International Journal of Mathematical Archive-7(6), 2016, 8-17
IJMA Available online through www.ijma.info ISSN 2229 – 5046

Between α-closed sets and $\tilde{g}_\alpha$ -closed sets

N. BALAMANI*
Assistant Professor of Mathematics,
Avinashilingam Institute for Home Science and Higher Education for Women University,
Coimbatore - 641043, Tamil Nadu, India.

A. PARVATHI
Professor of Mathematics,
Avinashilingam Institute for Home Science and Higher Education for Women University,
Coimbatore - 641043, Tamil Nadu, India.

(Received On: 27-05-16; Revised & Accepted On: 13-06-16)

ABSTRACT

In this paper we introduce and study a new class of generalized closed sets called $\psi^*\alpha$ -closed sets in topological spaces. We analyze the relations between $\psi^*\alpha$ -closed sets with already existing closed sets. We discuss some basic properties of $\psi^*\alpha$ -closed sets The class of $\psi^*\alpha$ -closed sets is properly placed between the class of α-closed sets and the class of $\tilde{g}_\alpha$ (resp. ψ)-closed sets. We prove that the class of $\psi^*\alpha$- closed sets form a topology.

Keywords: *α -closed sets, ψ-closed sets, ψg-closed sets and $\psi^*\alpha$ -closed sets*

1. INTRODUCTION

Njastad [18] introduced the concept of an α-open sets. Levine [13] introduced the notion of g-closed sets in topological spaces and studied their basic properties. Veerakumar [22] introduced and studied ψ-closed sets in topological spaces. Ramya and Parvathi [20] introduced a new concept of generalized closed sets called $\psi\hat{g}$ -closed sets and ψg-closed sets in topological spaces. Jafari *et.al*[10] introduced the class of $\tilde{g}_\alpha$-closed sets. In this paper we introduce a new class of generalized closed sets called $\psi^*\alpha$ -closed sets in topological spaces. This class is obtained by generalizing α -closed sets via ψg-open sets.

2. PRELIMINARIES

Throughout this paper (X, τ) represents non-empty topological space on which no separation axioms are defined, unless otherwise mentioned. The interior, closure and complement of a subset A of a space (X, τ) are denoted by int(A), cl(A) and A^c respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called
(i) Semi-open set [12] if A ⊆ cl(int(A))
(ii) α -open set [18] if A ⊆ int(cl(int(A)))
(iii) Pre-open set [17] if A ⊆ int(cl(A))
(iv) semi pre-open set [3] if A ⊆ cl(int(cl(A)))

The complements of the above mentioned sets are called semi-closed, α-closed, pre-closed and semi pre-closed sets respectively

The intersection of all semi-closed (resp.α-closed, pre-closed and semi pre-closed) subsets of (X,τ) containing A is called the semi-closure (resp. α-closure, pre-closure and semi pre-closure) of A and is denoted by scl(A) (resp. αcl(A), pcl(A) and spcl(A)). A subset A of (X, τ) is called nowhere dense if int(cl(A))=ϕ. A subset A of a topological space (X, τ) is called semi-closed (resp.α-closed) if and only if scl(A)=A(resp. αcl(A)=A)

Corresponding Author: N. Balamani*, Assistant Professor of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women University, Coimbatore - 641043, Tamil Nadu, India.

Definition 2.2: A subset A of a topological space (X, τ) is called
(a)generalized closed set (briefly g-closed) [13] if cl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(b) generalized semi-closed set (briefly gs-closed) [4] if scl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(c) semi-generalized closed set (briefly sg-closed) [5] if scl(A) ⊆ U whenever A⊆U and U is semi- open in (X, τ).
(d) generalized α-closed set(briefly gα-closed) [14] if αcl(A) ⊆ U whenever A⊆U and U is α-open in (X, τ).
(e) α-generalized closed set (briefly αg-closed) [15] if αcl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(f) generalized semi-pre -closed set(briefly gsp-closed) [7] if spcl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(g) $\hat{g}$ - closed set [24] if cl(A) ⊆ U whenever A⊆U and U is semi-open in (X, τ).
(h) g^*-closed set[23] if cl(A) ⊆ U whenever A⊆U and U is g-open in (X, τ).
(i)*g- closed set [30] if cl(A) ⊆ U whenever A⊆U and U is $\hat{g}$ -open in (X, τ).
(j) gp-closed set [16] if pcl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(k) g^*p -closed set [25] if pcl(A) ⊆ U whenever A⊆U and U is g-open in (X, τ).
(l) α$\hat{g}$ - closed set [1] if αcl(A) ⊆ U whenever A⊆U and U is $\hat{g}$ -open in (X, τ).
(m) αgs -closed set[19] if αcl(A) ⊆ U whenever A⊆U and U is semi-open in (X, τ).
(n) $g^\#$s-closed set [26] if scl(A) ⊆ U whenever A⊆U and U is αg-open in (X, τ).
(o) $^\#$gs-closed set [29] if scl(A) ⊆ U whenever A⊆U and U is *g -open in (X, τ).
(p) $\tilde{g}$-closed set [9] if cl(A) ⊆ U whenever A⊆U and U is $^\#$ gs-open in (X, τ).
(q) $\tilde{g}_\alpha$-closed set [10] if αcl(A) ⊆ U whenever A⊆U and U is $^\#$ gs-open in (X, τ).
(r) $\tilde{g}$-semi-closed set[21] if scl(A) ⊆ U whenever A⊆U and U is $^\#$ gs-open in (X, τ).
(s) $\tilde{g}$-pre closed set[8] if pcl(A) ⊆ U whenever A⊆U and U is $^\#$ gs-open in (X, τ).
(t) $g^\#$- closed set[27] if cl(A) ⊆ U whenever A⊆U and U is αg - open in (X, τ).
(u) $g^\#p^\#$ -closed set[2] if cl(A) ⊆ U whenever A⊆U and U is $g^\#$ - open in (X, τ).
(v) ψ-closed set [22] if scl(A) ⊆ U whenever A⊆U and U is sg-open in (X, τ).
(w) ψg-closed set [20] if ψcl(A) ⊆ U whenever A⊆U and U is open in (X, τ).
(x) g^*ψ -closed set[28] if ψcl(A) ⊆ U whenever A⊆U and U is g-open in (X, τ).
(y) ψ$\hat{g}$ - closed set[20] if ψcl(A) ⊆ U whenever A⊆U and U is $\hat{g}$ -open in (X, τ).
(z) αψ- closed set [6] if ψcl(A) ⊆ U whenever A⊆U and U is α-open in (X, τ).
The complements of the above mentioned sets are called their respective open-sets.

