

CHAPTER 1

Preliminaries

This chapter has got an overture to our study which includes the requirements of versatile sets which are of great use in the sub sequel of the thesis.

1.1 Basic Concepts

Definition 1.1.1. [96]

Let \tilde{X} be a non empty set. A fuzzy set (FS) \tilde{A} in \tilde{X} is denoted by

$$\tilde{A} = \{(\check{p}, \mu_{\tilde{A}}(\check{p})) | \check{p} \in \tilde{X}\}$$

where $\mu_{\tilde{A}}: \tilde{X} \rightarrow [0,1]$ is the membership function of the FS \tilde{A} .

Definition 1.1.2 [8]

Let \tilde{X} be a non empty set. An intuitionistic fuzzy set (IFS) \tilde{A} in \tilde{X} is denoted by

$$\tilde{A} = \{(\check{p}, \mu_{\tilde{A}}(\check{p}), \nu_{\tilde{A}}(\check{p})) | \check{p} \in \tilde{X}\}$$

where $\mu_{\tilde{A}}: \tilde{X} \rightarrow [0,1]$ and $\nu_{\tilde{A}}: \tilde{X} \rightarrow [0,1]$ such that $0 \leq \mu_{\tilde{A}}(\check{p}) + \nu_{\tilde{A}}(\check{p}) \leq 1$ for any $\check{p} \in \tilde{X}$. where, $\mu_{\tilde{A}}(\check{p})$ and $\nu_{\tilde{A}}(\check{p})$ are the degree of membership and non-membership functions.

Definition 1.1.3. [74]

Let \tilde{X} be a non empty set. A neutrosophic set (NS), \tilde{A} in \tilde{X} is denoted by

$$\tilde{A} = \{(\check{p}, (T_{\tilde{A}}(\check{p}), (I_{\tilde{A}}(\check{p}), (F_{\tilde{A}}(\check{p}))) : \check{p} \in \tilde{X}, \}$$

where $(T_{\tilde{A}}(\check{p}), (I_{\tilde{A}}(\check{p}), (F_{\tilde{A}}(\check{p})) \in]^{-0}, 1^{+}[$ such that

$$0^- \leq (T_{\tilde{A}}(\check{p})) + (I_{\tilde{A}}(\check{p})) + (F_{\tilde{A}}(\check{p})) \leq 3^+$$

for any $\check{p} \in \tilde{X}$. Where, $T_{\tilde{A}}$, $I_{\tilde{A}}$ and $F_{\tilde{A}}$ are truth, indeterminacy and a falsity membership functions.

Definition 1.1.4. [75]

If $\tilde{A} = \{(\check{p}, T_{\tilde{A}}(\check{p}), I_{\tilde{A}}(\check{p}), F_{\tilde{A}}(\check{p})) | \check{p} \in X\}$ and $\tilde{B} = \{(\check{p}, T_{\tilde{B}}(\check{p}), I_{\tilde{B}}(\check{p}), F_{\tilde{B}}(\check{p})) | \check{p} \in X\}$ are any two NSs of X then

