A Study On Separation Axioms In Soft Bitopological Spaces

Thesis submitted in

Partial Fulfilment of the

Degree of Master of Philosophy (M.Phil)

By

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July 2019

DECLARATION

I declare that the dissertation entitled **" A Study On Separation Axioms In Soft Bitopological Spaces "** submitted by me for the degree of **Master of Philosopy (M.Phil.)** is the record of work carried out by me during the period from August 2018 to July 2019 under the guidance of **Dr.(Tmt).A.Kalaichelvi, M.Sc., M.Phil., Ph.D.,** Professor, Department of Mathematics , Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore and has not formed the basis for the award of any Degree, Diploma , Associateship, Fellowship ,Titles in this University or any other similar institution of Higher Learning .

Signature of the Candidate

CERTIFICATE

This is to certify that the dissertation entitled **" A Study On Separation Axioms In Soft Bitopological Spaces "** submitted for the degree of **Master of Philosopy (M.Phil.)** by Ramya. M.A., is the record of work carried out by her during the period from August 2018 to July 2019 under my guidance and supervision , and that this work has not formed the basis for the award of any Degree , Diploma ,Associateship, Fellowship ,Titles in this University or any other Universities or other similar institution of Higher Learning .

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Signature of the Head of the Department

A. Kalaiselvi 31.7.19

Signature of the Supervisor

ACKNOWLEDGEMENT

First and foremost, I am extremely thankful to the **LORD ALMIGHTY** for his graces and blessings showered on me.

I take immense pleasure in thanking **Dr.P.R.KRISHNAKUMAR,** Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for providing the conducive infrastructure for the conduct of the research study.

I would like to thank **Dr. (Tmt.) PREMAVATHY VIJAYAN**, Vice Chancellor, Avinashilingam Institute for Home Science and Higher Education forWomen, Coimbatore, for the encouragement and for providing the opportunity todevelop and establish my skills.

I extend my heartfelt thanks to **Dr. (Tmt.) S. KOWSALYA,** Registrar, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for the encouragement given by her during the investigation.

I express my deep sense of gratitude to **DR.G.P. Jeyanthi,** Director (Research and Consultancy), Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her commendable advice and guidance in each and every process involved in doing the research work in a successful manner.

I express my heartfelt thanks to **Dr. (Tmt.) K. UDAYA CHANDRIKA,** Professor of Mathematics, Dean, School of Physical and Computational Sciences, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her excellent support, unflinching encouragement and guidance during the course of the investigation

I express my heartfelt thanks to **Dr. (Tmt.) P. JEYALAKSHMI,** Professor and Head, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her support and guidance during the course of the investigation.

I thank my supervisor **Dr.(Tmt.)A. KALAICHELVI,** Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her inspiring guidance, innovative ideas, critical suggestions and constant encouragement throughout the completion of this work.

I would like to express my sincere thanks to all the **STAFF MEMBERS OF THE DEPARTMENT OF MATHEMATICS** who were responsible for the good finish of this dissertation.

I owe my special thanks to my **BELOVED PARENTS, FRIENDS AND ALSO THE GRACEFUL RELATIVES,** who helped me by providing full strength, support and encouragement to complete my project successfully.

CONTENT

INTRODUCTION

Soft set theory is one of the recent topics gaining significance in finding rational and logical solutions to various real life problems which involve uncertainty, impreciseness and vagueness. In 1999, Molodstov initiated a novel concept of soft set theory, which is completely a new approach for modeling vagueness and uncertainty.

"Given an initial universe U and a set of parameters E, a pair (F, A) is called a Soft set over U if F is a mapping given by F: A→ P(U) where A ⊆ E and P(U) is the power set of U ".

The topological structures of set theories dealing with uncertainties were first studied by Chang in 1968.

Shabir and Naz (2011) introduced the notion of Soft Topology (Definition1.15) which is defined over an initial universe with a fixed set of parameters. They studied some basic concepts of Soft topological spaces and also some related concepts such as soft interior, soft closure, soft subspace and soft separation axioms.

In 1963, Kelly, first initiated the concept of bitopological space. He defined a bitopological space (X, τ_1, τ_2) to be a set X equipped with two topologies. Also he studied τ_1 and τ_2 on X and initiated the systematic study of bitopological space.

Ittangi (2014) introduced the notion of soft bitopological space which is defined over an initial universal set with fixed set of parameters . Also he introduced some types of soft separation axioms.

Separation axioms play an important role in topology. Many mathematicians threw light on soft separation axioms in Soft topological spaces with respect to soft open set, soft b-open set, soft semi-open set, soft α-open set, soft pre-open set, soft β-open set, etc.

The main aim of this thesis is to study different soft seperation axioms in soft bitopological spaces. The plan of study is as follows:

1) Soft Seperation Axioms in soft Bitopological Spaces
2) Soft b- Seperation Axioms in soft Bitopological Spaces
3) Soft αW- Hausdorff axiom in soft Bitopological Spaces
4) Soft βW- Hausdorff axiom in soft Bitopological Spaces
5) Soft Pre-W-Hausdorff axiom in Soft Bitopological Spaces.
6) Soft gW- Hausdorff axiom in soft Bitopological Spaces.

Preliminary definitions and properties regarding soft sets, soft topological spaces and soft bitopological spaces are collected in the chapter I.

Chapter II is denoted to the study of separation axioms in soft bitopological spaces . Here pairwise soft T_0 , pairwise soft T_1, pairwise soft T_2 spaces are introduced by Ittangi are studied with their basic properties. Important results proved here are given in theorems 2.2, 2.3, 2.4, 2.5, 2.8 and 2.9.

Chapter III deals with soft b-separation axioms in soft bitopological Spaces. A new class of separatiom axioms called (1, 2)* - soft b- separation axioms introduced by Revathy et-al is studied. These separation axioms are introduced using (1,2)* - soft b-open set .Also the properties of (1,2)* -soft bT_i –spaces (i=0,1,2) under the bijection and irresolute open soft mapping are analysed. Further it is shown that the properties of (1,2)*- soft bT_i - spaces(i=0,1,2) are hereditary properties .

In chapter IV Soft αW – Hausdorff axiom is introduced both in soft topological spaces and in soft bi – topological spaces .In the first section of this chapter , soft αW–Hausdorff axiom in soft topological spaces is introduced in two different ways by referring the definition of fuzzy Hausdorffness introduced by Warren(1978) and it is extended to soft bitopological spaces and studied in the second section of this chapter. Here it is proved that each of the four different Hausdorff axioms is hereditary and productive.

The concept of βW- Hausdorff axiom is introduced both in soft topological spaces and soft bitopological spaces in chapter V. Main results proved regarding this axiom is given in theorems 5.1.2 ,5.1.3, 5.1.5, 5.1.6 and 5.1.9.

Chapter VI deals with soft Pre W – Hausdorff axiom .The first section of this chapter soft Pre W- Hausdorff axiom introduced by Khattak –et-al is studied .Here this axiom is introduced using soft pre-open sets. Soft pre W- Hausdorff axiom in soft bitopological spaces is introduced in this study and the definition and results are presented in the second section of this chapter. Here also it is proved that each of the four different Hausdorff axioms is hereditary and productive.

Chapter VII deals with the introduction and analysis of another new Hausdorff axiom called gW-Hausdorffness both in soft topological spaces and in soft bitopological spaces. This concept is introduced in different ways using generalized open sets .Soft gW- Hausdorffness in soft topological spaces is studied in the first section and in soft bitopological spaces in the second section of this chapter. Here also it is proved that all the different type of this axiom is hereditary and productive.

It is worth mentioning that the author of this thesis published two articles related to Soft gW – Hausdorff axiom in soft topological Spaces

(1) "gW- Hausdorffness in soft topological Spaces", *International Journal of Mathematics Trends and Technology,* Vol. 65, No. 6, (2019), pp. 92-97.

(2) " gW – Hausdorffness in soft bitopological spaces " , *International Journal of Research and Analytical Reviews* , Vol – 6 , No.2 , (2019) ,pp.500-506 .

REVIEW OF LITERATURE

Uncertain or imprecise data are inherent and pervasive in many important applications in the areas such as Economics, Engineering, Environment, Social Science, Medical Science and Business Management. There have been a great amount of research and applications in the literature concerning some special tools like probability theory, fuzzy set theory, intuitionistic fuzzy set theory, soft set theory, rough set theory, vague set theory and interval mathematics for modeling uncertain data.

Initially, fuzzy set theory was proposed by Zadeh (1965) as a mean of representing mathematically imprecise or vague system of informations in the real world.

In 1999, Russian researcher Molodtsov initiated a novel concept of soft set theory, which is completely a new approach for modeling vagueness and uncertainties.

Maji et al. (2002) gave first practical application of soft sets in decision making problems. Pei and Miao (2005) investigated the relationship between soft sets and information systems.

Shabir and Naz (2011) introduced the notion of Soft topological spaces which are defined over an initial universe with a fixed set of parameters. They studied some basic concepts of Soft topological spaces and also some related concepts such as soft interior, soft closure, soft subspace and soft separation axioms.

In 1963, Kelly ,first initiated the concept of bitopological space. He defined a bitopological space (X, τ_1 , τ_2) to be a set X equipped with two topologies. Also he studied τ_1 and τ_2 on X and initiated the systematic study of bitopological space.

Ittangi (2014) introduced the notion of soft bitopological space which defined over an initial universal set with fixed set of parameters. Also he introduced some types of soft separation axioms.

Separation axioms play an important role in topology. Many mathematicians contributed a lot towards Separation axioms in Topology, Fuzzy Topology, Soft Topology, Fuzzy soft Topology and Bitopology.

Following are some of the articles published on different separation axioms:

1) “W-Hausdorffness in Soft Bitopological Spaces”, Sasikala, D., Vijayalakshmi, V.M. and Kalaichelvi, A., 2017 [34].
2) “Soft separation axioms in Soft topological spaces”, Hussain, S. and Ahmad, B., 2015 [18].
3) “On Soft Preopen sets and Soft Pre Separation Axioms”, Akdag, M. and Ozkan, A., 2014 [3].
4) “Characterizations of b-Soft Separation Axioms in Soft Topological Spaces”, El-Sheikh, S.A., Hosny, R.A. and Abd El-latif, A.M., 2015 [15].
5) “On soft separation axioms via β-open soft sets”, Abd El-latif, A.M. and Hosny, R.A., 2015 [1].
6) “Characterization of Soft α- Separation axioms and Soft β- Separation Axioms in Soft single point spaces and in Soft ordinary spaces”, Arif Mehmood Khattak., Gulzar Ali Khan., Muhammad Ishfaq and Fahad Jamal., 2017 [6].
7) “Soft semi separation axioms and some types of soft functions”, Kandil, A., Tantawy, O.A.E., El-Sheikh, S.A. and Abd El-latif, A.M., 2014 [23].
8) “Soft Generalized Separation Axioms in Soft Generalized Topological Spaces”, Jyothis Thomas and Sunil Jacob John., 2015 [21].
9) “Soft W-Hausdorff Spaces”, Sruthi, P., Vijayalakshmi, V.M. and Kalaichelvi, A., 2017 [36].
10) “On Soft Hausdorff Spaces”, Banu Pazar Varol and Halis Aygun., 2013 [11].
11) “Separation axioms on soft topological spaces”, Tantawy, O., El-Sheikh, S.A. and Hamde, S., 2015 [41].
12) “Soft Hausdorff spaces and their some properties”, Izzettin Demirand Oya Bedre Ozbakir., 2014 [19].
13) “Soft β-Hausdorff and Soft β-Regular Spaces via Soft ideals”, Brindha Devi, S., Inthumathi, V. and Chitra, V., 2016 [12].

14) “Soft BW- Hausdorff space in Soft Bi Topological spaces”, Muhammad Ishfaq., Arif Mehmood Khattak., Gulzar Ali Khan., Zaheer Anjum., Zia Ullah., Rashid Ullah And Fahad Jamal., 2018 [29].

15) “Soft β- Hausdorff spaces via soft β-open sets”, Subhashini, J., Sumithra Devi, J. and Sekar, C., 2015 [37].

16) “Pairwise open (closed)soft sets in soft Bitopology spaces”A.Kandil, O.A.E.Tantaway , EL – Sheikh , Shawqi A .Hazza ,2015[24].

17) “Some Contribution of soft pre – open sets to soft W- Hausdorff space in soft topological spaces” Arif Mehmood Khattak, Ishaq Ahmed, ZaheerAnjum ,Muhammad Zamir , Fahad Jamal ,2018 [8].

18) “Soft Bitoplogical Spaces” , Basavaraj , M.Ittangi ,2014 [10].

19) “Generalized seperation axioms in bitopological spaces”,O.A.El-Tantaway and H.M.Abu – Donia ,2005 [40].

20) “Seperation axioms in bi- soft topological spaces”,Munazza Naz ,Muhammad Shabir and Muhammad Irfan Ali,2015 [30].

21) “Seperation axioms in bitopological spaces”, S.Selvanayaki and N.Rajesh, 2011 [38].

22) “New Seperation Axioms in soft bitopological Space” N.Revathi and K. Bageerathi , 2017 [32].

In this Review of Literature a brief survey of some of the articles published on soft sets, Soft topological spaces and Separation axioms in Soft topological spaces are given.

1. **“Soft set theory – First results”**

   Molodstov, D. (1999)

   The soft set theory offers a general mathematical tool for dealing objects. The basic notions of the theory of soft sets are introduced, the first results of the theory are presented and some problems of the future are discussed.

2. **“From soft sets to information system”**

   Daowu Pei and Duoqian Miao (2005)

   This paper discusses the relationship between soft sets and information systems. It is showed that soft sets are a class of special information systems. After soft sets are extended to several classes of general cases, the more general results also show that partition- type soft sets and information systems have the same formal structures and that fuzzy soft sets and fuzzy information systems are equivalent.

3. **"Generalized Seperation Axioms in bitopological Spaces"**

   O .A. El-Tantway and H .M. Abu- Donia(2005)

   In this paper we introduce some classes of sets in a bitopological Space (X, τ_1, τ_2).We show that some of these classes are infra topologies and some are supra topologies . Also ,We use these classes to introduce new bitopological properties and new types of continuous function between bitopological spaces. We prove that some of the introduced bitopological separation properties are preserved under some types of continuous functions.

4. **"On some structures of Soft topology"**

   Ahmad,B. and Hussain, S. (2012)

   In this paper, soft exterior is defined and its basic properties are studied. Several important results relating soft interior, soft exterior, soft closure and soft boundary in soft topological spaces are established.

5. **"On soft topological spaces"**

   Shabir, M., and Naz, M. (2011)

   In this paper, it is shown that a soft topological space gives a parameterized family of topological spaces. Furthermore, with the help of an example it is established that the converse does not hold. The soft subspace of a soft topological space are defined and inherent concepts as well as the characterization of soft open and soft closed sets in soft subspaces are investigated. Finally, soft T_1-spaces and notions of soft normal and soft regular spaces are discussed. A sufficient condition for a soft topological space to be a soft T_1-space is also presented.

6. **"Seperation Axioms in Bitopological Spaces"**
S.Sevanayaki and N.Rajesh (2011).

In this paper ,We introduced a new type of separation axiom in bitopological spaces called quasi $T_{1/2}$ space in terms of the concept of quasi open sets and quasi kernel and investigate some of their fundamental properties . Also we introduced and studied some new notions in bitopological Spaces by utilizing quasi open sets.

7. **"On soft Hausdorff spaces"**
Banu Pazar Varol and Halis Aygun (2012)

The aim of this paper is to study some properties of soft Hausdorff space introduced by Shabir and Naz. Firstly, they give a representation of soft sets and soft topological spaces. Secondly, they introduce some new concepts in soft topological space such as convergence of sequence, homeomorphism and investigate the relation between these concepts and Hausdorff axioms in soft topological space.

8. **"On soft topological space via semiopen and semiclosed soft sets"**
Mahanta, J. and Das, P.K. (2012)

This paper introduces semiopen and semiclosed soft sets in soft topological spaces. The notions of interior and closure are generalized using these sets. A detail study is carried out on properties of semiopen, semiclosed soft sets, semi interior and semiclosure of a soft set in a soft topological space. Various forms of soft functions, like semicontinuous, irresolute, semiopen soft functions are introduced and characterized. Further soft semicompactness, soft semiconnectedness and soft semiseparation axioms are imtroduced and studied.

9. **"Soft Generalized Closed sets in Soft topological spaces"**
Kannan, K. (2012)

In this paper, it is shown that the Soft Generalized closed sets in soft topological spaces which are defined over an initial universe with a fixed set of parameters. A sufficient condition for a soft g-closed set to be a soft closed set is also introduced. Moreover, the union and intersection of two soft g-closed sets are discussed. Finally, the new soft separation axiom, namely soft $T_{1/2}$ – space is introduced and its basic properties are investigated.

10. "On Soft Preopen Sets in Soft Topological Spaces"

Gnanambal Ilango and Mrudula Ravindran (2013)

In this paper, they introduced a soft topology via soft preopen sets. Also they state and prove the condition for collection of soft preopen sets to be a soft topology.

11. "On soft semi-open sets and soft semi-topology"

Sai, B.V.S.T. and Srinivasa Kumar, V. (2013)

In this paper, some interesting properties of soft semiopen sets are studied. Soft semitopology on the collection of all soft semiopen sets over a fixed universe set is introduced.

12. "Soft Hausdorff spaces and their some properties"

Izzettin Demir and Oya Bedre Ozbakir (2014)

In this paper they have contributed to the progress of the Soft topological strucures. They introduced Soft Hausdorff spaces using the definitions of soft points and investigated some of their properties. They have given some new concepts such as cluster soft point, soft point, soft net and have supported them with examples. Then by using these concepts, they have obtained some properties with respect to soft Hausdorff spaces which are important for further research on soft topology.

13. "Soft β- Hausdorff spaces Via soft β-open sets"

Subhashini, J., Sumithra Devi, J. and Sekar, C. (2015)

In this paper they introduced soft β- Hausdorff spaces in soft topological spaces via soft β-open sets. The notions of interior and closure are generalized using these sets. A detail study is carried out on the soft β- Hausdorff spaces.

14. "Separation axioms on soft topological spaces"

Tantawy, O., El-Sheikh, S.A. and Hamde, S. (2015)

In this paper they introduced the separation axioms soft T_i (i=0,1,2,3,4,5) by using the concept of soft points and have studied some of their properties. They observed that in a soft T_1-space, the soft point x_e may be not closed soft set, so there are spaces which are soft T_1 but not Soft T_{i-1} (i=3,4,5). In order to overcome this problem they have presented the necessary condition for a soft space to be soft T_1-space. Finally they have discussed the hereditary and some soft topological properties for such spaces.

15. "Characterizations of b-soft separation Axioms in Soft Topological Spaces"

S.A.El – Sheikh , Rodyna A. Hosny and A. M.Abd El- latif (2015)

Many scientists have studied and improved the soft set theory, which is initiated by Molodtsov and easily applied to many problems having uncertainties from social life. The main purpose of our paper, is to introduce new soft separation axioms based on the b-open soft sets which are more general than of the open soft sets. We show that, the properties of soft b-Ti-spaces (i =1,2) are soft topological properties under the bijection and irresolute open soft mapping. Also, the property of being soft b-regular and soft b-normal are soft topological properties under bijection, irresolute soft and irresolute open soft functions. Further, we show that the properties of being soft b-Ti-spaces (i = 1,2,3,4) are hereditary properties.

16. "Soft β- Hausdorff and Soft β- Regular Spaces via Soft ideals"

Brindha devi, S., Inthumathi, V. and Chitra, V. (2016)

In this paper they introduced the notions of soft β-I-Hausdorff spaces which is weaker than soft semi-I-Hausdorff spaces. Also they established the relationships between the existing spaces. Further they define soft β-I-regular spaces and investigate some of their properties.