3. $\psi^*\alpha$ -CLOSED SETS

Definition 3.1: A subset A of a topological space (X, τ) is said to be $\psi^*\alpha$ -closed set if αcl(A) ⊆ U whenever A⊆U and U is ψg -open in (X, τ).

The class of all $\psi^*\alpha$ -closed sets of (X,τ) is denoted by $\psi^*\alpha$ C(X, τ).

Proposition 3.2: Every closed set in (X, τ) is $\psi^*\alpha$ -closed but not conversely.

Proof: Let A be a closed set and U be any ψg -open set containing A in X. Since every closed set is α-closed, αcl(A) ⊆ cl(A) =A⊆ U. Therefore A is $\psi^*\alpha$ - closed.

Example 3.3: Let X={a, b, c}, τ ={ϕ,{a},{a, b},X}. Then the subset {b} is $\psi^*\alpha$ -closed but not closed in (X, τ).

Proposition 3.4: Every α -closed set in (X, τ) is $\psi^*\alpha$ -closed but not conversely.

Proof: Let A be an α-closed set and U be any ψg -open set containing A in X. Since A is α-closed, αcl(A) =A, αcl(A) =A⊆ U. Therefore A is $\psi^*\alpha$ -closed.

Example 3.5: Let X={a, b, c}, τ ={ϕ,{a, b},X}. Then the subset {a, c} is $\psi^*\alpha$ -closed but not α -closed in (X, τ).

Lemma 3.6: Every $^\#$gs -closed set in (X, τ) is ψg - closed but not conversely.

Proof: Let A be a $^\#$gs -closed set and U be any open set containing A in X. Since every open set is *g-open and A is $^\#$gs -closed, scl(A)⊆U. For every subset A of X, ψcl(A)⊆scl(A) and so ψcl(A)⊆U. Hence A is ψg -closed.

Example 3.7: Let X={a, b, c}, τ ={ϕ,{a}, X}.Then the subset {a, b} is ψg -closed but not $^\#$gs -closed in (X, τ).

Proposition 3.8: Every $\psi^*\alpha$ -closed set in (X, τ) is $\tilde{g}_\alpha$- closed but not conversely.

Proof: Let A be a $\psi^*\alpha$-closed set and U be any $^\#$gs - open set containing A in X. Since every $^\#$gs -open set is ψg -open and A is $\psi^*\alpha$ -closed, αcl(A) ⊆ U. Hence A is $\tilde{g}_\alpha$- closed.

Example 3.9: Let X={a, b, c, d}, τ ={ϕ,{d},{a, b},{a, b, d},X}. Then the subset {b, c, d} is $\tilde{g}_\alpha$- closed but not $\psi^*\alpha$ -closed in (X, τ).

Proposition 3.10: Every $\psi^*\alpha$ -closed set in (X, τ) is gα(resp.αg, sg, gs, $\tilde{g}_s$)-closed but not conversely.

Proof: By [10], every $\tilde{g}_\alpha$- closed set is gα (resp. αg, sg, gs, $\tilde{g}_s$)- closed set. Hence it holds.

Example 3.11: Let X={a, b, c, d}, τ ={ϕ,{d}, {a, b}, {a, b, d}, X}.Then the subset {a, c, d} is gα-closed αg-closed, sg-closed, gs-closed and $\tilde{g}_s$- closed but not $\psi^*\alpha$ -closed in (X, τ).

Proposition 3.12: Every $\psi^*\alpha$ -closed set in (X, τ) is $\tilde{g}$- pre closed but not conversely.

Proof: Follows from the fact that every $\tilde{g}_\alpha$- closed is $\tilde{g}$- pre closed.

Example 3.13: Let X={a, b, c}, τ ={ϕ,{a, b},X}. Then the subset {a} is $\tilde{g}$- preclosed but not $\psi^*\alpha$ -closed in (X, τ).

Lemma 3.14: Every semi -closed set in (X, τ) is ψg -closed but not conversely.

Proof: Let A be a semi- closed set and U be any open set containing A in X. Since A is semi- closed, scl(A)=A. For every subset A of X, ψcl(A)⊆scl(A) and so we have ψcl(A)⊆U. Hence A is ψg -closed.

Example 3.15: Let X={a, b, c}, τ ={ϕ,{a},X}. Then the subset {a, b} is ψg -closed but not semi -closed in (X, τ).

Proposition 3.16: Every $\psi^*\alpha$ -closed set in (X, τ) is αgs-closed but not conversely.

Proof: Let A be a $\psi^*\alpha$ -closed set and U be any semi-open set containing A in X. Since every semi-open set is ψg-open and A is $\psi^*\alpha$ -closed, αcl(A) ⊆ U. Hence A is αgs -closed.

Example 3.17: Let X={a, b, c}, τ ={ϕ,{a},{b, c},X}.Then the subset {a, c} is αgs-closed but not $\psi^*\alpha$ -closed in (X, τ).

Lemma 3.18: Every g-closed set in (X, τ) is ψg-closed but not conversely.

Proof: Let A be a g-closed set and U be any open set containing A in X. Since A is g-closed, cl(A)⊆U. For every subset A of X, ψcl(A)⊆cl(A) and so we have ψcl(A)⊆U. Hence A is ψg-closed.

Example 3.19: Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b},X}. Then the subset {a} is ψg-closed but not g-closed in (X, τ).

Proposition 3.20: Every $\psi^*\alpha$ -closed set in (X, τ) is gp - closed (g^*p-closed) but not conversely.

Proof: Let A be a $\psi^*\alpha$ -closed set and U be any open (g-open) set containing A in X. Since every open (g-open) set is ψg-open and A is $\psi^*\alpha$ -closed, αcl(A) ⊆ U. For every subset A of X, pcl(A)⊆αcl(A)and so we have pcl(A)⊆U. Hence A is gp -closed (g^*p-closed).