1. $(\tilde{A}) \subseteq (\tilde{B})$ iff $T_{\tilde{A}}(\check{p}) \leq T_{\tilde{B}}(\check{p}); I_{\tilde{A}}(\check{p}) \leq I_{\tilde{B}}(\check{p}); F_{\tilde{A}}(\check{p}) \geq F_{\tilde{B}}(\check{p}), \forall \check{p} \in X$
2. $(\tilde{A}) = (\tilde{B})$ iff $T_{\tilde{A}}(\check{p}) = T_{\tilde{B}}(\check{p}); I_{\tilde{A}}(\check{p}) = I_{\tilde{B}}(\check{p}); F_{\tilde{A}}(\check{p}) = F_{\tilde{B}}(\check{p}), \forall \check{p} \in X$
3. $\tilde{A} \cap \tilde{B} = \{(\check{p}, T_{\tilde{A} \cap \tilde{B}}(\check{p}), I_{\tilde{A} \cap \tilde{B}}(\check{p}), F_{\tilde{A} \cap \tilde{B}}(\check{p})) | \check{p} \in X\}$ where,
 - i. $T_{\tilde{A} \cap \tilde{B}}(\check{p}) = \min\{T_{\tilde{A}}(\check{p}), T_{\tilde{B}}(\check{p})\}$
 - ii. $I_{\tilde{A} \cap \tilde{B}}(\check{p}) = \min\{I_{\tilde{A}}(\check{p}), I_{\tilde{B}}(\check{p})\}$
 - iii. $F_{\tilde{A} \cap \tilde{B}}(\check{p}) = \max\{F_{\tilde{A}}(\check{p}), F_{\tilde{B}}(\check{p})\}$
4. $\tilde{A} \cup \tilde{B} = \{(\check{p}, T_{\tilde{A} \cup \tilde{B}}(\check{p}), I_{\tilde{A} \cup \tilde{B}}(\check{p}), F_{\tilde{A} \cup \tilde{B}}(\check{p})) | \check{p} \in X\}$ where,
 - i. $T_{\tilde{A} \cup \tilde{B}}(\check{p}) = \max\{T_{\tilde{A}}(\check{p}), T_{\tilde{B}}(\check{p})\}$
 - ii. $I_{\tilde{A} \cup \tilde{B}}(\check{p}) = \max\{I_{\tilde{A}}(\check{p}), I_{\tilde{B}}(\check{p})\}$
 - iii. $F_{\tilde{A} \cup \tilde{B}}(\check{p}) = \min\{F_{\tilde{A}}(\check{p}), F_{\tilde{B}}(\check{p})\}$
5. The complement of the set \tilde{A} , $(\mathcal{C}(\tilde{A}))$ may be defined as

$$\mathcal{C}(\tilde{A}) = \{(\check{p}, F_{\tilde{A}}(\check{p}), 1 - I_{\tilde{A}}(\check{p}), T_{\tilde{A}}(\check{p})) | \check{p} \in X\}$$

Definition 1.1.5. [89]

A single-valued neutrosophic set (SVNS) \tilde{A} in \tilde{X} is a NS which is denoted by

$$\tilde{A} = \{(\check{p}, (T_{\tilde{A}}(\check{p})) + (I_{\tilde{A}}(\check{p})) + (F_{\tilde{A}}(\check{p}))), \check{p} \in \tilde{X}\}$$

and characterized by the degree of membership $(T_{\tilde{A}}(\check{p}))$, indeterminacy $(I_{\tilde{A}}(\check{p}))$ and non-membership $(F_{\tilde{A}}(\check{p}))$ functions, where $T_{\tilde{A}}(\check{p}), I_{\tilde{A}}(\check{p}), F_{\tilde{A}}(\check{p}) \in [0,1]$ such that

$$0 \leq (T_{\tilde{A}}(\check{p})) + (I_{\tilde{A}}(\check{p})) + (F_{\tilde{A}}(\check{p})) \leq 3, \text{ for all } \check{p} \in \tilde{X}, \text{ respectively.}$$

Definition 1.1.6. [78]

Let U be a non empty set and the NS $\tilde{A} \subset U$. Let $T_{\tilde{A}}(\tilde{p}), I_{\tilde{A}}(\tilde{p}), F_{\tilde{A}}(\tilde{p})$ represent the functions that describe the degrees of membership, indeterminate-membership, and non-membership respectively, of a generic element $\tilde{p} \in U$, with respect to the NS \tilde{A}

$$T_{\tilde{A}}(\tilde{p}), I_{\tilde{A}}(\tilde{p}), F_{\tilde{A}}(\tilde{p}): U \rightarrow P[0, \Omega]$$

where $0 < 1 < \Omega$, and Ω is called over limit. $T_{\tilde{A}}(\tilde{p}), I_{\tilde{A}}(\tilde{p}), F_{\tilde{A}}(\tilde{p}) \subseteq [0, \Omega]$, and $P[0, \Omega]$ is the set of all subsets of $[0, \Omega]$.

