterministic	$1/rp_1$	$1/rp_1^2$
	R_2	Exponential	$1/rp_2$	$2/rp_2^2$
	R_3	Erlang-2 type	$1/rp_3$	$3/2rp_3^2$
Life time of the server		Exponential	$1/a$	$2/a^2$
Batch size (X)		Geometric (Geo(p_1))	$1/1-p_1$	$(p_1+1)/(1-p_1)^2$

As in Chapter II, the optimal threshold values are calculated using the solution procedure given in Section 3.2 and the bilevel policy is compared with the corresponding N-policy. The influence of the dormant cost and holding cost on performance measures are also studied. The behaviour of system size probabilities corresponding to different arrival rates is observed and the significant effect of probability q_1 (with which the customers would like to repeat their service when the service is interrupted) on the mean system size is also established.

The values of the total expected cost function $T_C(m, N)$ for a given set of parameters are tabulated in Table 3.1 and the optimal cost $T_C(m^*, N^*)$ = 148.319 is observed at m^* = 5 and N^* = 6. The graphical representation of the surface of $T_C(m, N)$ and $T_C(m, N^*(m))$ are portrayed in Figures 3.1a and 3.1b.

Table 3.1 The Expected Cost $T_C(m, N)$ for Different Values of m and N Corresponding to J = 15

$(C_h, C_{set}, C_{dor}, C_{busy}, C_y, C_{VI}, C_{br}, C_{build}) = (10, 150, 100, 10, 1000, 8, 20, 8)$

$(a_1, \mu_1, \mu_2, J) = (0.3, 1.5, 2, 15)$

$(p_1, \lambda, \gamma, \eta, a, rp_1, rp_2, rp_3, q_1, q_2, q_3) = (0.60, 0.48, 1, 0.5, 5, 2, 5, 4, 0.2, 0.5, 0.3)$

m \ N	1	3	4	5	6	8	14	17	21	23
1	152.992	154.577	153.138	152.098	151.551	152.018	163.500	172.865	187.398	195.257
3		153.115	149.701	149.207	148.951	149.478	159.673	168.338	182.067	189.576
5				154.874	148.319	148.682	156.990	164.685	177.301	184.324
6					148.640	148.894	156.126	163.257	175.217	181.952
7						149.473	155.588	162.115	173.365	179.785
9							155.478	160.709	170.413	176.140
11							156.581	160.460	168.504	173.481
13							158.770	161.303	167.631	171.823
15								163.143	167.741	171.132