Example 3.21: Let X={a, b, c}, τ ={ϕ,{a, b}, X}. Then the subset {a} is gp -closed (g^*p-closed) but not $\psi^*\alpha$ -closed in (X, τ).

Lemma 3.22: Every sg -closed set in (X, τ) is ψg - closed but not conversely.

Proof: Let A be a sg -closed set and U be any open set containing A in X. Since every open set is semi-open and A is sg-closed, scl(A)⊆U. For every subset A of X, ψcl(A)⊆scl(A) and so we have ψcl(A)⊆U. Hence A is ψg -closed.

Example 3.23: Let X={a, b, c}, τ ={ϕ,{a}, {a, b},X}. Then the subset {a, c} is ψg -closed but not sg -closed in (X, τ).

Proposition 3.24: Every $\psi^*\alpha$ -closed set in (X, τ) is ψ-closed but not conversely.

Proof: Let A be a $\psi^*\alpha$ - closed set and U be any sg -open set containing A in X. Since every sg-open set is ψg-open and A is $\psi^*\alpha$ -closed set, αcl(A) ⊆ U. For every subset A of X, scl(A)⊆αcl(A) and so we have scl(A)⊆U. Hence A is ψ -closed.

Example 3.25: Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b},X}.Then the subset {a} is ψ-closed but not $\psi^*\alpha$ -closed in (X, τ).

Proposition 3.26: Every $\psi^*\alpha$ -closed set in (X, τ) is $\psi\hat{g}$ (resp. ψg, gsp)-closed but not conversely.

Proof: By [20], every ψ-closed set is $\psi\hat{g}$ (resp. ψg, gsp)-closed. Therefore it holds

Example 3.27: Let X={a, b, c, d}, τ ={ϕ,{a},X}.Then the subset {a, b} is $\psi\hat{g}$ -closed, ψg-closed, and gsp-closed but not $\psi^*\alpha$ -closed in (X, τ).

Lemma 3.28: Every αg -closed set in (X, τ) is ψg - closed but not conversely.

Proof: Let A be an αg -closed set and U be any open set containing A in X. Since A is αg -closed, αcl(A) ⊆ U. For every subset A of X, ψcl(A)⊆αcl(A) and so ψcl(A)⊆U. Hence A is ψg -closed.

Example 3.29: Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b},X}. Then the subset {a} is ψg -closed but not αg -closed in (X, τ).

Lemma 3.30: Every gα -closed set in (X, τ) is ψg - closed but not conversely.

Proof: Let A be a gα -closed set and U be any open set containing A in X. Since every open set is α-open and A is gα -closed, αcl(A) ⊆ U. For every subset A of X, ψcl(A)⊆αcl(A) and so ψcl(A)⊆U. Hence A is ψg -closed.

Example 3.31: Let X={a, b, c}, τ ={ϕ,{a},{a, b},X}.Then the subset {a, c} is ψg -closed but not gα -closed in (X, τ).

Proposition 3.32: Every $\psi^*\alpha$ -closed set in (X, τ) is $g^{\#}s$ -closed but not conversely.

Proof: Let A be a $\psi^*\alpha$ -closed set and U be any αg -open set containing A in X. Since every αg open set is ψg -open and A is $\psi^*\alpha$-closed, αcl(A) ⊆ U. For every subset A of X, scl(A)⊆αcl(A) and so scl(A)⊆U. Hence A is $g^{\#}s$ -closed.

Example 3.33: Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b},X}.Then the subset {a} is $g^{\#}s$ -closed but not $\psi^*\alpha$ -closed in (X, τ).

Lemma 3.34: Every $\hat{g}$ -closed set in (X, τ) is ψg -closed but not conversely.

Proof: Let A be a $\hat{g}$ -closed set and U be any open set containing A in X. Since every open set is semi open and A is $\hat{g}$ -closed, cl(A)⊆U. For every subset A of X, ψcl(A)⊆cl(A) and so we have ψcl(A)⊆U. Hence A is ψg -closed.

Example 3.35: Let X={a, b, c}, τ ={ϕ,{a},X}. Then the subset {a, b} is ψg -closed but not $\hat{g}$ - closed in (X, τ).

Proposition 3.36: Every $\psi^*\alpha$ -closed set in (X, τ) is $\alpha\hat{g}$ - closed but not conversely.

Proof: Let A be a $\psi^*\alpha$ -closed set and U be any $\hat{g}$ -open set containing A in X. Since every $\hat{g}$ - open set is ψg-open and A is $\psi^*\alpha$ -closed, αcl(A) ⊆ U. Hence A is $\alpha\hat{g}$-closed.

Example 3.37: Let X={a, b, c}, τ ={ϕ,{a},{a, b}, X}. Then the subset {a, c} is $\alpha\hat{g}$-closed but not $\psi^*\alpha$ -closed in (X, τ).

Lemma 3.38: Every $^*g$-closed set in (X, τ) is ψg-closed but not conversely.

Proof: Let A be a $^*g$ -closed set and U be any open set containing A in X. Since every open set is $\hat{g}$ -open and A is $^*g$ -closed, cl(A)⊆U. For every subset A of X, ψcl(A)⊆cl(A) and so we have ψcl(A)⊆U. Hence A is ψg-closed.

Example 3.39: Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b}, X}.Then the subset {b} is ψg -closed but not $^*g$ -closed in (X, τ).

Proposition 3.40: Every $\psi^*\alpha$ -closed set in (X, τ) is $^{\#}gs$- closed but not conversely.

Proof: Let A be a $\psi^*\alpha$ -closed set and U be any $^*g$ -open set containing A in X. Since every $^*g$ -open set is ψg-open and A is $\psi^*\alpha$ -closed, αcl(A) ⊆ U. For every subset A of X, scl(A)⊆αcl(A) and so we have scl(A)⊆U. Hence A is $^{\#}gs$ -closed.

Example 3.41: Let X={a, b, c}, τ ={ϕ,{a},{a, b},X}.Then the subset {a, c} is $^{\#}gs$ -closed but not $\psi^*\alpha$ -closed in (X, τ).

Proposition 3.42: Every $\psi^*\alpha$ -closed set in (X, τ) is $g^*\psi$ -closed but not conversely.

