METHODOLOGY

CHAPTER: III

METHODOLOGY

3.1 UNIFIED FISSION MODEL

3.1.1 Introduction:

Alpha emission is one of the important decay modes for heavy nuclei. The reliable information on the nuclear structure including the ground state energy, the ground state half life nuclear spin and parity the nuclear deformation, shell effects, and the nuclear interaction provided through measurements on alpha decay. Since alpha decays used to extract the reliable and useful information about the parent nucleus, measurements on alpha decay used to identify the new nuclides or new heavy elements.

The α decay half-lives are calculated with the Q_a values obtained by using Audi et al [42] in the frame work of the unified fission model (UFM) with the coulomb repulsion, nuclear attraction with the proximity potential. Since UFM is efficient to deal with the alpha decay it is necessary to ensure with the best accurate theoretical energy decay values provided to get accurate alpha decay half lives in Q value.

3.1.2 Gamow's theory of alpha decay:

The quantum mechanical explanation was given by Gurney, Condon and Gamow, that a particle inside the nucleus may escape from the nucleus with energy much less than its potential barrier. These experiments connected the possibility of the particle to go out of the nucleus with the Schrödinger wave function ψ whose square measures the density of probability of the particle in a certain region. The Schrodinger wave function generally does not vanish in the regions where the potential energy (U) is higher than the total energy that is E-U is negative, but through decreasing exponentially, it maintains certain finite value. This enables the particle to leak the potential barrier. It is based on the concept of a preformed alpha particle tunneling through essentially a coulomb barrier due to the α - particle and the daughter nucleus. The decay constant λ_G is defined as,

$$\lambda_G = \nu P \text{--------->} 3.1$$

Where ν is the assault frequency with which alpha particle bombarded the walls of the barrier. P is the frequency of existence of alpha particle at the barrier, i.e. the alpha preformation probability factor.

3.1.3 Q- value of the reaction:

Consider the reaction

$$a + X \text{--------}\rightarrow Y + b$$

When an incident projectile hits the target nucleus, a nuclear reaction takes place and as a result there is a new nucleus Y and an outgoing particle b. The above reaction can also be written in short form as X (a, b). Since the total mass and energy is conserved, we have

$$(E_a + M_a{}^2) + (M_x c^2) = (E_y + M_y c^2) + (E_b + M_b c^2) \text{------>} 3.2$$

Where, E_a = Kinetic energy of the projectile, $M_a c^2$ = rest energy of the projectile

And similarly E_b and $M_a c^2, E_y, M_y c^2$ and $M_x c^2$ the target nucleus X is assumed to be at rest. The Q value is expressed as,

$$Q = E_y + E_b - E_a \text{------>} 3.3$$

I.e. it is the change in the total kinetic energy. This change in the total kinetic energy in a nuclear reaction is clearly the nuclear disintegration energy. So Eqn. (3.2), eqn.(3.3) becomes

$$Q = E_y + E_b - E_a = \left[(M_x + M_X) - (M_Y + M_y)\right]c^2 \text{------>} 3.4$$

Thus we see that Q is also the change in the total rest mass. The nuclear reaction is an exoergic reaction, if Q is positive. The nuclear reaction is an endoergic reaction, if Q is negative [34]. For the present calculations, Q_α is calculated using,

$$Q_{\alpha} = [M(A, Z)-M(A-4)-M(\alpha)]\, c^2$$

3.1.4 Theoretical frame work on the UFM:

The half-life of a parent nucleus decaying via α emission can be calculated by means of the WKB barrier penetration probability. In the UFM the decay constant is simply defined as,

$$\lambda = v_0 P \text{------------------} > (3.5)$$

Where ν_0 is the assault frequency with which the cluster hits the barrier, P is the probability of penetrating. Apparently for a pure coulomb potential, the above equation will give the Gamow factor λ_G. Thus, in UFM the alpha fragments are said to be formed at a relative separation co-ordinate R. The assault frequency ν_0 in this model is calculated simply by assuming that the total K.E in the ground state is positive Q value, which is shared between the two fragments. Thus

$$\nu_0 = \frac{velocity}{R_0} = \frac{\left(\frac{2E_2}{\mu}\right)^{\frac{1}{2}}}{R_0} \text{--------------} > (3.6)$$

where R_0 is the radius of the parent nucleus and $E_2 = (1/2)\mu\mu^2$ is the kinetic energy of the alpha particle inside the nucleus. Where E2 is given by

$$E_2 = \left(\frac{A_1}{A}\right)Q$$

Similarly knowing the decay constant, λ the half-life time can be obtained

$$T_{1/2} = \frac{\ln 2}{\lambda} \text{-----------} > (3.7)$$

P in eqn. (3.5) is given by

$$P = P_{ov} P_{nov} \text{--------------} > (3.8)$$

Where P_{ov} is the probability of penetration for the over lapping potential from parent nucleus radius to touching point configuration and it is given by

$$P_{ov} = \exp\left[-\frac{2}{\hbar} \int_{R_0}^{R_t} \{2\mu[V(R) - Q]\}^{1/2} dR \right] \text{------} > (3.9)$$

This P_{ov} is represented as P_o in further discussions.