Definition 1.1.7. [19]

Let X be a non empty set. An intuitionistic gradation of openness of fuzzy subsets of X is an ordered pair (τ, τ^*) of functions from I^X to I such that

- i. $\tau(\lambda) + \tau^*(\lambda) \leq 1, \forall \lambda \in I^X$
- ii. $\tau(\tilde{0}) = \tau(1) = 1, \tau^*(\tilde{0}) = \tau^*(1) = 0$
- iii. $\tau(\lambda_1 \cap \lambda_2) \geq \tau(\lambda_1) \wedge \tau(\lambda_2), \tau^*(\lambda_1 \cap \lambda_2) \leq \tau^*(\lambda_1) \vee \tau^*(\lambda_2),$
 $\lambda_i \in I^X, i = 1, 2$
- iv. $\tau(\bigcup_{i \in \Delta} \lambda_i) \geq \bigwedge_{i \in \Delta} \tau(\lambda_i), \tau^*(\bigcup_{i \in \Delta} \lambda_i) \leq \bigvee_{i \in \Delta} \tau^*(\lambda_i), \lambda_i \in I^X, i \in \Delta.$

Then the pair (τ, τ^*) is an intuitionistic gradation of openness on X . The triplet (X, τ, τ^*) is called an intuitionistic fuzzy topological spaces. τ and τ^* may be interpreted as gradation of openness and gradation of non-openness, respectively.

Definition 1.1.8. [19]

Let X be a non empty set and $F, F^*: I^X \rightarrow I$ be two mappings satisfying

- i. $F(\lambda) + F^*(\lambda) \leq 1, \forall \lambda \in I^X$
- ii. $F(\tilde{0}) = F(1) = 1, F^*(\tilde{0}) = F^*(1) = 0$
- iii. $F(\lambda_1 \cup \lambda_2) \geq F(\lambda_1) \wedge F(\lambda_2), F^*(\lambda_1 \cup \lambda_2) \leq F^*(\lambda_1) \vee F^*(\lambda_2),$
 $\lambda_i \in I^X, i = 1, 2$
- iv. $F(\bigcap_{i \in \Delta} \lambda_i) \geq \bigwedge_{i \in \Delta} F(\lambda_i), F^*(\bigcap_{i \in \Delta} \lambda_i) \leq \bigvee_{i \in \Delta} F^*(\lambda_i), \lambda_i \in I^X, i \in \Delta.$

Then the pair (F, F^*) is an intuitionistic gradation of closedness on X .

Definition 1.1.9. [31]

Consider \tilde{X} be a non empty set. \tilde{A} is a Pythagorean neutrosophic set on \tilde{X} is denoted by

$$\tilde{A} = \{(\check{p}, T_{\tilde{A}}(\check{p}), I_{\tilde{A}}(\check{p}), F_{\tilde{A}}(\check{p})) | \check{p} \in \tilde{X}\},$$

where $T_{\tilde{A}}(\check{p}), I_{\tilde{A}}(\check{p}), F_{\tilde{A}}(\check{p}) \in [0,1]$,

$$0 \leq (T_{\tilde{A}}(\check{p}))^2 + (I_{\tilde{A}}(\check{p}))^2 + (F_{\tilde{A}}(\check{p}))^2 \leq 2,$$

for every $\check{p} \in \tilde{X}$ where, $T_{\tilde{A}}(\check{p}), I_{\tilde{A}}(\check{p})$ and $F_{\tilde{A}}(\check{p})$ are degree of membership, indeterminacy, and non- membership funtions. Here $T_{\tilde{A}}(\check{p})$ and $F_{\tilde{A}}(\check{p})$ are dependent components and $I_{\tilde{A}}(\check{p})$ is an independent.