17. "New Seperation Axioms in soft bitopolgical space"

N. Revathi and K. Bageerathi (2017).

The Present Paper introduces a new class of separation axioms called (1,2)* - soft b-separation axioms using (1,2)*-soft b-open set .Also the properties of (1,2)*- soft bT_i–spaces (i=0,1,2) are soft bitopological properties under the bijection and irresolute open soft mapping .Further ,We show that the properties of (1,2)* - soft bT_i – spaces (i= 0,1,2) are hereditary properties.

18. "Soft BW-Hausdorff space in soft bitopological spaces"

Muhammed Ishfaq, Arif mehmood khattak, Gulzar Ali khan, Zaheer Anjum , Zia ullah, Rashid ullah, Fahad jamal (2018) .

In this article the concept of soft βW-T_2 structure in soft bitopological spaces is introduced in different ways . Fleix Hausdorff was a German Mathematician Who is supported to be the forefather of up -to-the –minute Topology .There are many topological Structures in soft topology but Hausdorff topological structure is interesting and more practical ,that is way it catches our attention to the best.

CHAPTER- I

SOFT BITOPOLOGICAL SPACES

Definition 1.1.

Let X be an initial universe and E be a set of parameters. Let P(X) denotes the power set of X and A be a non empty subset of E. A pair (F, A) denoted by F_A is called a **soft set** over X, where F is a mapping given by F: A→ P(X). In other words, a soft set over X is parameterized family of subsets of the universe X. For a particular e ∈ A, F(e) may be considered the set of e-approximate elements of the soft set (F,A) and if e ∉ A, then F(e)= ϕ

i.e. F_A = { F(e): e∈ A⊆ E; F:A→P(X) }

The family of all these soft sets over X with respect to the parameter set E is denoted by $SS(X)_E$.

Example 1.2.

Let X ={ x_1,x_2 } , E = { e_1,e_2 } then $\tilde{X}=\{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2\})\}$.The possible soft sub-sets are $F_{E_1}=\{(e_1,\{x_1\})\}$, $F_{E_2}=\{(e_1,\{x_2\})\}$, $F_{E_3}=\{(e_1,\{x_1,x_2\})\}$, $F_{E_4}=\{(e_2,\{x_1\})\}$, $F_{E_5}=\{(e_2,\{x_2\})\}$, $F_{E_6}=\{(e_2,\{x_1,x_2\})\}$, $F_{E_7}=\{(e_1,\{x_1\}),(e_2,\{x_1\})\}$, $F_{E_8}=\{(e_1,\{x_1\}),(e_2,\{x_2\})\}$, $F_{E_9}=\{(e_1,\{x_1\}),(e_1,\{x_1\})\}$, $F_{E_{10}}=\{(e_1,\{x_2\}),(e_2,\{x_1\})\}$, $F_{E_{11}}=\{(e_1,\{x_2\}),(e_2,\{x_2\})\}$, $F_{E_{12}}=\{(e_1,\{x_2\}),(e_2,\{x_1,x_2\})\}$, $F_{E_{13}}=\{(e_1,\{x_1,x_2\}),(e_2,\{x_1\})\}$, $F_{E_{14}}=\{(e_1,\{x_1,x_2\}),(e_2,\{x_2\})\}$, $F_{E_{15}}=\phi$, $F_{E_{16}}=\tilde{X}$.

Definition 1.3.

Let $F_A, G_B \in SS(X)_E$ .Then F_A is soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$,if

(1). $A \subseteq B$, and

(2). $F(e) \subseteq G(e), \forall e \in A$.

In this case , F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A , $G_B \tilde{\supseteq} F_A$

Definition 1.4.

Two soft subsets F_A and G_B over a common universe X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A .

Definition 1.5.

The complement of a soft set (F, A) denoted by $(F, A)' = (F', A)$, $F' : A \to P(X)$ is a mapping given by $F'(e) = X - F(e); \forall e \in A$ and F' is called the **soft complement** function of F. Clearly $(F')'$ is same as F and $((F, A)')' = (F, A)$.

Definition 1.6.

A soft set (F, A) over X is said to be a **null soft set** denoted by $\tilde{\phi}$ or ϕ_A if for all $e \in A$, $F(e) = \phi$ (null set).

Definition 1.7.

A Soft set (F,A) over X is said to be an **absolute soft set** denoted by $\tilde{A}$ or X_A if for all $e \in A$, F(e)=X. Clearly we have $X'A = \phi_A$ and $\phi\acute{A} = X_A$.

Definition 1.8.

Let $F_E \tilde{\in} S(X)$ .We say that $x_e = (e, \{x\})$ is a **soft point** of F_E , if $e \in E$ and $x \in F(e)$.

Definition 1.9.

The Soft point x_e said to be belonging to the soft set F_E , denoted by $x_e \tilde{\in} F_E$.

Definition 1.10.

The **union** of two soft sets F_A and G_B over the common universe X is the soft set H_C , where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), e \in A - B \\ G(e), e \in B - A \\ F(e) \cup G(e), e \in A \cap B \end{cases}$$

Definition 1.11.

The **intersection** of two soft sets F_A and G_B over the common universe X is the soft set H_C ,where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

Definition 1.12.

Let (X, τ, E) be a soft topological spaces, $(F,E) \in SS(X)_E$ and Y be a non null subset of X. Then the soft subset of X. Then the soft subset of (F, E) over Y denoted by (F_Y, E) is defined as follows:

$$F_Y(e) = Y \cap F(e), \forall\ e \in E$$

In other words , $(F_Y, E) = Y_E \cap (F,E)$.

Definition 1.13.

Let (X, τ, E) be a soft topological spaces and Y be a non null subset of X and Y be a non null subset of X. Then $\tau_Y = \{ (F_Y, E) : (F,E) \in \tau \}$ is said to be the **relative soft topology** on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

Definition 1.14.

Let $F_A \in SS(X)_E$ and $G_B \in SS(Y)_K$. The **Cartesian product** $(F_A \otimes G_B)$ is defined by $(F_A \otimes G_B)$ (e,k) = $F_A(e) \times G_B(k)$, $\forall (e,k) \in A$ x B. According to this definition $F_A \otimes G_B$ is a soft set over X $\times$ Y and its parameter set is E x K.

Definition 1.15.

Let τ be the collection of soft sets over X with the fixed set of parameters A. Then τ is said to be a **soft topology** on X, if

1) ϕ , X ϵ τ
2) The union of any number of soft sets in τ belongs to τ,
3) The intersection of any two soft sets in τ belongs to τ.

Definition 1.16.

Let X be an initial universe and E be the non-empty set of parameters. Let (X, τ_1, E) and(X, τ_2, E) be the two different soft topologies on X. Then (X, τ_1, τ_2, E) is called a **soft bitopological** space.

The two soft topologies (X, τ_1, E) and (X, τ_2, E) are independently satisfy the axioms of soft topology. the axioms of soft topology. The members of τ_1 are called τ_1 soft open sets and the complement of τ_1 soft open sets are called τ_1 soft closed sets.

Similarly, The members of τ_2 are called τ_2 soft open sets and the complements of τ_2 soft open sets are called τ_2 soft closed sets.

Example 1.17. Let X={h_1,h_2,h_3},E={e_1,e_2}and

τ_1={ ϕ ,X,(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E)}and

τ_2={ ϕ ,X,(G_1,E),(G_2,E),(G_3,E),(G_4,E),(G_5,E),(G_6,E),(G_7,E)},where

(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E), (G_1,E),(G_2,E),(G_3,E),(G_4,E),(G_5,E),(G_6,E),(G_7,E) are soft sets over X, defined as follows

$F_1(e_1) = \{h_2\}$ $F_1(e_2) = \{h_1\}$

$F_2(e_1) = \{h_2,h_3\}$ $F_2(e_2) = \{ h_1, h_2\}$

$F_3(e_1) = \{h_1,h_2\}$ $F_3(e_2) = X$

$F_4(e_1) = \{h_1,h_2\}$ $F_4(e_2) = \{ h_1, h_3\}$

$F_5(e_1) = \{h_2\}$ $F_5(e_2) = \{ h_1, h_2\}$

$G_1(e_1) = \{h_1,h_2\}$ $G_1(e_2) = \{ h_1, h_2\}$

$G_2(e_1) = \{h_2\}$ $G_2(e_2) = \{ h_1, h_3\}$

$G_3(e_1) = \{h_2,h_3\}$ $G_3(e_2) = \{ h_1\}$

$G_4(e_1) = \{h_2\}$ $G_4(e_2) = \{ h_1\}$

$G_5(e_1) = \{h_1,h_2\}$ $G_5(e_2) = X$

$G_6(e_1) = X$ $G_6(e_2) = \{ h_1, h_2\}$

$G_7(e_1) = \{h_2,h_3\}$ $G_7(e_2) = \{ h_1, h_3\}$

Then (X, τ_1, τ_2, E) , is a soft bitopological space.

Example 1.18 . Let X be an initial universe set and E be the non- empty set of parameters. Soft indiscrete topology $\tau_1 = \{\phi, X\}$ and soft discrete topology τ_2 is the collection of all soft sets defined over X. Then (X, τ_1, τ_2, E) is a soft bitopological space.

Definition 1.19. Let (X, τ_1, τ_2, E) be a soft bitopological space over X and Y be a non-empty subset of X. Then $\tau_{1Y} = \{({}^{y}F, E) : (F,E) \in \tau_1\}$ and $\tau_{2Y} = \{({}^{y}G, E) : (G,E) \in \tau_2\}$ are said to be relative topologies on Y . Then$\{ Y, \tau_{1Y}, \tau_{2Y}, E \}$ is called a relative soft bitopological space of (X, τ_1, τ_2, E).

Theorem 1.20. Let (X, τ_1, τ_2, E) is a soft bitopological space then $\tau = \tau_1 \cap \tau_2$ is a soft topological space over X.

Proof . 1). ϕ, X belong to $\tau_1 \cap \tau_2 = \tau$

2). Let { (F_i, E) : $i \in I$ } be a family of soft sets in $\tau_1 \cap \tau_2 = \tau$.

Then { $(F_i, E) \in \tau_1$ } and { $(F_i, E) \in \tau_2$) for all $i \in I$.

Therefore $\bigcup_{i\in I} (F_i, E) \in \tau_1$ and $\bigcup_{i\in I} (F_i, E) \in \tau_2$.

Thus $\bigcup_{i\in I} (F_i E) \in \tau_1 \cap \tau_2 = \tau$.

3). Let (F,E) ,(G,E) $\in \tau_1 \cap \tau_2 = \tau$.

Then (F,E) ,(G,E) $\in \tau_1$ and (F,E) , (G,E) $\in \tau_2$.

Since (F, E) $\cap$ (G,E) $\in \tau_1$ and (F, E) $\cap$ (G, E) $\in \tau_2$.

Therefore (F, E) $\cap$ (G,E) $\in \tau_1 \cap \tau_2 = \tau$.

Thus $\tau_1 \cap \tau_2 = \tau$ defines a soft topology on X.

Remark 1.21. If (X, τ_1, τ_2,E) is a soft bitopological space then $\tau_1 \cup \tau_2$ is not a soft topological space over X .

Example 1.22 . Let X = $\{h_1, h_2, h_3\}$, E = $\{e_1, e_2\}$ and

$\tau_1 = \{\phi, X, (F_1,E), (F_2,E), (F_3,E), (F_4,E)\}$ and

$\tau_2 = \{\phi, X, (G_1,E), (G_2,E), (G_3,E), (G_4,E), (G_5,E)\}$

Where $(F_1,E),(F_2,E),(F_3,E),(F_4,E)$, $(G_1,E),(G_2,E),(G_3,E),(G_4,E),(G_5,E)$ are soft sets over X, defined as follows

$F_1(e_1) = \{h_1, h_2\}$ $\quad$ $F_1(e_2) = \{h_2, h_3\}$

$F_2(e_1) = \{h_2\}$ $\quad$ $F_2(e_2) = \phi$

$F_3(e_1) = \{h_2\}$ $\quad$ $F_3(e_2) = \{h_3\}$

$F_4(e_1) = \{h_1,h_2\}$ $F_4(e_2) = \{ h_2\}$

$G_1(e_1) = \{h_1,h_2\}$ $G_1(e_2) = \{ h_2, h_3\}$

$G_2(e_1) = \phi$ $G_2(e_2) = \{ h_2, h_3\}$

$G_3(e_1) = \{ h_1, h_2\}$ $G_3(e_2) = \{ h_3\}$

$G_4(e_1) = X$ $G_4(e_2) = \{ h_2,h_3\}$

$G_5(e_1) = \phi$ $G_5(e_2) = \{h_3\}$

Then (X, τ_1, τ_2, E) is a soft bitopological space. Now define

$\tau = \tau_1 \cup \tau_2 = \{(F_1,E),(F_2,E),(F_3,E),(F_4,E), (G_1,E),(G_2,E),(G_3,E),(G_4,E),(G_5,E) \}$.

It take $(F_2 ,E) \cup (G_2 ,E) = (H ,E)$, then

$H(e_1) = F_2(e_1) \cup G_2(e_1) = \{h_2\} \cup \phi = \{ h_2\}$ and

$H(e_2) = F_2(e_2) \cup G_2(e_2) = \phi \cup \{ h_2,h_3\}= \{ h_2,h_3\}$ but $(H,E) \notin \tau$.

Thus τ is not a soft topology on X.

CHAPTER – II

SEPARATION AXIOMS IN SOFT BITOPOLOGICAL SPACES

Definiton 2.1. In a soft bitopological space (X, τ_1, τ_2, E)

i) τ_1 is said to be soft T_0 space with respect to τ_2 if for each distinct points x, y of X then there exists a τ_1 soft open set F_A and a τ_2 soft open set G_A such that $x \in F_A$ and $y \notin F_A$ or $y \in G_A$ and $x \notin G_A$.

Similarly τ_2 is said to be soft T_0 space with respect to τ_1 if for each distinct points x, y of X then there exist a τ_2 soft open set F_A and a τ_1 soft open set G_A such that $x \in F_A$ and $y \notin F_A$ or $y \in G_A$ and $x \notin G_A$.(X, τ_1, τ_2 ,E) is said to be **pairwise soft T_0 space** if τ_1 is soft T_0 space with respect to τ_2 and τ_2 is soft T_0 space with respect to τ_1.

ii) τ_1 is said to be soft T_1 space with respect to τ_2 if for each distinct points x, y of X then there exists a τ_1 soft open set F_A and a τ_2 soft open set G_A such that $x \in F_A$ and $y \notin F_A$ and $y \in G_A$ and $x \notin G_A$.

Similarly τ_2 is said to be soft T_1 space with respect to τ_1 if for each distinct points x, y of X then there exist a τ_2 soft open set F_A and a τ_1 soft open set G_A such that $x \in F_A$ and $y \notin F_A$ and $y \in G_A$ and $x \notin G_A$.(X , τ_1, τ_2 ,E) is said to be **pairwise soft T_1 space** if τ_1 is soft T_1 space with respect to τ_2 and τ_2 is soft T_1 space with respect to τ_1.

iii) τ_1 is said to be soft T_2 space with respect to τ_2 if for each distinct points x, y of X then there exists a τ_1 soft open set F_A and a τ_2 soft open set G_A such that $x \in F_A$, $y \in G_A$ and $F_A \cap G_A = \phi$.

Similarly τ_2 is said to be soft T_2 space with respect to τ_1 if for each distinct points x, y of X then there exist a τ_2 soft open set F_A and a τ_1 soft open set G_A such that $x \in F_A$,

$y \in G_A$ and $F_A \cap G_A = \phi$.(X , τ_1, τ_2 ,E) is said to be **pairwise soft T_2 space** if τ_1 is soft T_2 space with respect to τ_2 and τ_2 is soft T_2 space with respect to τ_1.

Theorem 2.2. Let (X, τ_1, τ_2,E) be a soft bitopological space over X and Y be a non -empty subset of X. If (X, τ_1, τ_2,E) is pairwise soft T_0 space the (Y, τ_{1Y}, τ_{2Y} ,E) is pairwise soft T_0 space .

Proof. Let (X,τ_1, τ_2,E) be a soft bitopological space over X and $x, y \in Y$ such that $x \neq y$. If (X, τ_1, τ_2,E) is pairwise soft T_0 space, then there exist a $\tau_1$soft open set F_A and a τ_2 soft open set G_A such that $x \in F_A$ and $y \notin F_A$ or $y \in G_A$ and $x \notin G_A$. Now $x \in Y$ and $x \in F_A$ .Hence $x \in Y \cap F_A = {}^{Y}F_A$ where $F_A \in \tau_1$Consider $y \notin F_A$ this means that $\alpha \in E$ then $y \notin Y \cap F(\alpha)$ for some $\alpha \in E$. Therefore $y \notin Y \cap F_A = ({}^{y}F, E)$. Therefore τ_{1Y} is soft T_0 space with respect to τ_{2Y}.

Similarly it can be prove that τ_{2Y} is soft T_0 space with respect to τ_{1Y}, that is $y \in G_A$ and $x \notin G_A$ then $y \in {}^{Y}G_A$ and $x \notin {}^{Y}G_A$. Thus (Y, τ_{1Y}, τ_{2Y} ,E) is pairwise soft T_0 space.

Theorem 2.3. Let (X, τ_1, τ_2,E) be a soft bitopological space over X and Y be a non-empty subset of X. If (X,τ_1, τ_2,E) is pairwise soft T_1 space the (Y, τ_{1Y}, τ_{2Y} ,E) is pairwise soft T_1 space.

Proof . Let (X, τ_1, τ_2,E) be a soft bitopological space over X and $x, y \in Y$ such that $x \neq y$. If (X, τ_1, τ_2,E) is pairwise soft T_1 space, then there exist a $\tau_1$soft open set F_A and a τ_2 soft open set G_A such that $x \in F_A$ and $y \notin F_A$ and $y \in G_A$ and $x \notin G_A$. Now $x \in Y$ and $x \in F_A$. Hence $x \in Y \cap F_A = ({}^{y}F, E)$ where $F_A \in \tau_1$. Consider $y \notin F_A$ this means that $\alpha \in E$. Then $y \notin Y \cap F(\alpha)$ for some $\alpha \in E$.Therefore $y \notin Y \cap F_A = {}^{y}F_A$ Now $y \in Y$ and $y \in G_A$. Hence $y \in Y \cap G_A = {}^{Y}G_A$. Therefore τ_{1Y} is soft T_1 space with respect to τ_{2Y}.

Similarly it can be prove that τ_{2Y} is soft T_1 space with respect to τ_{1Y}, that is $y \in G_A$ and $x \notin G_A$ then $y \in ({}^{y}G, E)$ and $x \notin ({}^{y}G, E)$. Thus $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pairwise soft T_1 space.

Theorem 2.4. Let (X, τ_1, τ_2, E) be a soft bitopological space over X. if (x, E) is a soft closed set in τ_2 for each $x \in X$ and (y, E) is a soft closed set in τ_1 for each $y \in X$ then (X, τ_1, τ_2, E) is pairwise soft T_1 space.

Proof. Suppose that for each $x \in X$, (x, E) is a soft closed set in τ_2 then $(x, E)^c$ is a soft open set in τ_2. Let $x, y \in Y$ such that $x \neq y$. For each $x \in X$, $(x, E)^c$ is a soft open set in τ_2 such that $y \in (x, E)^c$.

Similarly for each $y \in X$, (y, E) is a soft closed set in τ_1 then $(y, E)^c$ is a soft open set in τ_1 such that $x \in (y, E)^c$ and $y \notin (y, E)^c$. Thus (X, τ_1, τ_2, E) is pairwise soft T_1 space.

Theorem 2.5. Every Pairwise soft T_1 space is pairwise soft T_0 space.

Proof. Let (X, τ_1, τ_2, E) be a soft bitopological space over X and $x, y \in Y$ such that $x \neq y$. If (X, τ_1, τ_2, E) is pairwise soft T_1 space. That is (X, τ_1, τ_2, E) is pairwise soft T_1 space if τ_1 is soft T_1 space with respect to τ_2 and τ_2 is soft T_1 space with respect to τ_1. If τ_1 is soft T_1 space with respect to τ_2, then there exist a τ_1 soft open set F_A and a τ_2 soft open set G_A such that $x \in F_A$ and $y \notin F_A$ and $y \in G_A$ and $x \notin G_A$. Obviously $x \in F_A$ and $y \notin F_A$ or $y \in G_A$ and $x \notin G_A$ Therefore τ_1 is soft T_0 space with respect to τ_2.