Where P_{nov} in eqn. (3.8) is for probability of penetration for non-overlapping region and this can be calculated by using WKB approximation from touching point configuration to R_b,

$$P_{nov} = \exp\left[-\frac{2}{\hbar} \int_{R_t}^{R_b} \{2\mu[V(R)]\}^{\frac{1}{2}} dR \right] \text{---------} > (3.10)$$

Where R_b is given as

$$R_b = \frac{Z_1 Z_2 e^2}{Q} \text{-----------} > (3.11)$$

Potential for the overlapping region:

For $R<R_t$ (over lapping region), V(R) is parameterized simply as a polynomial of degree two in R,

$$V(R) = a_1 R + a_2 R^2 \quad \text{for } R_0 \le R < R_t \text{----------}>(3.12)$$

The constants $a_{i(i=1,2)}$ occurring in the polynomial is determined by using the following boundary condition:

1. At $R=R_0$, V(R)=Q
2. At ,$V(R) = R_t$

For $R \ge R_t$ (non- over lapping region) V(R) is defined as a sum of the coulomb potential, proximity potential [37].

$$V(R) = V_c(R) + V_p(R) \text{ for } R \ge R_t \text{----------}>(3.13)$$

3.1.5 Coulomb potential:

A scalar point function equal to the work per unit charge done against the Coulomb force interesting in transferring an article bearing infinitesimal positive charge from infinity to a point in the field of a specific charge distribution is known as coulomb potential. It is given by

$$V_C(R) = \frac{Z_1 Z_2 e^2}{R} ------------->(3.14)$$

Here Z_1 and Z_2 are atomic number of daughter and alpha, R is the distance between the fragment centers. e^2=1.44 MeV fm^2.

3.1.6 Nuclear Proximity Potential $(V_P(R))$

When two surfaces approach within a small distance of $\sim$ 2 to 3 fm, comparable with the surface thickness of interacting nuclei, or when a nucleus is at the verge of dividing into two fragments, then the two surfaces actually face each other across a small gap or crevice. In both the cases, the surface energy term alone could not give rise to the strong attraction that is observed, when the two surfaces are brought in close proximity. Such a additional attractive forces are called proximity forces and the additional potential due to these forces is called the proximity potential. The V_P is an additional attraction due the nuclear proximity potential,

$$V_P(R) = 4\pi \bar{R} \gamma b \varphi(\delta) --------------->(3.15)$$

Where $\bar{R}$ and $\varphi(\delta)$ are respectively, the inverse of the root mean square radius of the Gaussian curvature and the universal function, which is independent of the geometry of the system given,

$$\varphi(\delta) = -1/2(\delta - 2.54)^2 - 0.0852(\delta - 2.54)^3 \text{ For } \delta \leq 1.2511 ----->(3.16a)$$

$$\varphi(\delta) = --3.437\exp\left(\frac{-\delta}{0.75}\right) \text{For } \delta \geq 1.2511 ---------------> (3.16b)$$

In eq. (3.16a, 3.16b), $\delta = \frac{R - C_t}{b}$ is the overlap distance and surface thickness parameter b=0.99 fm.

$$\bar{R} = \frac{C_1 C_2}{C_t} ------------------> (3.17)$$

Where $C_t = C_1 + C_2$. Here C_i are the sussmann central radii and it is given by

$$C_i = R_i - \left(\frac{b^2}{R_t}\right) --------------> (3.18)$$

Where i=0, 1, 2 refers to the parent, daughter and alpha particle. The standard expression for radius is used, which is given as,

$$R_i = 1.28 A_i^{1/3} - 0.76 + 0.8 / A_i^{\frac{1}{3}} fm --------------> (3.19)$$

With i=0, 1, 2 corresponding to parent, daughter nucleus and the alpha. And γ is the specific nuclear surface tension defined as,

$$\gamma = 0.9517\left[1 - 1.7826\left(\frac{N - Z}{A}\right)^2\right] MeV fm^{-2} ------------> (3.20)$$

3.2 DYNAMICAL CLUSTER DECAY MODEL:

The dynamical cluster decay model [8] for de-excitation of excited compound system is based on the Quantum Mechanical Fragmentation Theory (QMFT). The QMFT is worked out in terms of the collective co ordinate of mass and charge symmetry.

$$\eta = \frac{A_1 - A_2}{A_1 + A_2} ----------------------------> (3.21)$$

Similarly $$\eta_z = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

and the relative separation R. These two coordinates refer to the nucleon division or exchange between the outgoing fragments and the transfer of kinetic energy of incident channel to internal excitation (total excitation energy TXE) or total kinetic energy (TKE) of the outgoing channel, the compound nucleus decay cross sections is given, using partial wave analysis in terms of these coordinates and the cross section is given by

$$\sigma = \sum_{l=0}^{l_{max}} \sigma_l = \frac{\pi}{K^2} \sum_{l=0}^{l_{max}} (2l+1) P_0 P \text{--------->} (3.22)$$

$$K = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$$

Where σ refers to cross section, P_0 preformation probability, P represents the penetrability.