Figure 3.1.a. The expected cost $T_C(m, N)$ for different values of m and N

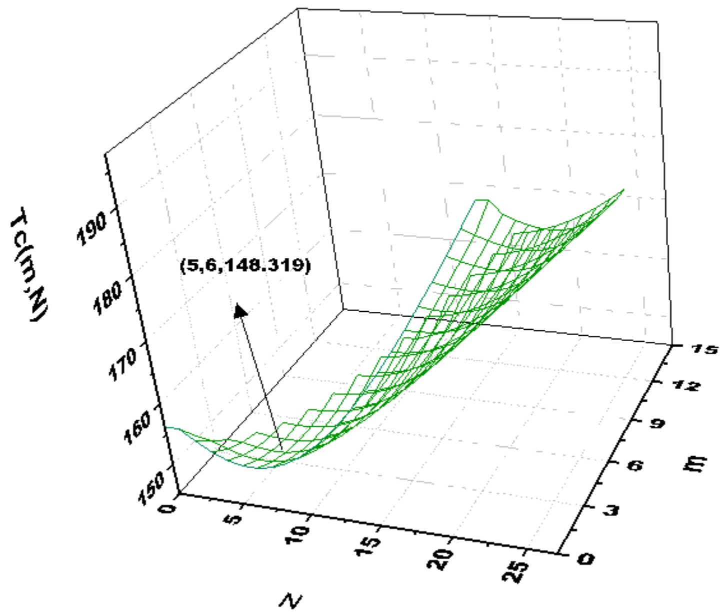


Figure 3.1.b. $T_C(m,\ N^*(m))$ Vs $(m,\ N^*(m))$

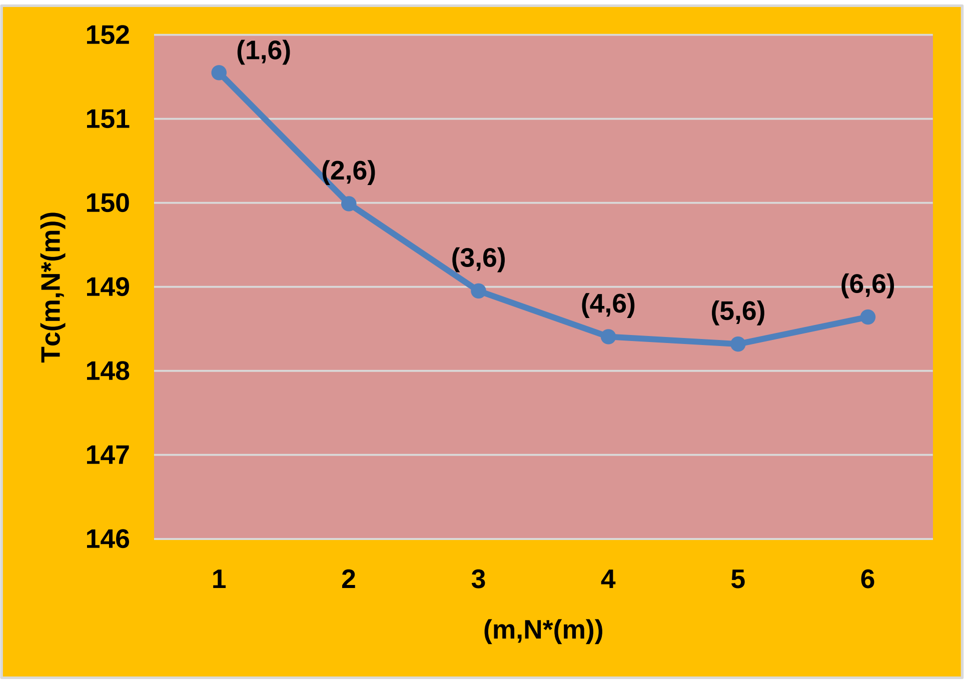


Table 3.2 gives the optimal values as the standby cost, C_{dor} changes. The table values show that as C_{dor} increases, the server starts the service earlier and the optimal values of (m, N)-policy approach the optimal value of (N, N)-policy. The parameters used to construct Table 3.2 are same as in Table 3.1 with γ = 0.15.

Table 3.2 Optimal Policy as Dormant Cost Changes

C_{dor}	(m^*, N^*)	$T_C(m^*, N^*)$	$L_{(m^*, N^*)}$	E(Cycle)	N^*	$T_C(N^*, N^*)$	$L_{(m^*, N^*)}$	E(Cycle)
1	(1, 13)	185.491	12.779	41.384	3	192.839	13.307	39.373
10	(1, 13)	185.955	12.779	41.384	3	192.839	13.307	39.373
50	(1, 12)	187.891	12.697	40.527	3	192.839	13.307	39.373
100	(1, 10)	189.911	12.597	38.943	3	192.839	13.307	39.373
200	(2, 7)	192.328	12.920	38.560	3	192.839	13.307	39.373
250	(3, 6)	192.757	13.256	39.831	3	192.839	13.307	39.373
300	(3, 4)	192.832	13.295	39.456	3	192.839	13.307	39.373

Table 3.3 presents optimal values as the holding cost increases. The values of the Table 3.3 show that as the holding cost increases, the system starts setup and service earlier than in the case of lower holding cost. The cost values and the parameters listed in Table 3.1 are used to construct the remaining Tables 3.3 to 3.6. The variant parametric values are mentioned in the corresponding tables.