Similarly if τ_2 is soft T_1 space with respect to τ_1, then there exists a τ_2 soft open set F_A and a τ_1 soft open set G_A such that $x \in F_A$ and $y \notin F_A$ and $y \in G_A$ and $x \notin G_A$. Obviously $x \in F_A$ and $y \notin F_A$ or $y \in G_A$ and $x \notin G_A$. Therefore τ_2 is soft T_0 space with respect to τ_1. Thus (X, τ_1, τ_2, E) pairwise soft T_0 space.

Remark 2.6. The converse of the Theorem 2.5 is not true.

Example 2.7. Let X = {h_1, h_2} , E = { e_1,e_2 } and τ_1= { ϕ ,X, F_A } and τ_2={ ϕ ,X}, where F_A is soft set over X, defined as follows F(e_1) = ϕ , F(e_2) = {h_1}

Then (X, τ_1, τ_2,E) be a soft bitopological space. Also (X, τ_1, τ_2,E) is a pairwise soft T_0 space but which is not a pairwise soft T_1 space ,because h_1, h_2 ∈ X with $h_1 \neq h_2$ and there do not exist $\tau_1$soft open set F_{A_1} and τ_2 soft open set G_{A_1} such that $h_1 \in F_{A_1}$ and $h_2 \notin F_{A_1}$ and $h_2 \in G_{A_1}$ and $h_1 \notin G_{A_1}$.

Theorem 2.8. Let (X, τ_1, τ_2,E) be a soft bitopological space over X and Y be a non-empty subset of X. If (X, τ_1, τ_2,E) is pairwise soft T_2 space the (Y, τ_{1Y}, τ_{2Y} ,E) is pairwise soft T_2 space.

Proof. Let (X, τ_1, τ_2,E) be a soft bitopological space over X and *x, y* ∈ Y such that $x \neq y$. If (X, τ_1, τ_2,E) is pairwise soft T_2 space, then there exist a $\tau_1$soft open set F_A and a τ_2 soft open set G_A such that $x \in F_A$, $y \in G_A$ and $F_A \cap G_A = \phi$. So for each $\alpha \in$E, $x \in$ F(α), $y \in$ G(α) and F(α) ∩ G(α) = ϕ.This implies that $x \in$ Y ∩ F(α) , $y \in$ Y∩G(α) and F(α) ∩G(α) = ϕ. Hence $x \in$ (^{Y}F,E), $y \in$ (^{Y}G,E) and (^{Y}F,E) ∩ (^{Y}G,E) = ϕ, where (^{Y}F,E) is soft open set in τ_{1Y},and (^{Y}G,E) is soft open set in τ_{2Y} Therefore $\tau_{1Y}$is soft T_2 space with respect to τ_{2Y}.

Similarly it can be prove that τ_{2Y} is soft T_2 space with respect to τ_{1Y}. Thus (Y, τ_{1Y}, τ_{2Y} ,E) is pairwise soft T_2 space **.**

Theorem 2.9. Every Pairwise soft T_2 space is pairwise soft T_1 space.

Proof. Let (X, τ_1, τ_2,E) be a soft bitopological space over X and *x, y* ∈Y such that $x \neq y$. If (X, τ_1, τ_2,E) is pairwise soft T_2 space. That is (X, τ_1, τ_2,E) is pairwise soft T_2 space if $\tau_1$is soft T_2 space with respect to τ_2 and τ_2 is soft T_2 space with respect to τ_1. If τ_1 is soft T_2 space with respect to τ_2, then there exist a $\tau_1$soft open set

F_A and a τ_2 soft open set G_A such that $x \in F_A$, $y \in G_A$ and $F_A \cap G_A = \phi$ Obviously $x \in F_A$ and $y \notin F_A$ and $y \in G_A$ and $x \notin G_A$. Therefore τ_1 is soft T_0 space with respect to τ_2 .

Similarly if τ_2 is soft T_2 space with respect to τ_1 , then there exists a τ_2 soft open set F_A and a τ_1 soft open set G_A such that $x \in F_A$, $y \in G_A$ and $F_A \cap G_A = \phi$ .Obviously $x \in F_A$ and $y \notin F_A$ and $y \in G_A$ and $x \notin G_A$. Therefore τ_2 is soft T_1 space with respect to τ_1.Thus (X, τ_1, τ_2, E) pairwise soft T_1 space .

CHAPTER - III

SOFT b – SEPERATION AXIOMS IN SOFT BITOPOLOGICAL SPACES

3.1. Soft b- Seperation Axioms in Soft topological Spaces

Definition.3.1.1 (El-Sheikh and Abd El-latif, 2015)

Let (X, τ, E) be a soft topological space and $F_A \in SS(X, E)$. Then F_A is called a **soft b-open set** if $F_A \mathrel{\tilde{\subseteq}} cl(int(F_A)) \mathrel{\tilde{\cup}} int(cl(F_A))$. The set of all soft b-open sets is denoted by BOS(X, τ, E), or BOS(X) and the set of all soft b-closed sets is denoted by BCS(X, τ, E), or BCS(X).

Definition 3.1.2 (El-Sheikh and Abd El-latif, 2015)

Let (X, τ, E) be a soft topological space and $F_A \in SS(X, E)$. Then, the **b-soft interior** of F_A is denoted by $bSintF_A$, where

$bSint\, F_A = \tilde{\cup}\{ G_A : G_A \mathrel{\tilde{\subseteq}} F_A , G_A \in BOS(X)\}$. Also, the b-soft closure of F_A is denoted by $bScl(F_A)$, where

$bScl(F_A) = \tilde{\cap}\{ H_E : H_E \in BCS(X), (F_A) \mathrel{\tilde{\subseteq}} H_E \}$.

Definition 3.1.3 (El-Sheikh et al., 2015)

Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. Then, (X, τ, E) is called a **soft b-T_o-space** if there exist soft b-open sets F_A and G_A such that either $x \in F_A$ and $y \notin F_A$ or $y \in G_A$ and $x \notin G_A$.

Definition 3.1.4 (El-Sheikh et al., 2015)

Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. Then, (X, τ, E) is called a **soft b-T_1-space** if there exist soft b-open sets F_A and G_A such that $x \in F_A$ and $y \notin F_A$ and $y \in G_A$ and $x \notin G_A$.

Definition 3.1.5 (El-Sheikh et al., 2015)

Let (X, τ, E) be a soft topological space and x, y ∈ X such that x ≠ y. Then (X, τ, E) is called a **soft b-Hausdorff space or soft b-T_2-space** if there exist soft b-open sets F_A and G_A such that $x \in F_A$, $y \in G_A$ and $F_A \tilde{\cap} G_A = \tilde{\phi}$.

Theorem 3.1.6

For a soft topological space (X, τ, E) we have:

soft b-T_2-space ⇒ soft b-T_1-space ⇒ soft b-T_o-space.

Proof:

1) Let (X, τ, E) be a soft b-T_2-space and x, y ∈ X such that x ≠ y. Then, there exist soft b-open sets F_A and (G, E) such that $x \in F_A$, $y \in G_A$ and $F_A \tilde{\cap} G_A = \tilde{\phi}$. Since $F_A \tilde{\cap} G_A = \tilde{\phi}$. Then, $x \notin G_A$, $y \notin F_A$. Therefore, there exist soft b-open sets F_A and G_A such that $x \in F_A$ and $y \notin F_A$ and $y \in G_A$ and $x \notin G_A$. Thus, (X, τ, E) is soft b-T_1-space.
2) Let (X, τ, E) be a soft b-T_1 -space and x, y ∈ X such that x ≠ y. Then, there exist soft b-open sets F_A and G_A such that $x \in F_A$ and $y \notin F_A$ and $y \in G_A$ and $x \notin G_A$. Obviously then we have, either $x \in F_A$ and $y \notin F_A$ or $y \in G_A$ and $x \notin G_A$. Thus, (X, τ, E) is soft b-T_o-space.

Remark 3.1.7

The converse of Theorem 3.1.6 is not true in general, as shown in the following examples.

Example 3.1.8

1) Let X = {a, b}, E = {e_1, e_2} and τ = { $\tilde{X}$, $\tilde{\phi}$, $(F_1, E), (F_2, E), (F_3, E)$} where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined as follows:

$F_1(e_1) = X$, $F_1(e_2) = \{b\}$,

$F_2(e_1) = \{a\}$, $F_2(e_2) = X$,

$F_3(e_1) = \{a\}$, $F_3(e_2) = \{b\}$.

Then, τ defines a soft topology on X. Also, (X, τ, E) is soft b-T_1-space, but it is not a soft b-T_2-space, for a, b $\in$ X and a $\neq$ b, but there is no soft b-open sets F_A and G_A such that a $\in F_A$, b $\in G_A$ and $F_A \tilde{\cap} G_A = \tilde{\phi}$

2) Let X = {a, b}, E = {e_1, e_2} and τ = {$\tilde{X}$, $\tilde{\phi}$, (F_{A_1})} where $(F_1, E)$is soft set over X defined as follows by $F_1(e_1)$ = X, $F_1(e_2)$ ={b}.

Then τ defines a soft topology on X. Also (X, τ, E) is soft b-T_o-space but not a soft b-T_1-space, since a, b $\in$ X, a $\neq$ b, but all the soft b-open sets which contains a also contains b.

Theorem 3.1.9

A soft subspace (Y, τ_Y, E) of a soft b-T_2-space (X, τ, E) is soft b-T_2.

Proof.

Let x, y $\in$ Y such that x $\neq$ y. Then x, y $\in$ X such that x $\neq$ y. Hence, there exist soft b-open sets F_A and G_A in X such that x $\in F_A$, y $\in G_A$ and $F_A \tilde{\cap} G_A = \tilde{\phi}$. It follows that, x $\in$ F(e), y $\in$ G(e) and F(e) $\cap$ G(e) = $\phi$for all e $\in$ E. This implies that, x $\in$ Y $\cap$ F(e), y $\in$ Y $\cap$ G(e) and F(e) $\cap$ G(e) = ϕ for all e $\in$ E.

Thus, x $\in \tilde{Y} \tilde{\cap} F_A = (F_Y, E)$, y $\in \tilde{Y} \tilde{\cap} G_A = (G_Y, E)$ and $(F_Y, E) \tilde{\cap} (G_Y, E) = \phi$, where (F_Y, E), (G_Y, E) are soft b-open sets in Y. Therefore, (Y, τ_Y, E) is soft b-T_2-space.

Definition 3.1.10 (El-Sheikh et al., 2015)

Let (X, τ, E) be a soft topological space, (G, E) be a soft b-closed set in X and x $\in$ X such that x $\notin$(G, E). If there exist soft b-open sets $(F_1, E)$and (F_2, E)such that x $\in(F_1, E)$, (G, E) $\tilde{\subseteq}$ (F_2, E) and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}$, then (X, τ, E) is called a **soft b-regular space**. A soft b-regular T_1-space is called a soft b-T_3-space.

Theorem 3.1.11

Let (X, τ, E) be a soft topological space, (G, E) be asoft b-closed set in X and x $\in$ X such that x $\notin$ (G, E). If (X, τ, E) is soft b-regular space, then there exists a soft b-open set (F, E) such that x $\in$ (F, E) and (F, E) $\tilde{\cap}$(G, E) = $\tilde{\phi}$.

Proof.

Obvious from definition 3.1.10

3.2. Soft b- Seperation Axioms in Soft Bitopological Spaces

Definition 3.2.1. Let (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) be a soft bitopological Space. Then a soft set F_A is called **soft** $\tilde{\tau}_{1,2}$ **- open** if $F_A = F_B \tilde{\cup} F_C$, where $F_B \in \tilde{\tau}_1$ and $F_C \in \tilde{\tau}_2$. The soft complement of $\tilde{\tau}_{1,2}$ - open set is called **soft** $\tilde{\tau}_{1,2}$ **- closed** .

Definition 3.2.2. Let X be a soft bitopological space .Then a soft set F_A is called **(1,2)*- soft-b-open set** (briefly (1,2)*-sb-open) if $F_A \tilde{\subseteq} \tilde{\tau}_{1,2}$-int ($\tilde{\tau}_{1,2}$-cl(F_A)) $\tilde{\cup}$ $\tilde{\tau}_{1,2}$-cl($\tilde{\tau}_{1,2}$-int (F_A)).

Definition 3.2.3. Let X be a soft bitopological space and a soft set F_A over X .

(1). **(1, 2)* -soft b- closure** (briefly (1,2)*-sbcl(F_A)) of a set F_A in *X* is defined by (1, 2)* - sbcl (F_A) = $\tilde{\bigcap}$ { $F_E \tilde{\supseteq} F_A$: F_E is a (1, 2)* - soft b- closed set in X }.

(2). **(1, 2)*- soft b- interior** (briefly (1,2)*-sbint(F_A)) of a set F_A in *X* is defined by (1, 2)*- sbint (F_A) = $\tilde{\bigcap}$ { $F_B \tilde{\subseteq} F_A$: F_B is a (1, 2)* -soft b- open set in *X* }.

Definition 3.2.4. A soft mapping $\tilde{f}$: (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) $\rightarrow$ (Y , $\tilde{\sigma}_1$, $\tilde{\sigma}_2$,E) is said to be **(1, 2)* -soft b- continuous** (briefly (1,2)*-sb-continuous) if the inverse image of each $\tilde{\sigma}_{1,2}$ -open set of Y is (1, 2)* -sb-open set in X .

Definition 3.2.5. A soft mapping $\tilde{f}$: (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) → (Y , $\tilde{\sigma}_1$, $\tilde{\sigma}_2$,E) is said to be **(1, 2)* -soft b- irresolute** (briefly (1,2)*-sb-irresolute) if $\tilde{f}^{-1}$(F_A) is a (1, 2)* -sb-closed set in X , for every (1, 2)* -sb-closed set F_A in Y .

Definition 3.2.6. Let (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) be a soft bitopological Space over X and F_E ∈ S(X) . $x_e \in \tilde{X}$ is said to be a **(1,2)*- soft-b-limit point** ((1,2)* -sb-limit point) of F_E if every (1,2)*- soft b – neighbourhood containing x_e contains a soft point of F_E other than x_e .

Definition 3.2.7. The collection of all(1,2)* - soft b –limit points of F_E is called the **(1,2)* - soft b – derived set** of F_E and is denoted by (1,2)* -sbD (F_E).

Definition 3.2.8. A soft mapping $\tilde{f}$: (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) → (Y, $\tilde{\sigma}_1$, $\tilde{\sigma}_2$,E) is said to be **(1, 2)* -soft b- open map** (briefly (1,2)*-sb-open) if the image of every $\tilde{\tau}_{1,2}$ - open setof $\tilde{X}$ is (1, 2)* -sb-open set in $\tilde{Y}$.

Definition 3.2.9. Let (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) be a soft bitopological Space over X and for every soft points x_e , y_e and for every soft points x_e , y_e ∈ X with $x_e \neq y_e$. Then the soft bitopological space (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) is said to be **((1,2)* -sbT$_0$ - space)** if there exists (1,2)* - soft b-open sets F_{E_1} and F_{E_2} such that either $x_e \in F_{E_1}$ but $y_e \tilde{\notin} F_{E_2}$ or $y_e \in F_{E_2}$ but $x_e \tilde{\notin} F_{E_2}$.

Example 3.2.10. Let X={ x, y }, E={ e_1 ,e_2 }, X = { (e_1 , { x, y }) , (e_2 , { x, y })} .The possible soft subsets are considered as in example 1.2. Define $\tilde{\tau}_1$ = { X , ϕ , F_{E_1} , F_{E_7} } and $\tilde{\tau}_2$ = { X , ϕ , F_{E_3} } .Then $\tilde{\tau}_{1,2}$ –open sets are (X , F_{E_1} , F_{E_3} , F_{E_7} , $F_{E_{13}}$ } and the collection of all (1,2)*- soft– b-open set is (1,2)*-sbO(X) ={ X , ϕ , F_{E_1} , F_{E_3} , F_{E_7} , F_{E_8} , F_{E_9} , $F_{E_{13}}$, $F_{E_{14}}$ }.Then (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) is a (1,2)* - soft bT$_0$ - space over X .

Remark 3.2.11. Every (1,2)*- soft bT_0 – space is soft bitopological space. But the following example shows that every soft bitopological space need not be (1,2)*- soft bT_0 – space.

Example 3.2.12. Consider the soft indiscrete bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ over X. The only (1,2)* - soft b-open sets are ϕ and $\tilde{X}$ .Now, the (1,2)*- soft b-open set X contains x_e but it also contains y_e . Thus there is no (1,2)*- soft b-open set which contains x_e but does not contain y_e Hence it is not a (1,2)* - soft bT_0 – space .

Theorem 3.2.13. Let $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ be a soft bitopological spaces over X and $x_e, y_e \tilde{\in}$ X with $x_e \neq y_e$, then there exists (1,2)* - soft b-open sets F_{E_1} and F_{E_2} such that either $x_e \in F_{E_1}$ and $y_e \in {F_{E_1}}^C$ or $y_e \in F_{E_2}$ and $x_e \in {F_{E_2}}^C$.Then, the soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a (1,2)*-soft bT_0 - space.

Proof. Let $x_e, y_e \in X$ with $x_e \neq y_e$ and let F_{E_1} and F_{E_2} be (1,2)* - soft b- open sets such that either $x_e \in F_{E_1}$ and $y_e \in {F_{E_1}}^{\tilde{C}}$ or $y_e \in F_{E_2}$ and $x_e \in F^{\tilde{C}}{}_{E_2}$. If $x_e \in F_{E_1}$ and $y_e \in F^{\tilde{C}}{}_{E_1}$, then $y_e \in (F(e))^{\tilde{C}}$ for all $e \in E$. Therefore $y_e \notin F_{E_1}$. Similarly ,if $y_e \in F_{E_2}$ and $x_e \in F^{\tilde{C}}{}_{E_2}$ then $x_e \notin F_{E_2}$ .Hence $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a (1,2)* - soft bT_0 - space.

Theorem 3.2.14 . A soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is (1,2)* -Soft bT_0 - space over X if and only if (1,2)*-sbcl $\{x_e\} \neq$ (1,2)*-sbcl $\{y_e\}$ for every pair of distinct soft point x_e, y_e of X .

Proof. Let $x_e, y_e \in X$ with $x_e \neq y_e$.Since X is (1,2)*- soft bT_0 - space, then there exists (1,2)*-soft b-open sets F_E and G_E such that either $x_e \in F_E$ but $y_e \notin F_E$ or $y_e \in G_E$ but $x_e \notin G_E$. Since $X \setminus F_E$ is a (1,2)*- soft b-closed set which does not contain x_e but y_e .By definition ,(1,2)*-sbcl(y_e) is the intersection of all (1,2)*-soft b-closed set containing y_e .Therefore (1,2)*-sbcl(y_e) $\tilde{\subseteq} X \setminus F_E$. Hence $x_e \notin X \setminus F_E$ implies that $x_e \notin$ (1,2)*-sbcl(y_e). Thus $x_e \in$ (1,2)*-sbcl(x_e) but $x_e \notin$ (1,2)*-sbcl (y_e) . Hence (1,2)* -sbcl {} $\neq$ (1,2)*-sbcl{ y_e }.