Proof: Follows from the fact that every ψ-closed is $g^*\psi$ -closed

Example 3.43: Let X={a, b, c, d}, τ ={ϕ,{a}, {a, b},X}. Then the subset{a, c, d} is $g^*\psi$ -closed but not $\psi^*\alpha$ -closed in (X, τ).

Proposition 3.44: Every $\psi^*\alpha$ -closed set in (X, τ) is $\alpha\psi$ -closed but not conversely.

Proof: Let A be a $\psi^*\alpha$ -closed set and U be any α -open set containing A in X. Since every α -open set is ψg -open and A is $\psi^*\alpha$ -closed, $\alpha cl(A) \subseteq U$. For every subset A of X, $\psi cl(A) \subseteq \alpha cl(A)$. and so we have $\psi cl(A) \subseteq U$. Hence A is αψ-closed.

Example 3.45: Let X={a, b, c}, τ ={ϕ,{a},{b},{a, b},X}.Then the subset {a} is $\alpha\psi$ -closed but not $\psi^*\alpha$ -closed in (X, τ).

Remark 3.46: The following example shows that $\psi^*\alpha$ -closedness is independent from g-closedness, g^*-closedness, $^*g$-closedness and $g^\# p^\#$- closedness.

Example 3.47: Let X={a, b, c}, τ ={ϕ,{a},{a, b},X}.In this topology the set {a, c} is g-closed, g^*-closed, $^*g$ - closed and $g^\# p^\#$- closed but not $\psi^*\alpha$ -closed. The set {b} is $\psi^*\alpha$ -closed but not g-closed, g^*-closed, $^*g$ - closed and $g^\# p^\#$- closed.

Remark 3.48: The following examples show that $\psi^*\alpha$ -closedness is independent from semi-closedness.

Example 3.49: Let X={a, b, c}, τ ={ϕ,{a, b},X}.In this topology the set {b, c} is $\psi^*\alpha$ -closed but not semi-closed.

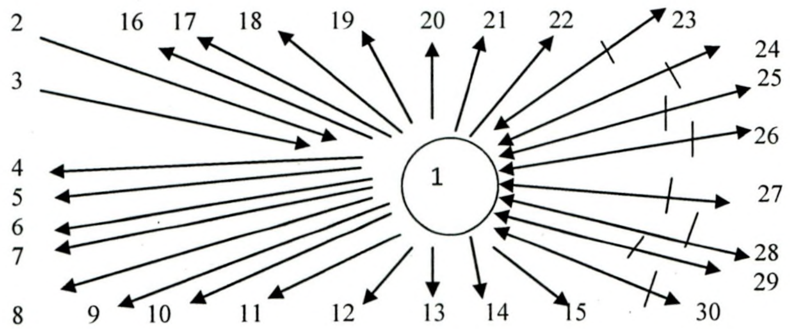
Example 3.50: Let X={a, b, c}, τ ={ϕ.{a},{b},{a, b},X}. In this topology the set{b} is semi-closed but not $\psi^*\alpha$ -closed.

Remark 3.51: The following examples show that $\psi^*\alpha$ -closedness is independent from ĝ-closedness, $g^\#$-closedness and g̃ -closedness.

Example 3.52: Let X={a, b, c} with τ ={ϕ,{a},{a, b},X}. In this topology the set{b} is $\psi^*\alpha$ -closed but not ĝ - closed, $g^\#$-closed and g̃-closed.

Example 3.53: Let X={a, b, c, d} with τ ={ϕ.{d},{a, b},{a, b, d}, X}. In this topology the set {a, c, d} is ĝ - closed, $g^\#$-closed and g̃-closed but not $\psi^*\alpha$ -closed.

Remark 3.54: The following diagram has shown the relationship of $\psi^*\alpha$ -closed sets with already existing various closed sets. where A → B represents A implies B but not conversely. where A ←+→ B represents A and B are independent of each other.



1. $\psi^*\alpha$ -closed 2. closed 3. α-closed 4. $\tilde{g}_\alpha$ -closed 5. gα-closed 6. αg -closed 7.sg-closed 8. gs-closed 9. g̃-semi- closed 10. g̃-pre-closed 11.αgs-closed 12.gp-closed 13. g^*p-closed 14. ψ-closed 15. ψĝ-closed 16.ψg- closed 17. gsp-closed 18. $g^\#s$-closed 19. αĝ -closed 20. $^\#gs$ -closed 21. $g^*\psi$-closed 22.αψ-closed 23.g- closed 24. g^*-closed 25.$^*g$-closed 26 $g^\# p^\#$-closed 27. semi-closed 28. ĝ -closed 29. $g^\#$-closed 30. g̃ -closed

Definition 3.55: A subset A of a topological space (X, τ) is said to be $\psi^*\alpha$ -open if its complement A^c is $\psi^*\alpha$ -closed.

The class of all $\psi^*\alpha$ -open sets in (X, τ) is denoted by $\psi^*\alpha O(X, \tau)$.

Proposition 3.56: Every open (respectively α-open) set is $\psi^*\alpha$ -open.

Proposition 3.57: Every $\psi^*\alpha$ -open set is $\tilde{g}_\alpha$ - open (respectively gα-open, αg-open, sg-open, gs-open, $\tilde{g}$-semi- open, $\tilde{g}$-pre- open, αgs-open, gp- open, g^*p-open, ψ-open, ψ$\hat{g}$- open, ψg-open, gsp- open, $g^\#$s-open, α$\hat{g}$ -open, $^\#$gs- open, $g^*\psi$-open and αψ-open)

4. PROPERTIES OF $\psi^*\alpha$ -CLOSED SETS AND $\psi^*\alpha$ -OPEN SETS

Theorem 4.1: If A and B are $\psi^*\alpha$ -closed sets in a topological space (X, τ), then A∪ B is $\psi^*\alpha$ -closed set in (X, τ).

Proof: Let A and B be any two $\psi^*\alpha$ -closed sets in (X, τ) and U be any ψg -open set containing A and B. We have αcl(A) ⊆ U and αcl(B) ⊆ U. Always αcl(A∪B) = αcl(A) ∪ αcl(B) ⊆ U. Hence A ∪ B is $\psi^*\alpha$ -closed in (X, τ).

Theorem 4.2: Let A be a $\psi^*\alpha$ -closed set in (X, τ). Then αcl(A)-A contains no non-empty closed set in (X, τ).