Table 3.3 Optimal Policy as Holding Cost Changes

C_h	(m^*, N^*)	$T_C(m^*, N^*)$	$L_{(m^*, N^*)}$	E(Cycle)	N^*	$T_C(N^*, N^*)$	$L_{(m^*, N^*)}$	E(Cycle)
20	(1, 6)	238.144	8.659	23.000	3	242.307	9.202	22.866
30	(1, 5)	324.259	8.608	21.856	2	332.724	8.984	20.838
50	(1, 5)	496.419	8.608	21.856	1	510.411	8.815	18.666
100	(1, 5)	926.821	8.608	21.856	1	951.159	8.815	18.666
500	(1, 5)	4370.034	8.608	21.856	1	4477.147	8.815	18.666

Table 3.4 and Figures 3.4.a. and 3.4.b. analyse the influence of the group arrival rate λ on the system size probabilities and mean system size when the server is in different states. For the Tables 3.4 to 3.6 the threshold values are chosen as m = 5 and N = 6.

Table 3.4 System Size Probabilities and Mean System Size with rest to ρ

λ	ρ	P_V / L_V	P_{set} / L_{set}	P_{dor} / L_{dor}	P_{busy} / L_{busy}	P_{br} / L_{br}
0.30	0.4	0.525	0.047	0.018	0.173	0.238
		1.707	0.417	0.089	1.365	1.829
0.35	0.5	0.462	0.046	0.013	0.202	0.277
		1.648	0.431	0.064	1.774	2.390
0.45	0.6	0.338	0.040	0.006	0.259	0.356
		1.422	0.410	0.032	2.906	3.952
0.50	0.7	0.276	0.035	0.004	0.288	0.396
		1.254	0.375	0.022	3.737	5.101
0.55	0.8	0.216	0.029	0.003	0.317	0.436
		1.050	0.323	0.015	4.918	6.733

System size probabilities   Mean system size

Figure 3.4.a System size probabilities Vs traffic intensity ρ

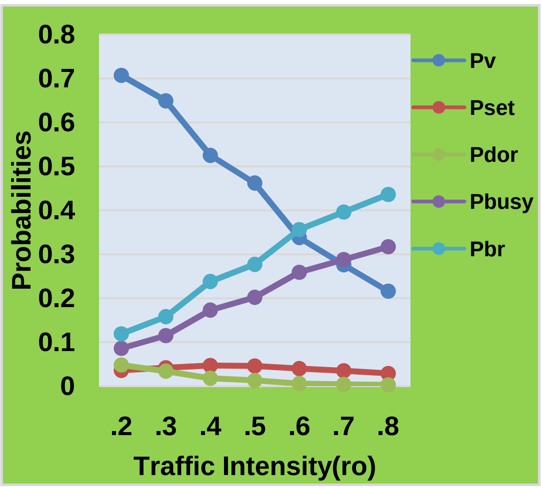
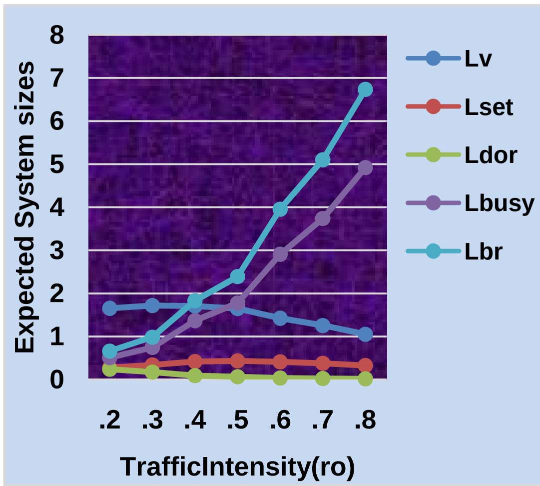


Figure 3.4.b. Expected System Size Vs Traffic Intensity ρ



In Table 3.5, we find that the mean system size increases when the probability q_1 with which the customer joins the waiting line to repeat the service soon after the breakdown increases. Also the system becomes significantly congested for higher breakdown rate **a**. Figure 3.5 shows its graphical representation.