Definition 1.1.10. [99]

Let \tilde{X} be a non empty set. An interval-valued neutrosophic set A in \tilde{X} is denoted by

$$A = \{\check{p}: [T_{A^-}(\check{p}), T_{A^+}(\check{p})], [I_{A^-}(\check{p}), I_{A^+}(\check{p})], [F_{A^-}(\check{p}), F_{A^+}(\check{p})] / \check{p} \in X\}$$

where $T_A(\check{p}), I_A(\check{p}), F_A(\check{p}) \in [0,1]$, $0 \leq [T_{A^-}(\check{p})] + [I_{A^-}(\check{p})] + [F_{A^-}(\check{p})] \leq 3$

$T_{A^+}(\check{p}), I_{A^+}(\check{p}), F_{A^+}(\check{p}) \in [0,1]$, $0 \leq [T_{A^+}(\check{p})] + [I_{A^+}(\check{p})] + [F_{A^+}(\check{p})] \leq 3$

Definition 1.1.11. [32]

Let \tilde{X} be a non empty set. \tilde{C} cubic set in \tilde{X} is denoted by

$$\tilde{C} = \{\check{p}, \tilde{A}(\check{p}), \lambda(\check{p}) / \check{p} \in X\}$$

where \tilde{A} is an interval-valued fuzzy set in \tilde{X} and λ is a FS in \tilde{X} .

Definition 1.1.12. [80]

Let \tilde{X} be a non empty set. A neutrosophic cubic set \tilde{C} in \tilde{X} is a pair $\tilde{C} = (\tilde{A}, \lambda)$ where $\tilde{A} = \{(\check{p}, \tilde{A}_T(\check{p}), \tilde{A}_I(\check{p}), \tilde{A}_F(\check{p})) / \check{p} \in X\}$ is an interval neutrosophic set in \tilde{X} and $\lambda = \{(\check{p}, \lambda_T(\check{p}), \lambda_I(\check{p}), \lambda_F(\check{p})) / \check{p} \in X\}$ is a neutrosophic set in \tilde{X} .

Definition 1.1.13. [55]

Let $\tilde{X} = \{\check{p}_1, \check{p}_2, \dots, \check{p}_n\}$ be a universal set. Let $\tilde{A} = \{\check{p}_i, \mu_{\tilde{A}}(\check{p}_i), \sigma_{\tilde{A}}(\check{p}_i), \gamma_{\tilde{A}}(\check{p}_i) : \check{p}_i \in \tilde{X}\}$ and $\tilde{B} = \{\check{p}_i, \mu_{\tilde{B}}(\check{p}_i), \sigma_{\tilde{B}}(\check{p}_i), \gamma_{\tilde{B}}(\check{p}_i) : \check{p}_i \in \tilde{X}\}$ be two SVNNSs on \tilde{X} . Then define a mapping $d: SVNNS(\tilde{X}) \times SVNNS(\tilde{X}) \rightarrow [0,1]$.

The Hamming distance between two SVNNSs \tilde{A} and \tilde{B} is denoted by

$$d_H(\tilde{A}, \tilde{B}) = \frac{1}{3} \sum_{j=1}^n \left(\begin{array}{l} |\mu_{\tilde{A}}(\check{p}_j) - \mu_{\tilde{B}}(\check{p}_j)| + \\ |\sigma_{\tilde{A}}(\check{p}_j) - \sigma_{\tilde{B}}(\check{p}_j)| + \\ |\gamma_{\tilde{A}}(\check{p}_j) - \gamma_{\tilde{B}}(\check{p}_j)| \end{array} \right) \quad (1.1.1)$$