Proof: Suppose that A is $\psi^*\alpha$-closed. Let F be a closed subset of αcl(A)-A. Then F^c is open and hence ψg -open such that A⊆ F^c. Since A is a $\psi^*\alpha$ -closed set, αcl(A) ⊆ F^c. Thus F⊆$(\alpha cl(A))^c$. Since every closed set is α -closed, F is α-closed. Hence F⊆αcl(A). Therefore F⊆ αcl(A) ∩ $(\alpha cl(A))^c$ =ϕ. Hence F=ϕ.

Remark 4.3: The converse of the above theorem is not true as seen from the following example.

Example 4.4: Let X={a, b, c}, τ={ϕ, {a}, {b, c}, X}. If A={b} then αcl(A)-A ={b, c}-{b}={c}does not contain non-empty closed set. However A is not a $\psi^*\alpha$-closed subset of (X, τ).

Theorem 4.5: A set A is $\psi^*\alpha$ –closed in (X, τ) if and only if αcl(A)-A contains no non-empty ψg -closed set in (X, τ).

Proof: (Necessity): Suppose that A is $\psi^*\alpha$ -closed. Let F be a ψg -closed set contained in αcl(A)-A. Now F^c is a ψg -open set in X such that A⊆ F^c. Since A is a $\psi^*\alpha$ -closed set in X, αcl(A) ⊆ F^c. Thus F⊆$(\alpha cl(A))^c$. Also F⊆αcl(A)-A. Therefore F⊆ αcl(A) ∩ $(\alpha cl(A))^c$ =ϕ. Hence F= ϕ.

Sufficiency: Suppose that αcl(A)-A contains no non empty ψg -closed set. Let A⊆G and G be ψg-open. If αcl(A) is not a subset of G then αcl(A) ∩ G^c is a non-empty ψg -closed subset of αcl(A)-A, which is a contradiction. Therefore αcl(A)⊆G and hence A is $\psi^*\alpha$ -closed.

Proposition 4.6: If A is ψg -open and $\psi^*\alpha$ -closed subset of (X, τ).Then A is an α-closed set of (X, τ).

Proof: Since A is ψg -open and $\psi^*\alpha$ -closed, αcl(A) ⊆ A. Hence A is α-closed.

Theorem 4.7: If a set A is $\psi^*\alpha$ -closed and ψg -open and F is α-closed in (X,τ), then A ∩ F is α-closed.

Proof: Since A is $\psi^*\alpha$ -closed and ψg -open, A is α-closed by **Proposition 4.6** Since F is α -closed in X, A ∩ F is α -closed in X.

Theorem 4.8: If A is a $\psi^*\alpha$ -closed set in (X,τ) and A⊆B⊆αcl(A) . Then B is also a $\psi^*\alpha$ -closed set in (X, τ).

Proof: Let U be a ψg -open set of (X, τ) such that B⊆U. Then A⊆U. Since A is a $\psi^*\alpha$ -closed set, αcl(A) ⊆ U. Also since B⊆αcl(A), αcl(B)⊆ αcl(αcl(A))= αcl(A). Hence αcl(B)⊆U. Therefore B is also a $\psi^*\alpha$ -closed set in (X, τ).

Theorem 4.9: Let A be a $\psi^*\alpha$ -closed set of (X,τ). Then A is α-closed if and only if αcl(A)-A is ψg -closed.

Proof: (Necessity): Let A be an α-closed subset of (X,τ). Then αcl(A)=A and therefore αcl(A)-A=ϕ which is ψg-closed in (X, τ).

Sufficiency: Let αcl(A)-A be a ψg -closed set. Since A is $\psi^*\alpha$ -closed by **theorem 4.5,** αcl(A)-A contains no non-empty ψg-closed set which implies αcl(A)-A=ϕ. That is αcl(A)=A. Hence A is α-closed.

Definition 4.10: Let (X,τ) be a topological space and let B⊆A⊆X Then B is $\psi^*\alpha$ -closed relative to A if $(\alpha cl)_A(B)$⊆U, whenever B⊆U, U is ψg-open in A.

Theorem 4.11: Let B⊆A⊆X and suppose that B is $\psi^*\alpha$ -closed in (X, τ), then B is $\psi^*\alpha$ -closed relative to A. The converse is true if A is α - open and $\psi^*\alpha$ -closed in (X, τ).

Proof: Suppose that B is a $\psi^*\alpha$ -closed in (X, τ). Let B⊆U, U is ψg - open in A. Since U is ψg - open set in A, U=V ∩ A, where V is ψg - open in X. Hence B⊆U⊆V. Since B is $\psi^*\alpha$ -closed in X. αcl(B)⊆V. Hence αcl(B) ∩ A ⊆ V ∩ A which in turn implies that $(\alpha cl)_A(B)$⊆V ∩ A=U. Therefore B is $\psi^*\alpha$ -closed relative to A.

Now, to prove the converse, assume that B⊆A⊆X where A is α- open and $\psi^*\alpha$ -closed in X and B is a $\psi^*\alpha$ -closed relative to A. Let B⊆U and U be ψg-open in X. Then A ∩ U is ψg- open in A. Since B⊆A and B⊆U, B⊆A ∩ U Since B is a $\psi^*\alpha$ -closed relative to A, $(\alpha cl)_A(B)$⊆A ∩ U. Since A is α-open, it is ψg-open in X. Since A⊆A and A is $\psi^*\alpha$ -closed in X, αcl(A)⊆A. Since B⊆A, αcl(B)⊆αcl(A).Hence αcl(B)⊆A. Therefore αcl(B)∩A⊆αcl(B) implies $(\alpha cl)_A(B)$=αcl(B). Hence αcl(B) ⊆A ∩ U=U. Thus B is $\psi^*\alpha$ -closed in X.

Theorem 4.12: In a topological space (X, τ), for each x∈X, either {x} is ψg-closed or X-{x} is $\psi^*\alpha$ closed set in (X,τ).

Proof: suppose that {x} is not ψg-closed in X. Then X-{x} is not ψg-open in X. Hence X is the only ψg-open set containing X-{x}. That is (X-{x})⊆X Therefore αcl(X-{x})⊆X which implies that X-{x} is $\psi^*\alpha$ -closed in (X, τ).

