Manimegalai

Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination – June 2021
IV Semester

Class : II M.Sc. **Time : 3 hours**
Major : Mathematics **Max. Marks : 100**

17MMAC23 Mathematical Methods

Part-A **10 x 1=10**
Choose the correct answer

1. Identify the Fourier cosine transform of $f(t)$: CO-1 K-1

   a. $\frac{2}{\pi}\int_0^\infty f(t)\cos(\xi t)\,dt$ b. $\sqrt{\frac{2}{\pi}}\int_0^\infty f(t)\cos(\xi t)\,dt$
   c. $\frac{\pi}{2}\int_0^\infty f(t)\cos(\xi t)\,dt$ d. $\sqrt{\frac{\pi}{2}}\int_0^\infty f(t)\cos(\xi t)\,dt$

2. Indicate the formula for $\mathcal{F}_s[te^{-at};\xi]$, where $a>0$, among the following:

   a. $\sqrt{\frac{\pi}{2}}\frac{2a\xi}{(a^2+\xi^2)^2}$ b. $\frac{2}{\pi}\frac{2a\xi}{(a^2+\xi^2)^2}$
   c. $\sqrt{\frac{2}{\pi}}\frac{2a\xi}{(a^2+\xi^2)^2}$ d. $\frac{2a\xi}{(a^2+\xi^2)^2}$

3. Select the name of the identity given by $\mathcal{F}[f\circ g;\xi]=F(\xi)G(\xi)$ where $F(\xi)$ and $G(\xi)$ are the Fourier transforms of f and g respectively. CO-2 K-2

   a. Parseval's identity b. Bessel's identity
   c. convolution theorem d. inversion theorem

4. Name of the equation given by $\Delta_2 u=\frac{1}{\kappa}\frac{\partial u}{\partial t},\ t>0$is _____. CO-2 K-1

   a. linear diffusion equation b. two-dimensional diffusion equation
   c. linear wave equation d. two-dimensional wave equation

5. Indicate the correct answer: A kernel $K(s,t)$ is called separable if it can be expressed as $K(s,t)=$_______ CO-3 K-2

   a. $\sum_{i=1}^{n} a_i(s)$ b. $\sum_{i=1}^{n} b_i(t)$
   c. $\sum_{i=1}^{n} a_i(s)b_i(t)$ d. $\sum_{i=1}^{n}[a_i(s)+b_i(t)]$

6. State which of the following theorem is concerned with the study of the homogeneous equation$g(s)=\lambda\int K(s,t)g(t)dt$ when $D(\lambda)=0$. CO-3 K-1

   a. Fredholm's first theorem b. Fredholm's second theorem
   c. Fredholm's third theorem d. Fredholm's kernel

7. Tell which type of integral equation arise from a boundary value problem in ordinary differential equation: CO-4 K-1

   a. Volterra-type b. Volterra equation of second kind
   c. Fredholm-type d.Singular type

8. Identify the type of the integral equation given by $\int_0^s \frac{g(t)}{(s-t)^{1/2}}dt$ CO-4 K-2

   a. Fredholm b. Volterra
   c. Abel d. Laplace

9. Select the Euler's equation among the following: CO-5 K-2

   a. $F_y-\frac{d}{dx}F_{y'}=0$ b. $F_y-\frac{d}{dx}F_{x'}=0$
   c. $F_y-\frac{d}{dy}F_{y'}=0$ d. $F_y-\frac{dy}{dx}=0$

10. Match the correct answer in the blank: The variation of a functional is the_____ part of the increment of the functional, which part is linear in δy. CO-5 K-1

    a. fractional b. principal c. real d. imaginary

Part B 5x6=30

Answer ALL questions

11.a. Write the statement of Fourier cosine inversion theorem and prove it. CO-1 K-3

(or)

11.b. Establish the formula $\mathcal{F}\left[e^{-a|t|};\xi\right]=\left(\frac{2}{\pi}\right)^{1/2}\frac{a}{a^2+\xi^2}$, $a>0$. CO-1 K-3

12.a. Write the Parseval's relation for Fourier transforms and prove it. CO-2 K-3

(or)

12.b. Compute a solution $u(x,y)$ for the Laplace's equation using the Fourier transform. CO-2 K-3

13.a. Solve the integral equation $g(s)=f(s)+\lambda\int_0^s e^{s-t}g(t)dt$ and evaluate the resolvent kernel. CO-3 K-3

(or)

13.b. Compute the resolvent for the integral equation $g(s)=f(s)+\lambda\int_0^1(s+t)g(t)dt$ CO-3 K-3

14.a. Modify the problem$y''+\lambda y=0, y(0)=0\,, y'(1)+\nu_2 y(1)=0$to a Fredholm integral equation. CO-4 K-3

(or)

14.b. Solve the integral equation $s=\int_0^s\frac{g(t)dt}{(s-t)^{1/2}}$CO-4 K-3

15.a. Sketch the proof of the fundamental lemma of the calculus of variations. CO-5 K-3

(or)

15.b. Show that the functional $v[y(x)]=\int_0^1[(y')^2+12xy]dx, y(0)=0, y(1)=1$ can be extremized only on the curve $y=x^3$. CO-5 K-2

Part C 5x12=60

Answer ALL questions

16.a. Summarize the proof of Fourier integral theorem given by CO-1 K-5
$\frac{1}{\pi}\int_{-\infty}^{\infty}f(t)\cos\{\xi(x-t)\}\,dt=\frac{1}{2}[f(x+)+f(x-)]$for all $x\in\mathbb{R}$where $f(t)\in\mathcal{P}^1(\mathbb{R}), f(t)\in\mathcal{A}_1(\mathbb{R})$.

(or)

16.b. Analyze in detail about Fourier transforms of derivatives. CO-1 K-4

17.a. Formulatethe method of solution of Laplace's equation in an infinite strip using Fourier transforms. CO-2 K-6

(or)

17.b. Estimate the solution of the linear diffusion equation $\frac{\partial^2 u}{\partial x^2}=\frac{1}{\kappa}\frac{\partial u}{\partial t}$ in the semi-infinite line $x\geq 0$ satisfying the boundary conditions $u(0,t)=f(t), t\geq 0, u(0,t)\to 0\quad as\ x\to\infty$ and the initial condition $u(x,0)=0$byapplying Fourier sine transform. CO-2 K-5

18.a. Summarize in detail about Fredholm's first theorem. CO-3 K-6

(or)

18.b. Devise a proof forFredholm's first theorem after stating it. CO-3 K-6

19.a. Explain the method of solving the following initial value problem using integral equations:
$y''+A(s)y'+B(s)y=F(s)$, $y(a)=q_0, y'(a)=q_1$where$A, B$and$F$are defined and continuous in the interval$a\leq s\leq b$. CO-4 K-4

(or)

19.b. Illustrate the method of finding the equivalent Fredholm integral equation for the problem
$y''+A(s)y'+B(s)y=F(s)$, $y(a)=y_0,\ y(b)=y_1.$ CO-4 K-4

20.a. Summarize the method in detail to derive Euler's equation which extremizes the functional
$v[y(x)]=\int_{x_0}^{x_1}F(x,y(x),y'(x)))dx,\ \ y(x_0)=y_0,\ \ y(x_1)=y_1.$ CO-5 K-5

(or)

20.b. Construct the Ostrogradsky equation which extremizes the functionals depending on the functions of several independent variables. CO-5 K-6

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