Conversely assume that $x_e, y_e \in X$ with $x_e \neq y_e$ and (1,2)*-sbcl { x_e } $\neq$ (1,2)*-sbcl { y_e }.Then by assumption ,there exists atleast one soft points $z_e \in X$ such that $z_e \in$ (1,2)*-sbcl ({ x_e }) but $z_e \notin$ (1,2)*-sbcl({ y_e }).Now we claim that $x_e \in$ (1,2)*-sbcl ({ y_e }).suppose not, $x_e \in$ (1,2)*-sbcl({ y_e }) then{ x_e } $\tilde{\subset}$ (1,2)*-sbcl({ y_e }) which implies that (1,2)* - sbcl ({ x_e }) $\tilde{\subset}$ (1,2)*- sbcl({ y_e }). Hence $z_e \in$ (1,2)*-sbcl({ x_e }) implies $z_e \notin$ (1,2)*-sbcl({ y_e }). This contradicts the fact that $z_e \notin$ (1,2)*-sbcl ({ y_e }).Therefore $x_e \notin$ (1,2)*-sbcl({ y_e }).Now $x_e \in$ [(1,2)*-sbcl({ y_e })] $\tilde{c}$ is a (1,2)* -soft b-open set containing x_e but not y_e.Hence (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) is (1,2)* -Soft bT_0 -space over X .

Theorem 3.2.15. A soft subspace of a (1,2)* -soft bT_0 -space is (1,2)*-soft bT_0-space.

Proof. Let $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ be a (1,2)* -soft bT_0-space over X and $(Y, \tilde{\tau}_{1y}, \tilde{\tau}_{2y}, E)$ be soft subspace of $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ over Y . Let $x_e, y_e \in Y$ such that $x_e \neq y_e$ and since $Y \tilde{\subseteq} X, x_e, y_e, \tilde{\in} \tilde{X}$. Since $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is (1,2)* -soft bT_0-space over X, there exists (1,2)* -soft b-open sets F_{E_1} and F_{E_2} such that either $x_e \in F_{E_1}$ but $y_e \notin F_{E_1}$ or $y_e \in F_{E_2}$ but $x_e \notin F_{E_2}$. Now $x_e \in Y \tilde{\cap} F_{E_1} = {}^Y F_{E_1}$ which is a (1,2)* -soft b-open set in $(Y, \tilde{\tau}_{1y}, \tilde{\tau}_{2y}, E)$. Consider $y_e \notin F_{E_1}$, this implies that $y_e \notin F(e)$ for some $e \in E$. Therefore $y_e \notin Y \tilde{\cap} F_{E_1} = {}^Y F_{E_1}$ .Similarly if $y_e \in F_{E_2}$ and $x_e \notin F_{E_2}$, then $y_e \in {}^Y F_{E2}$ and $x_e \notin {}^Y F_{E_2}$ .Thus $(Y, \tilde{\tau}_{1y}, \tilde{\tau}_{2y}, E)$ is also a (1,2)* -soft bT_0-space.

Theorem 3.2.16 . Let $\tilde{f} : (X, \tilde{\tau}_1, \tilde{\tau}_2, E) \to (Y, \sigma_1, \sigma_2, E)$ a bijective (1,2)* -soft b-open mapping and if X is a (1,2)* -soft T_0-space, then Y is a (1,2)* -soft bT_0-space.

Proof . Let y_{e_1}, y_{e_2} be two distinct soft points of Y. Since $\tilde{f}$ is bijective, there exists $x_{e_1}, x_{e_2} \in X$ such that $\tilde{f}(x_{e_1}) = y_{e_1}$ and $\tilde{f}(x_{e_2}) = y_{e_2}$. Since X is (1,2)* -soft T_0-space, then there exists $\tilde{\tau}_{1,2}$-open sets G_{E_1} and G_{E_2} of X such that $x_{e_1} \in G_{E1}$ but $x_{e_2} \notin G_{E_1}$ or $x_{e_2} \in G_{E_2}$ but $x_{e_1} \notin G_{E2}$. But $\tilde{f}$ is a (1,2)* -soft b-open mapping, then $\tilde{f}(F_{A_1}), \tilde{f}(F_{A_2})$ are (1,2)* -soft b-open sets in Y with $y_{e_1} \in \tilde{f}(F_{A_1})$ but $y_{e_2} \notin \tilde{f}(F_{A_1})$ or $y_{e_2} \in \tilde{f}(F_{A_2})$ but $y_{e_1} \notin \tilde{f}(F_{A_2})$. Therefore Y is a (1,2)* -soft bT_0-space.

Theorem 3.2.17. Let $\tilde{f} : (X, \tilde{\tau}_1, \tilde{\tau}_2, E) \rightarrow (Y, \sigma_1, \sigma_2, E)$ a injective (1,2)* -soft b-irresolute mapping and if Y is a (1,2)* -soft bT_0-space, then X is a (1,2)* -soft bT_0-space.

Proof. Let $x_e, y_e \in X$ with $x_e \neq y_e$. Since $\tilde{f}$ is injective and Y is (1,2)* -soft bT_0-space, then there exists (1,2)* -soft b-open sets F_{E_1} and F_{E_2} such that either $f(x_e) \in F_{E_1}$ but $f(y_e) \notin F_{E_1}$ or $f(y_e) \in F_{E_2}$ but $f(x_e) \notin F_{E_2}$ with $f(x_e) \neq f(y_e)$. Since $\tilde{f}$ is (1,2)* -soft b- irresolute mapping, $f^{-1}(F_{E_1})$ and $f^{-1}(F_{E_2})$ are in (1,2)* -soft b-open sets in X such that $x_e \in f^{-1}(F_{E_1})$ but $y_e \notin f^{-1}(F_{E_1})$ or $y_e \in f^{-1}(F_{E_2})$ but $x_e \notin f^{-1}(F_{E_2})$. Thus X is a (1,2)* -soft bT_0-space.

Definition 3.2.18. Let $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ be a soft bitopological space over X and for every soft points $x_e, y_e \in X$ with $x_e \neq y_e$. Then the soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is said to be (1,2)* -soft bT_1-space (**(1,2)* -s**bT_1**-space**) if there exists (1,2)* -soft b-open sets F_{E_1} and F_{E_2} such that either $x_e \in F_{E_1}$ but $y_e \notin F_{E_1}$ and $y_e \in F_{E_2}$ but $x_e \notin F_{E_2}$.

Example 3.2.19.

Let X = {x, y, z}, E = $\{e_1\}$ the soft subsets of X is $SS_E(X)$ and |S(X)|=8.They are G_{E_1}= {(e1 ,{x})}, G_{E_2}={(e1 ,{y})},G_{E_3}={(e1,{z})},G_{E_4}={(e1,{x, y})},G_{E_5}={(e1,{x, z})},G_{E_5}={(e1,{y, z})} Define $\tilde{\tau}_1 = \{X, \phi, G_{E_4}\}$ and $\tilde{\tau}_2 = \{X, \phi, G_{E_6}\}$ Then $\tilde{\tau}_{1,2}$-open sets are $\{X, \phi, G_{E_4}, G_{E_6}\}$.Then $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft bitopological space.The collection of (1,2)* -soft b-open sets are (1,2)*-SbO(X) = $\{X, \phi, G_{E_2}, G_{E_4}, G_{E_5}, G_{E_6}\}$ and (1,2)* -soft b-closed sets are (1,2)* -SbC(X) = $\{X, \phi, G_{E_5}, G_{E_3}, G_{E_2}, G_{E_1}\}$. Then this soft bitopological space is (1,2)* -soft bT_1-space .

Theorem 3.2.20. Let $(X, \tilde{\tau}_1, \tilde{\tau}_{2,} E)$ be a soft bitopological space over X and $x_{e,} y_e \in X$ such that $x_e \neq y_e$. If there exists (1,2)* -soft b-open sets F_{E_1} and F_{E_2} such that $x_e \in F_{E_1}$ but $y_e \in F_{E_1}^C$ and $y_e \in F_{E_2}$ but $x_e \in F_{E_2}^C$ .Then, the soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_{2,} E)$ is a (1,2)* -soft bT_1-space.

Proof. It is similar to the proof of proposition 3.2.13

The following theorem is a characterization for (1, 2)* -soft bT_1-space.

Theorem 3.2.21. Let $(X, \tilde{\tau}_1, \tilde{\tau}_{2,} E)$ be a $(1,2)^*$ -soft bT_1-space over X if and only if for each $x_{e,} \tilde{\in} \tilde{X}$, every soft singleton{ x_e } over X is $(1,2)^*$ -soft b-closed set.

Proof. Suppose that $(X, \tilde{\tau}_1, \tilde{\tau}_{2,} E)$ is a $(1,2)^*$ -soft bT_1-space over X and $x_{e,} \in X$. Now we have to prove that the soft singleton set { x_e } over X is $(1,2)^*$ -soft b-closed . Suppose { x_e } is not $(1,2)^*$ -soft b-closed . Then $(1,2)^*$ -sbcl({ x_e }) $\neq$ { x_e }. So there exists $y_e \neq x_e$, $y_e \in (1,2)^*$-sbcl({ x_e }). This contradicts the fact that $(X, \tilde{\tau}_1, \tilde{\tau}_{2,} E)$ be a $(1,2)^*$-soft bT_1-space. Therefore, soft singleton{ x_e } over X is $(1,2)^*$ -soft b-closed set.

Conversely, suppose the soft singleton { x_e } is $(1,2)^*$-soft b-closed for every $x_{e,} \in X$ .Since { x_e } is $(1,2)^*$-soft b-closed, $\{x_e\}^c$ is $(1,2)^*$-soft b-open set in X. Let $x_{e,} y_e \in X$ and $x_e \neq y_e$ such that { x_e } and{ y_e } are $(1,2)^*$-soft b-closed sets, then

$\{x_e\}^c$ and $\{y_e\}^c$ are $(1,2)^*$-soft b-open sets. Therefore $y_e \in \{x_e\}^c$ but $x_e \notin \{x_e\}^c$ and $x_e \in \{y_e\}^c$ but $y_e \notin \{y_e\}^c$.Thus $(X, \tilde{\tau}_1, \tilde{\tau}_{2,} E)$ is a $(1,2)^*$-soft bT_1-space over X.

Theorem 3.2.22. A soft subspace of a $(1,2)^*$-soft bT_1-space is $(1,2)^*$-soft bT_1-space.

Proof . Let $(X, \tilde{\tau}_1, \tilde{\tau}_{2,} E)$ be a $(1,2)^*$-soft bT_1-space over X and $(Y, \tilde{\tau}_1\tilde{y}, \tilde{\tau}_2\tilde{y}, E)$ be soft subspace of $(X, \tilde{\tau}_1, \tilde{\tau}_{2,} E)$ over Y. Let $x_{e,} y_e \in Y$ such that $x_e \neq y_e$. Since $Y \tilde{\subseteq} X$, $x_{e,} y_e \in X$ and $x_e \neq y_e$. Since $(X, \tilde{\tau}_1, \tilde{\tau}_{2,} E)$ is a $(1,2)^*$-soft bT_1-space over X, there exists $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} in X such that $x_e \in F_{E_1}$ but $y_e \notin F_{E_1}$ and $y_e \tilde{\in} F_{E_2}$ but $x_e \tilde{\notin} F_{E_2}$. Hence $x_e \in Y \tilde{\cap} F_{E_1} = {}^Y F_{E_1}$ which is a $(1,2)^*$-soft b-open set in $(Y, \tilde{\tau}_1\tilde{y}, \tilde{\tau}_2\tilde{y}, E)$. Since $y_e \notin F_{E_1}$, $y_e \notin Y \tilde{\cap} F_{E_1} = {}^Y F_{E_1}$. Similarly if $y_e \in F_{E_2}$ and $x_e \notin F_{E_2}$, then $y_e \in {}^Y F_{E_2}$ but $x_e \notin {}^Y F_{E_2}$. Thus $(Y, \tilde{\tau}_1\tilde{y}, \tilde{\tau}_2\tilde{y}, E)$ is also a $(1,2)^*$-soft bT_1-space.

Theorem 3.2.23. Every $(1,2)^*$-soft bT_1- space is $(1,2)^*$-soft bT_0- space.

Proof. Let $(X, \tilde{\tau}_1, \tilde{\tau}_{2,} E)$ be a $(1,2)^*$-soft bT_1- space. Then for every $x_{e,} y_e \in X$ with $x_e \neq y_e$, there exists $(1,2)^*$-soft b-open sets F_{E1} and F_{E2} such that $x_e \in F_{E1}$ but $y_e \notin F_{E1}$ and $y_e \in F_{E_2}$ but $x_e \notin F_{E2}$. Therefore $(X, \tilde{\tau}_1, \tilde{\tau}_{2,} E)$ is a $(1,2)^*$-soft bT_0- space.

The converse of the above preposition need not to be true.

Example 3.2.24. Let X = {x,y}, E = $\{e_1, e_2\}$, X = $\{(e_1,\{x,y\}),(e_2,\{x,y\})\}$. The possible soft subsets are considered as in Example 1.4 .

Define $\tilde{\tau}_1 = \{ X, \phi, F_{E_1}, F_{E_7} \}$ and $\tilde{\tau}_2 = \{ X, \phi, F_{E_3} \}$ .Then $\tilde{\tau}_{1,2}$ –open sets are$\{X, F_{E_1}, F_{E_3}, F_{E_7}, F_{E_{13}}\}$ and the collection of all (1,2)* - soft –b – open set are (1,2)* - sbO(X) = $\{X, \phi, F_{E_1}, F_{E_3}, F_{E_7}, F_{E_8}, F_{E_9}, F_{E_{13}}, F_{E_{14}}\}$.Then $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a (1,2)* - soft bT_0 -

space over X but not (1,2)* - soft –bT_1 –space over X. Since the soft singleton set F_{E_1} is not (1,2)* - soft b-closed set.

Theorem 3.2.25. If every finite soft subset of a soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is (1,2)* -soft b-closed set, then $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a (1,2)* - soft bT_1 - space.

Proof. Let $x_e, y_e \in X$ with $x_e \neq y_e$ .then by hypothesis ,{ x_e }, { y_e } are (1,2)* - soft b-closed sets, which implies that $\{x_e\}^c$ and $\{y_e\}^c$ are (1,2)*-soft b-open sets such that $x_e \in \{y_e\}^c$ and $y_e \in \{x_e\}^c$. Therefore $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a (1,2)* - soft bT_1 – space.

Theorem 3.2.26. Let $\tilde{f} : (X, \tilde{\tau}_1, \tilde{\tau}_2, E) \to (Y, \sigma_1, \sigma_2, E)$ a bijective (1,2)* -soft b- open mapping and if X is a (1,2)* -soft bT_1-space, then Y is a (1,2)* -soft bT_1-space.

Proof . Let y_{e_1}, y_{e_2} be two distinct soft points of Y . Since $\tilde{f}$ is bijective ,there exits $x_{e_1}, x_{e_2} \in X$ such that $\tilde{f}(x_{e_1}) = y_{e_1}$ and $\tilde{f}(x_{e_2}) = y_{e_2}$. Since X is (1,2)* - soft T_1-space, then there exists $\tilde{\tau}_{1,2}$-open sets G_{E_1} and G_{E_2} of X such that $x_{e_1} \in G_{E_1}$ but $x_{e_2} \in G_{E_1}$ and $x_{e_1} \notin G_{E_1}$ and $x_{e_2} \in G_{E_2}$ But $\tilde{f}$ is a (1,2)* -soft b-open mapping, then $\tilde{f}(F_{A_1}), \tilde{f}(F_{A_2})$ are (1,2)*-soft b-open sets in Y with $y_{e_1} \in \tilde{f}(F_{A_1})$ but $y_{e_2} \notin \tilde{f}(F_{A_1})$ and $y_{e_2} \in \tilde{f}(F_{A_2})$ but $y_{e_1} \notin \tilde{f}(F_{A_2})$. Therefore Y is a (1,2)* -soft bT_1-space.

Theorem 3.2.27. Let $\tilde{f} : (X, \tilde{\tau}_1, \tilde{\tau}_2, E) \to (Y, \sigma_1, \sigma_2, E)$ a injective (1,2)* -soft b- irresolute mapping and if, Y is a (1,2)* -soft bT_1-space, then X is a (1,2)* -soft bT_1-space.

Proof . The proof of the theorem is similar to the Theorem 3.2.17.

Definition 3.2.28. Let $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ be a soft bitopological Space over X and for every soft points $x_e, y_e \in X$ with $x_e \neq y_e$ .Then the soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is said to be (1,2)*- soft bT_2 – space **((1,2)*-sbT_2 – space)** or (1,2)*-

soft b – Hausdorff space if there exists (1,2)*- soft b-open sets F_{E_1} and F_{E_2} such that $x_e \in F_{E_1}$, $y_e \notin F_{E_2}$ and $F_{E_1} \tilde{\bigcap} F_{E_2} = \phi$.

Example 3.2.29. Consider a (1,2)* - soft discrete bitopological space (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E). Let x_e , y_e be two distinct soft points of X . And { x_e }, { y_e } are (1,2)* - soft b-open sets of x_e and y_e respectively such that { x_e } $\tilde{\bigcap}$ { y_e } = ϕ . Hence (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) is a (1,2)* -soft bT_2-space or (1,2)* -soft b-Hausdorff space.

Theorem 3.2.30 . A soft subspace of a $(1,2)^*$- soft bT_2-space is $(1,2)^*$-soft bT_2-space.

Proof . Let $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ be a $(1,2)^*$-soft bT_2-space over X and $(Y, \tilde{\tau}_1\tilde{y}, \tilde{\tau}_2\tilde{y}, E)$ be soft subspace of $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ over Y. Let $x_e, y_e \in Y$ such that $x_e \neq y_e$. Then x_e , y_e ∈ X and $x_e \neq y_e$. Since $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a $(1,2)^*$-soft bT_2-space over X, there exists $(1,2)^*$-soft b-open sets F_{E_1} and F_{E_2} in X such that $x_e \in F_{E_1}$ and $y_e \in F_{E_2}$ and $F_{E_1} \tilde{\bigcap} F_{E_2} = \phi$. It follows that $x_e \in F_{E_1}$ (e) , $y_e \in F_{E_2}$ (e) and F_{E_1}(e) $\tilde{\bigcap}$ F_{E_2}(e) = ϕ for all e $\in$ E Thus $x_e \in Y \tilde{\cap} F_{E_1} = {}^{Y}F_{E_1}$, $y_e \in Y \tilde{\cap} F_{E_2} = {}^{Y}F_{E_2}$ and ${}^{Y}F_{E_1} \tilde{\bigcap} {}^{Y}F_{E_2} = \phi$ where, ${}^{Y}F_{E_1}$, ${}^{Y}F_{E_2}$ are (1,2)* - soft b-open sets in Y .Therefore $(Y, \tilde{\tau}_1\tilde{y}, \tilde{\tau}_2\tilde{y}, E)$ is also a $(1,2)^*$-soft bT_2-space.

The Characterization for (1,2)* - soft b –hausdorff space is following.

Theorem 3.2.31 . A soft bitopological space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ be a $(1,2)^*$-soft bT_2-space over X if and only if for distinct points x_e, y_e of X ,there exists a (1,2)* - soft b-open set F_A containing x_e but not y_e such that $y_e \notin$ (1,2)*-sbcl (F_A).

Proof. Let x_e and y_e be two distinct points in (1,2)*- soft bT_2-space $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$. Then there exists disjoint (1,2)*- soft b-open sets F_A and G_B such that $x_e \in F_A$ and

$y_e \in G_B$.this implies that $x_e \in G_B^{\tilde{C}}$. so $G_B^{\tilde{C}} = F_A$ is a (1,2)*-soft b-closed containing x_e but not y_e and (1,2)*-sbcl(F_A) = F_A. Hence $y_e \notin$ (1,2)*-sbcl(F_A).

On the other hand , let x_e and y_e be two distinct soft points (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E). Then there exists a (1,2)* - soft b-open set F_A containing but not y_e such that $y_e \notin$ (1,2)*-sbcl (F_A). This implies the $y_e \in \left[(1,2)^* - sbcl(\{F_A\})\right]^{\tilde{C}}$. Hence F_A and $\left[(1,2)^* - sbcl(\{F_A\})\right]^{\tilde{C}}$ are two disjoint (1,2)*-soft b-open sets containing x_e and y_e respectively. Thus (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) is a $(1,2)^*$-soft bT_2-space over X.

Theorem 3.2.32. Let (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) be a $(1,2)^*$-soft bT_2-space over X and $x_e \tilde{\in} X$. Then every soft singleton { x_e } is (1,2)* - soft b-closed.