Definition 4.13: The intersection of all ψg-open subsets of (X, τ) containing A is called ψg- kernal of A and is denoted by ψg-ker(A)
i.e ψg-ker(A)=∩{U / U is ψg-open in (X, τ) and A ⊆U}

Theorem 4.14: A subset A of a topological space (X, τ) is $\psi^*\alpha$ -closed in (X,τ) if and only if αcl(A) ⊆ψg-ker(A).

Proof: (Necessity): Suppose that A is $\psi^*\alpha$ -closed set in (X,τ) and x∈αcl(A). If x∉ψg-ker(A), then there exists a ψg - open set U in (X,τ) such that A⊆U and x∉U. Since U is ψg - open set containing A and A is $\psi^*\alpha$-closed, we have αcl(A) ⊆U, which is a contradiction to x∈αcl(A) and x∉U.

Sufficiency: Suppose that αcl(A) ⊆ψg-ker(A). If U is any ψg-open set containing A, then ψg-ker(A) ⊆U so we have αcl(A) ⊆U. Hence A is $\psi^*\alpha$ -closed.

Remark 4.15: Jankovic and Reilly [11] stated that "If x is any point in a topological space (X, τ), then every singleton {x} is either nowhere dense or preopen in (X, τ)". Also this provides another decomposition namely $X=X_1 \cup X_2$ where X_1={x∈X:{x} is nowhere dense} and X_2={ x∈X:{x} is preopen}.

Proposition 4.16: For any subset A of a topological space (X,τ), X_2 ∩ αcl(A) ⊆ψg-ker(A).

Proof: Let x∈ X_2∩ αcl(A) and if x∉ψg-ker(A).Then there is a ψg-open set U containing A such that x∉U. Then U^c is ψg-closed set containing x. Since x∈αcl(A), αcl({x})⊆αcl(A). Since x∈X_2, {x}⊆int(cl({x}), hence int(cl({x}) ≠ϕ. Also x∈αcl(A), so A ∩ int(cl({x}))≠ϕ. Hence there is some point y∈ A ∩ int(cl({x})) and therefore y∈A ∩ U^c , which is a contradiction.

Theorem 4.17: A subset A of a topological space (X,τ) is $\psi^*\alpha$ -closed in (X,τ) if and only if X_1 ∩ αcl(A) ⊆A

Proof: (Necessity): Suppose that A is $\psi^*\alpha$ -closed in (X,τ) and x∈ X_1∩ αcl(A) but x∉A. Since x∈X_1, int(cl({x})) =ϕ so we have int(cl({x})) =ϕ ⊆{x}. Therefore {x} is semi-closed. Since every semi-closed set is ψg -closed, {x} is ψg -closed and hence U=X-{x} is ψg-open set containing A and so αcl(A) ⊆U. Since x∈αcl(A) so we have x∈U, which is a contradiction.

Sufficiency: Suppose that X_1 ∩ αcl(A) ⊆A. Since A ⊆ψg-ker(A), X_1 ∩ αcl(A) ⊆ψg-ker(A). Therefore αcl(A)= X∩ αcl(A)=($X_1 \cup X_2$) ∩ αcl(A)=(X_1∩ αcl(A)) ∪ (X_2∩ αcl(A)).By hypothesis X_1 ∩ αcl(A) ⊆ψg-ker(A) and by

Proposition 4.16: X_2∩ αcl(A) ⊆ψg-ker(A).Hence αcl(A) ⊆ψg-ker(A). Therefore by **Theorem 4.14** A is $\psi^*\alpha$ -closed.

Theorem 4.18: Arbitrary intersection of $\psi^*\alpha$ -closed sets in a topological space (X, τ) is $\psi^*\alpha$- closed in (X, τ).

Proof: Let F={A_i : i∈ Λ }be a family of $\psi^*\alpha$ -closed sets and $A=\cap_{i\in\Lambda} A_i$ Since A ⊆A_i for each i∈ Λ , $X_1 \cap \alpha cl(A) \subseteq X_1 \cap \alpha cl(A_i)$ for each i∈ Λ , using **theorem 4.17** for each $\psi^*\alpha$ -closed set A_i, we have $X_1 \cap \alpha cl(A) \subseteq X_1 \cap \alpha cl(A_i) \subseteq A_i$, for each i∈ Λ. Thus $X_1 \cap \alpha cl(A) \subseteq \cap_{i\in\Lambda} A_i$ = A That is $X_1 \cap \alpha cl(A) \subseteq A$ and so by **theorem 4.17** A is $\psi^*\alpha$-closed in (X,τ)

Remark 4.19: Thus from **theorem 4.1**and **theorem 4.18** leads us into another class of closed sets namely $\psi^*\alpha$ -closed sets which are closed under finite union and arbitrary intersection. Hence the class of $\psi^*\alpha$ -closed sets form a topology.

Lemma 4.20: For a subset A of (X, τ), αcl(X-A) =X-αint(A)

Theorem 4.21: A subset A of a topological space (X, τ) is $\psi^*\alpha$ -open if and only if U⊆αint(A) whenever U⊆A and U is ψg -closed.

Proof: (Necessity) Assume that A is $\psi^*\alpha$ -open. Then A^c is $\psi^*\alpha$ -closed. Let U be a ψg -closed set in (X, τ) contained in A. Then U^c is a ψg -open set in (X, τ) containing A^c. Since A^c is $\psi^*\alpha$ -closed, $\alpha cl(A^c) \subseteq U^c$ equivalently U⊆αint(A).

Sufficiency: Assume that U is contained in αint(A) whenever U is contained in A and U is ψg -closed in (X, τ). Let A^c be contained in U, where U is ψg -open. Then U^c is contained in A. By criteria, $U^c \subseteq \alpha int(A)$. This implies $(\alpha int(A))^c \subseteq U$ that is $\alpha cl(A^c) \subseteq U$. Therefore A^c is $\psi^*\alpha$ -closed. Hence A is $\psi^*\alpha$ -open in (X, τ).

Proposition 4.22: If αint(A) ⊆B⊆A and A is $\psi^*\alpha$ –open, then B is $\psi^*\alpha$ –open.