3.2.1 Preformation probability(P_0):

The preformation probability gives the information about the decaying nucleus, it is obtained by solving the Schrödinger equation in η at a fixed $R=R_a$

$$\left[-\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial\eta} \frac{1}{\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial\eta} + V_R(\eta T) \right] \psi_\eta^{(\omega)} = E_\eta^{(\omega)} \psi_\eta^{(\omega)} \text{--------->} (3.23$$

Where ω is the vibrational quantum number, with ω=0, 1, 2, 3...., $\omega=0$ refers to ground state and $\omega = 1,2,3...$ refers to excited state, P_0 is given as

$$P_0 = |\psi(\eta(A_i))|^2 \frac{2}{A} \sqrt{B_{\eta\eta}} \text{--------->} (3.24)$$

and for Boltzmann like function

$$|\psi|^2 = \sum_{\omega=0}^{\infty} |\psi^{\omega}|^2 \exp(-E^{\omega}/T) \text{------------} > (3.25$$

The mass parameters $B_{\eta\eta}$ refers to the smooth hydrodynamical masses [35]. The compound nucleus structure information enters the preformation probability P_0 through the fragmentation potential $V_R(\eta, T)$

3.2.2 Fragmentation potential V (η):

The collective potential energy or the fragmentation potential V (η, R) is calculated as

$$V(\eta,T,l) = -\sum_{i=1}^{2} B_i(A_i, Z_i) + \sum_{i=1}^{2} \delta U_i(T) + \frac{Z_1 Z_2 e^2}{V_c(T)} + V_p(T) + \frac{\hbar^2 l(l+1)}{2I_s(T)} \text{---} > (3.26)$$

Where $B_i(A_i, Z_i)$ refers to temperature dependent binding energies, V_P is temperature dependent proximity potential and V_ℓ represents the rotational energy due to angular momentum. And e^2 =1.44 MeV fm^2 and the relative separation between the centers of the fragments and at touching point is given by R (T). Where R (T) =R_1 (T) +R_2 (T) with that radius expression is defined in Ref [36] given as.

$$R_i(T) = 1.16(1 + 7063 \times 10^{-4} T^2) A_i^{1/3} \text{ fm} \text{-------------------}(3.27)$$

Here i=1, 2 corresponds to light and heavy fragments respectively. Whenever two nuclear surfaces touch each other an additional attraction is created so as to give the total energy which is called as nuclear proximity potential. And by using Blocki et al [37] nuclear proximity potential is determined as

$$V_p(T) = 4\pi \bar{R}(T) \gamma b(T) \phi[s(T) \text{--------------} > (3.28)]$$

Where $\bar{R}(T)=\frac{R_1(T)R_2(T)}{R_t(T)}$ which defines the inverse of the root mean square radius of the Gaussian surface. Similarly $\phi[s(T)]$ and $\gamma$are defined as universal functions which is independent of the geometry of the system and nuclear surface energy coefficient.

Where s (T) is given by

$$s(T)=\frac{R(T)-[R_1(T)+R_2(T)]}{b(T)} \text{-----------------} > (3.29)$$

Andb (T) in eq. (3.29) is given by

$$b(T)=0.68(1+7.37\times10^{-3}T^2 \text{fm}$$

Furthermore the moment of inertia in the Eq. (3.26) with complete sticking limit is given by

$$I_s(T) = \mu Rt^2(T) + \frac{2}{5}A_1 mR_1^2(T) + \frac{2}{5}A_2 mR_2^2(T)$$

Here $\mu=\frac{A_1A_2}{A_1+A_2}$, this is the reduced mass of the nucleon

The charges Z_i are fixed by minimizing the potential V (η), in the coordinate at each η value. B's are the experimental binding energies. Since the binding energies used here contain both the macroscopic part (liquid drop part) and the microscopic part (shell correction) for the study of excited systems, where the nuclear temperature effects also come in to picture, the fragmentation potential will take the following form (at fixed R)

Here V_{LDM} $(= B - \delta U)$ is the liquid drop part of the binding energies [47] and δU , the Shell corrections which is calculated using the analytical form of Myers and Swiatecki [38]. The nuclear temperature T (in MeV) is related approximately to the excitation energy E^*, through a semi- empirical statistical relation, as

$$E^* = \frac{1}{9}AT^2 - T\ (\text{MeV}) \text{--------------} (3.30)$$

The shell correction δU in the fragmentation potential equations is considered to vanish exponentially for E* ≥ 60 MeV, giving T_0=1.5 MeV. At higher excitation the shell correction vanish completely and only the liquid drop part of energy is present.