Proof . Let (X, $\tilde{\tau}_1$, $\tilde{\tau}_2$, E) be a $(1,2)^*$-soft bT_2-space over X **.** Let $x_e, y_e \in$ X and $x_e \neq y_e$,then there exists (1,2)*- soft b-open sets F_{E_1} and F_{E_2} such that $x_e \in F_{E_1}$, $y_e \in F_{E_2}$ and $F_{E_1} \tilde{\cap} F_{E_2} = \phi$. Since F_{E_2} is a (1,2)* - soft b –open set containing y_e such that F_{E_2} does not contain x_e or F_{E_2} does not contain any other soft point of{ x_e }. Hence a soft point y_e of X distinct from x_e cannot be a (1,2)* - soft b-limit point of { x_e }.Hence (1,2)* - soft b-derived set of x_e is (1,2)*-sbD { x_e } = ϕ and since (1,2)* - sbcl { x_e } = { x_e } $\tilde{\cup}$ ϕ = { x_e } . Hence { x_e } is (1,2)*- soft b-closed.

Theorem 3.2.33 . Every $(1,2)^*$-soft bT_2-space is (1,2)* -soft bT_1-space.

Proof . Let (X, $\tilde{\tau}_1$, $\tilde{\tau}_2$, E) be a $(1,2)^*$-soft bT_2-space. Then , for every $x_e, y_e \in$ X and $x_e \neq y_e$,then there exists (1,2)*- soft b-open sets F_{E_1} and F_{E_2} of x_e and y_e such that $F_{E_1} \tilde{\cap} F_{E_2} = \phi$. $x_e \tilde{\in} F_{E_1} \Rightarrow x_e \notin F_{E_1}$ as $F_{E_1} \tilde{\cap} F_{E_2} = \phi$.Similarly, $y_e \in F_{E_2}$ .This implies that $y_e \notin F_{E_1}$ Hence, $x_e \in F_{E_1}$ butt $y_e \notin F_{E_1}$ and $y_e \in F_{E_2}$ but $x_e \notin F_{E_2}$ .Therefore, the soft bitopological space (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) is a (1,2)* - soft bT_1 – space.

The converse of the above proposition is not true is shown in the following example.

Example 3.2.34 . Let X = { *x* ,*y*, *z* } , E = { e_1 } the soft subsets of X is given as in the example 3.2.29. Define $\tilde{\tau}_1 = \{X, \phi, G_{E_4}\}$ and $\tilde{\tau}_2 = \{X, \phi, G_{E_6}\}$. Then $\tilde{\tau}_{1,2}$ -open sets are $\{X, \phi, G_{E_4}, G_{E_6}\}$. Then $(X, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a soft bitopological space. The collection of (1,2)* -soft b-open sets are (1,2)* -SbO(X) =$\{X, \phi, G_{E_2}, G_{E_4}, G_{E_5}, G_{E_6}\}$ and (1,2)*-soft b-closed sets are (1,2)*-SbC(X) = $\{X, \phi, G_{E_5}, G_{E_3}, G_{E_2}, G_{E_1}\}$. Then this soft bitopological space is (1,2)* -soft bT_1 -space . Since every soft singleton set is (1,2)* -soft b-closed set.

Consider the soft points (e_1 , {x}),(e_1 , {z}) $\in$ X and (e_1 , {x}) $\neq$ (e_1 , {y}) ; there does not exists disjoint (1,2)*- soft b-open sets. Then (X , $\tilde{\tau}_1$, $\tilde{\tau}_2$,E) is not a (1,2)*-soft bT_2 – space.

Theorem 3.2.35 . Let $\tilde{f} :(X, \tilde{\tau}_1, \tilde{\tau}_2, E) \to (Y, \tilde{\sigma}_1, \tilde{\sigma}_2, E)$ be a bijective (1,2)* - soft b-open mapping and if $\tilde{X}$ is a (1,2)* - soft bT_2 –space , then Y is a (1,2)* - soft bT_2 –space .

Theorem 3.2.36 . Let $\tilde{f} :(X, \tilde{\tau}_1, \tilde{\tau}_2, E) \to (Y, \tilde{\sigma}_1, \tilde{\sigma}_2, E)$ is a injective (1,2)* - soft b-irresolute mapping and if is a (1,2)* - soft bT_2 –space ,then X is a (1,2)* - soft -bT_2 – space.

CHAPTER - IV

Soft αW − Hausdorff Axiom in Soft Bitopological Spaces

4.1. Soft αW − Hausdorffness in Soft Topological Spaces

Definition 4.1.1.

A soft topological space (X, τ, E) is said to be **Soft W-Hausdorff space of type 1** denoted by $(SW - H)_1$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$, there exists $F_A, G_B \in \tau$ such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Definition 4.1.2.

A soft topological space (X, τ, E) is said to be **Soft W-Hausdorff space of type 2** denoted by $(SW - H)_2$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$, there exists $F_E, G_E \in \tau$ such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$

Definition 4.1.3.

A soft set F_A is called a **soft α- closed** (soft α-closed) in a soft topological space (X, τ, E) if $Cl(F_A) \subseteq G_B$ when ever $F_A \subseteq G_B$ and G_B is soft open in X .

Definition 4.1.4.

A soft set F_A is called a **Soft α − open** (soft α - open) in a soft topological space (X, τ, E) if the relative complement F_A' is soft α- closed in X .

Equivalently, a soft set F_A is called a **Soft α − open** set (soft α-open) in a soft topological space (X, τ, E) if and only if $F_A \subseteq int(G_B)$ whenever $F_A \subseteq G_B$ and G_B is soft closed in X .

Definition 4.1.5.

A soft topological space (X, τ, E) is said to be **soft αW – Hausdorff space of type 1** denoted by $(s\alpha W - H)_1$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$, there exists soft α-open sets F_A, G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Theorem 4.1.6.

Soft subspace of a $(s\alpha W\text{-}H)_1$ space is $(s\alpha W\text{-}H)_1$.

Proof.

Let (X, τ, E) be a $(s\alpha W\text{-}H)_1$ space. Let Y be a non null subset of X. Let (Y, τ_y, E) be a soft subspace of X. Let (X, τ, E) where $\tau_y = \{(F_Y, E) : (F, E) \in \tau\}$ is the relative soft topology on Y. Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exists soft α - open sets F_A, G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$

Here $(F_A)_Y$, $(G_B)_Y$ are soft α – open sets .

Also $(F_A)_Y(e_1) = Y \cap F_A(e_1)$

$= Y \cap X$

$= Y$

$(G_B)_Y(e_2) = Y \cap G_B(e_2)$

$= Y \cap X$

$= Y$

$((F_A)_Y \cap (G_B)_Y)(e) = ((F_A \cap G_B)_Y)(e)$

$= Y \cap (F_A \cap G_B)(e)$

$= Y \cap \tilde{\phi}(e) = \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence (Y, τ_y, E) is $(s\alpha W - H)_1$

Theorem 4.1.7.

Product of two $(s\alpha W\text{-}H)_1$ spaces is $(s\alpha W\text{-}H)_1$.

Proof.

Let (X, τ_x, E) and (Y, τ_y, K) be two spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E \times K$.

Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since (X, τ_x, E) is $(s\alpha W\text{-}H)_1$, there exist α - soft open sets F_A, G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $F_A \otimes Y_K$, $G_B \otimes Y_K$ are soft α - open sets

$(F_A \otimes Y_K)(e_1, k_1) = F_A(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_B \otimes Y_K)(e_2, k_2) = G_B(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any $(e, k) \in (E \times K)$, $(F_A \otimes Y_K)(e, k) \neq \phi$

$\Rightarrow F_A(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_A(e) \times Y \neq \phi$

$\Rightarrow F_A(e) \neq \phi$

$\Rightarrow G_B(e) = \phi$ (since $F_A \cap G_B = \phi \Rightarrow F_A(e) \cap G_B(e) = \phi$)

$\Rightarrow G_B(e) \times Y_K(k) = \phi$

$\Rightarrow (G_B \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_A \otimes Y_K) \cap (G_B \otimes Y_K) = \tilde{\phi}$

Assume $k_1 \neq k_2$. Since (Y, τ_y, K) is $(s\alpha W\text{-}H)_1$, there exist soft α -open sets F_A, G_B , such that $F_A(k_1) = Y$, $G_B(k_2) = Y$ and $F_A \cap G_B = \tilde{\phi}$.

Here $X_E \otimes F_A$, $X_E \otimes G_B$ are soft α - open sets

$(X_E \otimes F_A)(e_1, k_1) = X_E(e_1) \times F_A(k_1)$

$= X \times Y$

$(X_E \otimes G_B)(e_2, k_2) = X_E(e_2) \times G_B(k_2)$

$= X \times Y$

If for any $(e, k) \in E \times K$, $(X_E \otimes F_A)(e, k) \neq \phi$

$\Rightarrow X_E(e) \times F_A(k) \neq \phi$

$\Rightarrow X \times F_A(k) \neq \phi$

$\Rightarrow F_A(k) \neq \phi$

$\Rightarrow G_B(k) = \phi$ (Since $F_A \cap G_B = \tilde{\phi} \Rightarrow F_A(k) \cap G_B(k) = \phi$)

$\Rightarrow (X_E)(e) \times G_B(k) = \phi$

$\Rightarrow (X_E \otimes G_B)(e, k) = \phi$

$\Rightarrow (X_E \otimes F_A) \cap (X_E \otimes G_B) = \tilde{\phi}$

Hence $(X \times Y, \tau_X \otimes \tau_y, E \times K)$ is $(s\alpha W\text{-}H)_1$.

Definition 4.1.8.

A soft topological space (X, τ, E) is said to be **soft αW-Hausdorff space of type 2** denoted by $(s\alpha W\text{-}H)_2$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$ there exists soft α -open sets F_E , G_E such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$

Theorem 4.1.9.

Soft subspace of a $(s\alpha W\text{-}H)_2$ space is $(s\alpha W\text{-}H)_2$.

Proof.

Let (X, τ, E) be a $(s\alpha W\text{-}H)_2$ space. Let Y be a non-null subset of X. Let (Y, τ_y , E) be a soft subspace of (X, τ, E) where $\tau_y = \{ (F_Y, E) : (F, E) \in \tau\}$ is the relative soft topology on Y. Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist soft α -open sets F_E, G_E such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$.

Here $((F_E)_Y, E)$, $((G_E)_Y, E)$ are soft α-open sets.

Also $(F_E)_Y(e_1) = Y \cap F_E(e_1)$

$= Y \cap X$

$= Y$

$(G_E)_Y(e_2) = Y \cap G_E(e_2)$

$= Y \cap X$

$= Y$

$((F_E)_Y \cap (G_E)_Y)(e) = ((F_E \cap G_E)_Y)(e)$

$= Y \cap (F_E \cap G_E)(e)$

$= Y \cap \tilde{\phi}(e)$

$= Y \cap \phi$

$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence (Y, τ_y, E) is $(s\alpha W\text{-}H)_2$.

Theorem 4.1.10.

Product of two $(s\alpha W\text{-}H)_2$ spaces is $(s\alpha W\text{-}H)_2$.

Proof.

Let (X, τ_X, E) and (Y, τ_y, K) be two $(s\alpha W\text{-}H)_2$ spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E\times K$.

Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since (X, τ_X, E) is $(s\alpha W\text{-}H)_2$, there exist soft α-open sets F_E, G_E such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$.

Here $F_E \otimes Y_K$, $G_E \otimes Y_K$ are soft α-open sets

$(F_E \otimes Y_K)(e_1, k_1) = F_E(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_E \otimes Y_K)(e_2, k_2) = G_E(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any $(e, k) \in (E\times K)$, $(F_E \otimes Y_K)(e, k) \neq \phi$

$\Rightarrow F_E(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_E(e) \times Y \neq \phi$

$\Rightarrow F_E(e) \neq \phi$

$\Rightarrow G_E(e) = \phi$ (Since $F_E \cap G_E = \tilde{\phi} \Rightarrow F_A(e) \cap G_E(e) = \phi$)

$\Rightarrow G_E(e) \times Y_K(k) = \phi$

$\Rightarrow (G_E \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_E \otimes Y_K) \cap (G_E \otimes Y_K) = \tilde{\phi}$

Similarly, one can prove the case when $k_1 \neq k_2$.

Hence $(X \times Y, \tau_X \otimes \tau_y, E \times K)$ is $(s\alpha W\text{-}H)_2$.

Definition 4.1.11.

Let (X, τ, E) be a soft topological space and $H \subseteq E$. Then(X, τ_H, H) is called **Soft p- subspace** of (X, τ, E) relative to the parameter set H where $\tau_H = \{ (F_A) / H : H \subseteq A \subseteq E, F_A \in \tau \}$ and $(F_A) / H$ is the restriction map on H.

Theorem 4.1.12.

Soft p-subspace of a $(s\alpha W\text{-}H)_1$ space is $(s\alpha W\text{-}H)_1$.

Proof .

Let (X, τ, E) be a $(s\alpha W\text{-}H)_1$ space. Let $H \subseteq E$. Let (X, τ_H, E) be a soft p-subspace of (X, τ, E) relative to the parameter set H where $\tau_H = \{(F_A)/H : H \subseteq A \subseteq E, F_A \in \tau\}$.Consider $h_1, h_2 \in E$. Therefore, there exists soft α-open sets F_A, G_B such that $F_A(h_1) = X$, $G_B(h_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $(F_A)/H, (G_B)/H \in \tau_H$.

Also$((F_A)/H)(h_1) = F_A(h_1) = X$

$((G_B)/H)(h_2) = G_B(h_2) = X$ and

$$((F_A)/H) \cap ((G_B)/H) = (F_A \cap G_B)/H = \tilde{\phi}/H = \tilde{\phi}$$

Hence (x, τ_H, H) is $(s\alpha W\text{-}H)_1$.

4.2. Soft $\alpha W -$ Hausdorffness in Soft Bitopological Spaces

Definition 4.2.1.

A soft bitopological space $(X, \tau_{1X}, \tau_{2X}, E)$ is said to be **Soft bi αW-Hausdorff space of type 1** denoted by $(sb\alpha W - H)_1$ if it is $(s\alpha W - H)_1$ with respect to τ_{1X} or $(s\alpha W - H)_1$ with respect to τ_{2X}.

Definition 4.2.2.

A soft bitopological space $(X, \tau_{1X}, \tau_{2X}, E)$ is said to be **Soft bi αW- Hausdorff space of type 2** denoted by $(sb\alpha W - H)_2$ if for every $e_1, e_2 \in E$ $e_1 \neq e_2$, there exist soft α-open sets F_A with respect to τ_{1X} , G_B with respect to τ_{2X} such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Theorem 4.2.3.

Soft subspace of a $(sb\alpha W- H)_1$ space is $(sb\alpha W - H)_1$.

Proof.

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(sb\alpha W - H)_1$ space. Then it is $(s\alpha W\text{-}H)_1$ with respect to τ_{1X} or $(s\alpha W\text{-}H)_1$ with respect to τ_{2X} . Let Y be a non null subset of X.Let $\{Y, \tau_{1Y} , \tau_{2Y} , E\}$ be a soft subspace of $(X, \tau_{1X}, \tau_{2X},E)$. From the theorem 4.1.6, a soft subspace of $(s\alpha W\text{-}H)_1$ space is $(s\alpha W\text{-}H)_1$. Therefore, $\{Y, \tau_{1Y} , \tau_{2Y} , E\}$ is $(s\alpha W\text{-}H)_1$ with respect to τ_{1Y} or $(s\alpha W\text{-}H)_1$ with respect to τ_{2Y}. Hence $\{Y, \tau_{1Y} , \tau_{2Y} , E\}$ is $(sb\alpha W - H)_1$.

Theorem 4.2.4

Soft subspace of a $(sb\alpha W - H)_2$ space is $(sb\alpha W - H)_2$.

Proof.

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(sb\alpha W - H)_2$ space. Let Y be a non null subset of X.
Let $\{Y, \tau_{1Y} , \tau_{2Y} , E\}$ be a soft subspace of $(X, \tau_{1X}, \tau_{2X}, E)$ where
$\tau_{1Y} = \{(F_Y, E) : (F, E) \in \tau_{1X}\}$ and
$\tau_{2Y} = \{(G_Y, E) : (G, E) \in \tau_{2X}\}$ are said to be the relative topologies on Y.
Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist soft α-open sets F_A , G_B such that

$F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$

Here $((F_A)_Y, E) \in \tau_{1Y}$, $((G_B)_Y, E) \in \tau_{2Y}$

Also $(F_A)_Y(e_1) = Y \cap F_A(e_1)$

$= Y \cap X$

$= Y$

$(G_B)_Y(e_2) = Y \cap G_B(e_2)$

$= Y \cap X$

$= Y$

$((F_A)_Y \cap (G_B)_Y)(e) = ((F_A \cap G_B)_Y)(e)$

$= Y \cap (F_A \cap G_B)(e)$

$= Y \cap \tilde{\phi}(e)$

$= Y \cap \phi$

$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence $\{Y, \tau_{1Y}, \tau_{2Y}, E\}$ is $(sb\alpha W - H)_2$.

Definition 4.2.5

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a soft topological space over X and $H \subseteq E$. Then $\{X, \tau_{1H}, \tau_{2H}, H\}$ is called soft p – subspace of $(X, \tau_{1X}, \tau_{2X}, E)$ relative to the parameter set H where

$\tau_{1H} = \{ (F_A)/H : H \subseteq A \subseteq E, F_A \in \tau_{1X} \}$,

$\tau_{2H} = \{ (G_B)/H : H \subseteq B \subseteq E, G_B \in \tau_{2X} \}$ and $(F_A)/H$, $(G_B)/H$ are the restriction maps on H .

Theorem 4.2.6.

Soft p- subspace of a $(sb\alpha W - H)_1$ space is $(sb\alpha W - H)_1$.

Proof.

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(sb\alpha W - H)_1$ space. Then it is $(s\alpha W\text{- }H)_1$ with respect to τ_{1X} or $(s\alpha W\text{- }H)_1$ with respect to τ_{2X}. Let $H \subseteq E$. Let $(X, \tau_{1H}, \tau_{2H}, H)$ be a soft p-subspace of $(X, \tau_{1X}, \tau_{2X}, E)$ relative to the parameter set H. From the theorem 4.1.12, the soft p- subspace of $(s\alpha W\text{- }H)_1$ space is $(s\alpha W\text{- }H)_1$. Therefore , the soft p-subspace of $(sb\alpha W - H)_1$ is $(s\alpha W - H)_1$ with respect to τ_{1H} or with respect to τ_{2H} . Hence $(X, \tau_{1H}, \tau_{2H}, H)$ is $(sb\alpha W - H)_1$.

Theorem 4.2.7.

Soft p –subspace of a $(sb\alpha W - H)_2$ space is $(sb\alpha W - H)_2$.

Proof.

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(sb\alpha W - H)_2$ space. Let $H \subseteq E$. Let $(X, \tau_{1H}, \tau_{2H}, H)$ be a soft p-subspace of $(X, \tau_{1X}, \tau_{2X}, E)$ relative to the parameter set H where

$$\tau_{1H} = \{ (F_A) /H : H \subseteq A \subseteq E , F_A \in \tau_{1X} \},$$

$$\tau_{2H} = \{ (G_B) /H : H \subseteq B \subseteq E , G_B \in \tau_{2X} \}.$$

Consider $h_1 , h_2 \in H$, $h_1 \neq h_2$ .Then $h_1 , h_2 \in E$. Therefore , there exist soft α-open sets F_A , G_B such that $F_A(e_1) = X$,$G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $(F_A) / H \in \tau_{1H}$, $(G_B) / H \in \tau_{2H}$

Also $((F_A) / H) (h_1) = F_A (h_1) = X$

$((G_B) / H) (h_2) = G_B (h_2) = X$ and

$$((F_A) / H) \cap (G_B) / H)) = (F_A \cap G_B) / H$$

$$= \tilde{\phi} / H$$

$$= \tilde{\phi}$$

Hence $(X, \tau_{1H}, \tau_{2H}, H)$ is $(sb\alpha W - H)_2$.