Proof: Follows from lemma **4.20 and Theorem 4.8**

Theorem 4.23: If A and B are $\psi^*\alpha$ -open sets in (X, τ), then A ∩ B is $\psi^*\alpha$ -open in (X, τ).

Proof: Let A and B be $\psi^*\alpha$-open sets in (X, τ). Then X-A and X-B are $\psi^*\alpha$-closed sets and (X-A)∪(X-B)= X-(A ∩B) is $\psi^*\alpha$-closed in (X, τ). Hence A ∩ B is $\psi^*\alpha$-open.

Theorem 4.24: If a set A is $\psi^*\alpha$-open in (X, τ) if and only if G=X whenever G is ψg-open and $\alpha int(A) \cup A^c \subseteq G$.

Proof: (Necessity): Let A be $\psi^*\alpha$-open and G is ψg-open and $\alpha int(A) \cup A^c \subseteq G$. This gives $G^c \subseteq (\alpha int(A) \cup A^c)^c = (\alpha int(A))^c \cap A = (\alpha int(A))^c - A^c = \alpha cl(A^c) - A^c$. Since A^c is $\psi^*\alpha$- closed and G^c is ψg -closed by **theorem 4.5,** it follows that $G^c=\phi$. Therefore G=X.

(Sufficiency): Suppose that F is ψg -closed and F⊆A. Then $\alpha int(A) \cup A^c \subseteq \alpha int(A) \cup F^c$. As open implies α-open implies ψg-open, we get αint(A) is ψg-open and F^c ψg-open. Henceαint(A) ∪ F^c ψg-open. It follows by the hypothesis that $\alpha int(A) \cup F^c = X$ and hence F⊆αint(A). Therefore by **theorem 4.21,** A is $\psi^*\alpha$- open in (X, τ).

5. $\psi^*\alpha$ –CLOSURE

Definition 5.1: The $\psi^*\alpha$ -closure of A (briefly $\psi^*\alpha cl(A)$) of a topological space (X,τ) is defined as follows.
$\psi^*\alpha cl(A) = \cap$ {F⊆X: A ⊆F and F is $\psi^*\alpha$ -closed in (X,τ)}

Proposition 5.2: For a subset A of a topological space (X, τ), $A \subseteq \psi^*\alpha cl(A) \subseteq cl(A)$

Proof: Follows from proposition **3.2**

Remark 5.3: If A is $\psi^*\alpha$ -closed in (X,τ), then $\psi^*\alpha cl(A)=A$.

Theorem 5.4: Let A be a subset of X and x∈X, then $x \in \psi^*\alpha cl(A)$ if and only if for every $\psi^*\alpha$-open set U containing x, U ∩ A ≠ϕ.

Proof: (Necessity): Let $x \in \psi^*\alpha cl(A)$ and there exists a $\psi^*\alpha$-open set U containing x such that U ∩ A = φ. Since $A \subseteq U^c$, $\psi^*\alpha cl(A) \subseteq U^c$ and hence $x \notin \psi^*\alpha cl(A)$,which is a contradiction. Hence U ∩ A ≠ϕ.
(Sufficiency): Assume the given condition. Suppose that $x \notin \psi^*\alpha cl(A)$. Then there exists a $\psi^*\alpha$ - closed set F containing A such that x∉F. Then $x \in F^c$ and F^c is $\psi^*\alpha$-open. By assumption, $F^c \cap A \neq \phi$. Since A ⊆ F, $F^c \cap A = \phi$, which is a contradiction. Therefore $x \in \psi^*\alpha cl(A)$.

Proposition 5.5: Let A and B be any two subsets of (X, τ). Then the following statements are true
(a) $\psi^*\alpha cl(\phi)=\phi$ and $\psi^*\alpha cl(X)=X$.
(b) If $A \subseteq B$, then $\psi^*\alpha cl(A) \subseteq \psi^*\alpha cl(B)$.
(c) $\psi^*\alpha cl(A) \cup \psi^*\alpha cl(B)= \psi^*\alpha cl(A \cup B)$
(d) $\psi^*\alpha cl(A\cap B) \subseteq \psi^*\alpha cl(A) \cap \psi^*\alpha cl(B)$
(e) $\psi^*\alpha cl(\psi^*\alpha cl(A)) =\psi^*\alpha cl(A)$.

Proof: (a) and **(b)** follow from the definition of $\psi^*\alpha$-closure.
(c) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, by **(b)** $\psi^*\alpha cl(A) \subseteq \psi^*\alpha cl(A \cup B)$ and $\psi^*\alpha cl(B) \subseteq \psi^*\alpha cl(A \cup B)$.Hence $\psi^*\alpha cl(A) \cup \psi^*\alpha cl(B) \subseteq \psi^*\alpha cl(A \cup B)$. To prove the reverse inclusion, let $x \in \psi^*\alpha cl(A\cup B)$ and suppose that $x \notin \psi^*\alpha cl(A) \cup \psi^*\alpha cl(B)$.Then $x \notin \psi^*\alpha cl(A)$ and $x \notin \psi^*\alpha cl(B)$. Therefore there exist a $\psi^*\alpha$ - closed sets U and V in X such that $A \subseteq U$, $B \subseteq V$, $x \notin U$ and $x \notin V$. Hence we have $A \cup B \subseteq U \cup V$ and $x \notin U \cup V$. By **theorem 4.1,** $U \cup V$ is a $\psi^*\alpha$-closed set and hence $x \notin \psi^*\alpha cl(A \cup B)$, which is a contradiction. Hence $\psi^*\alpha cl(A\cup B) \subseteq \psi^*\alpha cl(A) \cup \psi^*\alpha cl(B)$.Therefore $\psi^*\alpha cl(A) \cup \psi^*\alpha cl(B)= \psi^*\alpha cl(A \cup B)$.

(d) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, by **(b)** $\psi^*\alpha cl(A \cap B) \subseteq \psi^*\alpha cl(A)$ and $\psi^*\alpha cl(A \cap B) \subseteq \psi^*\alpha cl(B)$. Hence $\psi^*\alpha cl(A\cap B) \subseteq \psi^*\alpha cl(A) \cap \psi^*\alpha cl(B)$.

(e) Follows from the definition of $\psi^*\alpha$-closure.

Remark 5.6: The reverse inclusion of **(d)** is not true in general as seen from the following example.