3.2.3 Temperature dependent binding energies:

The mass defect of a nucleus is the differences between the mass of a nucleus and the mass of its constituent protons and neutrons [40]. The energy equivalent of this mass difference ($\Delta E=\Delta mc^2$) is known as the binding energy of the nucleus. The binding energy of a nucleus is defined as the work done to separate a nucleus into its constituent neutrons and protons. The magnitude of the binding energy of a nucleus determines its stability against disintegration. If binding energy greater is than zero, the nucleus is stable and energy must be supplied from outside to disrupt it into its constituents. If binding energy less than zero, the nucleus is unstable and it will disintegrate by itself [41].

The given formula represented by the binding energy (BE) is

$BE = (\Delta M)\ C^2$ MeV.

$$BE = (Zm_p+Nm_n-M) \times 931.494 \text{ MeV} \text{------------------} (3.31)$$

Masses are in atomic mass unit. Temperature dependent binding energies can be calculated using Guet temperature dependent binding energy formula [47] which is explained below.

$$F = F_{sym} + F_{as} + F_{coul} \text{-----------} > (3.32)$$

Where $F_{sym}, F_{as}, F_{coul}$ refers to symmetric binding energy, asymmetric binding energy, and Coulomb binding energy

$$F_{sym}(A) = a_v A + a_s A^{2/3} + a_c A^{1/3} + a_0 \text{---------} > (3.33)$$

Temperature dependence of these co-efficient are given by, [47]

$$a_i(T) = a_i(0)(1 - x_i T^2) ----------> (3.34) \text{ at } T=0$$

The values for the parameters are given in the Guet paper $a_i(0), x_i$ et al which are listen in table 3.1

$$F_{asy}(A,I) = \frac{JAI^2}{\left[1 + (9J/4Q_g)A^{-1/3}\right]} ----------------> (3.35)$$

$$\text{Where } I = \frac{N - Z}{A}, \text{ Neutron excess}$$

$$F_{coul}(A,Z) = c_1 Z^2 A^{-1/3} + c_2 Z^2 A^{-1} ---------------> (3.36)$$

Where c_1=c_1x (1-c_1T^2) and c_2=c_2x (1-c_2T^2)

The A, Z in eqn. (3.36) is termed as mass number and atomic number of the parent.

The A, Z in eqn. (3.36) is termed as mass number and atomic number

By substituting the Guet values in the concerned equation will give the temperature dependent binding energies. Hence forth the temperature dependent binding energies (Tdbe) is given as

$$B(A, Z) = F_{sym} + F_{aym} + F_{coul} + Sc --------> (3.37)$$

Where Sc denotes the shell correction.

3.2.4 Penetration Probability (P):

The penetration probability can be calculated by using WKB approximation, which is used to find the potential barrier with two turning points R_a and R_b. The penetration probability is given as

$$P = \exp\left[\frac{-2}{\hbar}\{2\mu[V(R) - Q_{eff}]\}^{1/2}\right] dR ----------> (3.38)$$

The sum of proximity potential and coulomb potential and rotational energy tends to give the secondary potential V(R). The first turning points for all compound nucleus is given as suggested in [44, 45, 46]

$$R_a(T) = R_t(T) + \Delta R(T).$$

Where the potentials V (R_a) act as an effective Q value. Let Q_{eff} and R_b be the Second turning points that satisfies the equation,

$$V(R_a) = V(R_b) = Q_{eff} = TKE(T) \text{--------} (3.39)$$

The free parameter $\Delta R(T)$ is taken as 1 fm, in which the proximity forces is said to exists

Therefore the excitation energy of the compound nucleus is due to the sum of the E_{cm} and Q value, Q_{in} for incoming channel. In order to calculate the Q_{in}, and the binding energies [42] is used.

Table 3.1 Liquid drop and droplet co-efficient of Guet's temperature dependent mass formula in MeV^{-1}, MeV^{-2} for $a_i(0)$ and x_i respectively.

	a_v	a_s	a_c	a_0	J	Q_g	c_1	c_2
$a_i(0)$	-15.77	17.22	10.24	-8.0	30.03	35.4	0.737	-1.28
$10^3 x_i$	-3.36	5.53	5.43	-12.3	1.04	12.8	0.763	-13.2