Theorem 4.2.8.

Product of two $(sb\alpha W - H)_1$ spaces is $(sb\alpha W - H)_1$.

Proof.

Let $(X, \tau_{1X}, \tau_{2X}, E)$ and $\{Y, \tau_{1Y} , \tau_{2Y} , K\}$ be two $(sb\alpha W - H)_1$ spaces. Then

$(X, \tau_{1X}, \tau_{2X}, E)$ is $(s\alpha W\text{-}H)_1$ with respect to τ_{1X} or $(s\alpha W\text{-}H)_1$ with respect to τ_{2X} and $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ is $(s\alpha W\text{-}H)_1$ with respect to τ_{1Y} or $(s\alpha W\text{-}H)_1$ with respect to τ_{2Y}. From theorem 4.1.7, the product of two $(s\alpha W\text{-}H)_1$ spaces is $(s\alpha W\text{-}H)_1$. Hence the product of two $(sb\alpha W - H)_1$ spaces is $(sb\alpha W - H)_1$.

Theorem 4.2.9.

Product of two $(sb\alpha W - H)_2$ spaces is $(sb\alpha W - H)_2$.

Proof.

Let $(X, \tau_{1X}, \tau_{2X}, E)$ and $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ be two $(sb\alpha W - H)_2$ spaces.

Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E \times K$. Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since $(X, \tau_{1X}, \tau_{2X}, E)$ is $(sb\alpha W - H)_2$, there exist soft α-open sets $F_A \in \tau_{1X}$, $G_B \in \tau_{2X}$ such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here soft α-open sets are $F_A \otimes Y_K$, $G_B \otimes Y_K$

$$(F_A \otimes Y_K)(e_1, k_1) = F_A(e_1) \times Y_K(k_1)$$
$$= X \times Y$$

$$(G_B \otimes Y_K)(e_2, k_2) = G_B(e_2) \times Y_K(k_2)$$
$$= X \times Y$$

If for any $(e, k) \in (E \times K)$, $(F_A \otimes Y_K)(e,k) \neq \phi$

$\Rightarrow F_A(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_A(e) \times Y \neq \phi$

$\Rightarrow F_A(e) \neq \phi$

$\Rightarrow G_B(e) = \phi$

(since $F_A \cap G_B = \phi \Rightarrow F_A(e) \cap G_B(e) = \phi$)

$\Rightarrow G_B(e) \times Y_K(k) = \phi$

$\Rightarrow (G_B \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_A \otimes Y_K) \cap (G_B \otimes Y_K) = \tilde{\phi}$.

Assume $k_1 \neq k_2$. Since $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ is $(sb\alpha W - H)_2$, there exist soft α-open sets F_A, G_B such that $F_A(k_1) = Y$, $G_B(k_2) = Y$ and $F_A \cap G_B = \tilde{\phi}$

Here soft α- open sets are $X_E \otimes F_A$, $X_E \otimes G_B$.

$(X_E \otimes F_A)(e_1, k_1) = X_E(e_1) \times F_A(k_1)$

$= X \times Y$

$(X_E \otimes G_B)(e_2, k_2) = X_E(e_2) \times G_B(k_2)$

$= X \times Y$

If for any $(e, k) \in E \times K$, $(X_E \otimes F_A)(e,k) \neq \phi$

$\Rightarrow X_E(e) \times F_A(k) \neq \phi$

$\Rightarrow X \times F_A(k) \neq \phi$

$\Rightarrow F_A(k) \neq \phi$

$\Rightarrow G_B(k) = \phi$

(Since$F_A \cap G_B = \phi \Rightarrow F_A(k) \cap G_B(k) = \tilde{\phi}$)

$\Rightarrow X_E(e) \times G_B(k) = \phi$

$\Rightarrow (X_E \otimes G_B)(e, k) = \phi$

$\Rightarrow (X_E \otimes F_A) \cap (X_E \otimes G_B) = \tilde{\phi}$

Hence $(X \times Y, \tau_{1X} \otimes \tau_{1Y}, \tau_{2X} \otimes \tau_{2Y}, E\times K)$ is $(sb\alpha W - H)_2$.

CHAPTER – V

SOFT βW- HAUSDORFF AXIOMS IN SOFT BITOPOLOGICAL SPACES

5.1. Soft βW – Hausdorffness in Soft Topological Spaces

Definition.5.1.1.

A Soft topological space (X, τ, E) is said to be **soft βW- Hausdorff space of type 1** denoted by $(S\beta W - H)_1$ if for every e_1, $e_2 \in E$, $e_1 \neq e_2$ there exists soft β-open sets F_A, G_B such that $F_A(e_1) = X, G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Theorem.5.1.2.

Soft subspace of a $(S\beta W - H)_1$space is $(S\beta W - H)_1$.

Proof.

Let (X, τ, E) be a$(S\beta W - H)_1$ space. Let Y be a non null subset of X. Let (Y, τ_Y, E) be a soft subspace of (X, τ, E) where $\tau_Y = \{(F_Y, E) : (F, E) \in \tau\}$ is the relative soft topology on Y. Consider e_1, $e_2 \in E$, $e_1 \neq e_2$ there exists soft β-open sets F_A ,G_B, such that $F_A(e_1) = X, G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $(F_A)_Y$, $(G_B)_Y$ are soft β-open sets.

$$
\begin{aligned}
\text{Also } (F_A)_Y (e_1) &= Y \cap F_A (e_1)\\
&= Y \cap X\\
&= Y\\
(G_B)_Y (e_2) &= Y \cap G_B (e_2)\\
&= Y \cap X\\
&= Y\\
((F_A)_Y \cap (G_B)_Y) (e) &= ((F_A \cap G_B)_Y)(e)\\
&= Y \cap (F_A \cap G_B) (e)\\
&= Y \cap \tilde{\phi} (e)\\
&= Y \cap \phi\\
&= \phi\\
(F_A)_Y \cap (G_B)_Y &= \tilde{\phi}
\end{aligned}
$$

Hence (Y,τ_Y, E) is $(S\beta W - H)_1$

Theorem.5.1.3.

Product of two $(S\beta W - H)_1$spaces is $(S\beta W - H)_1$.

Proof.

Let (X, τ_X, E) and (Y, τ_Y, K) be two $(S\beta W - H)_1$spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E \times K$.

Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since (X, τ_X, E) is$(S\beta W - H)_1$, there exist soft β-open sets F_A, G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \widetilde{\phi}$.

Here $F_A \otimes Y_K$, $G_B \otimes Y_K$ are soft β-open sets

$(F_A \otimes Y_K)(e_1, k_1) = F_A(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_B \otimes Y_K)(e_2, k_2) = G_B(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any $(e, k) \in (E \times K)$, $(F_A \otimes Y_K)(e, k) \neq \phi$

$\Rightarrow F_A(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_A(e) \times Y \neq \phi$

$\Rightarrow F_A(e) \neq \phi$

$\Rightarrow G_B(e) = \phi$ (since $F_A \cap G_B = \widetilde{\phi} \Rightarrow F_A(e) \cap G_B(e) = \phi$)

$\Rightarrow G_B(e) \times Y_K(k) = \phi$

$\Rightarrow (G_B \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_A \otimes Y_K) \cap (G_B \otimes Y_K) = \widetilde{\phi}$

Assume $k_1 \neq k_2$. Since (Y, τ_Y, K) is $(S\beta W - H)_1$, there exist soft β-open sets F_A, G_B, such that $F_A(k_1) = Y$, $G_B(k_2) = Y$ and $F_A \cap G_B = \widetilde{\phi}$.

Here $X_E \otimes F_A$, $X_E \otimes G_B$ are soft β-open sets

$(X_E \otimes F_A)(e_1, k_1) = X_E(e_1) \times F_A(k_1)$

$= X \times Y$

$(X_E \otimes G_B)(e_2, k_2) = X_E(e_2) \times G_B(k_2)$

$= X \times Y$

If for any $(e, k) \in E \times K$, $(X_E \otimes F_A)(e, k) \neq \phi$

$\Rightarrow X_E(e) \times F_A(k) \neq \phi$

$\Rightarrow X \times F_A(k) \neq \phi$

$\Rightarrow F_A(k) \neq \phi$

$\Rightarrow G_B(k) = \phi$ (Since $F_A \cap G_B = \widetilde{\phi} \Rightarrow F_A(k) \cap G_B(k) = \phi$)

$\Rightarrow X_E(e) \times G_B(k) = \phi$

$\Rightarrow (X_E \otimes G_B)(e, k) = \phi$

$\Rightarrow (X_E \otimes F_A) \cap (X_E \otimes G_B) = \widetilde{\phi}$

Hence $(X \times Y, \tau_X \otimes \tau_Y, E \times K)$ is $(S\beta W - H)_1$.

Definition.5.1.4.

A soft topological space (X, τ, E) is said to be **soft βW-Hausdorff space of type 2** denoted by $(S\beta W - H)_2$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$ there exists soft β-open sets F_E, G_E, such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \widetilde{\phi}$.

Theorem.5.1.5.

Soft subspace of a $(S\beta W - H)_2$ space is $(S\beta W - H)_2$.

Proof.

Let (X, τ, E) be a $(S\beta W - H)_2$ space. Let Y be a non-null subset of X. Let (Y, τ_Y, E) be a soft subspace of (X, τ, E) where $\tau_Y = \{ (F_Y, E) : (F, E) \in \tau\}$ is the relative soft topology on Y. Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist soft β-open sets F_E, G_E, such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \widetilde{\phi}$.

Here $((F_E)_Y, E)$, $((G_E)_Y, E)$ are soft β-open sets.

Also $(F_E)_Y(e_1) = Y \cap F_E(e_1)$

$= Y \cap X$

$= Y$

$(G_E)_Y(e_2) = Y \cap G_E(e_2)$

$= Y \cap X$

$= Y$

$((F_E)_Y \cap (G_E)_Y)(e) = ((F_E \cap G_E)_Y)(e)$

$= Y \cap (F_E \cap G_E)(e)$

$= Y \cap \tilde{\phi}(e)$

$= Y \cap \phi$

$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence (Y, τ_Y, E) is $(S\beta W - H)_2$.

Theorem.5.1.6

Product of two $(S\beta W - H)_2$ spaces is $(S\beta W - H)_2$.

Proof.

Let (X, τ_X, E) and (Y, τ_Y, K) be two $(S\beta W - H)_2$ spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E \times K$.

Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since (X, τ_X, E) is $(S\beta W - H)_2$, there exist soft β-open sets F_E, G_E, such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$.

Here $F_E \otimes Y_K$, $G_E \otimes Y_K$ are soft β-open sets

$(F_E \otimes Y_K)(e_1, k_1) = F_E(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_E \otimes Y_K)(e_2, k_2) = G_E(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any $(e, k) \in (E \times K)$, $(F_E \otimes Y_K)(e, k) \neq \phi$

$\Rightarrow F_E(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_E(e) \times Y \neq \phi$

$\Rightarrow F_E(e) \neq \phi$

$\Rightarrow G_E(e) = \phi$ (Since $F_E \cap G_E = \tilde{\phi} \Rightarrow F_A(e) \cap G_E(e) = \phi$)

$\Rightarrow G_E(e) \times Y_K(k) = \phi$

$\Rightarrow (G_E \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_E \otimes Y_K) \cap (G_E \otimes Y_K) = \tilde{\phi}$

Similarly, one can prove the case when $k_1 \neq k_2$.

Hence $(X \times Y, \tau_X \otimes \tau_Y, E \times K)$ is $(S\beta W - H)_2$.

Definition 5.1.7.

Let (X, τ, E) be a soft topological space and $H \subseteq E$. Then(X, τ_H, H) is called **Soft p- subspace** of (X, τ, E) relative to the parameter set H where $\tau_H = \{ (F_A) / H : H \subseteq A \subseteq E, F_A \in \tau \}$ and $(F_A)/H$ is the restriction map on H.

Theorem 5.1.8.

Soft p-subspace of a $(S\boldsymbol{\beta}W-H)_1$ space is $(S\boldsymbol{\beta}W-H)_1$.

Proof .

Let (X, τ, E) be a $(S\beta W-H)_1$ space . Let $H \subseteq E$. Let (X, τ_H, E) be a soft p-subspace of (X, τ, E) relative to the parameter set H where $\tau_H = \{(F_A)/H : H \subseteq A \subseteq E, F_A \in \tau\}$. Consider $h_1, h_2 \in E$. Therefore,there exists soft β- open sets F_A, G_B such that $F_A(h_1) = X$, $G_B(h_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $(F_A)/H$, $(G_B)/H \in \tau_H$.

Also$((F_A)/H)(h_1) = F_A(h_1) = X$

$((G_B)/H)(h_2) = G_B(h_2) = X$ and

$$((F_A)/H) \cap (G_B)/H)) = (F_A \cap G_B)/H$$

$$= \tilde{\phi} / H$$

$$= \tilde{\phi}$$

Hence (x, τ_H, H) is $(S\boldsymbol{\beta}W-H)_1$.

5.2. Soft β W- Hausdorff space in soft bitopological spaces

Definition 5.2.1 .

A soft bitopological space $(X, \tau_{1X}, \tau_{2X}, E)$ is said to be **Soft bi-βW-Hausdorff space of type 1** or soft βBW- T_2 space of type 1 denoted by $(S\beta BW - H)_1$ if it is $(S\beta W - H)_1$ with respect to τ_{1X} or $(S\beta W - H)_1$ with respect to τ_{2X}.

Definition 5.2.2.

A soft bitopological space $(X, \tau_{1X}, \tau_{2X}, E)$ is said to be **Soft bi- βW- Hausdorff space of type 2** signified by $(S\beta BW - H)_2$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$ there occurs soft β-open sets F_A, G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Theorem 5.2.3.

Soft subspace of a $(S\beta BW- H)_1$ space is $(S\beta BW - H)_1$.

Proof.

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(S\beta BW - H)_1$ space. Then it is $(S\beta W\text{-}H)_1$ with respect to τ_{1X} or $(S\beta W\text{-}H)_1$ with respect to τ_{2X}. Let Y be a non vacous subset of X. Let $\{Y, \tau_{1Y}, \tau_{2Y}, E\}$ be a soft subspace of $(X, \tau_{1X}, \tau_{2X}, E)$. From the theorem 5.1.2, a soft subspace of $(S\beta W\text{-}H)_1$ space is $(S\beta W\text{-}H)_1$. Therefore, $\{Y, \tau_{1Y}, \tau_{2Y}, E\}$ is $(S\beta W\text{-}H)_1$ with respect to τ_{1Y} or $(S\beta W\text{-}H)_1$ with respect to τ_{2Y}. Hence $\{Y, \tau_{1Y}, \tau_{2Y}, E\}$ is $(S\beta BW - H)_1$.

Theorem 5.2.4.

Soft subspace of a $(S\beta BW - H)_2$ space is $(S\beta BW - H)_2$.

Proof .

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(S\beta BW - H)_2$ space. Let Y be a non null subset of X.

Let $\{Y, \tau_{1Y}, \tau_{2Y}, E\}$ be a soft subspace of $(X, \tau_{1X}, \tau_{2X}, E)$ where

$\tau_{1Y} = \{(F_Y, E) : (F, E) \in \tau_{1X}\}$ and

$\tau_{2Y} = \{(G_Y, E) : (G, E) \in \tau_{2X}\}$ are said to be the relative topologies on Y.

Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there occur soft β-open sets F_A, G_B such that

$F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Hence also $((F_A)_Y, E) \in \tau_{1Y}$, $((G_B)_Y, E) \in \tau_{2Y}$

Also
$$\begin{aligned}(F_A)_Y(e_1) &= Y \cap F_A(e_1)\\ &= Y \cap X\\ &= Y\\ (G_B)_Y(e_2) &= Y \cap G_B(e_2)\\ &= Y \cap X\\ &= Y\end{aligned}$$

$((F_A)_Y \cap G_B)_Y)(e) = ((F_A \cap G_B)_Y)(e)$

$= Y \cap (F_A \cap G_B)(e)$

$= Y \cap \tilde{\phi}(e)$

$= Y \cap \phi$

$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence $\{Y, \tau_{1Y}, \tau_{2Y}, E\}$is $(S\beta BW - H)_2$.

Theorem 5.2.5

Product of two $(S\beta BW - H)_1$ spaces is $(S\beta BW - H)_1$.

Proof .

Let $(X, \tau_{1X}, \tau_{2X}, E)$ and $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ be two $(S\beta BW - H)_1$ spaces. Then $(X, \tau_{1X}, \tau_{2X}, E)$ is $(S\beta W\text{-} H)_1$ with respect to τ_{1X} or $(S\beta W\text{-}H)_1$ with respect to τ_{2X} and $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ is $(S\beta W\text{-}H)_1$ with respect to τ_{1Y} or $(S\beta W\text{-}H)_1$ with respect to τ_{2Y} . From theorem 5.1.3, the product of two $(S\beta W\text{-}H)_1$ spaces is $(S\beta W\text{-} H)_1$. Hence the product of two $(S\beta BW - H)_1$ spaces is $(S\beta BW - H)_1$.

Theorem 5.2.6

Product of two $(S\beta BW - H)_2$ spaces is $(S\beta BW - H)_2$.

Proof .

Let $(X, \tau_{1X}, \tau_{2X}, E)$ and $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ be two $(S\beta BW - H)_2$ spaces.

Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E \times K$. Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Suppose $e_1 \neq e_2$. Since $(X, \tau_{1X}, \tau_{2X}, E)$ is $(S\beta BW - H)_2$, there exist soft β-open sets F_A, G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $(F, A) \cap (G, B) = \tilde{\phi}$.

Therefore $F_A \otimes G_K$ with respect to $\tau_{1X} \otimes \tau_{1Y}$, $G_B \otimes Y_K \in \tau_{2X} \otimes \tau_{2Y}$

$(F_A \otimes Y_K)(e_1, k_1) = F_A(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_B \otimes Y_K)(e_2, k_2) = G_B(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any $(e, k) \in (E \times K)$, $(F_A \otimes Y_K)(e, k) \neq \phi$

$\Rightarrow F_A(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_A(e) \times Y \neq \phi$

$\Rightarrow F_A(e) \neq \phi$

$\Rightarrow G_B(e) = \phi$

(since $F_A \cap G_B = \phi \Rightarrow F_A(e) \cap G_B(e) = \phi$)

$\Rightarrow G_B(e) \times Y_K(k) = \phi$

$\Rightarrow (G_B \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_A \otimes Y_K) \cap (G_B \otimes Y_K) = \tilde{\phi}$.

Assume $k_1 \neq k_2$. Since $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ is $(S\beta BW - H)_2$, there exist soft β-open sets F_A, G_B such that $F_A(k_1) = X$, $G_B(k_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Therefore $X_E \otimes F_A \in \tau_{1X} \otimes \tau_{1Y}$, $X_E \otimes G_B \in \tau_{2X} \otimes \tau_{2Y}$

$(X_E \otimes F_A)(e_1, k_1) = X_E(e_1) \times F_A(k_1)$

$= X \times Y$

$(X_E \otimes G_B)(e_2, k_2) = X_E(e_2) \times G_B(k_2)$

$= X \times Y$

If for any $(e, k) \in E \times K$, $(X_E \otimes F_A)(e, k) \neq \phi$

$\Rightarrow X_E(e) \times F_A(k) \neq \phi$

$\Rightarrow X \times F_A(k) \neq \phi$

$\Rightarrow F_A(k) \neq \phi$

$\Rightarrow G_B(k) = \phi$

(Since $F_A \cap G_B = \phi \Rightarrow F_A(k) \cap G_B(k) = \tilde{\phi}$)

$\Rightarrow X_E(e) \times G_B(k) = \phi$

$\Rightarrow (X_E \otimes G_B)(e, k) = \phi$

$\Rightarrow (X_E \otimes F_A) \cap (X_E \otimes G_B) = \tilde{\phi}$

Hence $(X \times Y, \tau_{1X} \otimes \tau_{1Y}, \tau_{2X} \otimes \tau_{2Y}, E \times K)$ is $(S\beta BW - H)_2$.