Example 5.7: Let X={a, b, c, d}, τ={ϕ, {a}, {a, b, c}, X}.If A={a} and B={d}, then $\psi^*\alpha cl(A)=X$ and $\psi^*\alpha cl(B)=\{d\}$, $A \cap B=\phi$, $\psi^*\alpha cl(A \cap B)= \phi$. But $\psi^*\alpha cl(A) \cap \psi^*\alpha cl(B)=\{d\}$.

Theorem 5.8: The $\psi^*\alpha$ –closure is a Kuratowski closure operator on (X,τ).

Proof: From $\psi^*\alpha cl(\phi)=\phi$, $A \subseteq \psi^*\alpha cl(A)$, $\psi^*\alpha cl(A) \cup \psi^*\alpha cl(B)= \psi^*\alpha cl(A \cup B)$ and $\psi^*\alpha cl(\psi^*\alpha cl(A)) =\psi^*\alpha cl(A)$ we can say that $\psi^*\alpha$ –closure is a Kuratowski closure operator on (X,τ).

REFERENCES

1. Abd El-Monsef, M.E., Rose Mary,S. and Lellis Thivagar, M. On $\alpha\hat{g}$ -closed sets in topological spaces, Assiut University Journal of Mathematics and computer science, Vol.36 (2007), 43 – 51.
2. Alli, K.,Subramanian,A. and Pious Missier $g^{\#}p^{\#}$-closed sets in topological spaces, International Journal of Mathematics and Soft computing Vol.3No.3.(2013),55-60.
3. Andrijevic,D., Semi –preopen sets, Math.Vesnik, Vol.38(1)(1986),24-32.
4. Arya,S.P. and Nour,T.,Characterizations of S-normal spaces, Indian J.pure Appl. Math. Vol.21(1990),717-719
5. Bhattacharyya,P. and Lahiri, B.K., Semi-generalized closed sets in topology, Indian J. Math., Vol.29(1987), 376-382.
6. Devi, R. and Parimala,M. On quasi αψ -open functions in topological spaces. Applied Mathematical Sciences. Vol.3No.(58)(2009), 2881-2886
7. Dontchev, J., On generalizing semi –pre open sets, Mem. Fac. Sci. Kochi Univ. Ser.A. Math., Vol.16(1995), 35-48.
8. Ganesan.S., Ravi.O. and Chandrasekar.S., $\tilde{g}$ -preclosed sets in topology, International Journal of Mathematical Archive- Vol.2(2)(2011), 294-299.
9. Jafari,S.,Noiri,T.,Rajesh,N. and LellisThivagar,M., Another generalization of closed sets, Kochi. J. Math., Vol.3 (2008), 25-38.
10. Jafari,S.,LellisThivagar,M. and Nirmala Rebecca Paul., Remarks on $\tilde{g}_\alpha$-closed sets in topological spaces, International Mathematical Forum, Vol.5, No.24 (2010), 1167-1178.
11. Jankovic,D.and Reilly,I.L.,Semi-Separation properties, Indian J.Pure.App Math., Vol.16(1985)964-957.
12. Levine, N., Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, Vol.70, (1963)36-41.
13. Levine, N., Generalized closed sets in topological spaces, Rend. Circ. Mat. Palermo, Vol.19 (2) (1970), 89-96.
14. Maki,H., Devi,R.,and Balachandran,K.,, Generalized α-closed sets in topology, Bull.Fukuoka Univ, Ed.,Part III., 42(1993),13-21.
15. Maki,H., Devi,R. and Balachandran., Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac.Sci. Kochi Univ. Ser.A, Math., 15(1994), 51-63.
16. Maki., H., Umehara, J. and Noiri, T.,Every topological spaces is pre-T1/2Mem.Fac.Soc. Kochi Univ. math., Vol.17(1996),33 – 42.
17. Mashhour,A.S.,Abd EL-Monsef,M.E.,and EL-Deep,,S.N., On pre-continuous and weak continuous mapping ,Proc.Math.and Phys.Soc.Egypt,Vol.53(1982),47-53.

18. Njastad,O., On some classes of nearly open sets, Pacific J. Math. 15(1965), 961-970.
19. Rajamani.M. and Vishwanathan,K.,αgs-closed sets in topological spaces, Acta Cienia Indica Vol. XXXM(3) (2004), 521-526.
20. Ramya,N.,and Parvathi,A.(2011), $\psi\hat{g}$ -closed sets in topological spaces, International Journal of Mathematical Archive - Vol.2(10) (2011),1992 – 1996.
21. Sundram,P., Rajesh,N., LellisThivagar,M. and Duszynski.Z., $\tilde{g}$ -semi-closed sets in topological spaces, Mathematica Pannonica, 18/1(2007),51-61.
22. Veera kumar, M.K.R.S., Between semi-closed sets and semi-pre closed sets, Rend. Istit.Mat. Univ. Trieste, (ITALY) XXXXII (2000), 25-41.
23. Veera kumar, M.K.R.S., Between closed and g-closed sets, Mem.Fac Sci. Kochi Univ.Math.,Vol.21(2000) 1-19.
24. Veerakumar,M.K.R.S., $\hat{g}$-closed sets in topological spaces, Bull.Allah. Math. Soc, Vol.18 (2003), 99-112.
25. Veera Kumar, M.K.R.S., g^*-pre closed sets, Acta Ciencia India, Vol.28 (1) (2002), 51-60.
26. Veera kumar, M.K.R.S. $g^\#$-semi-closed sets in topological spaces Indian J. Math. Vol.44, No.1 (2002)73-87.
27. Veera kumar, M.K.R.S., $g^\#$-closed sets in topological spaces, Mem.Fac Sci. KochiUniv.Math., Vol.24 (2002), 1-13.
28. Veera kumar, M.K.R.S., Between ψ-closed sets and gsp-closed sets, Antarctica. J.Math., Vol.2(1), (2005), 123-141.
29. Veera kumar, M.K.R.S., $^\#$g-semi-closed sets in topological spaces, Antarctica. J.Math., Vol.2(2)(2005), 201-222.
30. Veera kumar, M.K.R.S., Between g^*-closed sets and g- closed sets, Antarctica. J.Math., Vol.3(1)(2006)48-65.

Source of support: Nil, Conflict of interest: None Declared