The Euclidean distance between two SVNNSs \tilde{A} and \tilde{B} is denoted by

$$d_E(\tilde{A}, \tilde{B}) = \left\{ \frac{1}{3} \sum_{j=1}^n \left(\begin{array}{l} (\mu_{\tilde{A}}(\check{p}_j) - \mu_{\tilde{B}}(\check{p}_j))^2 \\ + (\sigma_{\tilde{A}}(\check{p}_j) - \sigma_{\tilde{B}}(\check{p}_j))^2 \\ + (\gamma_{\tilde{A}}(\check{p}_j) - \gamma_{\tilde{B}}(\check{p}_j))^2 \end{array} \right) \right\}^{\frac{1}{2}} \quad (1.1.2)$$

The Normalized Hamming distance between two SVNNSs \tilde{A} and \tilde{B} is denoted by

$$d_{n-H}(\tilde{A}, \tilde{B}) = \frac{1}{3n} \sum_{j=1}^n \left(\begin{array}{l} |\mu_{\tilde{A}}(\check{p}_j) - \mu_{\tilde{B}}(\check{p}_j)| + \\ |\sigma_{\tilde{A}}(\check{p}_j) - \sigma_{\tilde{B}}(\check{p}_j)| + \\ |\gamma_{\tilde{A}}(\check{p}_j) - \gamma_{\tilde{B}}(\check{p}_j)| \end{array} \right) \quad (1.1.3)$$

The Normalized Euclidean distance between two SVNNSs \tilde{A} and \tilde{B} is denoted by

$$d_{n-E}(\tilde{A}, \tilde{B}) = \left\{ \frac{1}{3n} \sum_{j=1}^n \left(\begin{array}{l} (\mu_{\tilde{A}}(\check{p}_j) - \mu_{\tilde{B}}(\check{p}_j))^2 \\ + (\sigma_{\tilde{A}}(\check{p}_j) - \sigma_{\tilde{B}}(\check{p}_j))^2 \\ + (\gamma_{\tilde{A}}(\check{p}_j) - \gamma_{\tilde{B}}(\check{p}_j))^2 \end{array} \right) \right\}^{\frac{1}{2}} \quad (1.1.4)$$

Definition 1.1.17. [17]

Let $\tilde{X} = \{\check{p}_1, \check{p}_2, \dots, \check{p}_n\}$ be a universal set. Let $\tilde{A} = \{\check{p}_j, \mu_{\tilde{A}}(\check{p}_j), \sigma_{\tilde{A}}(\check{p}_j), \gamma_{\tilde{A}}(\check{p}_j) : \check{p}_j \in \tilde{X}\}$, $\tilde{B} = \{\check{p}_j, \mu_{\tilde{B}}(\check{p}_j), \sigma_{\tilde{B}}(\check{p}_j), \gamma_{\tilde{B}}(\check{p}_j) : \check{p}_j \in \tilde{X}\}$ be two SVNNSs on \tilde{X} . Then define a mapping $d: SVNNS(\tilde{X}) \times SVNNS(\tilde{X}) \rightarrow [0,1]$. Then the distance measures between two SVNNSs \tilde{A} and \tilde{B} are denoted by

$$D_1(\tilde{A}, \tilde{B}) = \frac{1}{3n} \sum_{j=1}^n \left(\begin{array}{l} |\mu_{\tilde{A}}(\check{p}_j)^2 - \mu_{\tilde{B}}(\check{p}_j)^2| \\ + |\sigma_{\tilde{A}}(\check{p}_j)^2 - \sigma_{\tilde{B}}(\check{p}_j)^2| \\ + |\gamma_{\tilde{A}}(\check{p}_j)^2 - \gamma_{\tilde{B}}(\check{p}_j)^2| \end{array} \right) \quad (1.1.5)$$

and

$$D_2(\tilde{A}, \tilde{B}) = \frac{1}{3n} \sum_{j=1}^n \left| \begin{array}{l} (\mu_{\tilde{A}}(\check{p}_j)^2 - \mu_{\tilde{B}}(\check{p}_j)^2) \\ - (\sigma_{\tilde{A}}(\check{p}_j)^2 - \sigma_{\tilde{B}}(\check{p}_j)^2) \\ - (\gamma_{\tilde{A}}(\check{p}_j)^2 - \gamma_{\tilde{B}}(\check{p}_j)^2) \end{array} \right| \quad (1.1.6)$$