CHAPTER -VI

PRE W – HAUSDORFF AXIOMS IN SOFT BITOPOLOGICAL SPACES

6.1. Pre W- Hausdorffness in Soft Topological Spaces

Definition 6.1.1.

Let F_A be any soft set of a soft topological space (X, τ, E) then F_A is called

i) F_A soft pre – open set of X if $F_A \subseteq$ int (cl(F_A) and

ii) F_A soft pre – Closed set of X if $F_A \supseteq$ int (cl(F_A) .

Definition 6.1.2.

A soft topological space (X, τ, E) is said to be **Soft Pre -W– Hausdorff space of type 1** denoted by $(PSW–H)_1$ if for every e_1 , $e_2 \in E$, $e_1 \neq e_2$,there exists soft pre-open sets F_A ,G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Theorem 6.1.3.

Soft subspace of a $(PSW\text{-}H)_1$ space is $(PSW\text{-}H)_1$.

Proof.

Let (X, τ, E) be a $(PSW\text{-}H)_1$ space. Let Y be a non null subset of X. Let (Y, τ_y, E) be a soft subspace of X. Let (X, τ, E) where $\tau_y = \{(F_Y, E) : (F, E) \in \tau\}$ is the relative soft topology on Y. Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exists soft pre - open sets F_A ,G_B, such that

$F_A(e_1) = X$, $G_B(e_2) = X$

Here $(F_A)_Y$, $(G_B)_Y$ are soft pre – open sets .

Also $(F_A)_Y(e_1) = Y \cap F_A(e_1)$

$= Y \cap X$

$= Y$

$(G_B)_Y(e_2) = Y \cap G_B(e_2)$

$= Y \cap X$

$= Y$

$((F_A)_Y \cap (G_B)_Y)(e) = ((F_A \cap G_B)_Y)(e)$

$= Y \cap (F_A \cap G_B)(e)$

$= Y \cap \tilde{\phi}(e)$

$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence (Y, τ_y, E) is $(PSW - H)_1$.

Theorem 6.1.4.

Product of two $(PSW\text{-}H)_1$ spaces is $(PSW\text{-}H)_1$.

Proof.

Let (X, τ_x, E) and (Y, τ_y, K) be two spaces. Consider two distinct points (e_1, k_1), $(e_2, k_2) \in E \times K$.

Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since (X, τ_x, E) is $(PSW\text{-}H)_1$, there exist pre -soft open sets F_A, G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $F_A \otimes Y_K$, $G_B \otimes Y_K$ are soft pre - open sets

$(F_A \otimes Y_K)(e_1, k_1) = F_A(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_B \otimes Y_K)(e_2, k_2) = G_B(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any $(e, k) \in (E \times K)$, $(F_A \otimes Y_K)(e, k) \neq \phi$

$\Rightarrow F_A(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_A(e) \times Y \neq \phi$

$\Rightarrow F_A(e) \neq \phi$

$\Rightarrow G_B(e) = \phi$ (since $F_A \cap G_B = \phi \Rightarrow F_A(e) \cap G_B(e) = \phi$)

$\Rightarrow G_B(e) \times Y_K(k) = \phi$

$\Rightarrow (G_B \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_A \otimes Y_K) \cap (G_B \otimes Y_K) = \tilde{\phi}$

Assume $k_1 \neq k_2$. Since (Y, τ_y, K) is $(PSW\text{-}H)_1$, there exist soft pre -open sets F_A, G_B, such that $F_A(k_1) = Y$, $G_B(k_2) = Y$ and $F_A \cap G_B = \tilde{\phi}$.

Here $X_E \otimes F_A$, $X_E \otimes G_B$ are soft pre - open sets

$(X_E \otimes F_A)(e_1, k_1) = X_E(e_1) \times F_A(k_1)$

$= X \times Y$

$(X_E \otimes G_B)(e_2, k_2) = X_E(e_2) \times G_B(k_2)$

$= X \times Y$

If for any $(e, k) \in E \times K$, $(X_E \otimes F_A)(e, k) \neq \phi$

$\Rightarrow X_E(e) \times F_A(k) \neq \phi$

$\Rightarrow X \times F_A(k) \neq \phi$

$\Rightarrow F_A(k) \neq \phi$

$\Rightarrow G_B(k) = \phi$ (Since $F_A \cap G_B = \tilde{\phi} \Rightarrow F_A(k) \cap G_B(k) = \phi$)

$\Rightarrow (X_E)(e) \times G_B(k) = \phi$

$\Rightarrow (X_E \otimes G_B)(e, k) = \phi$

$\Rightarrow (X_E \otimes F_A) \cap (X_E \otimes G_B) = \tilde{\phi}$

Hence $(X \times Y, \tau_X \otimes \tau_y, E \times K)$ is $(PSW\text{-}H)_1$.

Definition 6.1.5 .

A soft topological space (X, τ, E) is said to be **Soft Pre-W-Hausdorff space of type 2** denoted by $(PSW\text{-}H)_2$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$ there exists soft pre-open sets F_E, G_E such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$

Theorem 6.1.6.

Soft subspace of a $(PSW\text{-}H)_2$ space is $(PSW\text{-}H)_2$.

Proof.

Let (X, τ, E) be a $(PSW\text{-}H)_2$ space. Let Y be a non-null subset of X. Let (Y, τ_y, E) be a soft subspace of (X, τ, E) where $\tau_y = \{ (F_Y, E) : (F, E) \in \tau\}$ is the relative soft topology on Y. Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist soft pre-open sets F_E, G_E, such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$.

Here $((F_E)_Y, E)$, $((G_E)_Y, E)$ are soft pre-open sets.

Also $(F_E)_Y(e_1) = Y \cap F_E(e_1)$

$= Y \cap X$

$= Y$

$(G_E)_Y(e_2) = Y \cap G_E(e_2)$

$= Y \cap X$

$= Y$

$((F_E)_Y \cap (G_E)_Y)(e) = ((F_E \cap G_E)_Y)(e)$

$= Y \cap (F_E \cap G_E)(e)$

$= Y \cap \tilde{\phi}(e)$

$= Y \cap \phi$

$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence (Y, τ_y, E) is $(PSW\text{-}H)_2$

Theorem 6.1.7.

Product of two $(PSW\text{-}H)_2$ spaces is $(PSW\text{-}H)_2$.

Proof.

Let (X, τ_X, E) and (Y, τ_y, K) be two $(PSW\text{-}H)_2$ spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E\times K$.

Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since (X, τ_X, E) is $(sgW\text{-}H)_2$, there exist soft pre -open sets F_E, G_E, such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$.

Here $F_E \otimes Y_K$, $G_E \otimes Y_K$ are soft pre-open sets

$(F_E \otimes Y_K)(e_1, k_1) = F_E(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_E \otimes Y_K)(e_2, k_2) = G_E(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any $(e, k) \in (E\times K)$, $(F_E \otimes Y_K)(e, k) \neq \phi$

$\Rightarrow F_E(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_E(e) \times Y \neq \phi$

$\Rightarrow F_E(e) \neq \phi$

$\Rightarrow G_E(e) = \phi$ (Since $F_E \cap G_E = \tilde{\phi} \Rightarrow F_A(e) \cap G_E(e) = \phi$)

$\Rightarrow G_E(e) \times Y_K(k) = \phi$

$\Rightarrow (G_E \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_E \otimes Y_K) \cap (G_E \otimes Y_K) = \tilde{\phi}$

Similarly, one can prove the case when $k_1 \neq k_2$.

Hence $(X \times Y, \tau_X \otimes \tau_y, E \times K)$ is $(PSW\text{-}H)_2$.

Definition 6.1.8.

Let (X, τ, E) be a soft topological space and $H \subseteq E$. Then(X, τ_H, H) is called **Soft p- subspace** of (X, τ, E) relative to the parameter set H where $\tau_H = \{ (F_A) / H : H \subseteq A \subseteq E, F_A \in \tau \}$ and $(F_A)/H$ is the restriction map on H.

Theorem 6.1.9.

Soft p-subspace of a $(PSW\text{–}H)_1$ space is $(PSW\text{-}H)_1$.

Proof .

Let (X, τ, E) be a $(PSW - H)_1$ space . Let $H \subseteq E$. Let (X, τ_H, E) be a soft p-subspace of (X, τ, E) relative to the parameter set H where $\tau_H = \{(F_A)/H : H \subseteq A \subseteq E,\ F_A \in \tau\}$. Consider $h_1, h_2 \in E$. Therefore , there exists soft pre -open sets F_A, G_B such that $F_A(h_1) = X$, $G_B(h_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $(F_A)/H$, $(G_B)/H \in \tau_H$.

Also$((F_A)/H)(h_1) = F_A(h_1) = X$

$((G_B)/H)(h_2) = G_B(h_2) = X$ and

$$((F_A)/H) \cap (G_B)/H)) = (F_A \cap G_B)/H$$
$$= \tilde{\phi} / H$$
$$= \tilde{\phi}$$

Hence (x, τ_H, H) is $(PSW - H)_1$.

6.2.Pre – W- Haudorff Axiom in Soft Bitopological Spaces

Definition 6.2.1.

A soft bitopological space $(X, \tau_{1X}, \tau_{2X}, E)$ is said to be **Soft bi Pre-W-Hausdorff space of type 1** denoted by $(PBSW - H)_1$ if it is $(PSW - H)_1$ with respect to τ_{1X} or $(PSW - H)_1$ with respect to τ_{2X}.

Definition 6.2.2.

A soft bitopological space $(X, \tau_{1X}, \tau_{2X}, E)$ is said to be **Soft bi Pre-W-Hausdorff space of type 2** denoted by $(PBSW - H)_2$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist soft pre-open sets F_A with respect to τ_{1X} , G_B with respect to τ_{2X} such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Theorem 6.2.3.

Soft subspace of a $(PBSW - H)_1$ space is $(PBSW - H)_1$.

Proof.

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(PSW - H)_1$ space. Then it is $(PSW\text{-}H)_1$ with respect to τ_{1X} or $(PSW\text{-}H)_1$ with respect to τ_{2X} . Let Y be a non null subset of X.Let

$\{Y, \tau_{1Y}, \tau_{2Y}, E\}$ be a soft subspace of $(X, \tau_{1X}, \tau_{2X}, E)$. From the theorem 6.1.3, a soft subspace of $(PSW\text{-}H)_1$ space is $(PSW\text{-}H)_1$. Therefore, $\{Y,\tau_{1Y}, \tau_{2Y}, E\}$ is $(PSW\text{-}H)_1$ with respect to τ_{1Y} or $(PSW\text{-}H)_1$ with respect to τ_{2Y}. Hence $\{Y, \tau_{1Y}, \tau_{2Y}, E\}$ is $(PBSW - H)_1$.

Theorem 6.2.4.

Soft subspace of a $(PBSW - H)_2$ space is $(PBSW - H)_2$.

Proof .

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(PBSW - H)_2$ space. Let Y be a non null subset of X.
Let $\{Y, \tau_{1Y}, \tau_{2Y}, E\}$ be a soft subspace of $(X, \tau_{1X}, \tau_{2X}, E)$ where

$\tau_{1Y} = \{(F_Y, E) : (F, E) \in \tau_{1X}\}$ and

$\tau_{2Y} = \{(G_Y, E) : (G, E) \in \tau_{2X}\}$ are said to be the relative topologies on Y.

Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist soft pre-open sets F_A with respect to τ_{1X} , G_B with respect to τ_{2X} such that

$F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$

Here $((F_A)_Y, E) \in \tau_{1Y}$, $((G_B)_Y, E) \in \tau_{2Y}$

Also $(F_A)_Y(e_1) = Y \cap F_A(e_1)$

$= Y \cap X$

$= Y$

$(G_B)_Y(e_2) = Y \cap G_B(e_2)$

$= Y \cap X$

$= Y$

$((F_A)_Y \cap (G_B)_Y)(e) = ((F_A \cap G_B)_Y)(e)$

$= Y \cap (F_A \cap G_B)(e)$

$= Y \cap \tilde{\phi}(e)$

$= Y \cap \phi$

$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence $\{Y, \tau_{1Y}, \tau_{2Y}, E\}$ is $(PBSW - H)_2$.

Definition 6.2.5.

Let (X, τ_{1X}, τ_{2X}, E) be a soft topological space over X and H ⊆ E. Then { X, τ_{1H}, τ_{2H}, H } is called soft p – subspace of (X, τ_{1X}, τ_{2X}, E) relative to the parameter set H where τ_{1H} = { (F_A) /H : H ⊆ A⊆ E , F_A ∈ τ_{1X} }, τ_{2H} = { (G_B) /H : H ⊆B⊆ E , G_B ∈ τ_{2X} } and (F_A) / H , (G_B) / H are the restriction maps on H .

Theorem 6.2.6.

Soft p- subspace of a $(PSBW- H)_1$ space is $(PSBW - H)_1$.

Proof .

Let (X, τ_{1X}, τ_{2X}, E) be a $(PSBW- H)_1$ space. Then it is $(PSW- H)_1$ with respect to τ_{1X} or $(PSW- H)_1$ with respect to τ_{2X} . Let H ⊆ E . Let (X, τ_{1H}, τ_{2H}, H) be a soft p- subspace of (X, τ_{1X}, τ_{2X}, E) relative to the parameter set H. From the theorem 6.1.9, the soft p- subspace of $(PSW- H)_1$ space is $(PSW- H)_1$. Therefore , the soft p-subspace of $(PBSW- H)_1$ is $(PSW- H)_1$ with respect to τ_{1H} or with respect to τ_{2H} . Hence (X, τ_{1H}, τ_{2H}, H) is $(PBSW - H)_1$.

Theorem 6.2.7.

Soft p –subspace of a $(PSBW - H)_2$ space is $(PSBW - H)_2$.

Proof .

Let (X, τ_{1X}, τ_{2X}, E) be a $(PSBW - H)_2$ space. Let H ⊆ E. Let (X, τ_{1H}, τ_{2H}, H) be a soft p-subspace of (X, τ_{1X}, τ_{2X}, E) relative to the parameter set H where

$$\tau_{1H} = \{ (F_A) /H : H \subseteq A \subseteq E , F_A \in \tau_{1X} \},$$

$$\tau_{2H} = \{ (G_B) /H : H \subseteq B \subseteq E , G_B \in \tau_{2X} \}.$$

Consider h_1 , h_2 ∈ H , $h_1 \neq h_2$ .Then h_1 , h_2 ∈ E. Therefore , there exist soft pre -open sets F_A with respect to τ_{1x} , G_B with respect to τ_{2X} such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $(F_A) / H \in \tau_{1H}$, $(G_B) / H \in \tau_{2H}$

Also $((F_A) / H) (h_1) = F_A (h_1) = X$

$((G_B) / H)(h_2) = G_B(h_2) = X$ and

$((F_A) / H) \cap ((G_B) / H)) = (F_A \cap G_B) / H$

$= \tilde{\phi} / H$

$= \tilde{\phi}$

Hence $(X, \tau_{1H}, \tau_{2H}, H)$ is $(PSBW - H)_2$.

Theorem 6.2.8.

Product of two $(PSBW - H)_1$ spaces is $(PSBW - H)_1$.

Proof .

Let $(X, \tau_{1X}, \tau_{2X}, E)$ and $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ be two $(PSBW - H)_1$ spaces. Then $(X, \tau_{1X}, \tau_{2X}, E)$ is $(PSW\text{-}H)_1$ with respect to τ_{1X} or $(PSW\text{-}H)_1$ with respect to τ_{2X} and $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ is $(PSW\text{-}H)_1$ with respect to τ_{1Y} or $(PSW\text{-}H)_1$ with respect to τ_{2Y}. From theorem 6.1.3, the product of two $(PSW\text{-}H)_1$ spaces is $(PSW\text{-}H)_1$.Hence the product of two $(PSBW - H)_1$ spaces is $(PSBW - H)_1$.

Theorem 6.2.9.

Product of two $(PSBW - H)_2$ spaces is $(PSBW - H)_2$.

Proof .

Let $(X, \tau_{1X}, \tau_{2X}, E)$ and $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ be two $(PSBW - H)_2$ spaces.

Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E \times K$. Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since $(X, \tau_{1X}, \tau_{2X}, E)$ is $(PSBW - H)_2$, there exist soft pre-open sets F_A, G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here soft pre-open sets are $F_A \otimes Y_K$, $G_B \otimes Y_K$

$(F_A \otimes Y_K)(e_1, k_1) = F_A(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_B \otimes Y_K)(e_2, k_2) = G_B(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any $(e, k) \in (E \times K)$, $(F_A \otimes Y_K)(e,k) \neq \phi$

$\Rightarrow F_A(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_A(e) \times Y \neq \phi$

$\Rightarrow F_A(e) \neq \phi$

$\Rightarrow G_B(e) = \phi$

(since $F_A \cap G_B = \phi \Rightarrow F_A(e) \cap G_B(e) = \phi$)

$\Rightarrow G_B(e) \times Y_K(k) = \phi$

$\Rightarrow (G_B \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_A \otimes Y_K) \cap (G_B \otimes Y_K) = \tilde{\phi}$.

Assume $k_1 \neq k_2$. Since $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ is $(PSBW - H)_2$, there exist soft pre -open sets F_A with respect to τ_{1Y}, G_B with respect to τ_{2Y}, such that $F_A(k_1) = Y$, $G_B(k_2) = Y$ and $F_A \cap G_B = \tilde{\phi}$

Here soft pre - open sets are $X_E \otimes F_A$ wih respct to $\tau_{1X} \otimes \tau_{1Y}$, $X_E \otimes G_B$ with respect to $\tau_{2X} \otimes \tau_{2Y}$

$$(X_E \otimes F_A)(e_1, k_1) = X_E(e_1) \times F_A(k_1) = X \times Y$$

$$(X_E \otimes G_B)(e_2, k_2) = X_E(e_2) \times G_B(k_2) = X \times Y$$

If for any $(e,k) \in E \times K$, $(X_E \otimes F_A)(e,k) \neq \phi$

$\Rightarrow X_E(e) \times F_A(k) \neq \phi$

$\Rightarrow X \times F_A(k) \neq \phi$

$\Rightarrow F_A(k) \neq \phi$

$\Rightarrow G_B(k) = \phi$

(Since $F_A \cap G_B = \phi \Rightarrow F_A(k) \cap G_B(k) = \tilde{\phi}$)

$\Rightarrow X_E(e) \times G_B(k) = \phi$

$\Rightarrow (X_E \otimes G_B)(e, k) = \phi$

$\Rightarrow (X_E \otimes F_A) \cap (X_E \otimes G_B) = \tilde{\phi}$

Hence $(X \times Y, \tau_{1X} \otimes \tau_{1Y}, \tau_{2X} \otimes \tau_{2Y}, E \times K)$ is $(PSBW - H)_2$.

CHAPTER -VII

gW – HAUSDORFF AXIOMS IN SOFT BITOPOLOGICAL SPACES

7.1. gW – Hausdorffness in Soft Topological Spaces

Definition 7.1.1.

A soft set F_A is called a **Soft generalized closed** (soft g-closed) in a soft topological space (X, τ, E) if $cl(F_A) \subseteq G_B$ when ever $F_A \subseteq G_B$ and G_B is soft open in X .

Definition 7.1.2.

A soft set F_A is called a **Soft generalized open** (soft g- open) in a soft topological space (X, τ, E) if the relative complement $F_A{}'$ is soft g- closed in X .

Equivalently, a soft set F_A is called a **Soft generalized open** set (soft g-open) in a soft topological space (X, τ, E) if and only if $F_A \subseteq int(G_B)$ whenever $F_A \subseteq G_B$ and G_B is soft closed in X .

Definition 7.1.3.

A soft topological space (X, τ, E) is said to be **Soft gW – Hausdorff space of type 1** denoted by $(sgW-H)_1$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$,there exists soft g-open sets F_A, G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Theorem 7.1.4.

Soft subspace of a $(sgW-H)_1$ space is $(sgW-H)_1$.

Proof.