Table 3.5 Expected System Size Vs Breakdown Rate (a) as q_1 Changes

a \ (q_1,q_2,q_3)	(0, 0.5, 0.5)	(0.2, 0.4, 0.4)	(0.4, 0.3, 0.3)	(0.6, 0.2, 0.2)	(0.8, 0.1, 0.1)	(1, 0, 0)
0.1	0.539	2.027	3.132	4.003	4.720	5.332
0.2	5.375	6.209	6.908	7.537	8.138	8.739
0.3	6.924	7.646	8.339	9.056	9.843	10.748
0.4	7.657	8.395	9.190	10.111	11.241	12.709
0.5	8.067	8.870	9.816	11.017	12.652	15.070

Figure 3.5 Expected System Size Vs. q_1 for different values of a

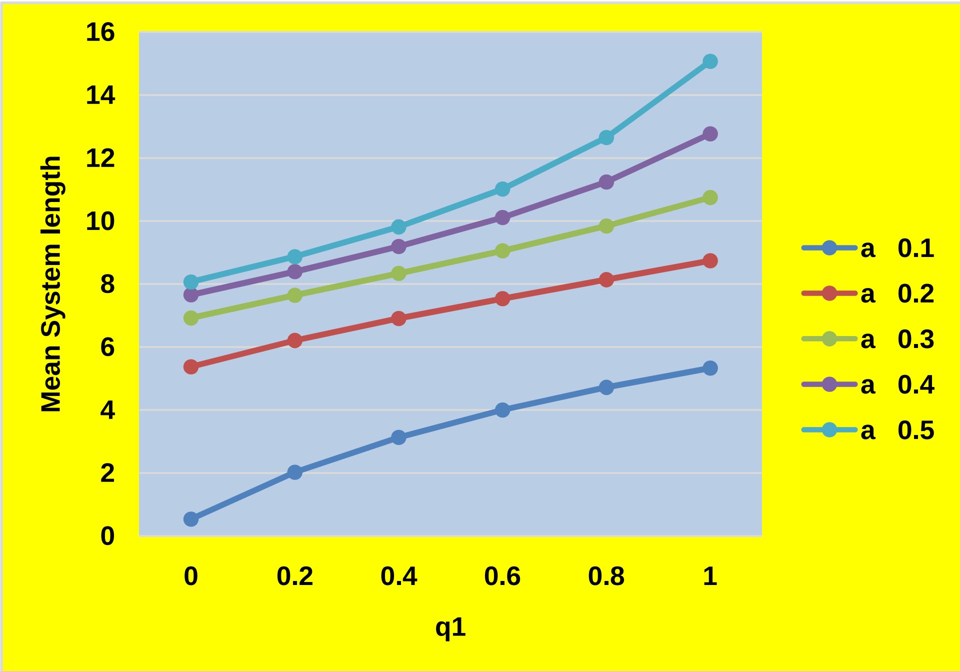


Table 3.6 shows that as the group arrival rate λ increases, expected queue length also increases and the results of the single vacation queueing model and multiple vacation model corresponding to J = 1 and $J \to \infty$.

Table 3.6. Mean System Size for Different Values of J and λ

λ \ J	1	5	10	15	17	20
0.15	2.837	3.137	3.307	3.346	3.350	3.352
0.30	4.417	5.289	5.403	5.407	5.408	5.408
0.45	7.473	8.674	8.721	8.721	8.721	8.721
0.55	11.718	13.013	13.038	13.038	13.038	13.038
0.65	24.900	26.227	26.241	26.241	26.241	26.241