Definition 1.1.18. [14]

A mapping $\tilde{S}: NS(\tilde{X}) \times NS(\tilde{X}) \rightarrow [0,1]$ is said to be a similarity measure between two NSs if it meets the axioms:

- i. $S(\tilde{U}, \tilde{V}) \geq 0$ for all $\tilde{U}, \tilde{V} \in NS(\tilde{X})$.
- ii. $S(\tilde{U}, \tilde{V}) = 0$ iff $\tilde{U} = \tilde{V}$ for all $\tilde{U}, \tilde{V} \in NS(\tilde{X})$.
- iii. $S(\tilde{U}, \tilde{V}) = S(\tilde{V}, \tilde{U})$ for all $\tilde{U}, \tilde{V} \in NS(\tilde{X})$.
- iv. If $\tilde{U} \subseteq \tilde{V} \subseteq \tilde{W}$ for all $\tilde{U}, \tilde{V}, \tilde{W} \in NS(\tilde{X})$, then $\tilde{S}(\tilde{U}, \tilde{W}) \geq \tilde{S}(\tilde{U}, \tilde{V})$ and $\tilde{S}(\tilde{U}, \tilde{W}) \geq \tilde{S}(\tilde{V}, \tilde{W})$

Definition 1.1.19. [7]

Let $\tilde{X} = \{\check{p}_1, \check{p}_2, \dots, \check{p}_n\}$ be a universal set. Let $\tilde{A} = \{\check{p}_i, \mu_{\tilde{A}}(\check{p}_i), \sigma_{\tilde{A}}(\check{p}_i), \gamma_{\tilde{A}}(\check{p}_i): \check{p}_i \in \tilde{X}\}$ and $\tilde{B} = \{\check{p}_i, \mu_{\tilde{B}}(\check{p}_i), \sigma_{\tilde{B}}(\check{p}_i), \gamma_{\tilde{B}}(\check{p}_i): \check{p}_i \in \tilde{X}\}$ be two SVNNSs on \tilde{X} . Then define a mapping $d: SVNNS(\tilde{X}) \times SVNNS(\tilde{X}) \rightarrow [0,1]$. Then the Sine Metric Single Valued Neutrosophic (SMSVNS) distance measure is denoted as $d_{SMSVNS}(\tilde{A}, \tilde{B})$

$$= \frac{5}{3n} \sum_{i=1}^n \frac{\sin \left\{ \frac{\pi}{6} |\mu_{\tilde{A}}(\check{p}_i) - \mu_{\tilde{B}}(\check{p}_i)| \right\} + \sin \left\{ \frac{\pi}{6} |\sigma_{\tilde{A}}(\check{p}_i) - \sigma_{\tilde{B}}(\check{p}_i)| \right\} + \sin \left\{ \frac{\pi}{6} |\gamma_{\tilde{A}}(\check{p}_i) - \gamma_{\tilde{B}}(\check{p}_i)| \right\}}{1 + \sin \left\{ \frac{\pi}{6} |\mu_{\tilde{A}}(\check{p}_i) - \mu_{\tilde{B}}(\check{p}_i)| \right\} + \sin \left\{ \frac{\pi}{6} |\sigma_{\tilde{A}}(\check{p}_i) - \sigma_{\tilde{B}}(\check{p}_i)| \right\} + \sin \left\{ \frac{\pi}{6} |\gamma_{\tilde{A}}(\check{p}_i) - \gamma_{\tilde{B}}(\check{p}_i)| \right\}} \quad (1.1.7)$$