Let (X, τ, E) be a $(sgW-H)_1$ space. Let Y be a non null subset of X. Let (Y, τ_y, E) be a soft subspace of X. Let (X, τ, E) where $\tau_y = \{(F_Y, E) : (F, E) \in \tau\}$ is the relative soft topology on Y. Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exists soft g- open sets F_A, G_B, such that $F_A(e_1) = X$, $G_B(e_2) = X$.

Here $(F_A)_Y$, $(G_B)_Y$ are soft g – open sets .

Also $(F_A)_Y(e_1) = Y \cap F_A(e_1)$

$= Y \cap X$

$= Y$

$(G_B)_Y(e_2) = Y \cap G_B(e_2)$

$= Y \cap X$

$= Y$

$((F_A)_Y \cap (G_B)_Y)(e) = ((F_A \cap G_B)_Y)(e)$

$= Y \cap (F_A \cap G_B)(e)$

$= Y \cap \tilde{\phi}(e)$

$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence (Y, τ_y, E) is $(sgW - H)_1$.

Theorem 7.1.5.

Product of two $(sgW\text{-}H)_1$ spaces is $(sgW\text{-}H)_1$.

Proof.

Let (X, τ_x, E) and (Y, τ_y, K) be two spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E \times K$.

Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since (X, τ_x, E) is $(sgW\text{-}H)_1$, there exist g -soft open sets F_A, G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $F_A \otimes Y_K$, $G_B \otimes Y_K$ are soft g -open sets

$(F_A \otimes Y_K)(e_1, k_1) = F_A(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_B \otimes Y_K)(e_2, k_2) = G_B(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any $(e, k) \in (E \times K)$, $(F_A \otimes Y_K)(e, k) \neq \phi$

$\Rightarrow F_A(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_A(e) \times Y \neq \phi$

$\Rightarrow F_A(e) \neq \phi$

$\Rightarrow G_B(e) = \phi$ (since $F_A \cap G_B = \phi \Rightarrow F_A(e) \cap G_B(e) = \phi$)

$\Rightarrow G_B(e) \times Y_K(k) = \phi$

$\Rightarrow (G_B \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_A \otimes Y_K) \cap (G_B \otimes Y_K) = \tilde{\phi}$

Assume $k_1 \neq k_2$. Since (Y, τ_y, K) is $(sgW\text{-}H)_1$, there exist soft g -open sets F_A, G_B , such that $F_A(k_1) = Y$, $G_B(k_2) = Y$ and $F_A \cap G_B = \tilde{\phi}$.

Here $X_E \otimes F_A$, $X_E \otimes G_B$ are soft g - open sets

$(X_E \otimes F_A)(e_1, k_1) = X_E(e_1) \times F_A(k_1)$

$= X \times Y$

$(X_E \otimes G_B)(e_2, k_2) = X_E(e_2) \times G_B(k_2)$

$= X \times Y$

If for any $(e, k) \in E \times K$, $(X_E \otimes F_A)(e, k) \neq \phi$

$\Rightarrow X_E(e) \times F_A(k) \neq \phi$

$\Rightarrow X \times F_A(k) \neq \phi$

$\Rightarrow F_A(k) \neq \phi$

$\Rightarrow G_B(k) = \phi$ (Since $F_A \cap G_B = \tilde{\phi} \Rightarrow F_A(k) \cap G_B(k) = \phi$)

$\Rightarrow (X_E)(e) \times G_B(k) = \phi$

$\Rightarrow (X_E \otimes G_B)(e, k) = \phi$

$\Rightarrow (X_E \otimes F_A) \cap (X_E \otimes G_B) = \tilde{\phi}$

Hence $(X \times Y, \tau_X \otimes \tau_y, E \times K)$ is $(sgW\text{-}H)_1$.

Definition 7.1.6 .

A soft topological space (X, τ, E) is said to be **Soft gW-Hausdorff space of type 2** denoted by $(sgW\text{-}H)_2$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$ there exists soft g-open sets F_E , G_E such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$

Theorem 7.1.7.

Soft subspace of a $(sgW\text{-}H)_2$ space is $(sgW\text{-}H)_2$.

Proof .

Let (X, τ, E) be a $(sgW\text{-}H)_2$ space. Let Y be a non-null subset of X. Let (Y, τ_y, E) be a soft subspace of (X, τ, E) where $\tau_y = \{ (F_Y, E) : (F, E) \in \tau\}$ is the relative soft topology on Y. Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist soft g-open sets F_E, G_E, such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$.

Here $((F_E)_Y, E)$, $((G_E)_Y, E)$ are soft g-open sets.

$$\text{Also } (F_E)_Y(e_1) = Y \cap F_E(e_1) = Y \cap X = Y$$

$$(G_E)_Y(e_2) = Y \cap G_E(e_2) = Y \cap X = Y$$

$$((F_E)_Y \cap (G_E)_Y)(e) = ((F_E \cap G_E)_Y)(e) = Y \cap (F_E \cap G_E)(e) = Y \cap \tilde{\phi}(e) = Y \cap \phi = \phi$$

$$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$$

Hence (Y, τ_y, E) is $(sgW\text{-}H)_2$.

Theorem 7.1.8.

Product of two $(sgW\text{-}H)_2$ spaces is $(sgW\text{-}H)_2$.

Proof.

Let (X, τ_X , E) and (Y, τ_y , K) be two $(sgW\text{-}H)_2$ spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E \times K$.

Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since (X, τ_X , E) is $(sgW\text{-}H)_2$, there exist soft g-open sets F_E, G_E, such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$.

Here $F_E \otimes Y_K$, $G_E \otimes Y_K$ are soft g-open sets

$$(F_E \otimes Y_K)(e_1, k_1) = F_E(e_1) \times Y_K(k_1) = X \times Y$$

$$(G_E \otimes Y_K)(e_2, k_2) = G_E(e_2) \times Y_K(k_2) = X \times Y$$

If for any $(e, k) \in (E \times K)$, $(F_E \otimes Y_K)(e, k) \neq \phi$

$$\Rightarrow F_E(e) \times Y_K(k) \neq \phi$$

$$\Rightarrow F_E(e) \times Y \neq \phi$$

$$\Rightarrow F_E(e) \neq \phi$$

$$\Rightarrow G_E(e) = \phi \qquad (\text{Since } F_E \cap G_E = \tilde{\phi} \Rightarrow F_A(e) \cap G_E(e) = \phi)$$

$$\Rightarrow G_E(e) \times Y_K(k) = \phi$$

$$\Rightarrow (G_E \otimes Y_K)(e, k) = \phi$$

$$\Rightarrow (F_E \otimes Y_K) \cap (G_E \otimes Y_K) = \tilde{\phi}$$

Similarly, one can prove the case when $k_1 \neq k_2$.

Hence $(X \times Y, \tau_X \otimes \tau_y , E \times K)$ is $(sgW\text{-}H)_2$.

Definition 7.1.9.

Let (X, τ, E) be a soft topological space and $H \subseteq E$. Then(X, τ_H, H) is called **Soft p- subspace** of (X, τ, E) relative to the parameter set H where $\tau_H = \{ (F_A) / H : H \subseteq A \subseteq E, F_A \in \tau \}$ and $(F_A)/H$ is the restriction map on H.

Theorem 7.1.10.

Soft p-subspace of a $(sgW-H)_1$ space is $(sgW-H)_1$.

Proof .

Let (X, τ, E) be a $(sgW-H)_1$ space . Let $H \subseteq E$. Let (X, τ_H, E) be a soft p-subspace of (X, τ, E) relative to the parameter set H where $\tau_H = \{ (F_A)/H : H \subseteq A \subseteq E, F_A \in \tau \}$. Consider $h_1, h_2 \in E$. Therefore,there exists soft g-open sets F_A, G_B such that $F_A(h_1) = X$, $G_B(h_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $(F_A)/H$, $(G_B)/H \in \tau_H$.

Also$((F_A)/H)(h_1) = F_A(h_1) = X$

$((G_B)/H)(h_2) = G_B(h_2) = X$ and

$$((F_A)/H) \cap (G_B)/H) = (F_A \cap G_B)/H$$

$$= \tilde{\phi} / H$$

$$= \tilde{\phi}$$

Hence (x, τ_H, H) is $(sgW-H)_1$.

7.2. gW – Hausdorff Axiom in Soft Bitopological Spaces

Definition 7.2.1.

A soft bitopological space $(X, \tau_{1X}, \tau_{2X}, E)$ is said to be **Soft bi gW-Hausdorff space of type 1** denoted by $(sbgW-H)_1$ if it is $(sgW-H)_1$ with respect to τ_{1X} or $(sgW-H)_1$ with respect to τ_{2X}.

Definition 7.2.2.

A soft bitopological space $(X, \tau_{1X}, \tau_{2X}, E)$ is said to be **Soft bi gW- Hausdorff space of type 2** denoted by $(sbgW - H)_2$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist soft g-open sets F_A with respect to τ_{1X} , G_B with respect to τ_{2X} such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Theorem 7.2.3.

Soft subspace of a $(sbgW- H)_1$ space is $(sbgW - H)_1$.

Proof.

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(sbgW - H)_1$ space. Then it is $(sgW\text{-}H)_1$ with respect to τ_{1X} or $(sgW\text{-}H)_1$ with respect to τ_{2X} . Let Y be a non null subset of X.

Let $\{Y, \tau_{1Y} , \tau_{2Y} , E\}$ be a soft subspace of $(X, \tau_{1X}, \tau_{2X},E)$. From the theorem 7.1.4, a soft subspace of $(sgW\text{-}H)_1$ space is $(sgW\text{-}H)_1$. Therefore, $\{Y, \tau_{1Y} , \tau_{2Y} , E\}$ is $(sgW\text{-}H)_1$ with respect to τ_{1Y} or $(sgW\text{-}H)_1$ with respect to τ_{2Y}. Hence $\{Y, \tau_{1Y} , \tau_{2Y} , E\}$ is $(sbgW - H)_1$.

Theorem 7.2.4.

Soft subspace of a $(sbgW - H)_2$ space is $(sbgW - H)_2$.

Proof .

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(sbgW - H)_2$ space. Let Y be a non null subset of X.

Let $\{Y, \tau_{1Y} , \tau_{2Y} , E\}$ be a soft subspace of $(X, \tau_{1X}, \tau_{2X}, E)$ where

$\tau_{1Y} = \{(F_Y, E) : (F, E) \in \tau_{1X}\}$ and

$\tau_{2Y} = \{(G_Y, E) : (G, E) \in \tau_{2X}\}$ are said to be the relative topologies on Y.

Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist soft g-open sets F_A with respect to τ_{1X} , G_B with respect to τ_{2X} such that

$F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$

Here $((F_A)_Y , E) \in \tau_{1Y}$, $((G_B)_Y , E) \in \tau_{2Y}$

Also $(F_A)_Y(e_1) = Y \cap F_A(e_1)$

$= Y \cap X$

$= Y$

$(G_B)_Y(e_2) = Y \cap G_B(e_2)$

$= Y \cap X$

$= Y$

$((F_A)_Y \cap (G_B)_Y)(e) = ((F_A \cap G_B)_Y)(e)$

$= Y \cap (F_A \cap G_B)(e)$

$= Y \cap \tilde{\phi}(e)$

$= Y \cap \phi$

$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence $\{Y, \tau_{1Y}, \tau_{2Y}, E\}$ is $(sbgW - H)_2$.

Definition 7.2.5.

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a soft topological space over X and $H \subseteq E$. Then $\{X, \tau_{1H}, \tau_{2H}, H\}$ is called soft p – subspace of $(X, \tau_{1X}, \tau_{2X}, E)$ relative to the parameter set H where

$\tau_{1H} = \{ (F_A)/H : H \subseteq A \subseteq E, F_A \in \tau_{1X} \}$,

$\tau_{2H} = \{ (G_B)/H : H \subseteq B \subseteq E, G_B \in \tau_{2X} \}$ and $(F_A)/H$, $(G_B)/H$ are the restriction maps on H .

Theorem 7.2.6.

Soft p- subspace of a $(sbgW- H)_1$ space is $(sbgW - H)_1$.

Proof .

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(sbgW- H)_1$ space. Then it is $(sgW- H)_1$ with respect to τ_{1X} or $(sgW- H)_1$ with respect to τ_{2X} . Let $H \subseteq E$. Let $(X, \tau_{1H}, \tau_{2H}, H)$ be a soft p- subspace of $(X, \tau_{1X}, \tau_{2X}, E)$ relative to the parameter set H. From the theorem7.1.10 ,the soft p- subspace of $(sgW- H)_1$ space is $(sgW- H)_1$. Therefore , the soft p-subspace of $(sbgW- H)_1$ is $(sgW- H)_1$ with respect to τ_{1H} or with respect to τ_{2H}. Hence $(X, \tau_{1H}, \tau_{2H}, H)$ is $(sbgW - H)_1$.

Theorem 7.2.7.

Soft p –subspace of a $(sbgW - H)_2$ space is $(sbgW - H)_2$.

Proof .

Let $(X, \tau_{1X}, \tau_{2X}, E)$ be a $(sbgW - H)_2$ space. Let $H \subseteq E$. Let $(X, \tau_{1H}, \tau_{2H}, H)$ be a soft p-subspace of $(X, \tau_{1X}, \tau_{2X}, E)$ relative to the parameter set H where

$$\tau_{1H} = \{ (F_A)/H : H \subseteq A \subseteq E , F_A \in \tau_{1X} \},$$

$$\tau_{2H} = \{ (G_B)/H : H \subseteq B \subseteq E , G_B \in \tau_{2X} \}.$$

Consider $h_1 , h_2 \in H$, $h_1 \neq h_2$ .Then $h_1 , h_2 \in E$. Therefore , there exist soft g-open sets F_A with respect to τ_{1x} , G_B with respect to τ_{2X} such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $(F_A) / H \in \tau_{1H}$, $(G_B) / H \in \tau_{2H}$

Also $((F_A) / H) (h_1) = F_A (h_1) = X$

$((G_B) / H) (h_2) = G_B (h_2) = X$ and

$$((F_A) / H) \cap ((G_B) / H)) = (F_A \cap G_B) / H$$

$$= \tilde{\phi} / H$$

$$= \tilde{\phi}$$

Hence $(X, \tau_{1H}, \tau_{2H}, H)$ is $(sbgW - H)_2$.

Theorem 7.2.8.

Product of two $(sbgW - H)_1$ spaces is $(sbgW - H)_1$.

Proof .

Let $(X, \tau_{1X}, \tau_{2X}, E)$ and $\{Y, \tau_{1Y} , \tau_{2Y} , K\}$ be two $(sbgW - H)_1$ spaces. Then $(X, \tau_{1X}, \tau_{2X}, E)$ is $(sgW- H)_1$ with respect to τ_{1X} or $(sgW-H)_1$ with respect to τ_{2X} and $\{Y, \tau_{1Y} , \tau_{2Y} , K\}$ is $(sgW-H)_1$ with respect to τ_{1Y} or $(sgW-H)_1$ with respect to τ_{2Y} .

From theorem 7.1.5, the product of two $(sgW\text{-}H)_1$ spaces is $(sgW\text{- }H)_1$.Hence the product of two $(sbgW - H)_1$ spaces is $(sbgW - H)_1$.

Theorem 7.2.9.

Product of two $(sbgW - H)_2$ spaces is $(sbgW - H)_2$.

Proof .

Let $(X, \tau_{1X}, \tau_{2X}, E)$ and $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ be two $(sbgW - H)_2$ spaces.

Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E \times K$. Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since $(X, \tau_{1X}, \tau_{2X}, E)$ is $(sgbW - H)_2$, there exist soft g-open sets $F_A \in \tau_{1X}$, $G_B \in \tau_{2X}$ such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here soft g-open sets are $F_A \otimes Y_K$ with respect to $\tau_{1X} \otimes \tau_{1Y}$, $G_B \otimes Y_K$ with respect to $\tau_{2X} \otimes \tau_{2Y}$

$(F_A \otimes Y_K)(e_1, k_1) = F_A(e_1) \times Y_K(k_1)$

$= X \times Y$

$(G_B \otimes Y_K)(e_2, k_2) = G_B(e_2) \times Y_K(k_2)$

$= X \times Y$

If for any $(e, k) \in (E \times K)$, $(F_A \otimes Y_K)(e,k) \neq \phi$

$\Rightarrow F_A(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_A(e) \times Y \neq \phi$

$\Rightarrow F_A(e) \neq \phi$

$\Rightarrow G_B(e) = \phi$

(since $F_A \cap G_B = \phi \Rightarrow F_A(e) \cap G_B(e) = \phi$)

$\Rightarrow G_B(e) \times Y_K(k) = \phi$

$\Rightarrow (G_B \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_A \otimes Y_K) \cap (G_B \otimes Y_K) = \tilde{\phi}$.

Assume $k_1 \neq k_2$. Since $\{Y, \tau_{1Y}, \tau_{2Y}, K\}$ is

$(sbgW - H)_2$, there exist soft g-open sets F_A with respect to τ_{1Y}, G_B with respect to τ_{2Y}, such that $F_A(k_1) = Y$, $G_B(k_2) = Y$ and $F_A \cap G_B = \tilde{\phi}$

Here soft g - open sets are $X_E \otimes F_A$ with respect to $\tau_{1X} \otimes \tau_{1Y}$, $X_E \otimes G_B$ with respect to $\tau_{2X} \otimes \tau_{2Y}$

$(X_E \otimes F_A)(e_1, k_1) = X_E(e_1) \times F_A(k_1)$

$= X \times Y$

$(X_E \otimes G_B)(e_2, k_2) = X_E(e_2) \times G_B(k_2)$

$= X \times Y$

If for any $(e,k) \in E \times K$, $(X_E \otimes F_A)(e,k) \neq \phi$

$\Rightarrow X_E(e) \times F_A(k) \neq \phi$

$\Rightarrow X \times F_A(k) \neq \phi$

$\Rightarrow F_A(k) \neq \phi$

$\Rightarrow G_B(k) = \phi$

(Since $F_A \cap G_B = \phi \Rightarrow F_A(k) \cap G_B(k) = \tilde{\phi}$)

$\Rightarrow X_E(e) \times G_B(k) = \phi$

$\Rightarrow (X_E \otimes G_B)(e, k) = \phi$

$\Rightarrow (X_E \otimes F_A) \cap (X_E \otimes G_B) = \tilde{\phi}$

Hence $(X \times Y, \tau_{1X} \otimes \tau_{1Y}, \tau_{2X} \otimes \tau_{2Y}, E \times K)$ is $(sbgW-H)_2$.

SUMMARY AND CONCLUSION

Soft set theory has become a very good source of research for many mathematicians of recent years because of its wide range of applicability.

Topological structure of soft sets has been explored in recent years. Ever since the introduction of Soft topological spaces by Shabir and Naz (2011), several authors have worked on this concept and the theory of Soft topological spaces has developed in many directions. In 1963, Kelly, first initiated the concept of bitopological space.Ittangi (2014) introduced the notion of soft bitopological space which defined over an initial universal set with fixed set of parameters.

In this thesis ,following soft separation axioms in soft bitopological spaces are studied :

1) Soft b- Seperation Axioms in soft Bitopological Spaces
2) Soft αW- Hausdorff axiom in soft Bitopological Spaces
3) Soft βW- Hausdorff axiom in soft Bitopological Spaces
4) Soft Pre-W-Hausdorff axiom in Soft Bitopological Spaces.
5) Soft gW- Hausdorff axiom in soft Bitopological Spaces.

Chapter I deals with fundamental definition and properties regarding soft sets and soft topological spaces.

Chapter II is devoted to the study of separation axioms in soft bitopological spaces.

Chapter III deals with (1,2)* soft b – separation axioms.

New types of Hausdorff Separation axiom called Soft αW-Hausdorff axiom, Soft βW- Hausdorff axiom , Soft Pre-W-Hausdorff axioms , Soft gW- Hausdorff axiom both in soft introduced and studied respectively in chapters IV ,V ,VI and VII .

These Hausdorff axioms widen the scope to do further research in the areas like supra topological spaces, Tri topological Spaces and Smooth topological spaces.

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